A Novel Approach to the Design of Controllers in an Automotive Cruise-Control System

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Outline I

Background

- Modern automobiles are complex electromechanical systems.
- They offer plenty of features like cruise-control, electronic braking systems, electronic stability, driver assistance systems, differential lock control and so on.
- These have resulted in reliability, increased safety, reduced congestion and emissions, improved driver comfort, and fuel economy.
- Each of these features, can be classified to belong to either powertrain control, vehicle control, or body control, and
- Can be compared to a typical feedback control system with a controller, actuator, controlled system and a sensor.

Background

- The most common strategies for control of these systems, include:
  1. Pole - placement,
  2. Linear Quadratic Regulators,
  3. IO Linearization, and
  4. Sliding Control to mention a few.
- These methods rely on an **accurate** model of the plant - linear or non-linear, and
- Use a fixed gain, or adopt a gain scheduling algorithm as is the need.
- Like in the case of all complicated systems, even for powertrain, engine, and body, not only is it difficult to determine a plant model,
- But also, the determined model is susceptible to uncertainity in its parameters.

Motivation

- The method adopted [1]provides for a model-free technique, which serves as a tangible alternative to the regular model-based design methods.
- The approach to synthesis and design of controllers is instead, data driven;
- With just, accurate readings of the plant data over a low-frequency band, the entire set of stabilizing PID gains is computed.
- This is in stark contrast with the conventional model based design methods, that require **complete** information of the plant and generally produce a **single** optimal controller.

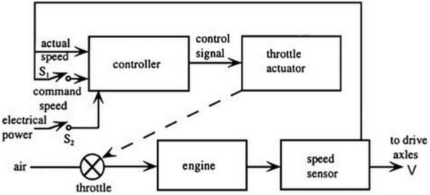
[1]Keel, Lee H., and Shankar P. Bhattacharyya. Controller synthesis free of analytical models: three term controllers. Automatic Control, IEEE Transactions on 53.6 (2008): 1353-1369.

Objectives

- To develop a complete stabilizing PID set, without the need for an identified vehicle model for an automotive cruise-control system from its frequency response.
- The resulting set is used to determine the PID gains satisfying certain performance indices.
- These performance indices are inturn derived from the need for efficient tracking and disturbance rejection.
- With this, the resulting PID controller is tuned with most appropriate gains, yielding a desired closed-loop response.
- This also serves as an analytical alternative to the otherwise - mostly iterative method of estimating the stabilizing PID gains.

Outline I

A Typical Cruise-Control System



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- The force-balance equation governing the plant dynamics is given by,

$$M\dot{v} = F_{tr} - \{F_d + F_{rr} + F_g\} \quad (1)$$

- Letting, $V(t) = V_c + \delta V$ and drag $D(t) = D_c + \delta D$ we have,

$$\delta D = \frac{dD}{dV}\delta V = \rho C_d A V_c \delta V = K_d \delta V \quad (2)$$

[2] Ribbens, William. Understanding automotive electronics: an engineering perspective. Butterworth-Heinemann, 2012.

A Typical Cruise-Control System

- Where D_c is the drag at speed V_c, and K_d is a constant for a given V_c and ρ .
- Also, for a sufficiently small slope,

$$sin\theta \approx \theta. \tag{3}$$

- With these simplifying assumptions, the linearized plant model is given by,

$$\frac{\delta V(s)}{U(s)} = \frac{K_a g_A / M r_w}{s + K_d / M} \tag{4}$$

- When the vehicle is on a downward slope of suitable grade (θ), the external forces alone are insufficient to decelerate it.
- A brake actuator in addition to the throttle actuator is required to maintain the desired speed.

PID gains for a desired response

- Pick a closed-loop transfer function with the same relative degree as that of the plant.
- The choice of the transfer function is also driven by the need for a certain set of performance indices.
- Let the general form of the controller be,

$$C(s) = \frac{K_d s^2 + K_p s + Ki}{s(1 + sT)} \tag{5}$$

- Let the closed-loop transfer function of the plant be,

$$H(j\omega) = \frac{N(s)}{D(s)} = K \frac{(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)} \tag{6}$$

with $m \leq n$ and K, a constant.

PID gains for a desired response

- The net change in phase is given by,

$$-\frac{\pi}{2}((n-m)+2z^{+}). \tag{7}$$

- It can be observed that, each time the controller (5) is cascaded with the plant (6), the magnitude of the closed-loop transfer function is modified while its phase remains unchanged.
- This forms the basis for the choice of a desired transfer function with the same order as that of the plant.
- With $z^{+}=0$ and $(n-m)$=1, the choice of the desired closed-loop transfer function is of the form,

$$H^{*}(j\omega)=\frac{K}{\tau s+1} \tag{8}$$

Optimization

- A frequency-response fitting using the method of least-squares, is performed on picking the desired closed-loop transfer function, $H^*(j\omega)$.
- This yields an estimate of the PID gains, K_p, K_i and, K_d, $\forall\ \omega_k$ such that,

$$H^*(j\omega_k) \approx \frac{K}{\tau(j\omega_k)+1},\ \forall\ k = 1, 2, \ldots, N \tag{9}$$

