From Particles to People: Vicsek-Inspired Behavioural Modelling Frameworks

PhD Thesis Defence

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Background

- "To demonstrate that people today are much closer than ever, a member of the group suggested a test. He offered a bet that we could name any person among the earth's one and a half billion inhabitants and through at most five acquaintances, one of which he knew personally, he could link to the chosen one", writes Frigyes Karinthy in *Igy irtok ti*, or *This is How You Write* [1]
- In 1967, Stanley Milgram's groundbreaking study on interconnectivity gave rise to the "six degrees of separation"[2]
- In 1998, NEC established that the Web had 18.59 degrees of separation[3]

[1] Karinthy, Frigyes. "Chain-links." Everything is different (1929): 21-26.

[2] Milgram, Stanley. "Six degrees of separation." Psychology Today 2 (1967): 60-64.

[3] Barabási, Albert-László. "Linked: The new science of networks." (2003): 409-410.

Background (Continued)

- Irreversible implosion of the world has paved way for intricate social networks, both physical and virtual, with tremendous **spreading and diffusive powers**;
  - Gaetan Dugas, believed to be **Patient Zero** of AIDS epidemic that has killed 20 million people worldwide[3]
  - Facebook users have been accused of **spreading fake news** that had a significant outcome on the Presidential elections of 2016[4]
  - "Marketing-induced **consumer-to-consumer** word of mouth generates more than twice the sales of paid advertising"[5]

[4] Allcott, Hunt, and Matthew Gentzkow. "Social media and fake news in the 2016 election." Journal of Economic Perspectives 31.2 (2017): 211-36.

[5] Bughin, Jacques, Jonathan Doogan, and Ole Jorgen Vetvik. "A new way to measure word-of-mouth marketing." McKinsey Quarterly 2.1 (2010): 113-116.

Objectives

Develop Vicsek-inspired *nearest-neighbour* models to capture the two human behavioural phenomena,

1. **Opinion Formation on Virtual Social Platforms**
   Influence of others with *similar opinions* primarily govern formation of opinions

2. **Disease Spread and Mitigation**
   Closeness of physical locations determines the extent of spread

Vicsek's Model

Update Rules

$$x_i(t+1) = x_i(t) + \vec{v}_i(t)\Delta t, \forall i, t$$
$$\theta_i(t+1) = \langle \theta(t) \rangle_r + \Delta\theta,$$

where,

$$\tan(\langle \theta_i(t) \rangle_r) = \frac{\sum_{j \in N_i(t)} v_j(t) \sin(\theta_j(t))}{\sum_{j \in N_i(t)} v_j(t) \cos(\theta_j(t))},$$
$$N_i(t) := \{j : |x_i(t) - x_j(t)| \leq r\}.$$

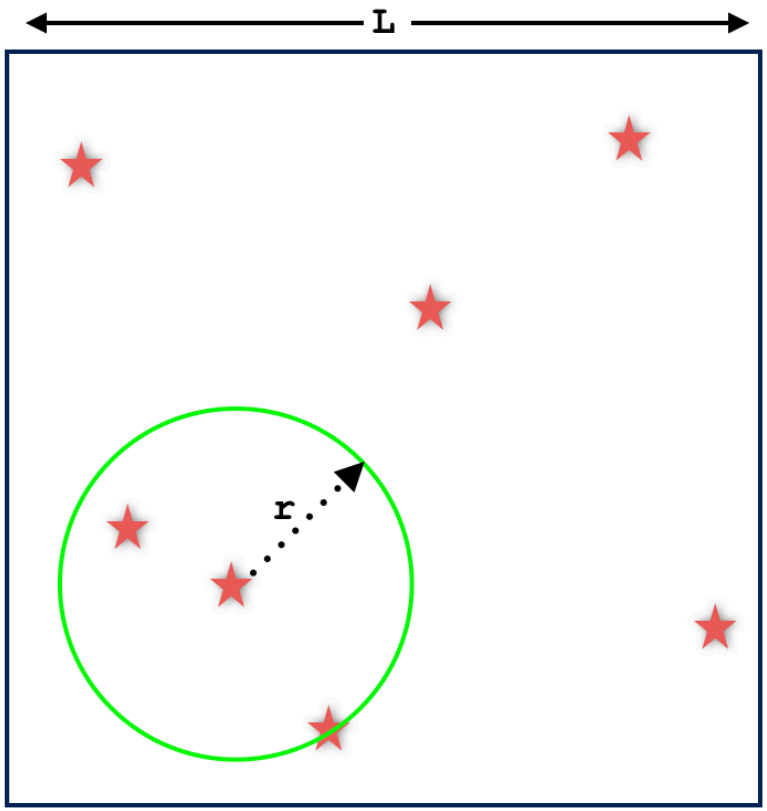

Figure: A typical layout

Vicsek's Model

Update Rules

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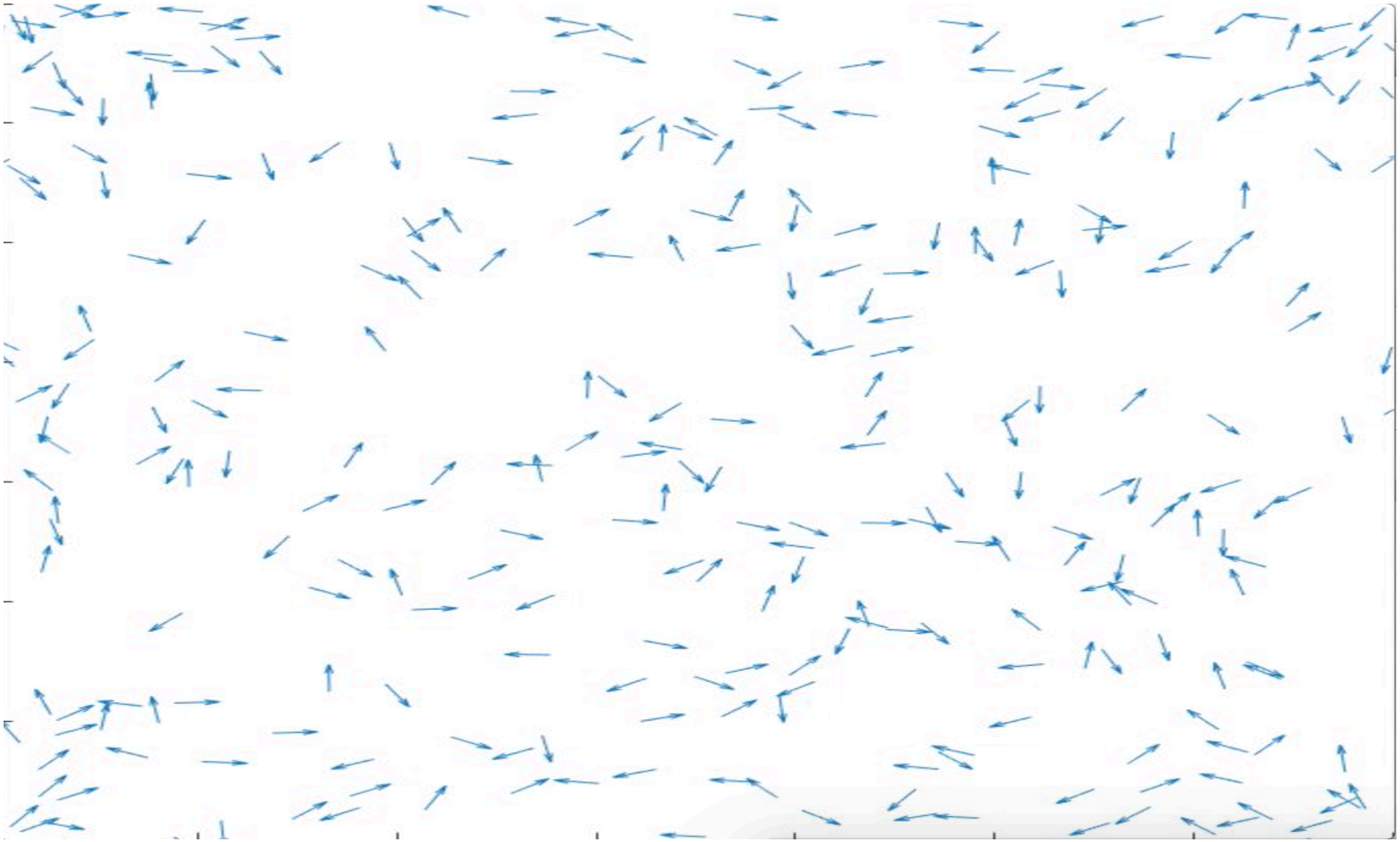
$$N_i(t) := \{j : |x_i(t) - x_j(t)| \leq r\}.$$

- N - Number of particles,
- L - Size of square cell,
- ρ - Density, $\frac{N}{L^2}$,
- r - Interaction radius,
- $\vec{v}_i(t)$ - Velocity,
- $x_i(t)$ - Position,
- $\theta_i(t)$ - Heading angle,
- $\langle \theta_i(t) \rangle_r$ - Average heading in r,
- $\Delta\theta$ - Extrinsic noise $[-\frac{\eta}{2}, \frac{\eta}{2}]$.

Vicsek's Model: Low-Density, Low-Noise

$(N = 300, L = 25 \text{ m}, v = 0.03 \text{ m/s}, r = 1 \text{ m}, \eta = 0.1 \text{ rad})$



Vicsek's Model: Observations

- Despite decentralized communication and dynamic neighbourhood, simple rules yield consensus about heading.
  - Consensus and localization in mobile and sensor networks[6]
  - Group behaviour in bird flocks, fish schools and bacterial colonies[7]
  - Political polarization in a group of finite size[8]

[6] Cao, Yongcan, et al. "An overview of recent progress in the study of distributed multi-agent coordination." IEEE Transactions on Industrial informatics 9.1 (2012): 427-438.

[7] Chazelle, Bernard. "The convergence of bird flocking." Journal of the ACM (JACM) 61.4 (2014): 1-35.

[8] Romenskyy, M., Spaiser, V., Ihle, T., & Lobaskin, V. (2018). Polarized Ukraine 2014: Opinion and territorial split demonstrated with the bounded confidence XY model, parametrized by Twitter data. Royal Society Open Science, 5(8), 171935.

Opinion Formation on Virtual Social Platforms: Objectives

1. Emulate a social network
   - Agents and their relevant psychological and sociological factors
   - Inter-agent interaction rules
   - Structure of the network
   - External influences
2. Evaluate the co-evolution of opinions and the underlying influence network
3. Discern factors affecting group behaviour

Opinion Formation on Virtual Social Platforms: Prevalent Approaches

1. **Network Topology**
   - *Static*[9], or
   - *Dynamic* - with state-based switching[10]
2. **Network Components**
   - *Nodes/Agents* - are prejudiced (random bias[9] or eigenvector centrality[10])
   - *Ties/Connections* - are lost or gained
   - *Contagion* - is directed, and is within a bounded confidence neighborhood[10]
3. **Update**
   - *Rule* - is DeGroot-Friedkin model[9], Hegselman-Krause model[10], Weibuch or their variants
   - *Nature* - is synchronous[9,10] or asynchronous

[9] Jia, P., MirTabatabaei, A., Friedkin, N. E., & Bullo, F. (2015). Opinion dynamics and the evolution of social power in influence networks. SIAM Review, 57(3), 367-397.

[10] Blondel, V. D., Hendrickx, J. M., & Tsitsiklis, J. N. (2009). On Krause's multi-agent consensus model with state-dependent connectivity. IEEE Transactions on Automatic Control, 54(11), 2586-2597.

Model-I: Opinion Formation with a Familiarity-based Bounded-Confidence Neighbourhood

1. **Network Topology**
   - *Dynamic* - **Opinion and Familiarity** dependent network
2. **Network Components**
   - *Nodes/Agents* - have an **individual popularity**, confirmation bias, **sociability** and an opinion bias
   - *Ties/Connections*- are lost or gained in a **controlled** manner within the bounded-confidence neighbourhood
   - *Influence* - is directed and is within an interaction neighbourhood
3. **Update**
   - *Rule* - is weighted average,
   - *Nature* - is asynchronous

Model-I: The Proposed Model

1 Opinion Update

$$\vec{x}_i(t) := \langle x_i(t), \theta_i(t) \rangle, \ \forall i, t,$$

$$x_i(t) = 1, \text{ and } \theta_i(t) \in (-\pi, \pi],$$

$$\theta_i(t+1) = \langle \theta_i(t) \rangle + \cancel{\Delta\theta},$$

$$\tan(\langle \theta_i(t) \rangle) = \frac{\sum_{j \in N_{I_i}(t)} w_{ij}(t) \sin(\theta_j(t))}{\sum_{j \in N_{I_i}(t)} w_{ij}(t) \cos(\theta_j(t))}$$

2 Neighbourhood

$$N_i(t) := \{j : j \neq i, ||\theta_i(t) - \theta_j(t)|| \leq \theta_{T_i}\},$$

$$\theta_{T_i} := \begin{cases} \theta_{T_R}, & \text{if i is Rigid,} \\ \theta_{T_F}, & \text{if i is Flexible} \end{cases},$$

$$N_i(t) := N_{D_i}(t) \cup N_{ND_i}(t),$$

$$N_{I_i}(t) := N_{D_i}(t) \cup \bar{N}_{ND_i}(t) \cup i$$

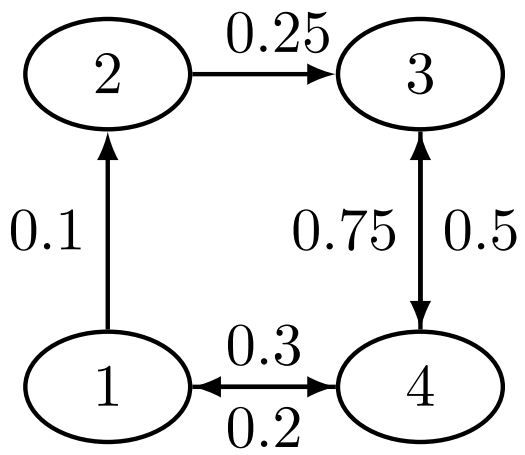
3 Influence Network

$$\mathrm{G(t)=(V,E(t))},$$

$$\mathrm{w}_{ii} = K_1 z_i,$$

$$\mathrm{Z} = \beta \left(I - \alpha D_{out}^{-1}(t) A(t)\right)^{-1},$$

$$\mathrm{w}_{ij}(t) = \frac{\sum_{p=1}^{p_{max}} \left(\prod_{(m,n) \in E_{Path}} w_{mn}(t) \right)}{p_{max}}$$



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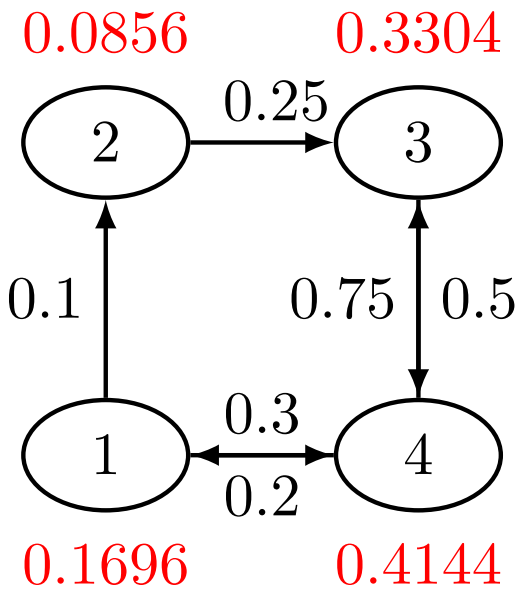


Model-I: Groups

1. **Opinion Distribution**[12]
   1. **Conservative**
      - Opinions closer to the mean,

$$\Theta(0) \sim \mathcal{N}(\mu, \sigma_c^2).$$

   2. **Liberal**
      - Opinions scattered away from the mean,

$$\Theta(0) \sim \mathcal{N}(\mu, \sigma_l^2),$$
$$\sigma_l > \sigma_c.$$

2. **Composition**
   1. **Homogeneous**
      - Same type of agents
   2. **Heterogeneous**
      - Agents can be Rigid or Flexible, with, $N_R > N_F$, or, $N_F > N_R$.
   3. **Balanced**
      - Agents can be Rigid or Flexible, with, $N_R = N_F$.

