Application of Nonlinear Controls to Automotive Cruise Control Systems

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I. Abstract

This paper develops a method of controlling automotive vehicles in a more robust manner than previously developed linear control systems. The nonlinear dynamics describing most automotive systems is considered, and the controller logic dictating the use of either adaptive cruise control or traditional cruise control is developed. Then, nonlinear controls are developed for each mode of cruise control using Input-Output Linearization, Control Lyapunov Functions, and Sliding Mode methods. The stability of these modes of operation are analyzed, and the developed controls are simulated using computational software.

II. Introduction

Cruise control systems, engine control systems which automatically control vehicle speeds, have been found in cars for the past 65 years. For the majority of the second half of the 20th century, cruise control systems were designed to simply maintain the speed of the automobile at a velocity set by the driver. However, by the mid-1990s, cars began to appear on the roadways with a new type of control system, Adaptive Cruise Control.

ACC is a radar-based system, which can reduce the speed of the commanded vehicle below the speed set by the driver to provide controlled following of sensed preceding targets. A speed command is generated based in part, on the relationship between the source vehicle's velocity and the velocity of the preceding target, and the driver's acceleration is limited accordingly, to adapt the source vehicle speed to that of the target [1]. ACC systems, instead of trying to match a specified velocity, attempt to maintain a constant headway between the driver's vehicle and a leading vehicle. While the operation of both types of cruise control is possible in any modern car, combining both methods into one unifying method of cruise control is problematic. Both traditional cruise control systems and ACC systems have been developed around the linearized equations of motion for the vehicle. Linear controllers using PID, optimal control, etc. [2], [3] and nonlinear controllers using techniques like the sliding control method [4], [5], multiple surface approach [5], [6], etc. have been adopted to deal with linearized models. However, these control methods may not apply under all conditions. These linearized equations are valid when inspecting each system separately because the driver must manually accelerate his car to his desired speed before turning on cruise control, the car will not see speeds far greater or less than the set value. These linear assumptions will not hold true when switching from one mode to another. For example, consider a driver who accelerates his car to 60 MPH and sets his cruise control at this speed. The car operates normally until a leading car is sensed, moving at a speed of 40 MPH. Adaptive Cruise Control dictates that a minimum headway is maintained, and the drivers car quickly matches the speed of the leading car. When the leading car switches lanes, ACC is turned off, and the drivers car attempts to reach its desired set speed. However, because the difference in the current speed of the car and the desired set speed is so great, the cruise control is not guaranteed to operate in a stable manner.Therefore, nonlinear controls need to be developed to meet both objectives. Both input-output linearization, control Lyapunov functions, and sliding mode controls, as detailed by Khalil, Sastry, Slotine, et al. [7]–[9] can be used to achieve these objectives.

III. Control Objectives

A. Position/Distance Tracking

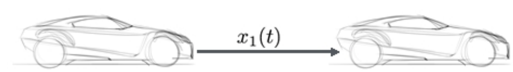


Fig. 1: A minimum distance should be maintained to ensure safety.

Figure 1 shows the case when there is a vehicle detected within a certain range. The objective is to maintain a constant headway between the controlled vehicle and the one immediately in front.

B. Velocity Tracking

In the absence of a leading vehicle, the adaptive cruise-control system will instead attempt to track a user-set speed as shown in Figure 2.

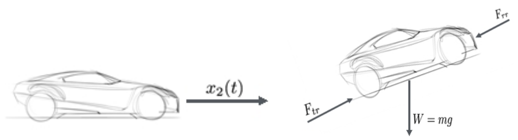
IV. Vehicle Dynamics

A. Nonlinear Model

The longitudinal vehicle dynamics can be expressed by the equation

$$M\dot{v} = F_{TR} - \{F_{rr} + F_g + F_{Drag}\} \quad (1)$$

Where v is the velocity of the vehicle, $\dot{v}$ is the acceleration of the vehicle, F_{rr} is the rolling resistance, F_g is the force due