- In matrix form,

$$\underbrace{\begin{bmatrix} (\Gamma_1)(j\omega_1)^2 & (\Gamma_1)(j\omega_1) & (\Gamma_1) \\ (\Gamma_2)(j\omega_2)^2 & (\Gamma_2)(j\omega_2) & (\Gamma_2) \\ \vdots & \vdots & \vdots \\ (\Gamma_N)(j\omega_N)^2 & (\Gamma_N)(j\omega_N) & (\Gamma_N) \end{bmatrix}}_{\mathcal{C}} \underbrace{\begin{bmatrix} \hat{K}_d \\ \hat{K}_p \\ \hat{K}_i \end{bmatrix}}_{\hat{x}} \approx \underbrace{\begin{bmatrix} (-H^*(j\omega_1)(j\omega_1(j\omega_1 T+1))) \\ (-H^*(j\omega_2)(j\omega_2(j\omega_2 T+1))) \\ \vdots \\ (-H^*(j\omega_N)(j\omega_N(j\omega_N T+1))) \end{bmatrix}}_{b} \tag{10}$$

Where $\Gamma_v = (H^*(j\omega_v)P(j\omega_v) - P(j\omega_v)),\ \forall\ v = 1, 2, \ldots, N$

Optimization

- The least-squares curve fitting problem for a fixed $K_p = K_p^*$ can be formulated as,

$$\begin{aligned} \chi = \min_{\hat{x}} \|\mathcal{C}\hat{x} - b\|_2^2 \\ \text{subject to } \ \mathcal{A}\bar{x} \leq c \end{aligned} \tag{11}$$

With the constraints,

$$\mathcal{A} = \begin{bmatrix} \omega_0^2 i_0 & 0 & -i_0 \\ \omega_1^2 i_1 & 0 & -i_1 \\ \vdots & \vdots & \vdots \\ \omega_l^2 i_l & 0 & -i_l \end{bmatrix} \tag{12}$$

$$\bar{x} = \begin{bmatrix} K_d & K_p^* & K_i \end{bmatrix}^T$$

$$c = \begin{bmatrix} i_0 \left(\frac{\omega_0 sin\phi(\omega_0) - \omega_0^2 T cos\phi(\omega_0)}{|P(j\omega_0)|^2} \right) \\ i_1 \left(\frac{\omega_1 sin\phi(\omega_1) - \omega_1^2 T cos\phi(\omega_1)}{|P(j\omega_1)|^2} \right) \\ \vdots \\ i_l \left(\frac{\omega_l sin\phi(\omega_l) - \omega_l^2 T cos\phi(\omega_l)}{|P(j\omega_l)|^2} \right) \end{bmatrix}$$

Optimization

- The distinct frequencies in (11), $0 = \omega_0 < \omega_1 < \ldots < \omega_{l-1} < \omega_l = \infty$ are solutions of,

$$K_p^* = -\frac{cos(\phi(\omega)) + \omega Tsin(\phi(\omega))}{(\|P(j\omega)\|)^2}. \tag{13}$$

  Also, $i_0, i_1, \ldots, i_l$ are the integers satisying (12).
- The minimization of (10) yields the best least-squares estimate over the region formed for a fixed K_p.
- This way, for a range of K_p, the best estimate for each region formed is determined.
- On comparing the estimates for all permissible values of K_p, the best global least-squares estimate is chosen such that,

$$x^* = \min_{K_p} (\chi) \tag{14}$$

Optimization

- The minimization yields a vector $x^* = \begin{bmatrix} \hat{K}_d^* & \hat{K}_p^* & \hat{K}_i^* \end{bmatrix}^T$
- These gains are the best global least-square estimates for all possible values of K_p.
- The desired controller $C_d(s)$ is then given by,

$$C_d(s) = \frac{\hat{K}_d^* s^2 + \hat{K}_p^* s + \hat{K}_i^*}{s(Ts+1)}. \tag{15}$$

- For the system under consideration, the above controller provides the best approximation to the desired response.
- Also, each estimate for a $K_p = K_p^*$ is contained in a stabilizing zone.
- This ensures that, each global least-squares estimate x^* will always stabilize the system.

Outline I

Procedure

- Approximate the frequency response of the system from its time-domain data.
- Synthesize the controller using the method in [3]
- Pick a desired closed-loop transfer function.
- Determine the PID gains through optimization to yield the desired closed-loop respose.

[3] Keel, Lee H., and Shankar P. Bhattacharyya. Controller synthesis free of analytical models: three term controllers. Automatic Control, IEEE Transactions on 53.6 (2008): 1353-1369.

Outline I

Result

Computation of stabilizing PID sets:

- The system (as in the figure below), is characterised by its frequency response.
- Let P represent the linearized version of the non-linear plant about an equillibrium point ($V = V_c$).
- It is assumed that the plant has no roots on the imaginary axis.