[12] De, A., Bhattacharya, S., Bhattacharya, P., Ganguly, N., & Chakrabarti, S. (2014, November). Learning a linear influence model from transient opinion dynamics. In Proceedings of the 23rd ACM International Conference on Information and Knowledge Management (pp. 401-410).

Model-I: Groups

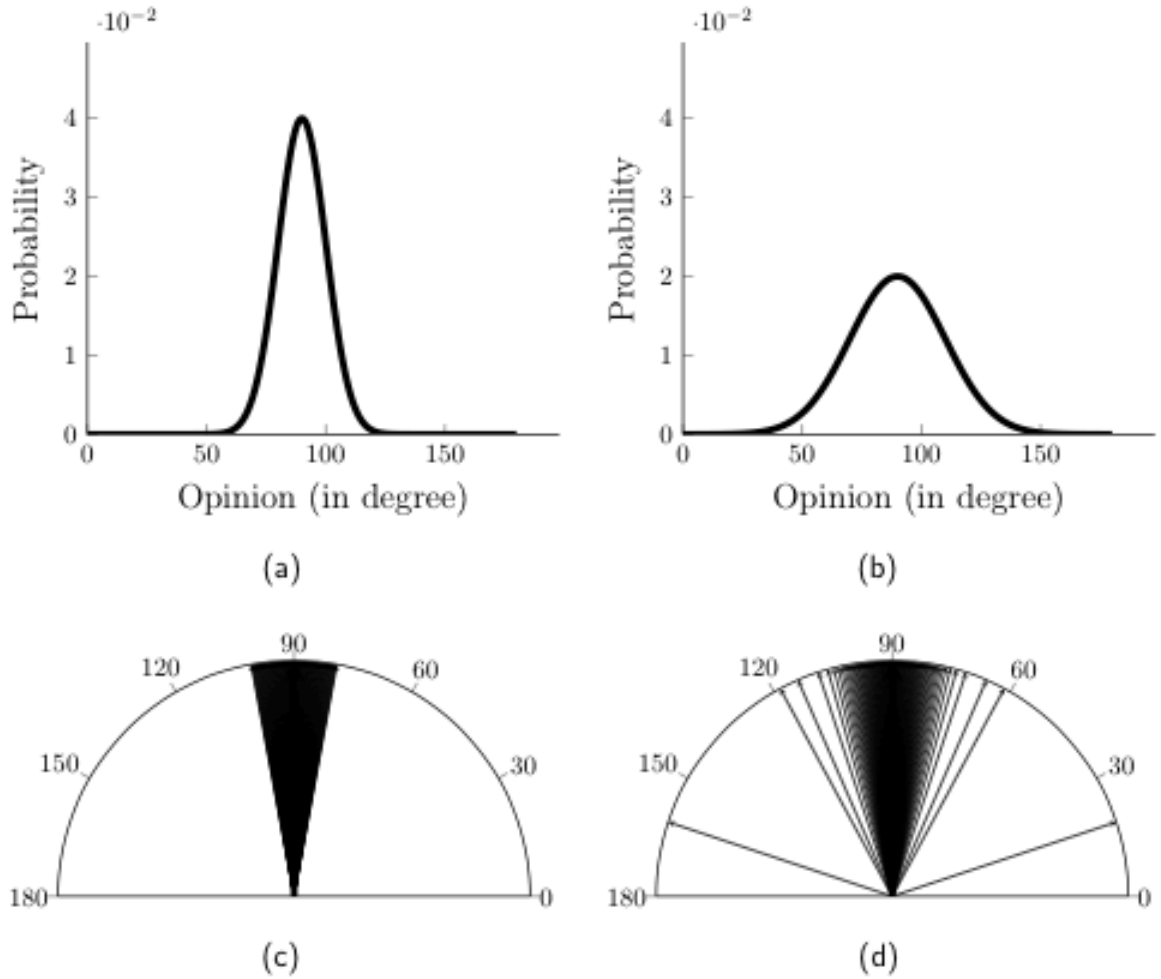


Figure: A Truncated Gaussian distribution of initial opinions. The probability density function and a set of opinions in a Conservative and Liberal group are in (a) and (b), and (c) and (d), respectively.

Model-I: Experiment-1

Determination of confirmation bias of Rigid (θ_{T_R}) and Flexible (θ_{T_F}) agents;

- Are they invariant of
  1. Group Type
  2. Group Size
  3. Density of connections in the network

Parameters	**Value**
Group Size	
N	$\{200, 400, 800\}$
Group Type	
μ	$90°$
σ_c	$10°$
σ_l	$20°$
Tolerance	
θ_{T_i}	$\{5°, 10°, 15°, 20°, 25°, 30°\}$
In-degree	
$D_{in_{max}}$	$\{0.25N, 0.5N, \cdots, N\}$
$D_{in_{min}}$	$0.05N$

Model-I: Definitions

Definition 1

A group of interacting agents ($i \in [1, N]$) is said to form an **opinion faction**, c_j, where, $j = 1, 2, \cdots$, when their opinions differ utmost by an opinion proximity measure (θ_{pm}), and the network (G_{c_j}) formed by them is at least weakly connected (wc).

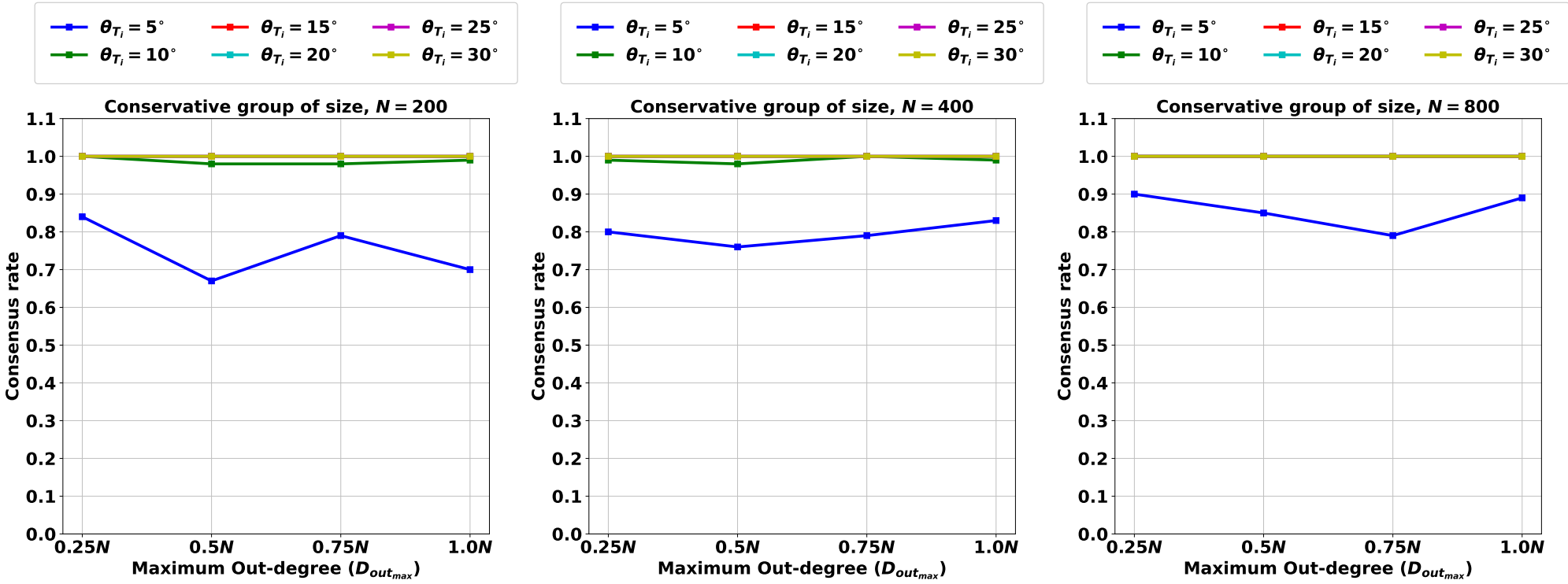
Definition 2

A group of N interacting agents, each with an opinion θ_i, is said to have reached consensus, **globally**, when there is one opinion faction.

Definition 3

A group of N interacting agents, each with an opinion θ_i, is said to have reached consensus, **locally**, when there is more than one opinion faction.

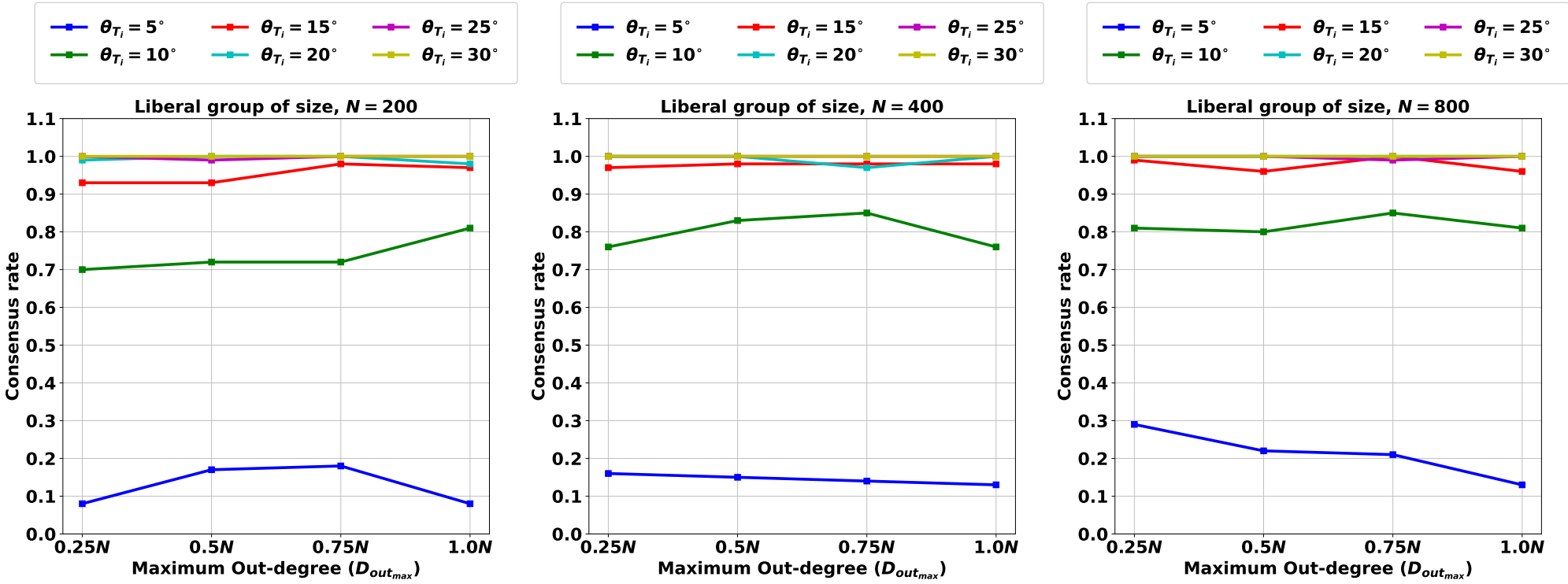
Model-I: Results of Experiment-1 in Conservative Groups



(a) (b) (c)

Figure: Variation in consensus rate with maximum in-degree for different opinion tolerances (θ_{T_i}); (a), (b) and (c) represent a Conservative group with population 200, 400 and 800, respectively.

Model-I: Results of Experiment-1 in Liberal Groups



(a) (b) (c)

Figure: Variation in consensus rate with maximum in-degree for different opinion tolerances (θ_{T_i}); (a), (b) and (c) represent a Liberal group with population 200, 400 and 800, respectively.

Model-I: Inferences from Experiment-1

1. Agents with a low opinion tolerance (5°) result in **opinion factions**
2. Agents with a high opinion tolerance (30°) almost always facilitate **global consensus**
3. Behaviours are **invariant** of group type, group size, and the density of connections

Confirmation Biases are $\theta_{T_R} = 5°$ and $\theta_{T_F} = 30°$

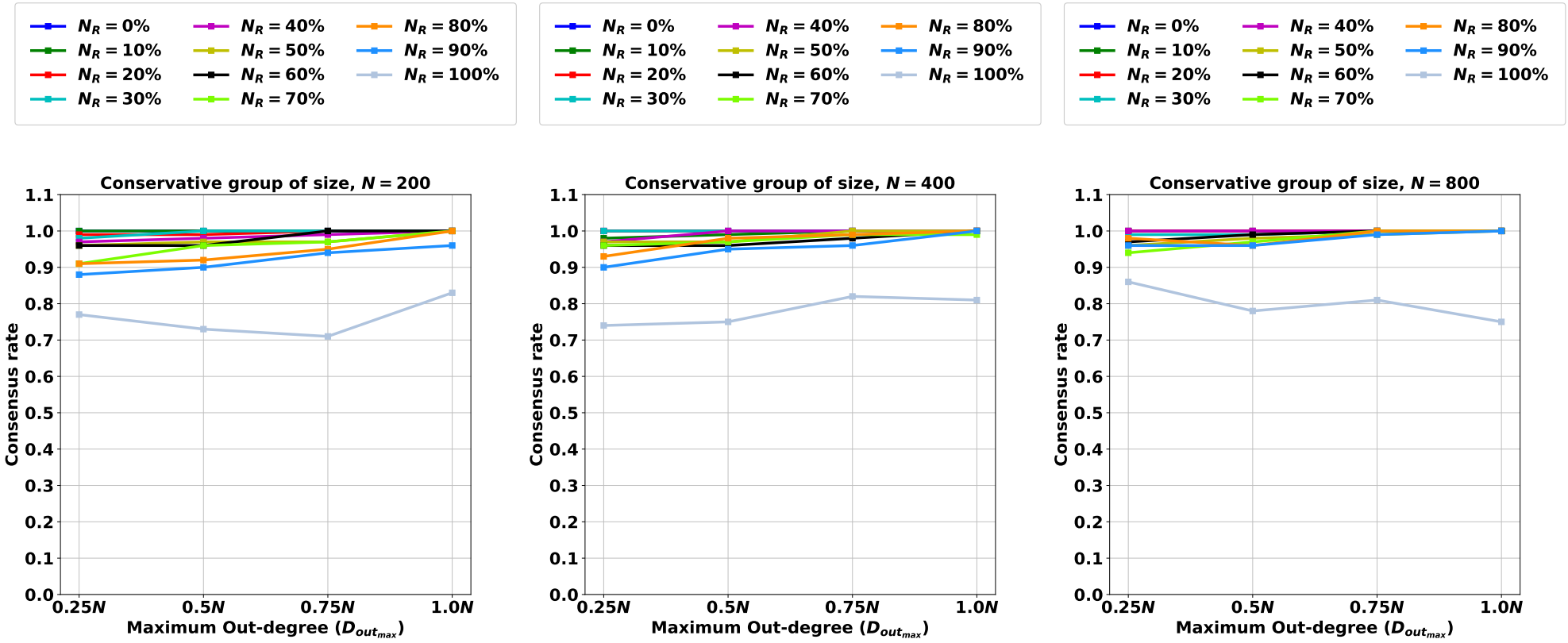
Model-I: Experiment-2

Evaluation of group behaviour

1. **Configuration 1**
   Conservative group with a Rigid majority
2. **Configuration 2**
   Liberal group with a Rigid majority
3. **Configuration 3**
   Conservative group with a Flexible majority
4. **Configuration 4**
   Liberal group with a Flexible majority