(a) Moving on flat ground (b) Moving on a slope

Fig. 2: Velocity tracking

to gravity, and F_{Drag} is the drag force. F_{TR} is the force being input into the system by the torque produced by the engine and is given by $F_{TR} = \frac{g_A T_b}{r_w}$. Where g_A is the gear ratio, r_w is the drive-wheel radius, and T_b is the brake torque. Then the dynamics can then be rewritten into the form

$$M\dot{v} = F_{TR} - \{f_0 + f_1 v + f_2 v^2\} \tag{2}$$

Setting the states to be $x = [x_1 x_2]^T$, the system state equations can be written as

$$\begin{aligned} \dot{x}_1 &= v_f - x_2 \\ \dot{x}_2 &= \tfrac{F_{TR}}{M} - \tfrac{\{f_0 + f_1 x_2 + f_2 {x_2}^2\}}{M} \end{aligned} \tag{3}$$

where x_1 is the vehicle's distance from a lead car, v_f is the velocity of the leading vehicle, and x_2 is the vehicle's velocity. The dynamics of x_2 can be rewritten as

$$\dot{x}_2 = \bar{F}_{TR} u - \{\bar{f}_0 + \bar{f}_1 x_2 + \bar{f}_2 {x_2}^2\} \tag{4}$$

where $\bar{F}_{TR} = \frac{g_A k_A}{M r_w} u$. Putting these equations into matrix form, yields

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} v_f - x_2 \\ -\{\bar{f}_0 + \bar{f}_1 x_2 + \bar{f}_2 {x_2}^2\} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{F}_{TR} \end{bmatrix} u \tag{5}$$

B. Linear Model

The nonlinear model given by (5) cannot be linearized because of the nonexistence of the equilibrium point. Instead, a common linear cruise control model was used for comparison. The linear dynamics of the system is given by

$$M\dot{v} = F_{TR} - f_1 v \tag{6}$$

V. Control Logic

This section details the development and analysis of the nonlinear controls developed for both modes of autonomous vehicle operation. Input-Output Linearization, Control Lyapunov Functions, and Sliding Mode Control are considered for the two modes.

The operation of the cruise control system is designed to follow a strict set of guidelines. First, the vehicle is started and driven to a desired velocity manually. Then, at the discretion of the operator, cruise control is turned on. The velocity of the vehicle at this instant is the velocity the controller will try to match. If no vehicle is sensed in front of the vehicle, the automobile will operate according to the velocity tracking control system. However, if at any point a vehicle is sensed in front of the control vehicle, the position tracking control system will take precedence, and a constant headway between the two vehicles will be matched, with this minimum headway being specified by the automotive manufacturer. If the front vehicle leaves the range of the sensors, the velocity tracking control system will once again take precedence, and will drive the vehicle back to its initial desired velocity.

A. Distance Tracking

The objective of this mode is to maintain a constant headway between the two vehicles. This is expressed mathematically as

$$y(t) = x_d(t) - x_1(t) \tag{7}$$

where x_1 denotes distance between the two vehicles, x_d denotes the prescribed safety critical distance, $y(t)$ denotes the output.

1) Controller for Linear Model: The dynamics with (7) can also be expressed in the control canonical form as

$$\begin{aligned} \dot{\xi}_1(t) &= v_f(t) - x_2(t) = \xi_2(t) \\ \dot{\xi}_2(t) &= \dot{v}_f(t) - \bar{f}_1 v_f(t) + \bar{f}_1 \xi_2(t) - \bar{F}_{TR} u(t) \end{aligned} \tag{8}$$

The control input is given by

$$u(t) = \frac{1}{\bar{F}_{TR}} \{\dot{v}_f(t) + \bar{f}_1 v_f(t) - \bar{f}_1 \xi_2(t) + k_1 \xi_1(t) + k_2 \xi_2(t)\} \tag{9}$$

Thus, the closed-loop dynamics is given by

$$\begin{aligned} \dot{\xi}_1(t) &= \xi_2(t) \\ \dot{\xi}_2(t) &= -k_1 \xi_1(t) - k_2 \xi_2(t) \end{aligned} \tag{10}$$

with $A_{cl} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$, and k_1 and k_2 can be tuned so that the desired closed-loop poles can be chosen .and A_{cl} can be made Hurwitz.