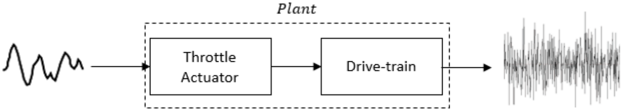


Figure : Open-loop System

Result

- The response approximated from time-domain data points, $P(j\omega)$ for $\omega \in (0, 100)$ is as below,

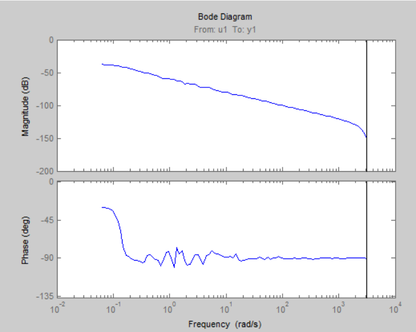


Figure : Approximated Frequency Response

- It can be observed from the magnitude plot, that the high-frequency slope is -20 db/decade $\rightarrow -20(n-m) = -20 \rightarrow (n-m) = 1$, first order.

Result

- From the phase plot, it can be observed that the total change in phase is -90 degrees,

$$-\frac{\pi}{2} = -((n-m) - 2(p^{+} - z^{+}))\frac{\pi}{2} \tag{16}$$

- The signature required for stability being,

$$\sigma(\bar{F}(s)) = (n-m) + 2z^{+} + 2 = 3. \tag{17}$$

- Since $(n-m)$ is odd, the following conditions must be satisfied:

$$[i_0 - 2i_1 - \cdots + (-1)^{(l-1)} 2i_{(l-1)}](-1)^{(l-1)} = 3 \tag{18}$$

$$j = sgn[\bar{F}_i(\infty^{-}, K_p)] = -sgn[\lim_{\omega \to \infty} g(\omega)] = 1 \tag{19}$$

- From (17) & (18), it can be observed that K_p should be chosen to satisfy, $\bar{F}_i(\omega, K_p^*) = 0$ and the feasible range of K_p is $(0, \infty)$.
- Picking $K_p = 6$ we have, $\omega = \{0, 8\}$ and $-[i_0 - 2i_1] = 3$. This implies, $I = \{1, -1\}$ is the string required for stability.

Result

- The linear inequalities for stability are,

$$K_i > 0 \tag{20}$$

$$-K_i + 64K_d > -3.05476 \tag{21}$$

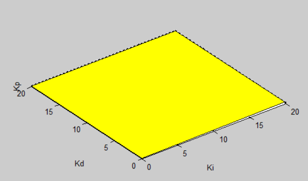


Figure : The complete set of stabilizing gains in (K_i, K_d) space for $K_p = 6$.

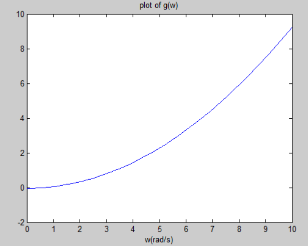


Figure : Feasbile range of K_p

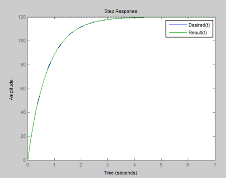
Outline I

Controller gains for a desired performance

Scenario:

- Cruise speed is set at 120 mph.
- With the required performance indices being:
  1. Zero steady-state error,
  2. Minimal or no-overshoot, and
  3. Settling time less than 4s.
- Let $\frac{1}{0.75s+1}$, be the chosen closed-loop transfer function. Then the resulting controller is of the form,

$$C_d(s) = \frac{0.124s^2 + 12.35s + 0.0693}{0.01s^2 + s}. \tag{22}$$



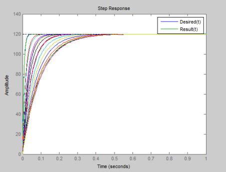


Figure : Step-response for a range of τ in (0.007,1)

Outline I

Observations

- The approximation of the response of the resulting system with that of the chosen closed-loop transfer function, is equivalent to enforcing a pole-zero cancellation; two closed-loop zeros with two closed-loop poles.
- The transfer function of the closed-loop system with a controller, is of relative degree one and of the form, $\frac{1}{\tau s+1}$.
- Since τ is a measure of speed of the response for a first order system. A range of τ can be prescribed by taking into account various vehicle design parameters.

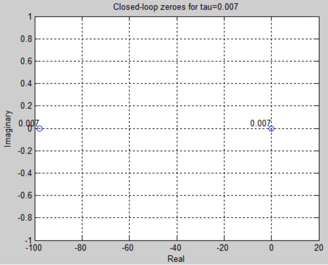


Figure : Zeros as a function of τ

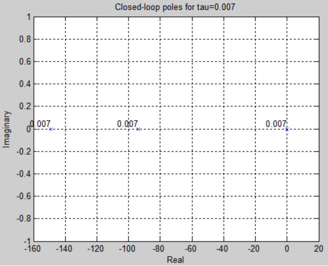


Figure : Poles as a function of τ

Thank you

Questions?