Parameters	Value
Population	
N	$\{200, 400, 800\}$
N_R	$\{0, 0.1N, 0.2N, \cdots, N\}$
N_F	$N - N_R$
Group Type	
μ	$90°$
σ_c	$10°$
σ_l	$20°$
Tolerance	
θ_{T_R}	$5°$
θ_{T_F}	$30°$
In-degree	
$D_{in_{max}}$	$\{0.25N, 0.5N, \cdots, N\}$
$D_{in_{min}}$	$0.05N$

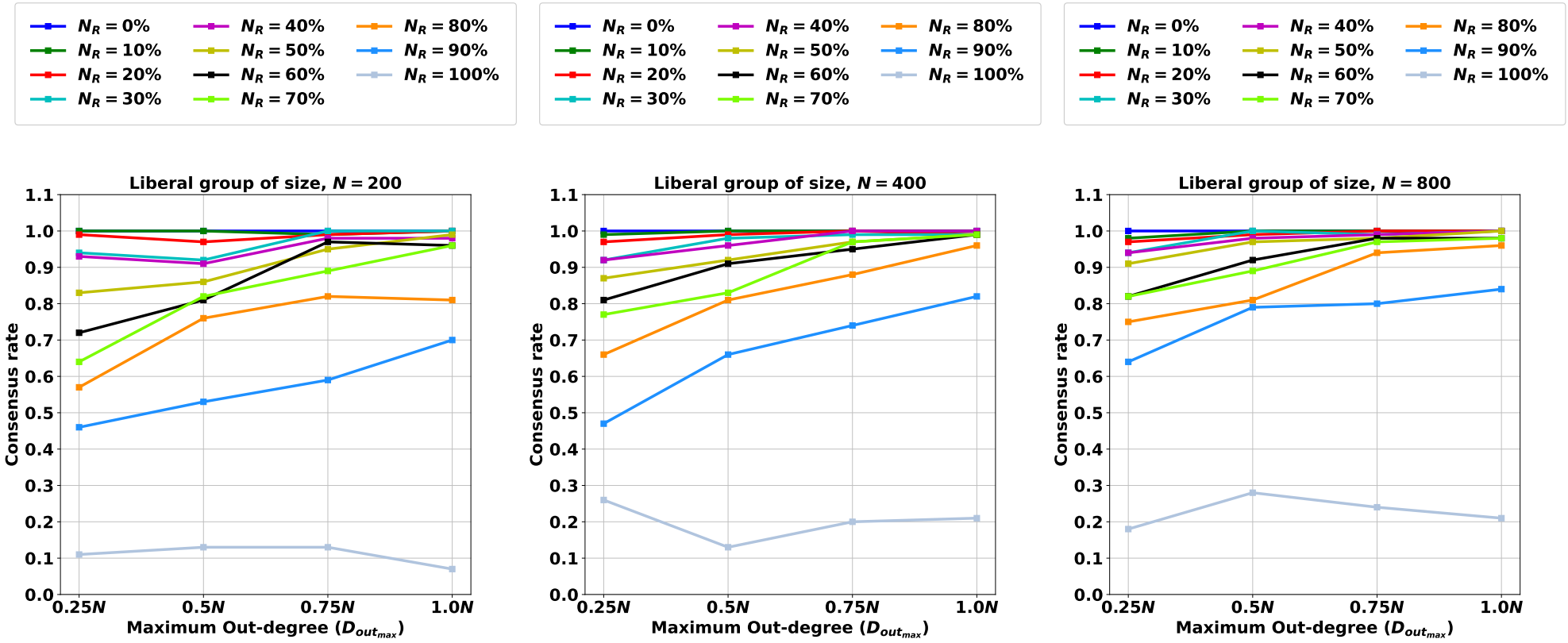
Model-I: Results of Experiment-2 in a Conservative Group



(a) (b) (c)

Figure: Variation in consensus rate with maximum in-degree for different opinion tolerances (θ_{T_i}); (a), (b) and (c) represent a Conservative group with population 200, 400 and 800, respectively.

Model-I: Results of Experiment-2 in a Liberal Group



(a) (b) (c)

Figure: Variation in consensus rate with maximum in-degree for different opinion tolerances (θ_{T_i}); (a), (b) and (c) represent a Liberal group with population 200, 400 and 800, respectively.

Model-I: Inferences from Experiment-2

1. A Conservative group with Flexible agents is the **most conducive** for global consensus
2. A Liberal group with Rigid agents is the **least suited** for both global consensus

	Rigid≥ Flexible	**Flexible≥Rigid**
Conservative	Low Tolerance	Moderately Tolerant
Liberal	Fragmented	Highly Tolerant

Model-I: Conclusion

Advantages

- Captures collective behaviour in different group configurations

Disadvantages

- The bounded-confidence assumption is far from reality;
  - Interactions are often due to familial ties, expertise, or popularity
- Opinion update, opinion bias and tolerance are not correlated;
  - Rigid agents **deviate** from their initial opinions
  - Flexible agents are **isolated** from the group
- Agents in different opinion groups do not interact

Model-II: Opinion Formation with Relative Agreement Neighborhood

1. **Network Topology**
   - *Dynamic* - Regular, Small-world and Random networks
2. **Network Components**
   - *Nodes/Agents* - have an opinion bias
   - *Ties/Connections* - are lost or gained based on **agent type**
   - *Contagion* - is directed
3. **Update**
   - *Rule* - is weighted average,
   - *Nature* - is **dependent on agent-type**

Model-II: Characterization of Agents

1. **Inclination** to alter their opinion,

$$P_{u_i} = (1 - w_{ii}),$$
$$w_{ii} \in (0, 1).$$

2. **Susceptibility** to the influence of peers with contrasting opinions,

$$w'_{ij}(t) = \exp\left(-\frac{k|\theta_i(t) - \theta_j(t)|}{1 - w_{ii}}\right),$$
$$k = 10.$$

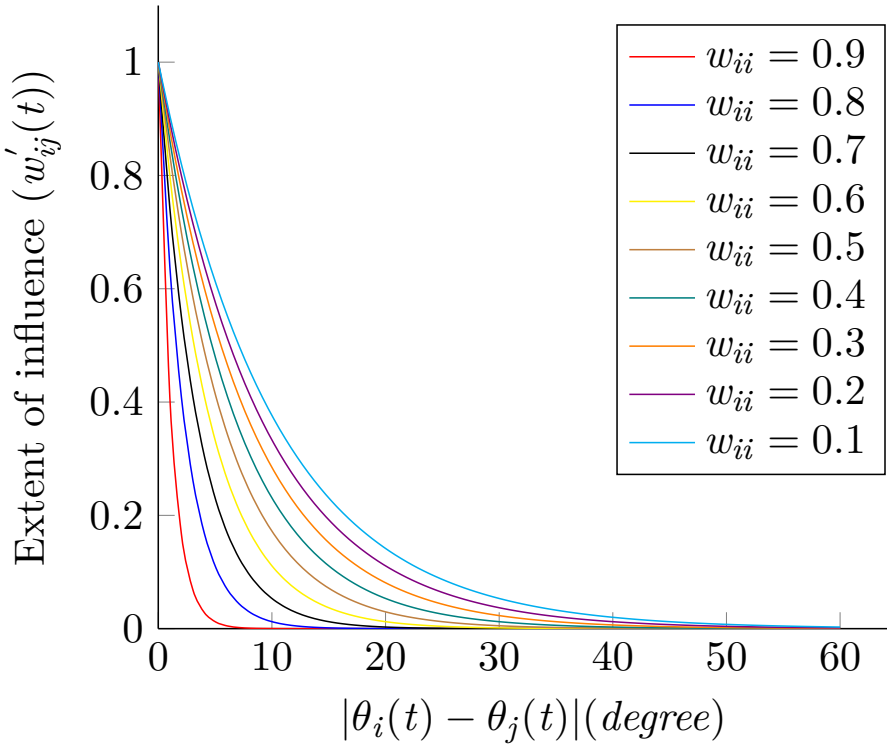


Figure: Extent of influence $(w'_{ij}(t))$ exercised by agent j on agent i as a function of difference in opinions $(|\theta_i(t) - \theta_j(t)|)$ and willingness to change (w_{ii})

Model-II: Proposed Model

1 Opinion Update

$$\vec{x}_i(t) := \langle x_i(t), \theta_i(t) \rangle, \ \forall i, t,$$

$$x_i(t) = 1, \text{ and } \theta_i(t) \in (-\pi, \pi],$$

$$\theta_i(t+1) = \langle \theta_i(t) \rangle + \cancel{\Delta\theta},$$

$$\tan(\langle \theta_i(t) \rangle) = \frac{\sum_{j \in N_{I_i}(t)} w_{ij}(t) \sin(\theta_j(t))}{\sum_{j \in N_{I_i}(t)} w_{ij}(t) \cos(\theta_j(t))}$$

2 Neighborhood

$$N_{D_i}(t) := \{j : (j,i) \in E(t), j \neq i\},$$

$$N_{ND_i}(t) := V - N_{D_i}(t),$$

$$N_{I_i}(t) \subseteq N_{D_i}(t) \cup N_{\bar{ND}_i}(t) \cup i$$

3 Influence Scores

$$w_{ij}(t) = \frac{\sum_{j \in N_{I_i}(t)} w'_{ij}(t)(1 - w_{ii})}{\sum_{j \in N_{I_i}(t)} w'_{ij}(t)},$$

$$\sum_{j \in N_{I_i}(t)} w_{ij}(t) = 1 - w_{ii},$$

Rigid: $w_{ii} > \sum_{j \in N_{I_i}(t)} w_{ij}(t),$

Flexible: $w_{ii} \leq \sum_{j \in N_{I_i}(t)} w_{ij}(t)$

4 Network Evolution

$$p_{\text{gain}_{(j,i)}}(t) = p_c w'_{ij}(t),$$

$$p_{\text{lose}_{(j,i)}}(t) = p_d(1 - w'_{ij}(t)),$$

$$0 \leq p_c, p_d \leq 1$$

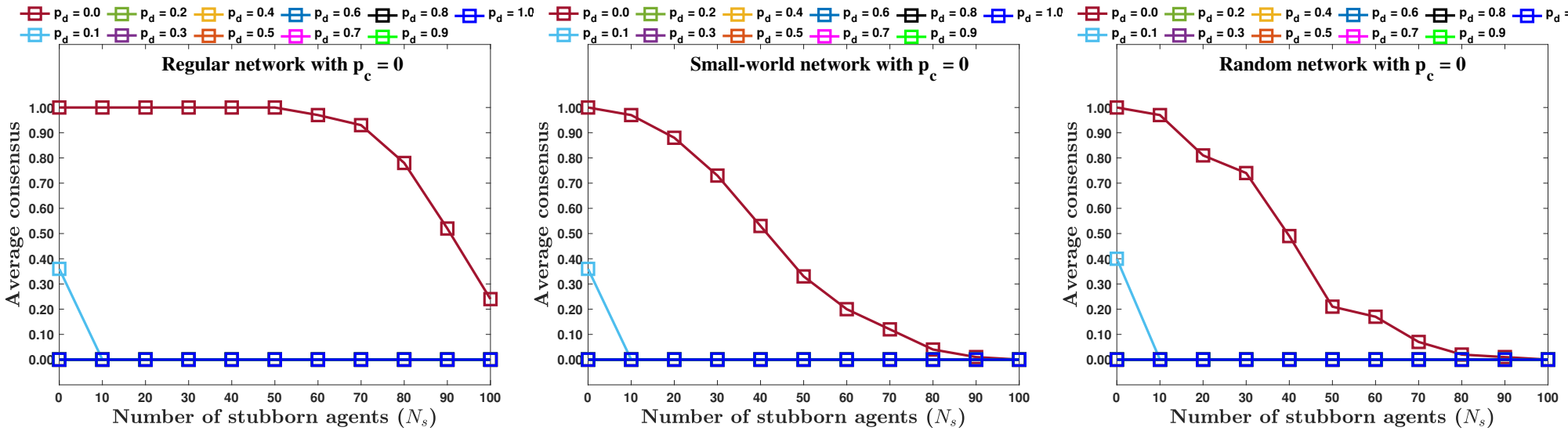
Model-II: Experiment-1

Evaluation of group behaviour in four configurations, subject to,

1. Group Composition
2. Group Type
3. Influence Network Structure
4. Rate of Network Evolution

Parameters	Value
Population	
N	200
N_R	$\{0, 0.1N, \cdots, N\}$
N_F	$N - N_R$
Group Type	
μ, σ_c, σ_l	90°, 10°, 20°
Opinion Bias (w_{ii})	
Rigid	$[0.6, 1)$
Flexible	$(0, 0.6)$
Influence Network	
Total Degree	$d = 0.25N$
Rewiring	$p \in \{0, 0.5, 1\}$
Connect Rate (p_c)	$\{0, 0.5, 1\}$
Disconnect Rate (p_d)	$\{0, 0.5, 1\}$

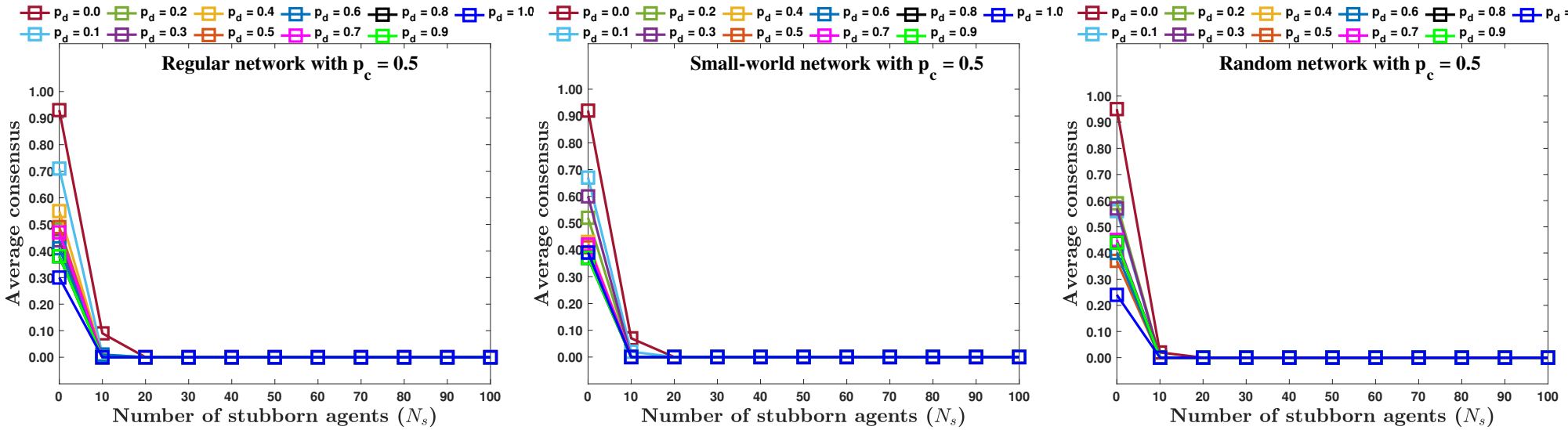
Model-II: Results of Experiment-1



(a) (b) (c)

Figure: Variation in consensus rate in a Liberal group with respect to the initial structure of the influence network, network evolution rate and group composition