2) Controller by Input-Output Linearization: It is helpful to convert the system of equations given in (5) into control canonical form. The system is of relative degree two, meaning that the nonlinear dynamics of the system can be cancelled out by the control input. Considering the control canonical dynamics

$$\begin{aligned} \xi_1 &= y \\ \xi_2 &= L_f y \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \dot{v}_f(t) + \bar{f}_0 + \bar{f}_1 x_2 + \bar{f}_2 x_2^2 - \bar{F}_{TR} u \end{aligned} \tag{11}$$

The control input u can be picked to negate nonlinear

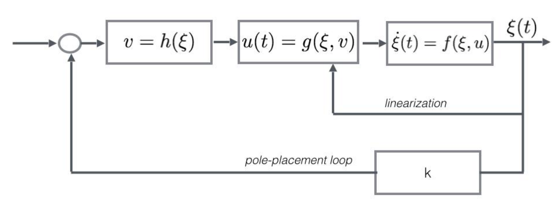


Fig. 3: Input-Output linearization block diagram

dynamics and place poles to the left half s-plane

$$u(t) = \frac{1}{\bar{F}_{TR}} \{\dot{v}_f(t) + \bar{f}_0 + \bar{f}_1 x_2 + \bar{f}_2 x_2^2 + k_1 \xi_1 + k_2 \xi_2\} \tag{12}$$

Thus, the closed-loop dynamics is given by

$$\begin{aligned}\dot{\xi}_1(t) &= \xi_2(t)\\ \dot{\xi}_2(t) &= -k_1\xi_1(t) - k_2\xi_2(t)\end{aligned} \tag{13}$$

with $A_{cl} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$, and k_1 and k_2 can be tuned so that the desired closed-loop poles can be chosen and A_{cl} can be made Hurwitz.

3) Controller by Control Lyapunov Function: The control canonical dynamics can also be used to construct a CLF controller as well. Consider a Lyapunov function candidate $V = \frac{1}{2}({\xi_1}^2 + {\xi_2}^2)$, with $\dot{V} = \xi_1\xi_2 + \xi_2(\dot{v}_f(t) + \bar{f}_0 + \bar{f}_1 x_2 + \bar{f}_2 x_2^2 - \bar{F}_{TR}u)$. Taking the Lie derivatives $L_f V$ and $L_g V$,

$$\begin{aligned}\phi_0 &= L_f V + \alpha V\\ &= \xi_1\xi_2 + \xi_2(\dot{v}_f + \bar{f}_0 + \bar{f}_1 x_2 + \bar{f}_2 x_2^2 + \alpha(\xi_1^2 + \xi_2^2))\\ \phi_1 &= L_g V = -\bar{F}_{TR}\xi_2\end{aligned} \tag{14}$$

The control input is given by

$$u = \begin{cases} 0, \phi_0 \leq 0 \\ -\frac{\phi_0}{\phi_1}, \phi_1 > 0 \end{cases} \tag{15}$$

Thus, for the closed-loop system, if $\phi_0 \leq 0$, $\dot{V} = \phi_0 - \alpha(\xi_1^2 + \xi_2^2) \leq -\alpha(\xi_1^2 + \xi_2^2)$; else, $\dot{V} = \phi_0 - \alpha(\xi_1^2 + \xi_2^2) + \phi_1\frac{\phi_0}{\phi_1} = -\alpha(\xi_1^2 + \xi_2^2)$. Therefore, the equilibrium point $[\xi_1, \xi_2]^T = [0, 0]^T$ is exponentially stable.

