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Application of Nonlinear Controls to Automotive Cruise Control Systems

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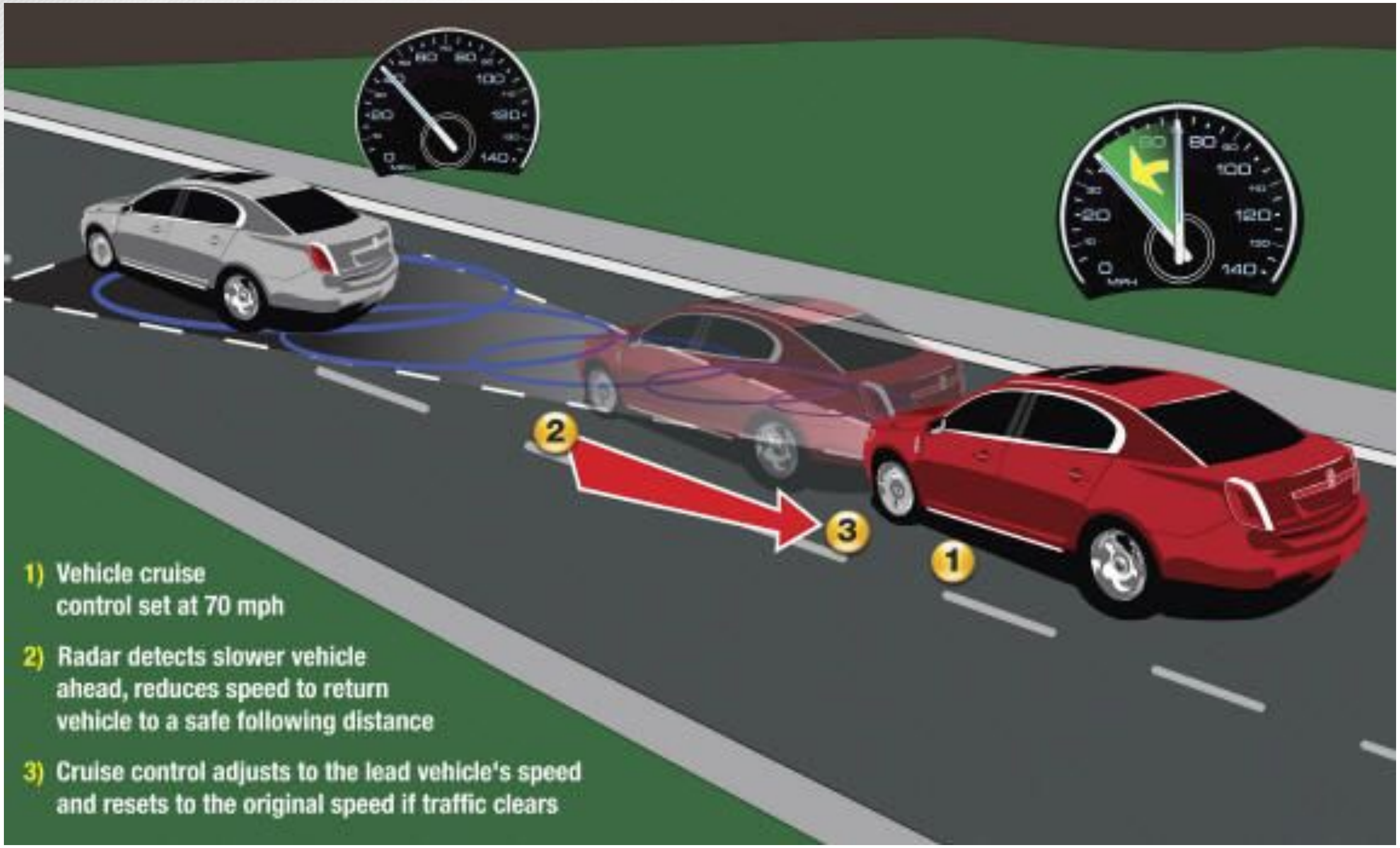
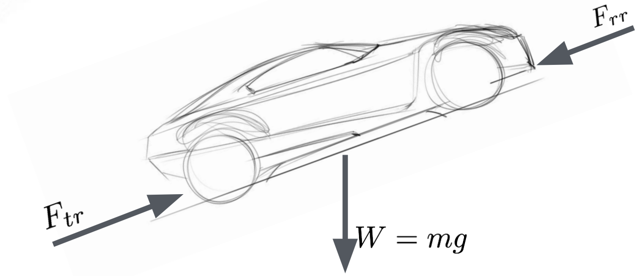


http://www.autointhenews.com/wp-content/uploads/2010/04/Adaptive-Cruise-Control-image.jpg