Model-II: Results of Experiment-1



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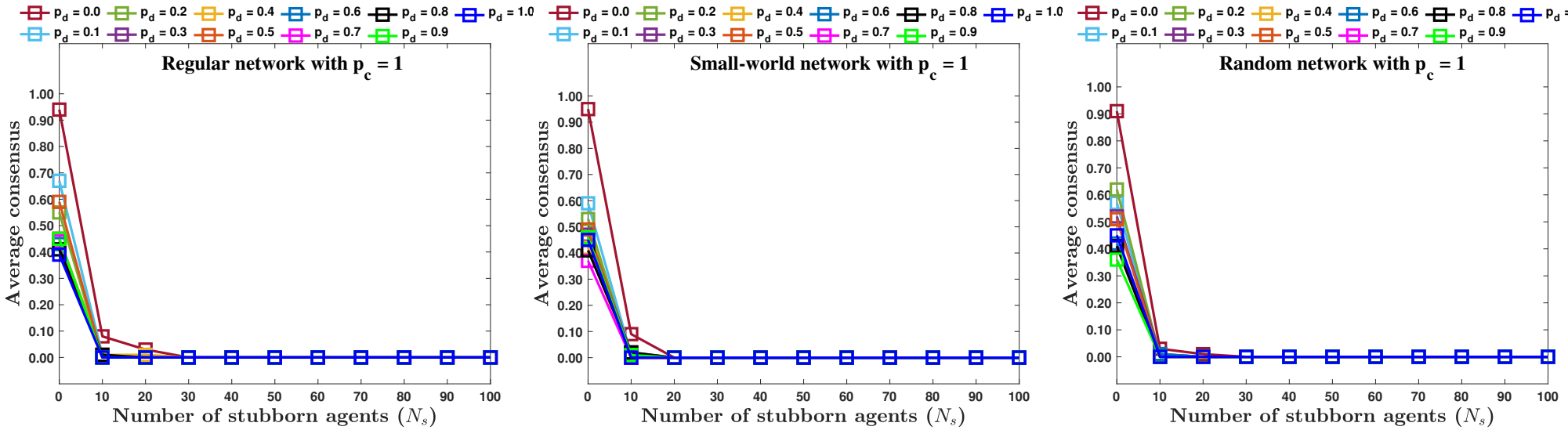

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Figure: Variation in consensus rate in a Liberal group with respect to the initial structure of the influence network, network evolution rate and group composition

Model-II: Inferences from Experiment-1

- The observations regarding the impact of **group size, types and compositions** are same as in Model-I
- The structure of the initial network affects the outcome only when the network is **static**
  - A regular network is more conducive for global consensus than small-world and random networks
- The formation of ties ($p_c = 1$) **does not** facilitate global consensus

Model-II: Conclusion

Advantages

- The relative agreement neighborhood appropriately **captures** group behaviour
- The model also provides for a more **accurate** representation of individual behaviour

Disadvantages

- The structure and evolution of the underlying influence network can be **tuned with data** gathered from virtual social platforms

Disease Spread and Mitigation: Objectives

1. Model disease spread as a consequence of human movement and contact;
   - A **synthetic populace** with relevant individual attributes
   - Disease-specific **transmission** and **evolution** probabilities
   - Rules governing **human movement** and **inter-personal contact**
2. Design **scenarios** mandating different movement patterns
3. Evaluate possible **mitigatory** measures

Disease Spread and Mitigation: Prevalent Approaches

1. **Population Attributes**
   - Age-structure, Population size[14], Ethnicity, Marital Status, Spatial Location[15]
2. **Population Mixing and Contact**
   - *Networks* - Home, School, Work-place, Transportation, Markets, Special-Care Centres
   - *Shifting Positions* - Movement along a lattice, GPS data-based[16]
   - *Motion Rules* - Social-Force model
3. **Disease Transmission and Evolution**
   - *Age-specific* transmission and evolutionary dynamics
   - *Characterization* - S-E-P-A-R-D, S-I-D-A-R-T-H-E

[14] Kerr, C. C., Stuart, R. M., Mistry, D., Abeysuriya, R. G., Rosenfeld, K., Hart, G. R.,& Klein, D. J. (2021). Covasim: an agent-based model of COVID-19 dynamics and interventions. PLOS Computational Biology, 17(7), e1009149.

[15] Koo, J. R., Cook, A. R., Park, M., Sun, Y., Sun, H., Lim, J. T., Dickens, B. L. (2020). Interventions to mitigate early spread of SARS-CoV-2 in Singapore: a modelling study. The Lancet Infectious Diseases, 20(6), 678-688.

[16] Silva, P. C., Batista, P. V., Lima, H. S., Alves, M. A., Guimarães, F. G., & Silva, R. C. (2020). COVID-ABS: An agent-based model of COVID-19 epidemic to simulate health and economic effects of social distancing interventions. Chaos, Solitons & Fractals, 139, 110088.

Model-III: Human Motion Patterns and Disease Spread

1. **Population Attributes**
   - Age-stratified population
   - Country-specific population density
2. **Population Mixing and Contact**
   - *Motion Rules* - Vicsek-inspired motion rules with physical distancing and goal-following
3. **Disease Transmission and Evolution**
   - *Age-specific* transmission and evolutionary dynamics
   - *Characterization* - severity, infection status and cognizance

Model-III: The Proposed Model

1. **Demography**

Age: $a_i \in \{Y, M, O\}, \forall i,$

Density: $N/km^2,$

Health Status: $\theta_i(t),$

Positions: $x_i, y_i \in (0, L)$

2. **Motion Patterns**

Stationary
$(W_{GF} = 1)$
Short-range
$(W_{GF} = 1)$
Long-range
$(W_{GF} = 0)$

3. **Disease Spread**

$$j := \underset{z \in N_{I_i}(t+1)}{\arg\min}\ d_{iz}(t+1),$$

$$N_{I_i}(t+1) := \{j : d_{ij}(t) \leq r_I : \theta_j(t) \text{ is infected}\}$$

4. **Motion Rules**

$$v_{\mathrm{T}_i}(t+1) = v_{\mathrm{T}_i}(t) + v_{CA_i}(t+1)\, W_{CA} + v_{GF_i}(t+1)\, W_{GF},$$

$$x_i(t+1) = x_i(t) + v_{\mathrm{Total}_i}(t+1)\Delta t$$

$$N_{C_i}(t+1) := \{j : d_{ij}(t) \leq r_{C_i}, j \neq i\}$$

$$v_{CA_i}(t+1) = -\sum_{j \in N_{C_i}(t+1)} \frac{\frac{x_i(t) - x_j(t)}{||x_i(t) - x_j(t)||^2}}{\Delta t},$$

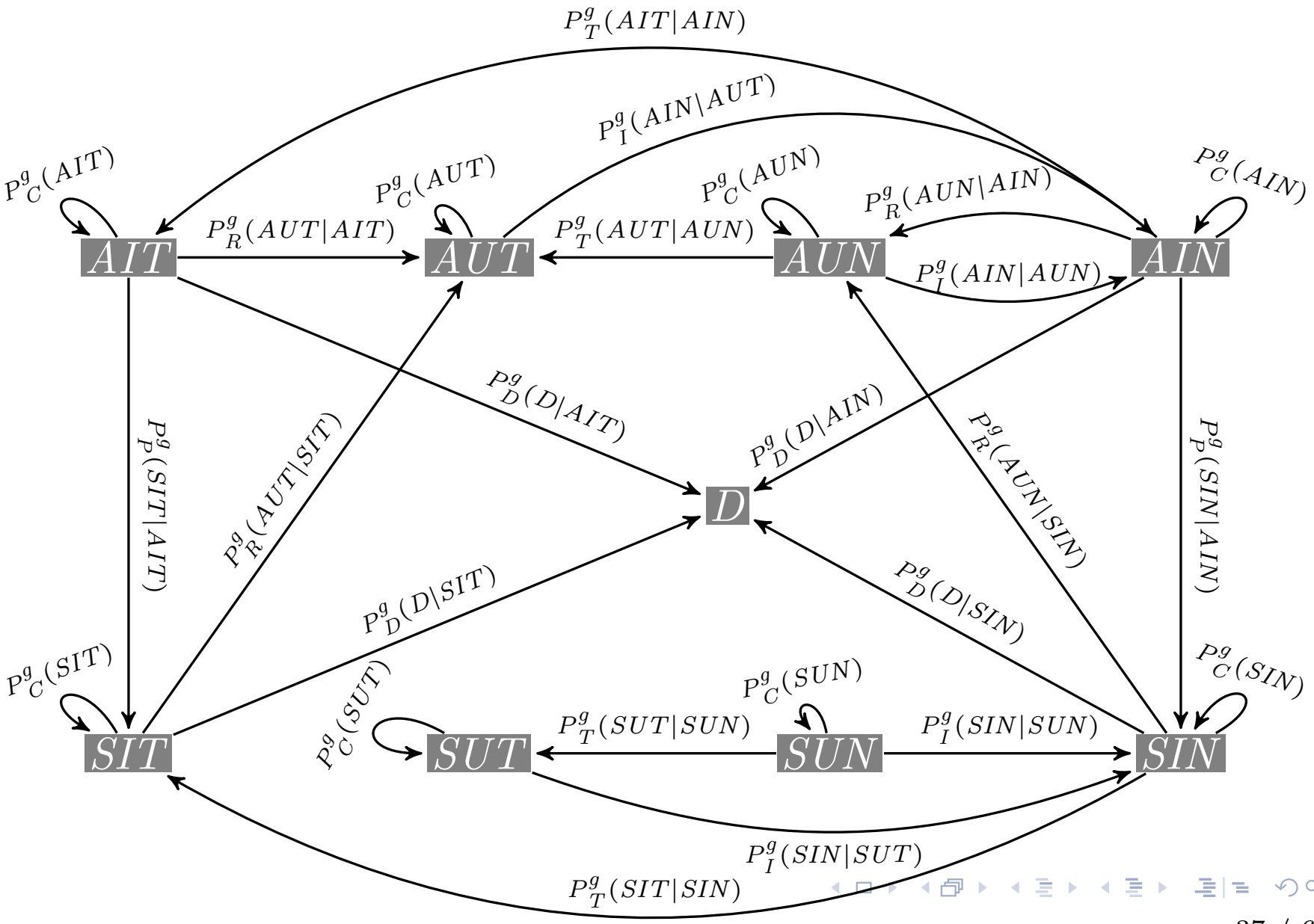
$$v_{GF_i}(t+1) = -\left(\frac{x_i(t) - x_{goal_i}(t)}{\Delta t}\right)$$

Model-III: The Proposed Disease Characterization

1. **Infection Status:** Infected (I), Uninfected (U)
2. **Severity:** Symptomatic (S), Asymptomatic (A)
3. **Cognizance:** Tested (T), Not-Tested (N)

Infection Status & Severity \ Cognizance		Tested (T)	Not Tested (N)
Symptomatic (S)	**Infected (I)**	SIT	SIN
	Uninfected (U)	SUT	SUN
Asymptomatic (A)	**Infected (I)**	AIT	AIN
	Uninfected (U)	AUT	AUN

Model-III: Disease Transmission, Progression and Evolution



Model-III: Experiment-1

1. **Pattern 1:** All agents are stationary
2. **Pattern 2:** Some (or all) agents exhibit short-range movement while the rest are stationary
3. **Pattern 3:** Some (or all) agents exhibit long-range movement while the rest are stationary
4. **Pattern 4:** Some (or all) agents exhibit short-range movement while others move long distances.

Parameters	Value
N	400
I	$0.01N$
L	1000 m
V_{min}	-5 m/Δt
V_{max}	5 m/Δt
r_I	10 m
r_C	5 m, 25 m
r_{GF}	100 m
t_f	21 days
Δt	15 mins
M	100

Model-III: Disease Transmission, Progression and Evolution

		P_S	P_P	P_R	P_D	P_C
SIT	Children	-	-	0.12495	0.00005	0.875
	Adults	-	-	0.124	0.001	0.875
	Seniors	-	-	0.065	0.06	0.875
SIN	Children	-	-	0.09994	0.00006	0.9
	Adults	-	-	0.06	0.04	0.9
	Seniors	-	-	0.02	0.08	0.9
AIT	Children	-	0.00998	0.19	0.00002	0.8
	Adults	-	0.03993	0.16	0.00007	0.8
	Seniors	-	0.08	0.1	0.02	0.8
AIN	Children	-	0.02996	0.12	0.00004	0.85
	Adults	-	0.0493	0.1	0.0007	0.85
	Seniors	-	0.1	0.01	0.04	0.85
SUT	Children	0.2537	-	-	-	0.7463
	Adults	0.5	-	-	-	0.5
	Seniors	0.5951	-	-	-	0.4049
SUN	Children	0.2537	-	-	-	0.7463
	Adults	0.5	-	-	-	0.5
	Seniors	0.5951	-	-	-	0.4049
AUT	Children	0.2537	-	-	-	0.7463
	Adults	0.5	-	-	-	0.5
	Seniors	0.5951	-	-	-	0.4049
AUN	Children	0.2537	-	-	-	0.7463
	Adults	0.5	-	-	-	0.5
	Seniors	0.5951	-	-	-	0.4049

Progress

$P_P(SIN|AIN)$

$> P_P(SIT|AIT)$

Recover

$P_R(AUT|AIT)$

$> P_R(AUN|AIN)$

$> P_R(AUT|SIT)$

$> P_R(AUN|SIN)$

Decease

$P_D(D|SIN) > P_D(D|SIT)$

$> P_D(D|AIN)$

$> P_D(D|AIT)$

Delay

$P_C(SIT) > P_C(AIT)$

Model-III: Definitions

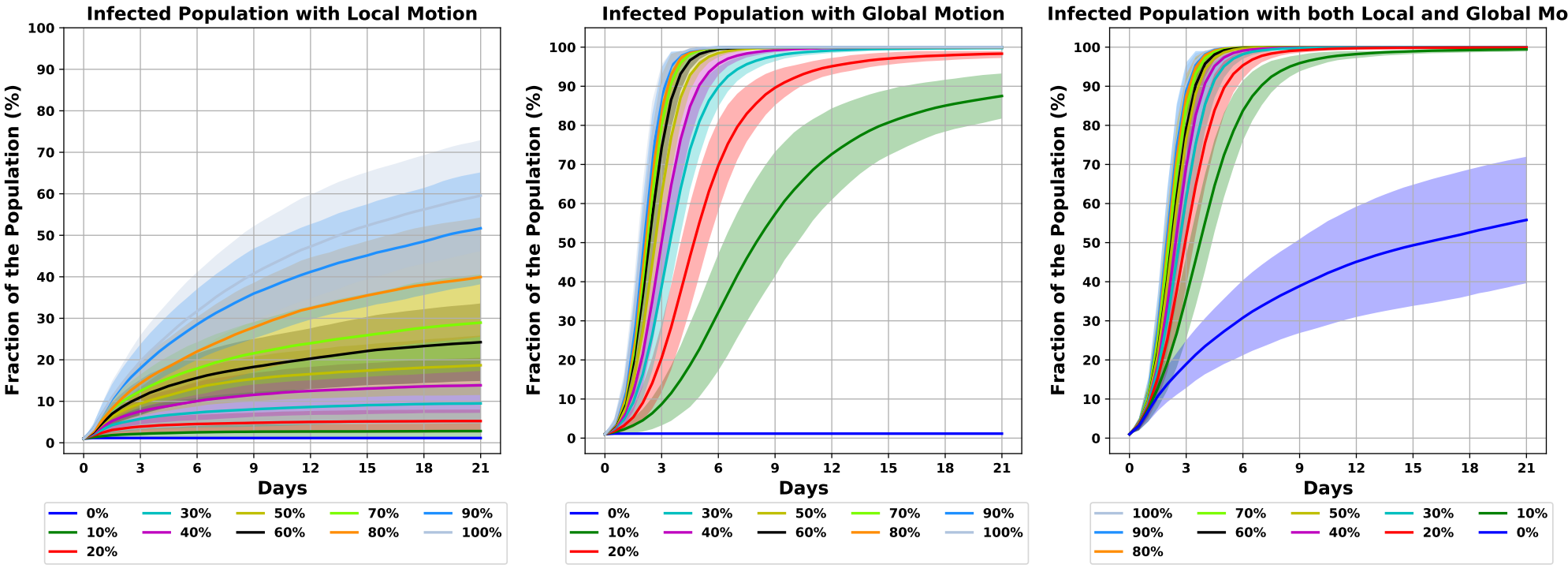
Definition 4

Average Cumulative Infections, is the daily average of the total number of infected agents in states $\{SIT, SIN, AIT, AIN\}$ obtained from M MonteCarlo runs.