4) Sliding mode Controller and saturation improvement: The sliding surface is defined as

$$S(\xi(t)) = \dot{\xi}_1(t) + \lambda\xi_2(t) \tag{16}$$

Taking the derivative,

$$\begin{aligned}\dot{S}(\xi(t)) = \dot{v}_f(t) - \bar{F}_{TR}u(t) + \{\bar{f}_0 + \bar{f}_1(v_f(t) - \xi_2(t))\\ + \bar{f}_2(v_f(t) - \xi_2^2(t))\} + \lambda\xi_2(t)\end{aligned} \tag{17}$$

For $\dot{S} \leq -\mu sgn(\frac{s}{0.01})$, the control input is found to be

$$\begin{aligned}u(t) = \tfrac{1}{\bar{F}_{TR}}\{\lambda\xi_2(t) + \dot{v}_f(t) + \bar{f}_0 + \bar{f}_1(v_f(t) - \xi_2(t))\\ + \bar{f}_2(v_f(t) - \xi_2(t))^2 + 1.1 sign(\tfrac{s}{0.01})\}\end{aligned} \tag{18}$$

To improve the controller, the saturation function is introduced to smooth and bound the response by replacing the sign function. The sign and saturation functions are defined the same as in [7].

B. Velocity Tracking

The objective of this mode is to track the user-set speed. This is expressed mathematically as

$$y(t) = x_2(t) - v_d \tag{19}$$

where $x_2(t)$ denotes velocity of the controlled vehicle, $v_d(t)$ is the desired/user-set speed, $y(t)$ is the output.

1) Controller for Linear Model: The dynamics can also be expressed in the control canonical form as

$$\dot{\xi}(t) = -\bar{f}_1\xi(t) - \bar{f}_1 v_d + \bar{F}_{TR}u(t) \tag{20}$$

And the control law is given by

$$u(t) = \frac{1}{\bar{F}_{TR}}\{\bar{f}_1\xi(t) - \bar{f}_1 v_d - k\xi(t)\} \tag{21}$$

Then the closed-loop dynamics is

$$\dot{\xi}(t) = -(\bar{f}_1 + k)\xi(t) \tag{22}$$

. The value of k can be suitably tuned to get any desired performance characteristics.

2) Controller by Input-Output Linearization:

$$\dot{\xi}(t) = \bar{f}_0 + \bar{f}_1 v(t) + \bar{f}_2 v(t) - \bar{F}_{TR}u(t) \tag{23}$$

The relative degree is 1, the control input is

$$u(t) = \frac{1}{\bar{F}_{TR}}\{\bar{f}_0 + \bar{f}_1 v(t) + \bar{f}_2 v(t) - k\xi(t)\} \tag{24}$$

3) Controller by Control Lyapunov Function: Consider a Lyapunov function candidate $V = \frac{1}{2}\xi^2$, thus $\dot{V} = \xi(-\bar{f}_0 - \bar{f}_1 v - \bar{f}_2^2 + \bar{F}_{TR}u)$. Taking the Lie derivatives $L_f V$ and $L_g V$,

$$\begin{aligned}\phi_0 &= L_f V + \alpha V = \xi(-\bar{f}_0 - \bar{f}_1 v - \bar{f}_2 v^2) + \alpha\xi^2\\ \phi_1 &= L_g V = -\bar{F}_{TR}\xi\end{aligned} \tag{25}$$

The control input is given by

$$u = \begin{cases} 0, \phi_0 \leq 0 \\ -\frac{\phi_0}{\phi_1}, \phi_1 > 0 \end{cases} \tag{26}$$

Thus, for the closed-loop system, if $\phi_0 \leq 0$, $\dot{V} = \phi_0 - \alpha\xi^2 \leq -\alpha\xi^2$; else, $\dot{V} = \phi_0 - \alpha\xi^2 + \phi_1\frac{\phi_0}{\phi_1} = -\alpha\xi^2$. Therefore, the equilibrium point $\xi = 0$ is exponentially stable.