$$\text{Infected}_{Avg}(t) := \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \mathbb{1}_{\mathbb{I}\mathbb{n}_{\mathbb{j}}}(\mathbb{t})}{M}, \ \forall t,$$

$$\mathbb{1}_{\mathbb{I}\mathbb{n}_{\mathbb{j}}}(\mathbb{t}) := \begin{cases} 1, & \text{if } \theta_j(t) \in \{SIT, SIN, AIT, AIN\} \\ 0, & \text{otherwise} \end{cases}.$$

Model-III: Results of Experiment-1



(a) (b) (c)

Figure: Cumulative infections with short-range, long-range, and both short-range and long-range motion patterns

Model-III: Experiment-2

1. **Containment Zone**

- **Infections** are concentrated within a bounded region
- The **porosity** of the boundary indicates the probability with which an agent outside the zone can enter
- Movement of agents residing in this region is **forbidden**

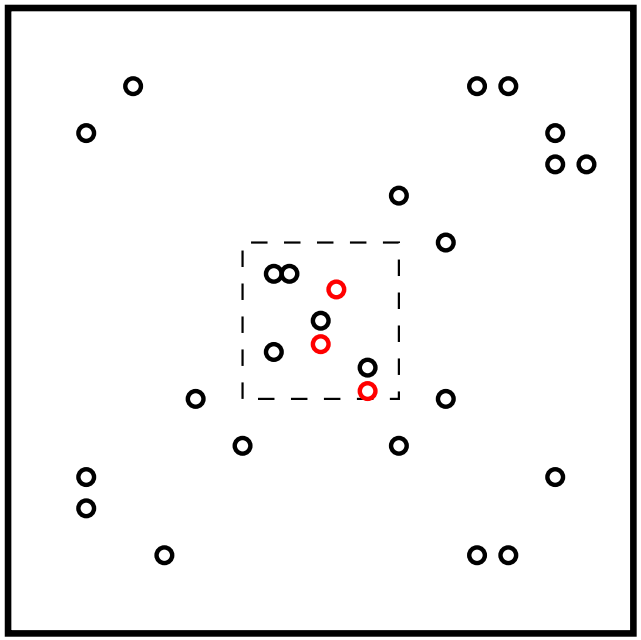


Figure: A typical layout with infected agents (in red) and uninfected agents (in black); the dashed box represents a containment zone

Model-III: Results of Experiment-2

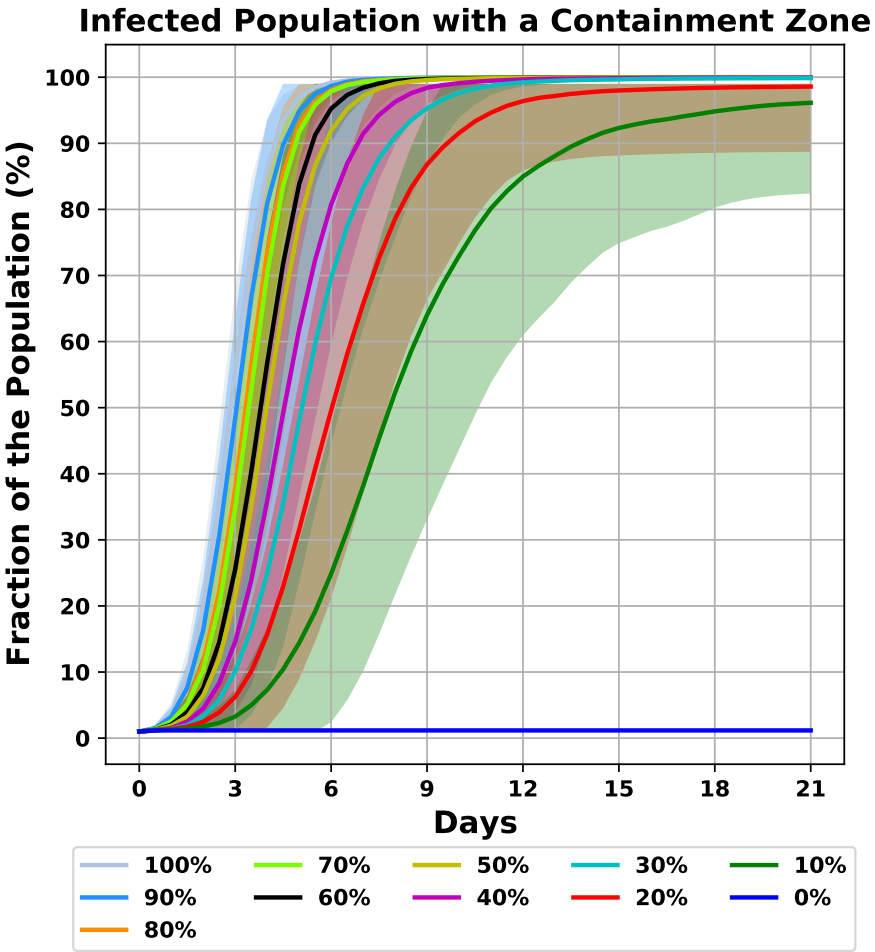
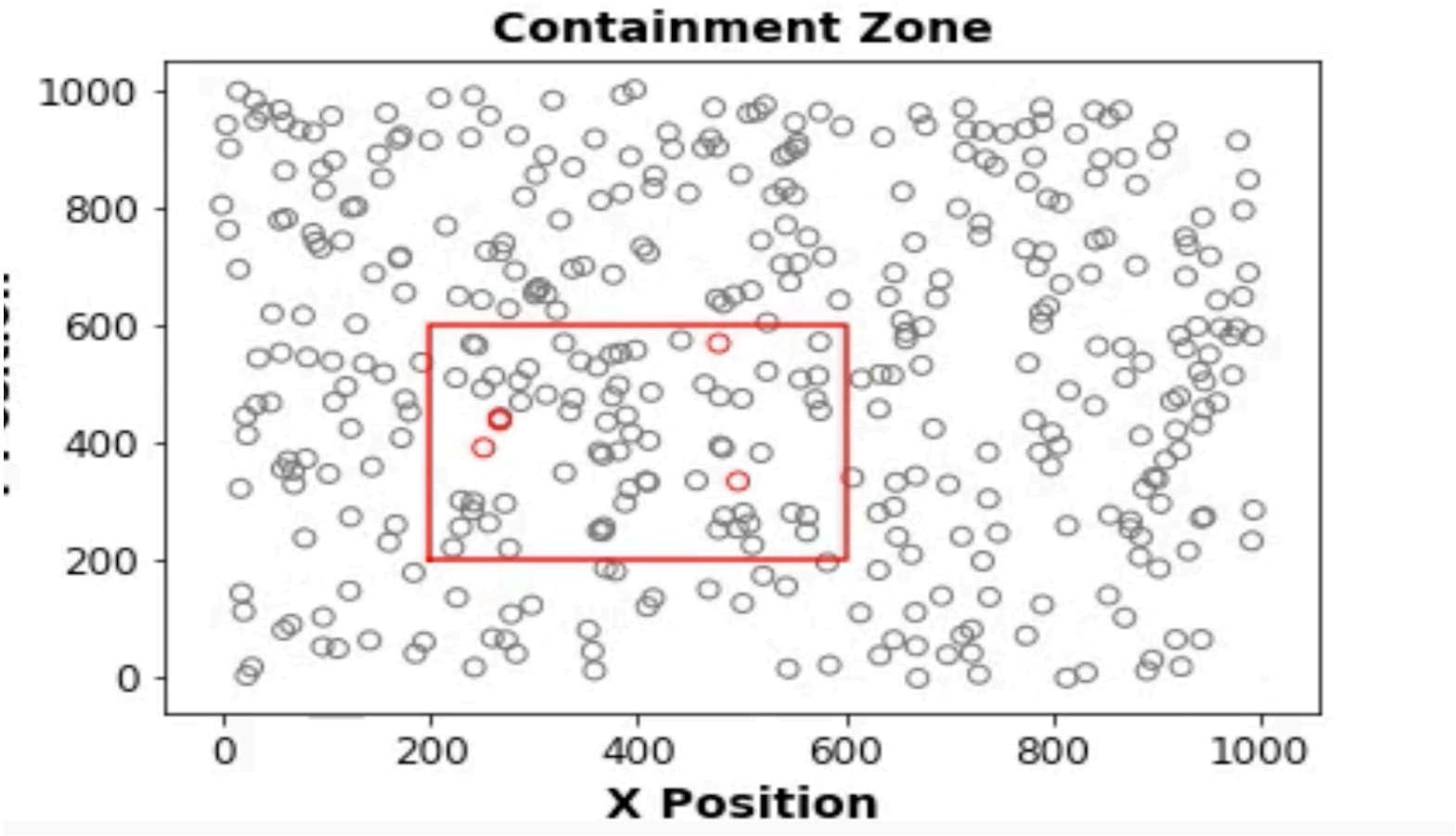


Figure: Cumulative infections with a containment zone of varying porosity

Model-III: Results of Experiment-2



Model-III: Conclusion

Advantages

- Simple human movement patterns demonstrate **logical trends** in disease outbreak

Disadvantages

- Spatial distribution is **random** without any household structure
- Persistent and repetitive motion **unlike** time-appropriate human movement

Model-IV: Activity-based Human Movement and Disease Spread

1. **Population Attributes**
   - Age, Household Structure and Occupation
   - City-specific population and household density
2. **Population Mixing and Contact**
   - *Motion Rules* - Vicsek-inspired motion rules with physical distancing and goal-following
   - Time-appropriate, Activity-driven motion
   - Activity-specific contact patterns
3. **Disease Transmission and Evolution**
   - *Age-specific* transmission and evolutionary dynamics
   - *Characterization* - severity, infection status and cognizance

Model-IV: Synthetic City

1. **Total Grids**

$$G_T = \frac{L_{x_{Max}} \times L_{y_{Max}}}{L_G^2}$$

2. **Occupied Grids**

$$G_O \in [G_{Min}, G_{Max}], \; G_{Max} \leq G_T$$

3. **Household Density**

$$\sum_{k=1}^{G_O} h_{d_k} = H, \; h_{d_k} > 0 \; \forall k$$

4. **Household Position**

$$h_{x_j} \in \left[G_{x_{Min_k}}, G_{x_{Max_k}}\right],$$

$$h_{y_j} \in \left[G_{y_{Min_k}}, G_{y_{Max_k}}\right],$$

$$j = 1, 2, \ldots, h_{d_k}, \; k = 1, 2, \ldots, G_O$$

5. **Age**

$$a_i \in \{Y, M, O\}, \forall i \in N$$

6. **Household Matching**

Iterative Proportional Fitting[17]

7. **Spatial Co-ordinates**

$$x_i(0) = h_{x_j} + b_{x_i},$$
$$y_i(0) = h_{y_j} + b_{y_i},$$
$$\forall i \in h_j, \; j = 1, 2, \ldots, H$$

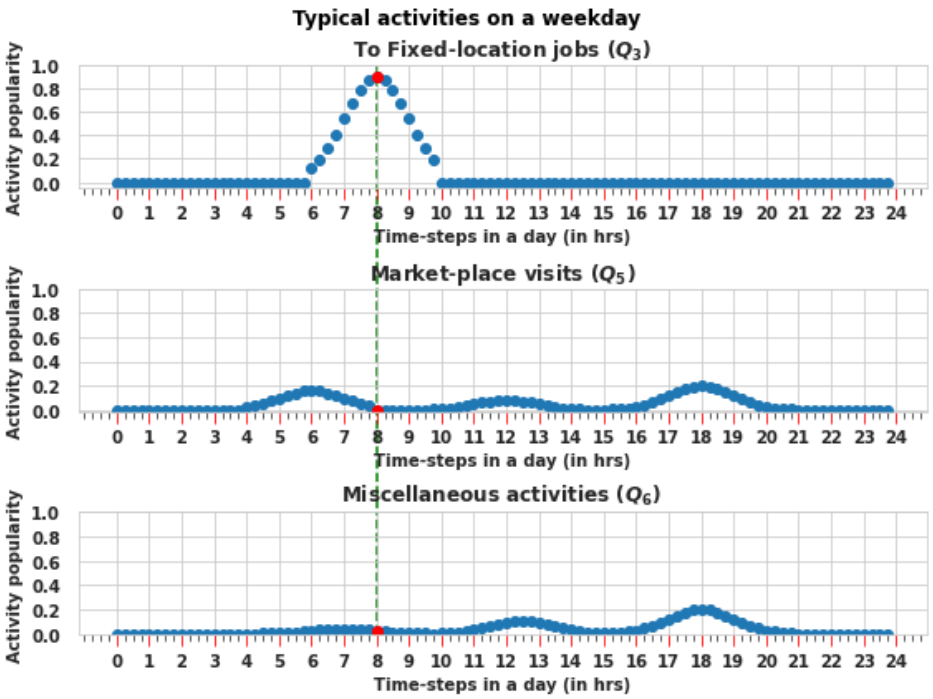
[17] N. Lomax and P. Norman, Estimating population attribute values in a table:get me started in iterative proportional fitting, The Professional Geographer, vol. 68, no. 3, pp. 45

Model-IV: Activity Schedule

1. **Activities**
   - Home (Q_1)
   - School (Q_2)
   - Fixed-location Job (Q_3)
   - Errands (Q_4)
   - Nomadic Jobs (Q_5)
   - Market Visits (Q_6)
2. **Probability Mass Function**

$$\sum_{l=1}^{6} P_t^g(Q = Q_l | Q_i) \;=\; 1,$$

$\forall g = \{Y, M, O\}, t,$ and $i = 1, 2, \ldots, 6.$



1- $\mathrm{P}_t^{M/O}(Q_3|Q_1) - P_t^{M/O}(Q_5|Q_1)$
- $\mathrm{P}_t^{M/O}(Q_6|Q_1) = P_t^{M/O}(Q_1|Q_1)$

Model-IV: Daily-Contacts Matrix

Matrix Entries

$$c_{ij}(t) = \frac{\sum_{k=1}^{N} \mathbb{1}_{\mathbb{k}}(\mathbb{t})}{\sum_{l=1}^{N} \mathbb{1}_{\mathbb{l}}(\mathbb{t})},$$

$$i, j = 1, 2, \ldots, 6.$$

$$\mathbb{1}_{\mathbb{l}}(\mathbb{t}) := \begin{cases} 1, & \text{if } q_l(t) == q_i, \\ 0, & \text{otherwise.} \end{cases},$$

$$\mathbb{1}_{\mathbb{k}}(\mathbb{t}) := \begin{cases} 1, & \text{if } (q_i(t), q_j(t)) \text{ is allowed,} \\ & j \in N_{D_i}(t+1), \\ 0, & \text{otherwise} \end{cases}.$$

		$q_j(t)$					
		Home	**School**	**Workplace**	**Nomadic Jobs**	**Market Visit**	**Misc. Activities**
$q_i(t)$	**Home**	✓	-	-	✓	-	✓
	School	-	✓	-	✓	-	✓
	Workplace	-	-	✓	✓	-	✓
	Nomadic Jobs	✓	✓	✓	✓	✓	✓
	Market Visit	-	-	-	✓	✓	-
	Misc. Activities	✓	✓	✓	✓	-	✓

Model-IV: Experiment-1

1. **Unlock Phase-1**
   - Phase in India between 1st and 30th June 2020
   - Public spaces were re-opened
   - Schools remained shut
   - All other activities were permitted during the day
   - Night and weekend lockdowns were enforced

Parameters	**Value**
Total Time (t_f)	31 days
Time Step (Δt)	15 mins
MonteCarlo Runs (M)	10
Initial Infections	0.5% (Children) 49% (Adults) 50.5% (Seniors)
Infection Severity	80% (Asymp.) 20% (Symp.)
Velocity	-0.6 m/s (V_{Min}) 0.6 m/s (V_{Max})
Radii	1 m (r_{GF}) 2 m (r_I) 5 m (r_{SD_i})

Model-IV: Definitions

Definition 5

Average Active Infections, is the daily average of the total number of tested and infected agents in states $\{AIT, SIT\}$ obtained from M MonteCarlo runs.

Definition 6

Average Recoveries, is the daily average of the total number of recovered agents in state $\{AUT\}$ obtained from M MonteCarlo runs.

Definition 7

Average Mortalities, is the daily average of the total number of fatalities $\{D\}$ obtained from M MonteCarlo runs.

Model-IV: Definitions

Definition 8

Time-to-Infection, is the average time taken for an uninfected agent to contract the disease (t_I) obtained from M MonteCarlo runs. Higher the value of this measure better is the effectiveness of the intervention strategy.

Definition 9

Time-to-Recover, is the average time taken by agents to recover from the disease (t_R) obtained from M MonteCarlo runs.

Definition 10

Time-of-Infectiousness, is the average time for which an infected agent remains infectious (t_{OI}) obtained from M MonteCarlo runs.

Model-IV: City-specific Demographic Details

		Parameters				
		Population per 10 sq.Km	Age-Split			Households per 10 sq.Km
			Children	Adults	Seniors	
Cities	Mumbai	2,06,340	22.06%	72.12%	5.82%	46,102
	Bengaluru	1,08,720	22.78%	72.39%	4.83%	27,094
	Lucknow	64,560	25.16%	68.35%	6.49%	12,269
	Diu	20,580	30.8%	63.4%	5.8 %	4,502

		Age Group		
		Children	Adults	Seniors
Activities	Home (Q_1)	25%	20%	60%
	Schools (Q_2)	75%	5%	2%
	Workplace (Q_3)	-	60%	33%
	Nomadic Jobs (Q_4)	-	15%	5%
	Market Visits (Q_5)	Applicable		
	Misc. Activities (Q_6)	Applicable		

Model-IV: City-specific Synthetic Layouts

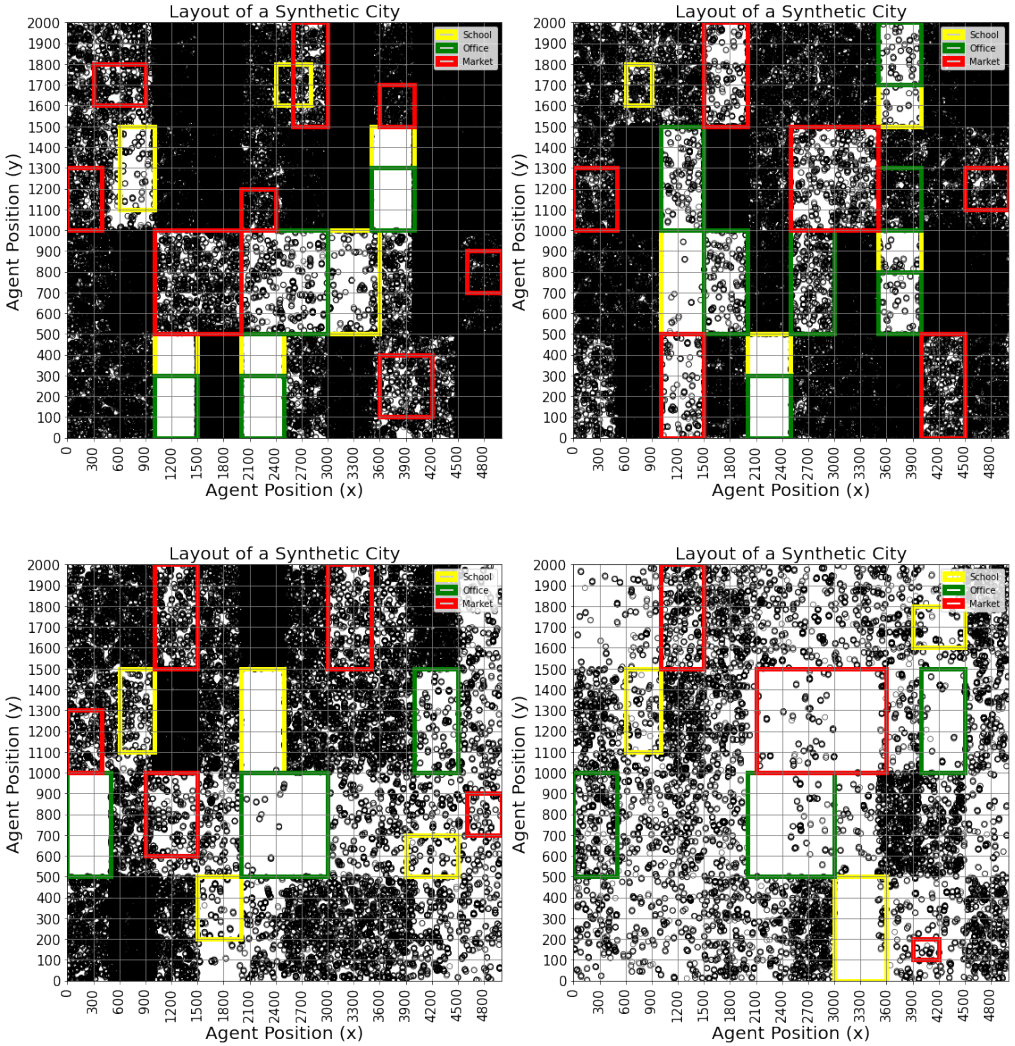


Figure: Layouts generated for all four cities; (a), (b), (c) and (d) represent the cities of Mumbai, Bangalore, Lucknow and Diu over a 10 sq.Km area.

Model-IV: Activity Schedule of Children, Adults and Seniors

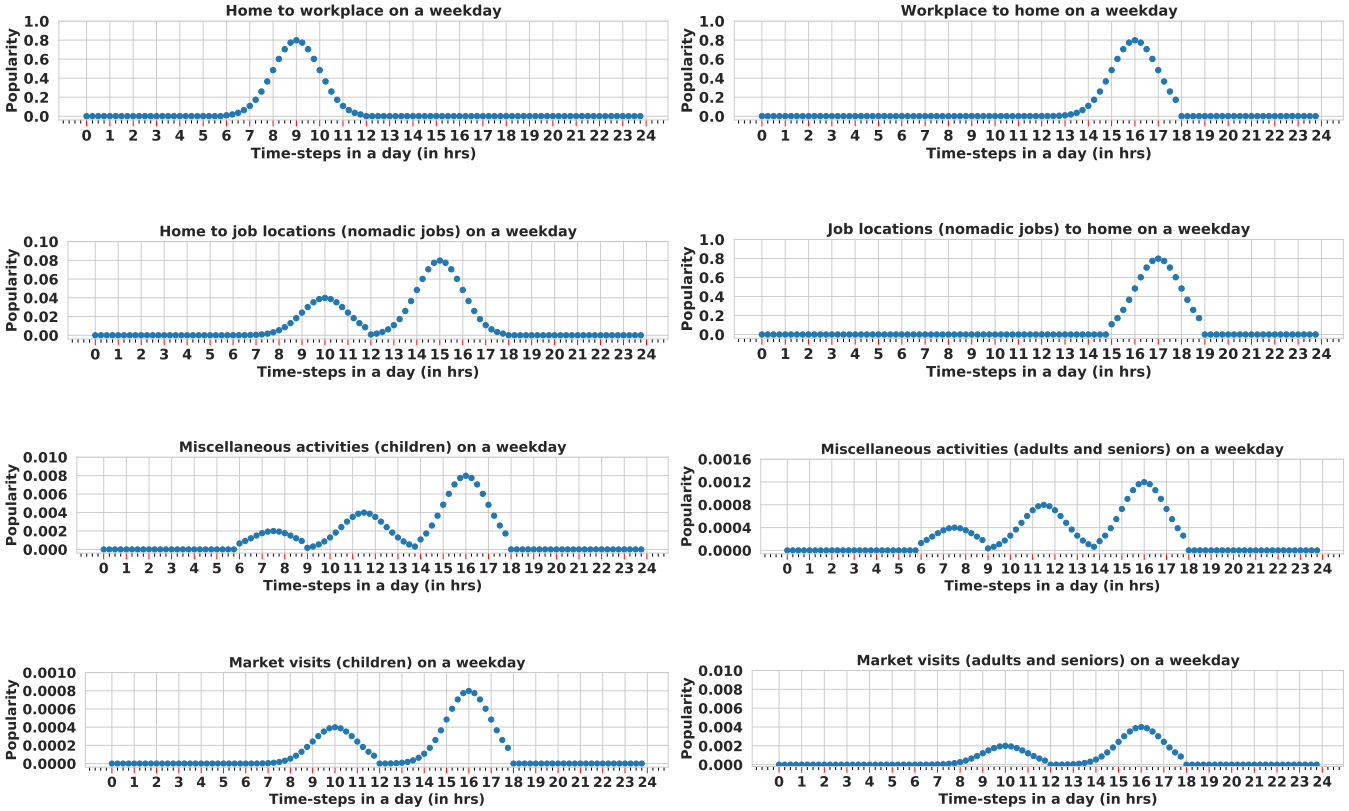


Figure: Activity-schedule of children, adults and seniors during Unlock Phase-1, in Mumbai; (a), (b), (c) and (d) represent the popularity of fixed-location and nomadic jobs, while (e), (g) and (f), (h) correspond to miscellaneous activities and market-place visits by children and adults, respectively.

Model-IV: City-specific Disease Evolution

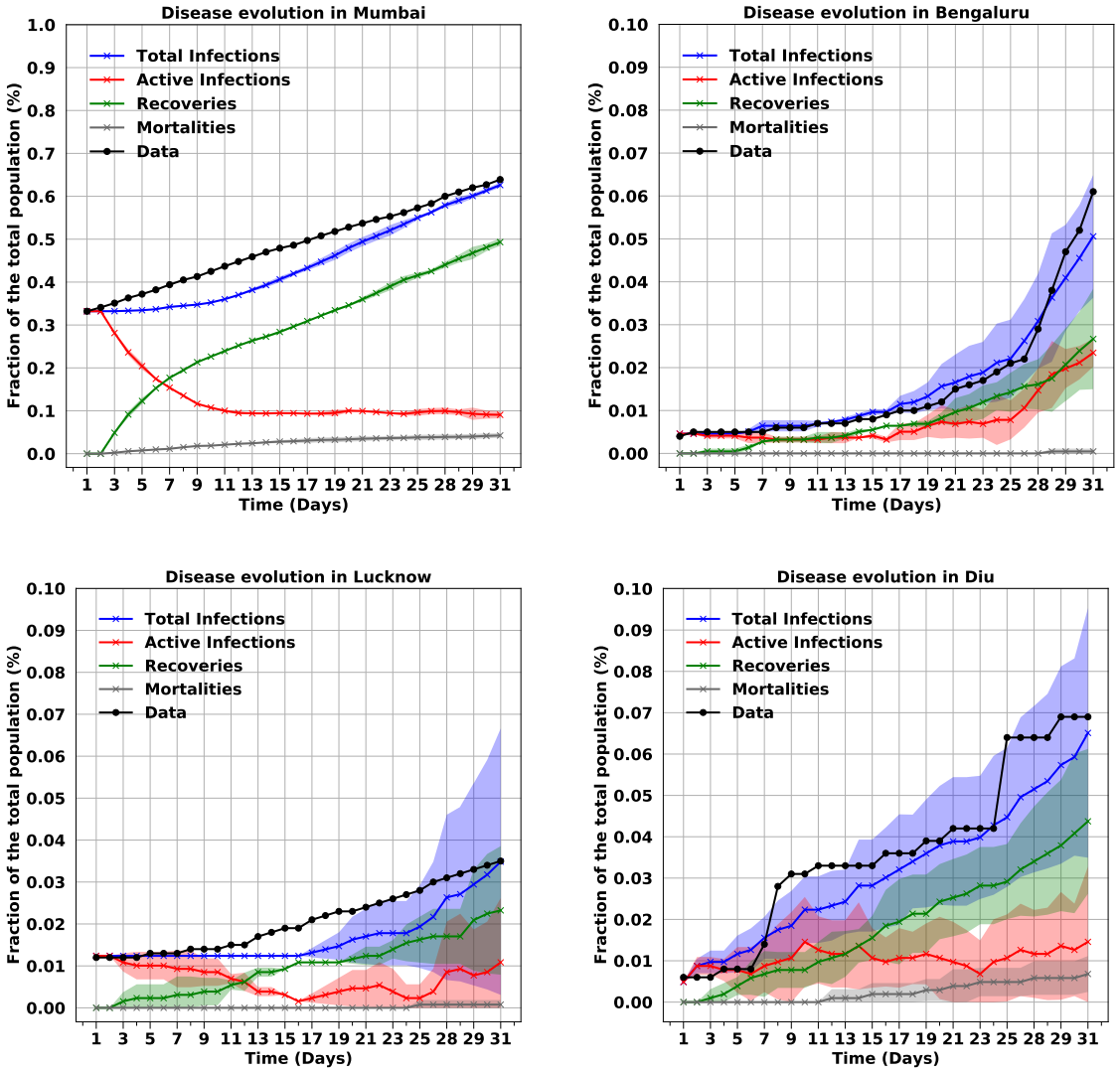


Figure: Disease evolution observed in four cities during Unlock Phase-1.