VI. Experimental Results

The various methods of controls are put into MATLAB in order to confirm their efficacy and visually understand how they operate. The various parameters used in the simulations are shown in Tables 1.

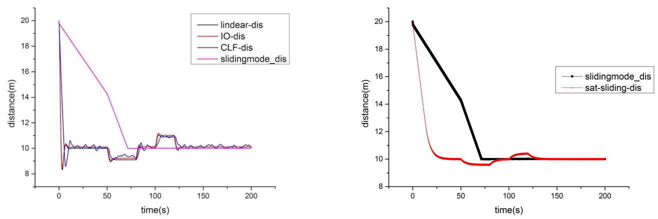
TABLE I: Parameter Values

g_A	k_a	M	r_w	f_0/M	f_1/M
3.4	10	1650kg	1	4	4
f_2/M	x_d	$x_1(0)$	$x_2(0)$	v_d	
10	10m	20m	35mph	40mph	

A. Distance Tracking

The distance tracking portion of the simulations involves generating a signal that would approximate a moving lead car that would both accelerate and deaccelerate. The accuracy of the various methods in maintaining the minimum distance can be seen from Figures 4-7.

As shown in Figure 4, both the linear controller and the IO controller are able to match the input from the lead vehicle smoothly and accurately. As the lead vehicle speeds up and slows down, the distance between the vehicles does



(a) All controllers (b) Sign and saturated sliding mode

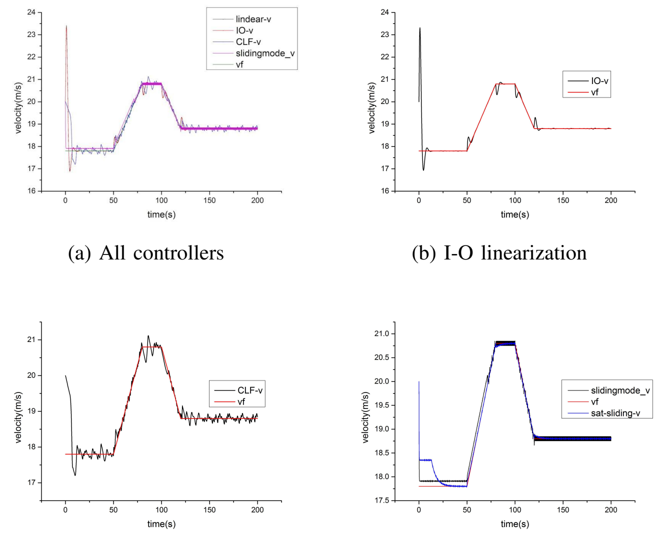
Fig. 4: Distance evolving process

vary, but the desired distance is recovered quickly. The CLF controller also is able to match the input of the lead vehicle, but shows to vary around the desired distance even when the lead vehicle is travelling at a constant speed. The sliding mode controller converges at the desired distance steadily, and is extremely stable once it reaches this value, but reaches this value very slowly. If the inputs to the sliding mode controller are saturated, the settling time is dramatically decreased, but will lose some of the stability seen in the unsaturated controller once the desired distance is reached.

The plots of the velocity of the driver vehicle controlled by the various controllers is compared to the velocity of the lead vehicle in Figure 5. All the controller methods are able to follow the velocity profile with relative degrees of accuracy. The IO controller matches the velocity profile with much greater accuracy than the CLF controller. Of particular interest is the velocity profile of the sliding mode controller. It follows the lead vehicle velocity with great accuracy, but shows to have high frequency jitters along the duration of its operation. These jitters are a direct result of the input demanded by the controller, as shown in Figure 5(d). The high frequency switches between torque output from the engine and the application of breaks is neither implementable nor efficient. By introducing the saturation curve into the controller design, these oscillations in vehicle speed can be significantly reduced.