Model-IV: City-specific Daily-Contacts Matrix

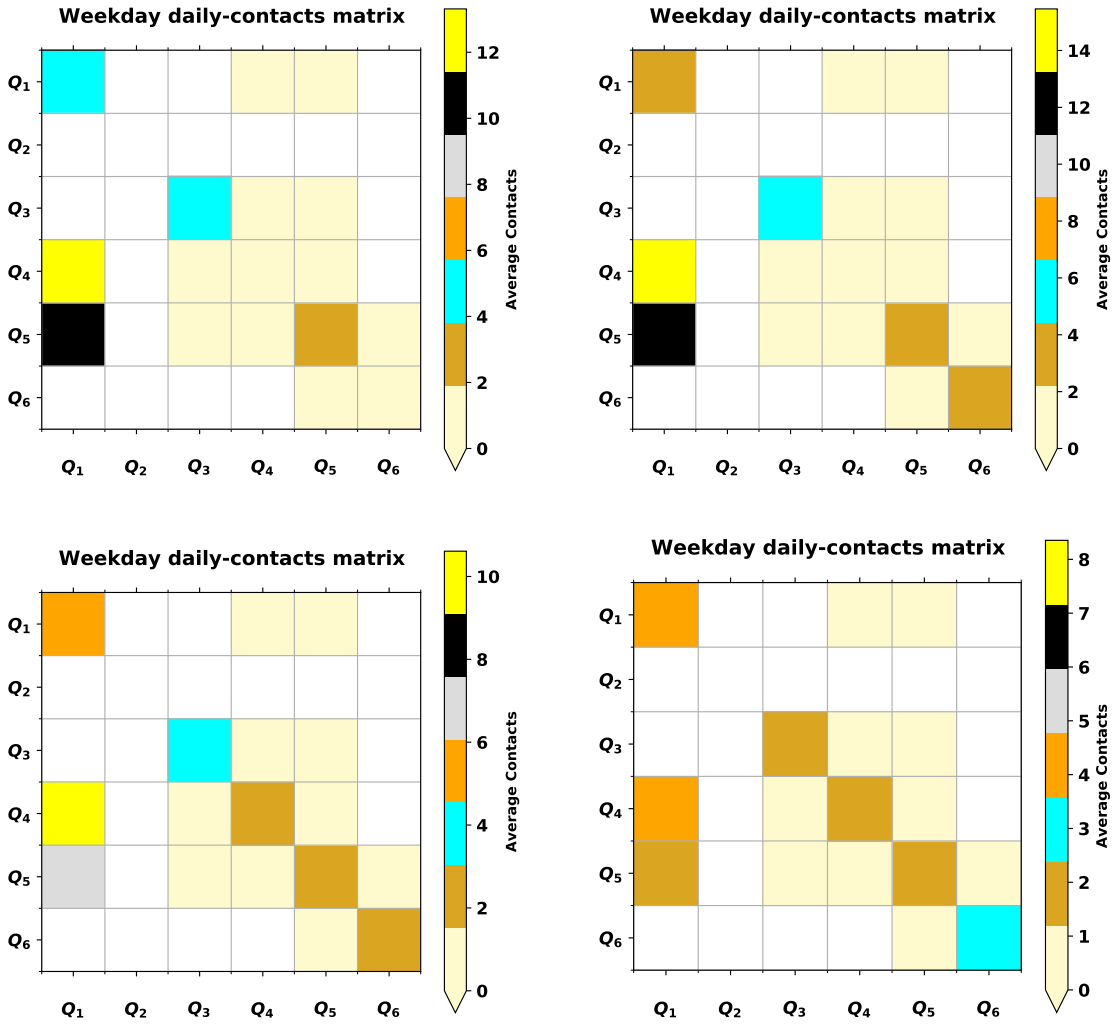
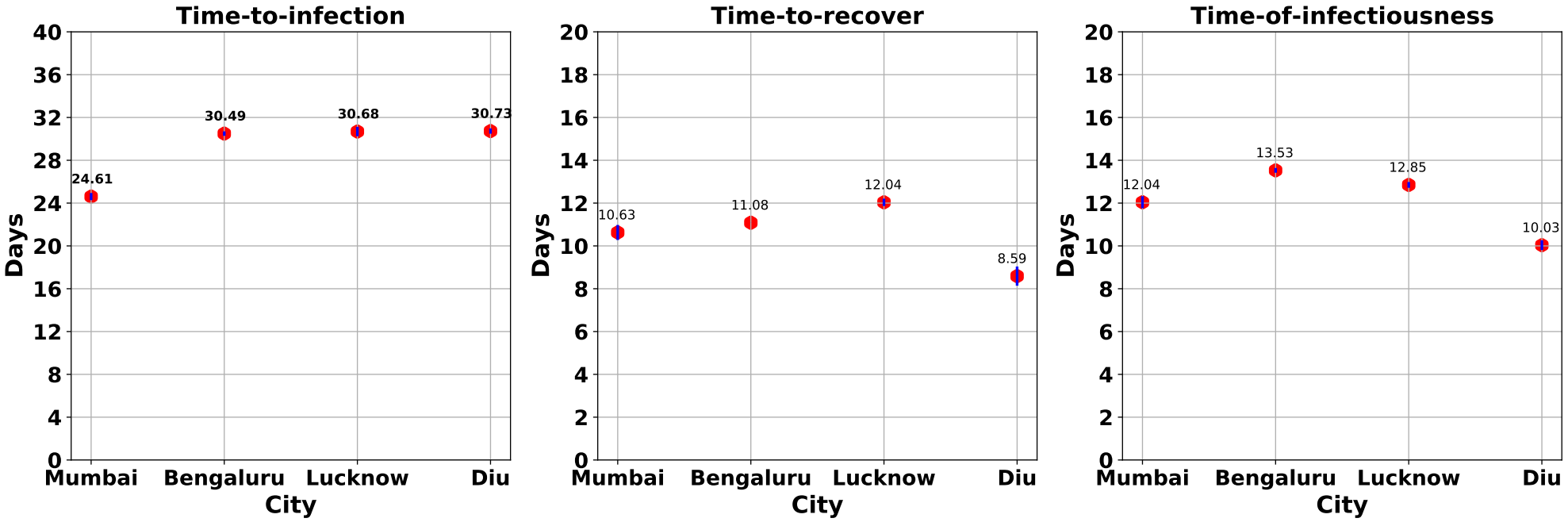


Figure: Weekday daily-contacts matrix during Unlock Phase-1

Model-IV: Average Time to Infection, Recovery and of Infectiousness



(a) (b) (c)

Figure: (a) Time-to-infection to quantify the extent of disease spread due to activities in Unlock Phase-1 and parameters to quantify the impact of testing, (b) is the average time to recover, and (c) is the average time for which the infection lasts, in all the four cities.

Model-IV: Conclusion

Advantages

- Calibrating the extent of activity yields **matching trends** of disease spread with real-world data.

Disadvantages

- **Does not** account for lowered infectivity of confirmed infections due to awareness and isolation.
- Possibility of infection spread during **transit** has also been overlooked.
- Activity schedules are based on common knowledge, and **may not match** with real-time data.

Conclusion

- Vicsek-inspired models as applied to two human behavioural phenomena were discussed
- The models are **bottom-up** approaches accounting for several psychological and sociological aspects of human behaviour
- In future, we would like to supplement them with data to evaluate their accuracy

Publications Based on This Work

1. Vedam, Narayani, and Ghose, Debasish. "Vectorial-Opinion Dynamics With Familiarity Neighborhoods in Virtual Social Groups." IEEE Transactions on Computational Social Systems (2021).
2. Vedam, Narayani, and Ghose, Debasish. "The Impact of Activity-based Movement on the Spread of COVID-19; An Agent-based Model." (under review)
3. Vedam, Narayani, and Ghose, Debasish. "The Impact of Mobility and Interventions on the Spread of Diseases" (under review)
4. Vedam, Narayani, and Ghose, Debasish. "Influence Dynamics and Consensus in an Opinion-Neighborhood based Modified Vicsek-like Social Network." arXiv preprint arXiv:1808.10716 (2018).

Thanks!

Model-I: Agents' Attributes

1. Opinions

$$\vec{x}_i(t) := \langle x_i(t), \theta_i(t) \rangle, \ \forall i, t,$$
$$x_i(t) = 1, \text{ and } \theta_i(t) \in (-\pi, \pi].$$

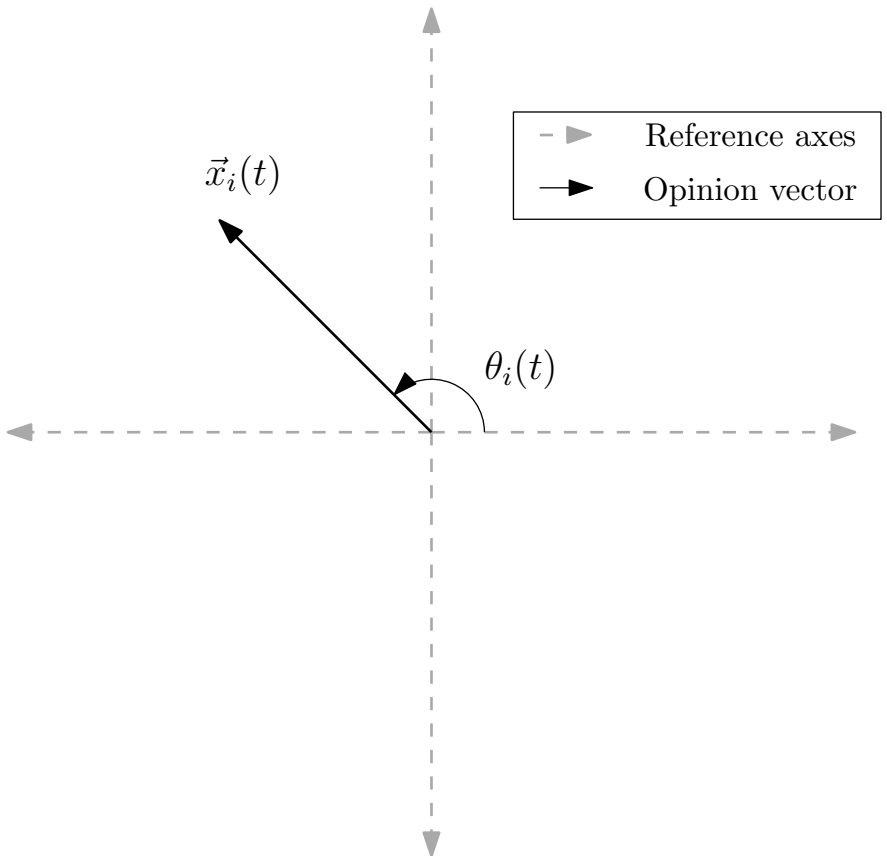


Figure: Figure 2: Opinion of Agent i

1. Opinions

$$\vec{x}_i(t) := \langle x_i(t), \theta_i(t) \rangle, \ \forall i, t,$$
$$x_i(t) = 1, \text{ and } \theta_i(t) \in (-\pi, \pi].$$

2. Confirmation Bias

$$N_i(t) := \{j : j \neq i, d_{ij}(t) \leq \theta_{T_i}\}, \forall i, t$$
$$d_{ij}(t) := \theta_i(t) - \theta_j(t), \ \forall t$$

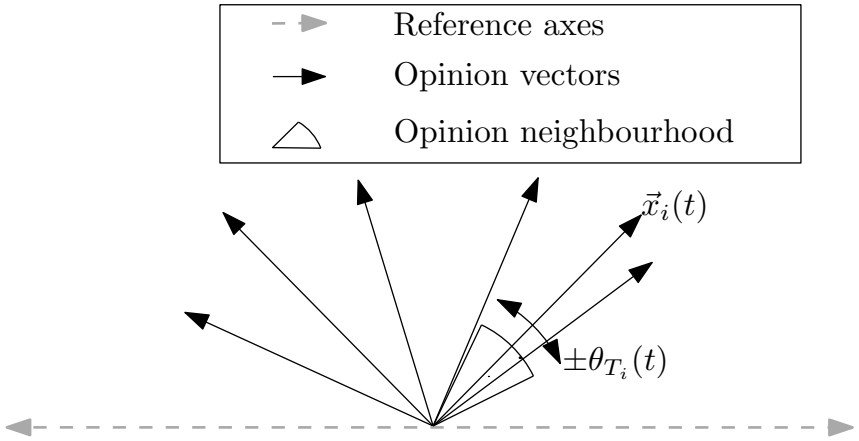


Figure: Figure 3: Confirmation bias of Agent i indicated by the sector

Model-I: Agents' Attributes

1. Opinions

$$\vec{x}_i(t) := \langle x_i(t), \theta_i(t) \rangle, \ \forall i, t,$$
$$x_i(t) = 1, \text{ and } \theta_i(t) \in (-\pi, \pi].$$

2. Confirmation Bias

$$N_i(t) := \{j : j \neq i, d_{ij}(t) \leq \theta_{T_i}\}, \forall i, t$$
$$d_{ij}(t) := \theta_i(t) - \theta_j(t), \ \forall t$$

3. Agent Types

- Rigid (θ_{T_R})
- Flexible (θ_{T_F})
  $\theta_{T_R} < \theta_{T_F}$

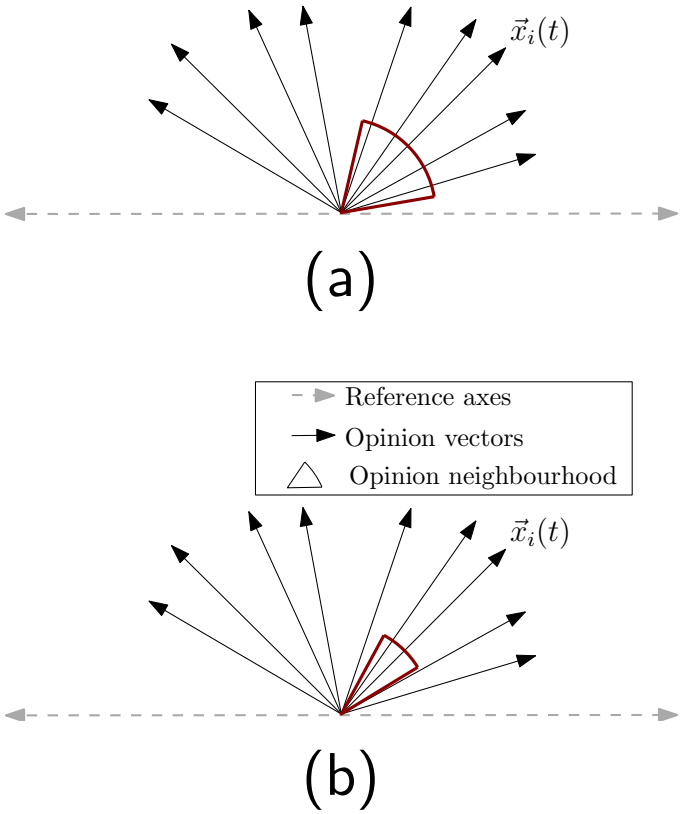


Figure: Figure 4: Opinion tolerance of (a) Flexible and (b) Rigid agents

4. Neighbourhoods

- **Opinion Neighbourhood:** $N_i(t)$
- **Direct Contacts**

$$N_{D_i}(t) \subseteq N_i(t)$$

- **Non-Direct Contacts**

$$N_{ND_i}(t) \subseteq N_i(t) - N_{D_i}(t), \ \forall i, \ t$$

- **Interactions**

$$N_{I_i}(t) \subseteq N_{D_i}(t) \cup N_{ND_i}(t) \subseteq N_i(t) \ , \forall i, \ t$$

Model-I: Groups (Continued)

Conservative Group	Only Stubborn Populace	Only Flexible Populace	Stubborn = Flexible Populace	Stubborn > Flexible Populace	Stubborn < Flexible Populace
Liberal Group	Only Stubborn Populace	Only Flexible Populace	Stubborn = Flexible Populace	Stubborn > Flexible Populace	Stubborn < Flexible Populace
	Homogeneous		**Balanced**	**Heterogeneous**	

Figure: Classification of Liberal and Conservative groups based on the composition of agents

Model-I: Influence Network

Network

$$G(t) = (V, E(t)),$$
$$|V| = N,$$
$$E(t) := \{(j,i) : j \in N_{I_i}(t), \forall i, j \neq i\},$$
$$D_{in_i}(t) := |N_{I_i}(t)|,$$
$$D_{out_i}(t) := |\{j : i \in N_{D_j}(t)\}|, \forall i, t,$$
$$A(t) = \begin{cases} a_{ij}(t) > 0, & \text{if } (j,i) \in E(t) \\ a_{ij}(t) = 0, & \text{otherwise} \end{cases}.$$

Influence Score

$$w_{ij}(t) = \frac{\sum_{p=1}^{p_{max}} \left(\prod_{(m,n) \in E_{Path}} w_{mn}(t) \right)}{p_{max}},$$
$$E_{Path} = \{(m,n) | \ (m,n) \in (j,i)\},$$
$$p_{max} = |E_{Path}|.$$

Individual Popularity

$$Z = \beta \left(I - \alpha D_{out}^{-1}(t) A(t) \right)^{-1}, Z \in \Re^N$$
$$\beta > 0,$$
$$0 < \alpha < 1/N.$$

Model-I: Opinion and Familiarity Dependent Network

Algorithm 1: Algorithm to generate the initial influence network

```
Inputs: N, Θ(0), Θ_T, D_{in_max}, μ, σ_c, σ_l;
not_connected ← True;
not_influenced ← True;
while not_connected do
    not_influenced = True ;
    while not_influenced do
        Θ(0) ← Initialize_Opinions(μ, σ_c, σ_l);
        D_in(0) ← Initialize_In_Degrees(N, D_{in_max});
        A(0) ← Neighbourhood(N, Θ_T, D_in(0));
        if row_sum(A(0))!=0 then
            not_influenced = False;
        end
    end
    not_connected ← Is_Connected(A(0));
end
```