The acceleration profiles of the various controllers can be seen in Figure 6. Both the IO and linear controllers follow a very similar profile, with accelerations never rising over 0.5 g. These accelerations are felt directly by the driver, so it is important that the acceleration of the vehicle stays small in magnitude and relatively smooth. The CLF controller shows higher magnitudes of acceleration, constant oscillations between positive and negative values, and high jerk, all undesirable traits when considering passenger comfort. The sliding mode controller shows high frequency jitters in acceleration, but once again, the saturation curve can be used to virtually eliminate this. However, the saturated sliding mode controller shows an initial acceleration much greater than any other mode.

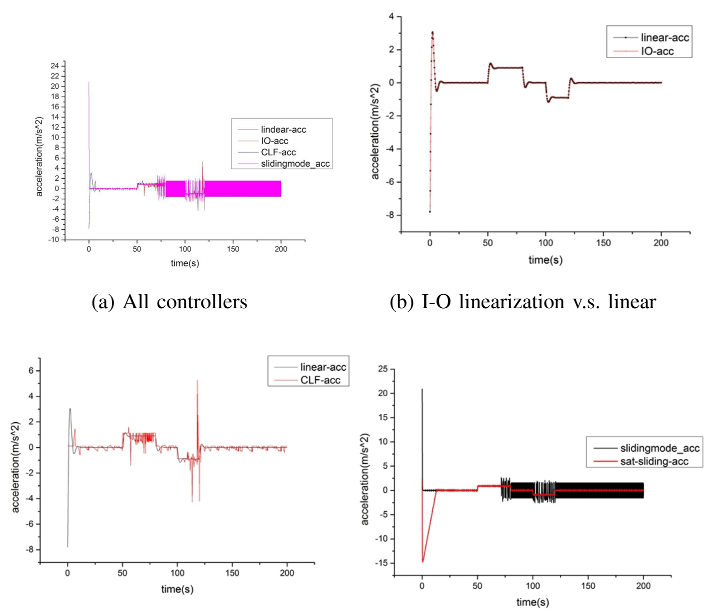
The control inputs required can be seen in Figure 7. Both the IO and linear controller are both implementable and relatively efficient. The CLF controller alternates between positive and negative inputs with a much greater frequency, and shows



(a) All controllers (b) I-O linearization

(c) CLF (d) Sign and saturated sliding mode

Fig. 5: Velocity evolving process, v_f is the front car's velocity



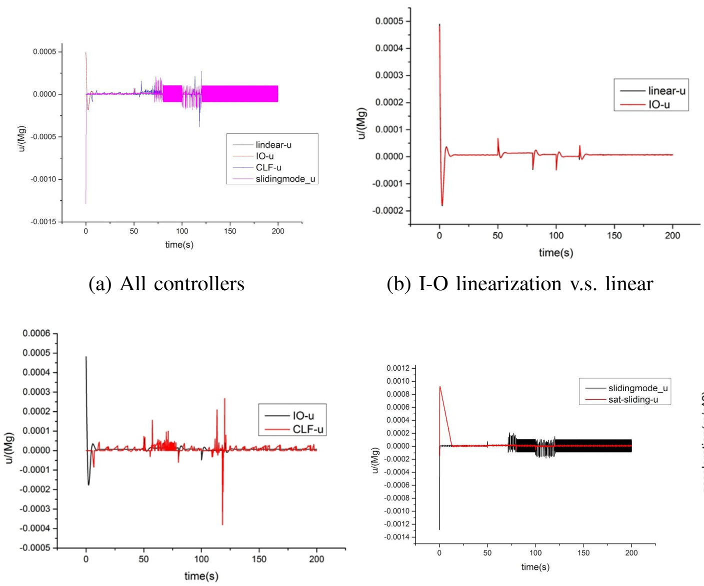
(a) All controllers (b) I-O linearization v.s. linear

(c) CLF v.s. linear (d) Sign and saturated sliding mode

Fig. 6: Acceleration evolving process

a few spikes where high torque input is necessary. Even if an engine can produce this output, it will consume a lot more fuel while in this mode of operation. The sliding mode controller shows high frequency oscillations between positive and negative torque, which is neither implementable nor fuel efficient, but this can be eliminated with the introduction of the saturation curve.