Update Rules

$$\theta_i(t+1) = \langle \theta_i(t) \rangle, \forall i, t,$$

$$\tan(\langle \theta_i(t) \rangle) = \frac{\sum_{j \in N_{I_i}(t) \cup i} w_{ij}(t) \sin(\theta_j(t))}{\sum_{j \in N_{I_i}(t) \cup i} w_{ij}(t) \cos(\theta_j(t))},$$

$$w_{ii} := K_1 z_i, \forall i, K_1 = 0.1K,$$

$$\sum_{j, j \neq i} w_{ij}(t) = 1 - w_{ii}, \ \forall i.$$

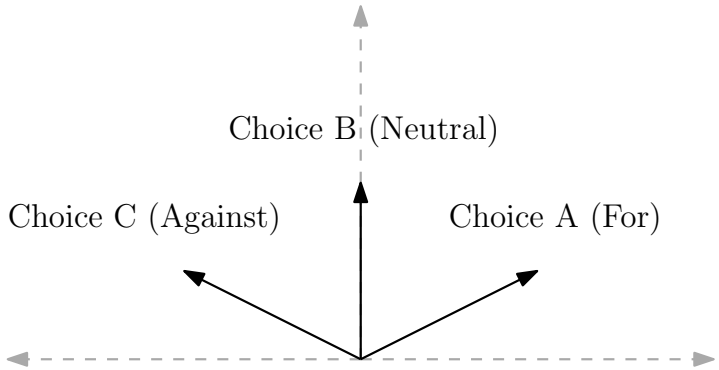


Figure: Unit vectors subtending 45°, 90° and 135° indicating opinions of three agents

Losing a Tie

$$E(t+1) \;=\; \begin{cases} E(t) - \{(j,i)\}, & \text{if } d_{ij}(t) > \theta_{T_i}, \forall i, \forall j \in N_{D_i}(t), \\ E(t), & \text{otherwise.} \end{cases}, \;\; t \geq 0.$$

Gaining a Tie

$$E(t+1) = \begin{cases} E(t) \cup \{(j,i)\}, & \text{if } \sum_{0}^{t} \mathbb{1}_{\mathbb{ij}}(\mathbb{t}) \geq s_i, \forall i, \forall j \in N_{ND_i}(t) \cap N_{I_i}(t), \\ E(t), & \text{otherwise.} \end{cases},$$

where,

$$\mathbb{1}_{\mathbb{ij}}(\mathbb{t}) = \begin{cases} 1, & \text{if } j \in N_{ND_i}(t), \\ 0, & \text{otherwise.} \end{cases}, \;\; t \geq 0,$$

$$s_i := Kz_i, \forall i$$

Model-II: Influence Network

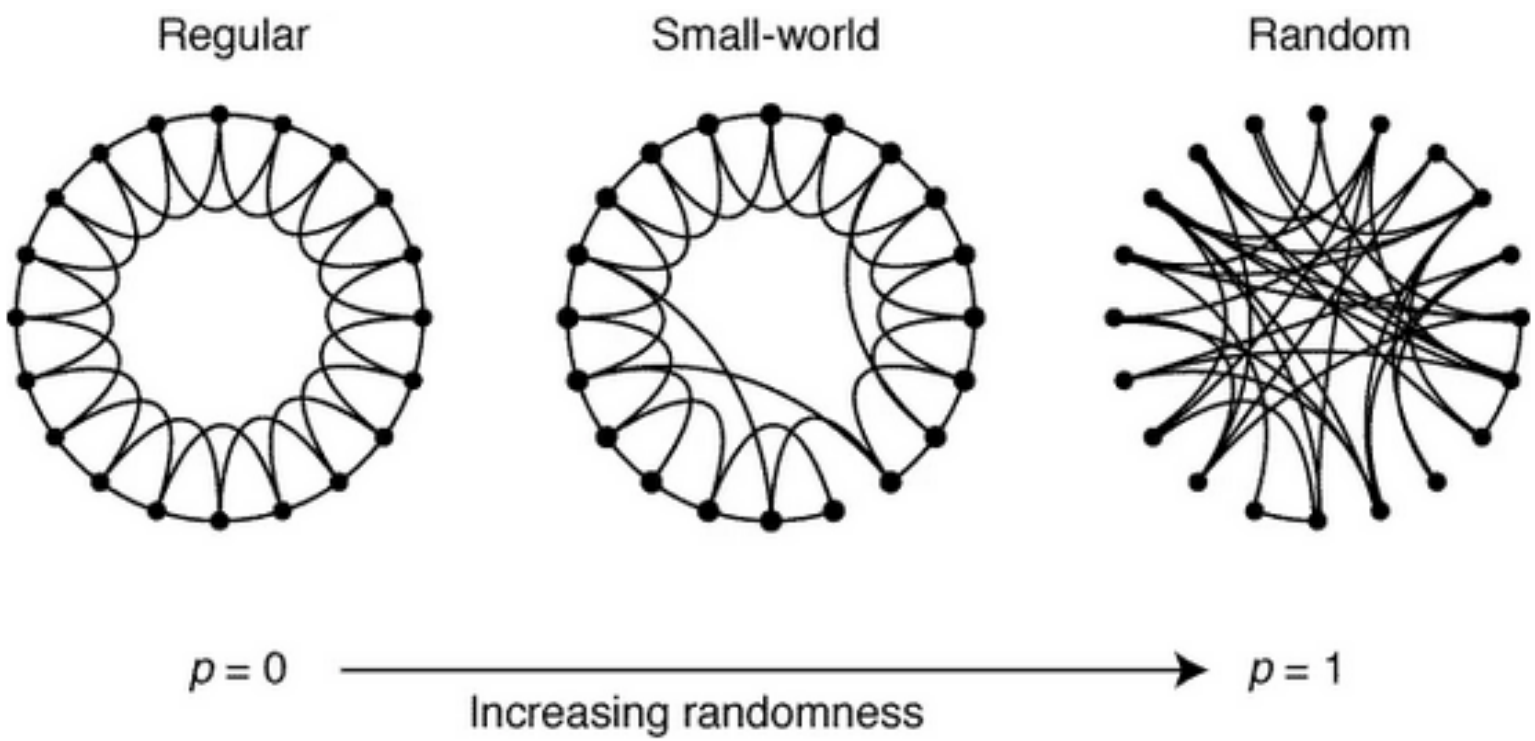
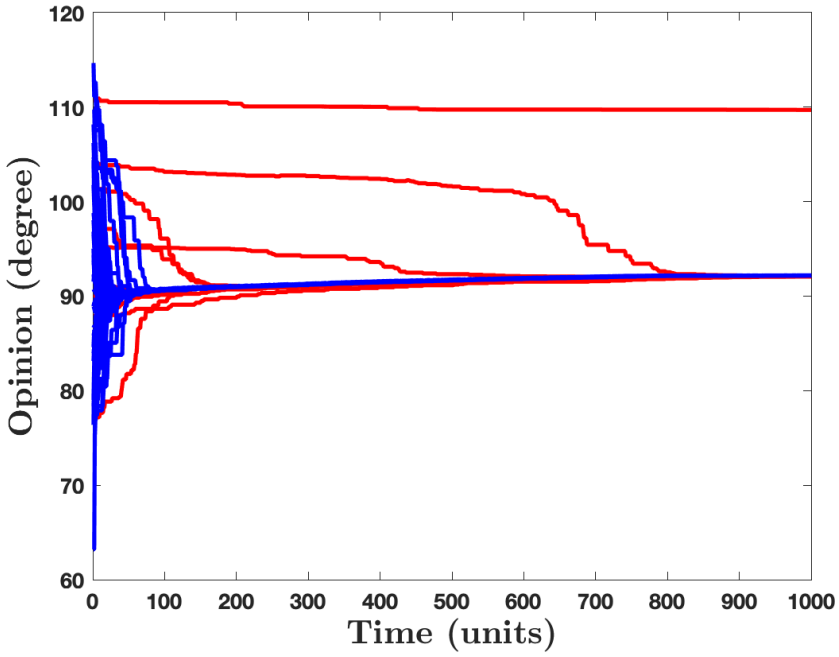


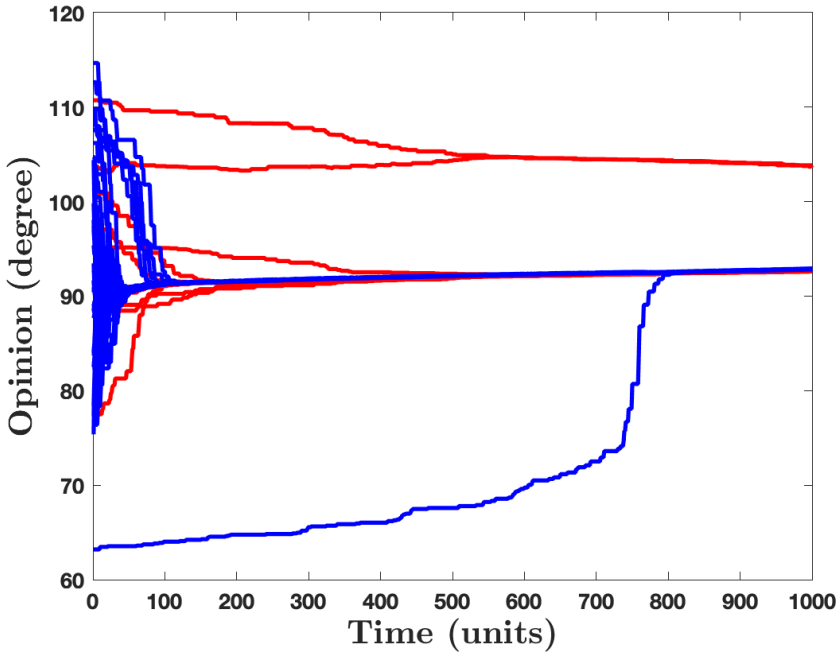
Figure: Illustration of the impact of rewiring a Regular network (left-most, $p = 0$) to generate a Small-World (middle, $p = 0.5$) and a Random network (right-most, $p = 1$) [13]

[13] Watts, D. J. (1998). Watts-Strogatz small world network Nature. Nature, 393, 440-442.

Model-II: Interplay



(a)



(b)

Figure: Evolution of opinions of Rigid (in red) and Flexible (in blue) agents in a Conservative group, with a Small-world influence network; (a) and (b) are two separate instances with Flexible agents in the majority

Model-III: Disease Transmission

Disease Neighbourhood

$$N_{D_i}(t+1) := \{j : x_i(t) - x_j(t) \leq r_I, \\ j \in [1, N], j \neq i\}$$

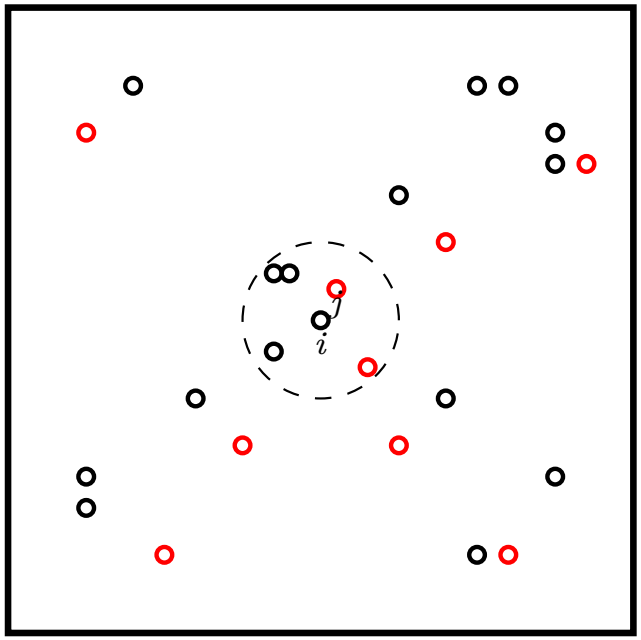


Figure: A typical layout with infected (red) and uninfected (black) agents; the dashed circle indicates the disease neighbourhood of i

Transmission

$$d_{ij}(t+1) = \min_{z \in N_{I_i}(t+1)} d_{iz}(t+1), \forall i, j \neq i, t$$

$$d_{ij}(t+1) := x_i(t) - x_j(t),$$

$$N_{I_i}(t+1) := \{j \in N_{D_i}(t+1) : \\ \theta_j(t) \in \{SIN, AIN, SIT, AIT\}\}.$$

Definition 11

Average Active Infections, is the daily average of the total number of tested and infected agents in states $\{AIT, SIT\}$ obtained from M MonteCarlo runs.

$$\mathrm{AI}_{Avg}(t) \;:=\; \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \mathbb{1}_{\mathbb{AI}_{\mathbb{j}}}(\mathbb{t})}{M}, \;\forall t,$$

where,

$$\mathbb{1}_{\mathbb{AI}_{\mathbb{j}}}(\mathbb{t}) \;:=\; \begin{cases} 1, & \text{if } \theta_j(t) \in \{AIT, SIT\}, \\ 0, & \text{otherwise.} \end{cases}.$$

Definition 12

Average Recoveries, is the daily average of the total number of recovered agents in state $\{AUT\}$ obtained from M MonteCarlo runs.

$$\mathrm{R}_{Avg}(t) \;:=\; \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \mathbb{1}_{\mathbb{R}_{\mathbb{j}}}(\mathbb{t})}{M}, \;\forall t,$$

where,

$$\mathbb{1}_{\mathbb{R}_{\mathbb{j}}}(\mathbb{t}) \;:=\; \begin{cases} 1, & \text{if } \theta_j(t) = AUT, \\ & \theta_j(t-k) \in \{AIT, SIT\}, \\ & 0 < k \leq t, \\ 0, & \text{otherwise.} \end{cases} .$$

Definition 13

Average Mortalities, is the daily average of the total number of fatalities $\{D\}$ obtained from M MonteCarlo runs.

$$\mathrm{F}_{Avg}(t) \;:=\; \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \mathbb{1}_{\mathbb{F}_{\mathbb{j}}}(\mathbb{t})}{M}, \;\forall t,$$

where,

$$\mathbb{1}_{\mathbb{F}_{\mathbb{j}}}(\mathbb{t}) \;:=\; \begin{cases} 1, & \text{if } \theta_j(t) = D, \\ 0, & \text{otherwise.} \end{cases}.$$

Definition 14

Time-to-Infection, is a measure of the time taken for an uninfected agent to contract the disease (t_I). Higher the value of this measure better is the effectiveness of the intervention strategy.

$$t_I := \frac{\sum_{i=1}^{N} \sum_{t=1}^{tf} \mathbb{1}_{\mathbb{I}}}{N},$$

where,

$$\mathbb{1}_{\mathbb{I}} := \begin{cases} 1, & \text{if } \theta_i(t) \in \{SUT, SUN, AUT, AUN\}, \\ 0, & \text{otherwise.} \end{cases}$$

$$t_{I_{Avg}} := \frac{t_I}{M}$$

Definition 15

Time-to-Recovery, is a measure of the time taken by agents to recover from the disease (t_R).

$$t_R := \frac{\sum_{i=1}^{N}\left(\mathbb{1}_{\mathbb{i}} \sum_{t=1}^{tf} \mathbb{1}_{\mathbb{R}}\right)}{N},$$

where,

$$\mathbb{1}_{\mathbb{i}} := \begin{cases} 1, & \text{if } \theta_i(t_f) = AUT, \\ 0, & \text{otherwise.} \end{cases},$$

$$\mathbb{1}_{\mathbb{R}} := \begin{cases} 1, & \text{if } \theta_i(t) \in \{SIT, AIT\}, \\ 0, & \text{otherwise.} \end{cases}.$$

$$t_{R_{Avg}} := \frac{t_R}{M}$$

Definition 16

Time-of-Infectiousness, is a measure of the time for which an infected agent remains infectious (t_{OI}).

$$t_{OI} := \frac{\sum_{i=1}^{N} \sum_{t=1}^{tf} \mathbb{1}_{\mathbb{OI}}}{N},$$

where,

$$\mathbb{1}_{\mathbb{OI}} := \begin{cases} 1, & \text{if } \theta_i(t) \in \{SIT, AIT, AUT, D\}, \\ & \theta_i(t-k) \in \{SIT, AIT, SIN, AIN\}, \\ & 0 < k \leq t, \\ 0, & \text{otherwise.} \end{cases}.$$

$$t_{OI_{Avg}} := \frac{t_{OI}}{M}$$