The IO controller is the best fit for this mode of operation. It is able to successfully track distance with little error, follows



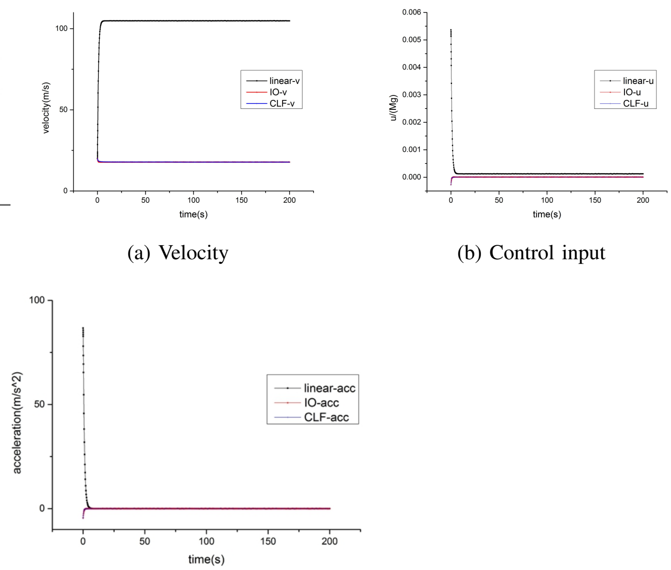
(a) All controllers

(b) I-O linearization v.s. linear

(c) CLF v.s. linear

(d) Sign and saturated sliding mode

Fig. 7: Control input u evolving process



(a) Velocity

(b) Control input

(c) Acceleration

Fig. 8: Velocity tracking

the velocity of the lead vehicle very well, and produces a desirable acceleration profile with minimum required input. The saturated sliding mode controller, while having good performance characteristics and having the ability to track distance better than the IO controller, experiences a maximum acceleration two times greater than that of the IO controller, which is highly undesirable for any human operated vehicle.

B. Velocity Tracking

The velocity tracking portion of the simulations involved setting a desired velocity for the vehicle and setting the initial velocity at an arbitrary value away from the desired speed. Simulations show that both the IO controller and the CLF controller track the desired velocity with great accuracy, and converge at the desired speed in a very short time, as seen in Figure 8(a). The linear controller also stabilizes in a very short time step; however, the steady state error is very large in magnitude in comparison to the nonlinear controllers. The inputs of the nonlinear controllers, shown in Figure 8(b), stabilize smoothly to a very small positive value, which could be easily implemented by the vehicles engine.

The vehicle also experiences very little acceleration, as shown in Figure 8(c). The driver will initially feel an acceleration of 0.5 g, but this acceleration will quickly die down as the speed stabilizes. The CLF controller shows a smoother progression to equilibrium, so the driver will experience less jerk in a vehicle with this controller than a vehicle controller by the IO controller.

Either the CLF controller or IO controller would work well for this mode of operation, but the CLF controller is the slightly more attractive choice in this case, as the smoother acceleration profiles will lead to greater driver comfort.

VII. Conclusion

In this paper, nonlinear controls were developed for an automobile that would be able to track a desired speed or maintain a minimum distance from a lead car. These control systems were then simulated with mathematical software, and the time response of the vehicles for the various control methods were analyzed. It was concluded that the IO controller was the optimal choice when tracking distance, and the CLF controller was the optimal choice when tracking velocity. While sliding mode control appears to be promising, further research is needed to develop a way to limit the accelerations experienced by the driver. A unifying control structure that meets both objectives could also be advantageous for automobile manufacturers, and should be explored.

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