System Dynamics

$$m\dot{v}(t) = F_{tr} - F_{rr}$$

$$F_{rr} = f_0 + f_1 v(t) + f_2 v^2(t)$$

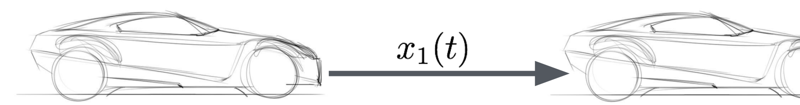


Case 1: Distance Tracking

$$\dot{x}_1(t) = v_f^*(t) - x_2(t)$$

$$\dot{x}_2(t) = \dot{v}_f(t) - \frac{F_{tr}}{m} + \frac{F_{rr}}{m}$$

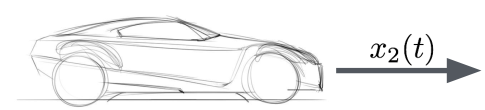
$$y(t) = x_1(t) - x_d$$ —> 0



Case 2: Velocity Tracking

$$\dot{x}_2(t) = \frac{F_{tr}}{m} - \frac{F_{rr}}{m}$$

$$y(t) = x_2(t) - v_d$$

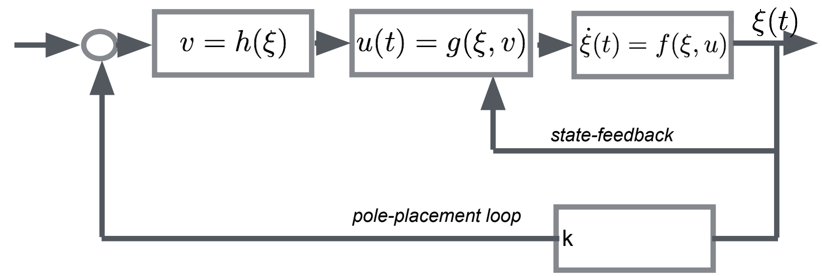


$v_f^*(t)$ *—> It is assumed that this data is constantly available to the vehicle of interest .The means of the same hasn't been worked out .*

Control : Linear Case

- System Dynamics :

$$m\dot{v}(t) = F_{tr} - f_1 v(t) \quad *$$



- Case 1: Distance Tracking

$$\dot{\xi}_1(t) = v_f(t) - x_2(t) = \xi_2(t)$$
$$\dot{\xi}_2(t) = \dot{v}_f(t) - \bar{f}_1\xi_2(t) + \bar{f}_1 v_f(t) - \bar{F}_{tr}u(t)$$
$$u(t) = \frac{1}{\bar{F}_{tr}}\{\dot{v}_f(t) + \bar{f}_1 v_f(t) - \bar{f}_1\xi_2(t) + k_1\xi_1(t) + k_2\xi_2(t)\}$$

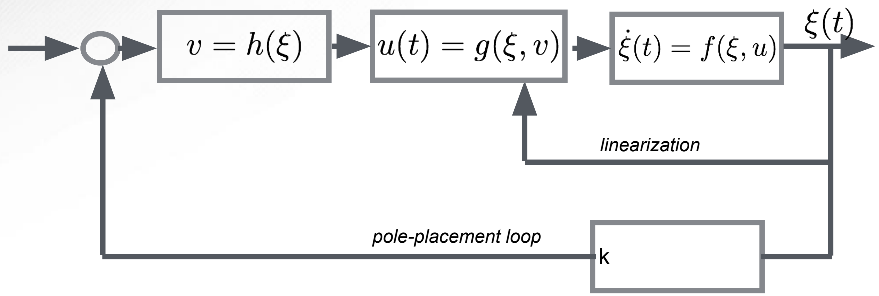
$\bar{f}_1$	0.003
$\bar{F}_{tr}$	0.020 6
k_1	4
k_2	4
k	1
x_d	10m

- Case 2: Velocity Tracking

$$\dot{\xi}(t) = -\bar{f}_1\xi(t) - \bar{f}_1 v_d + \bar{F}_{tr}u(t)$$
$$u(t) = \frac{1}{\bar{F}_{tr}}\{\bar{f}_1 v_d - k\xi(t)\}$$

** The non-linear terms have been ignored.*

Control Method - IO Linearization



Case 1: Distance Tracking

relative degree, γ =2

$$\dot{\xi}_1(t) = \dot{x}_1(t) = \xi_2(t)$$

$$\dot{\xi}_2(t) = \dot{v}_f(t) + (\bar{f}_0(t) + \bar{f}_1(t)(v_f - \xi_2(t)) + \bar{f}_2(v_f - \xi_2(t))^2) - \bar{F}_{tr}u(t)$$

$$u(t) = \frac{1}{\bar{F}_{tr}}\{\dot{v}_f(t) + \bar{f}_0 + \bar{f}_1(v_f(t) - \xi_2(t)) + \bar{f}_2(v_f(t) - \xi_2(t))^2 + k_1\xi_1(t) + k_2\xi_2(t)\}$$

Case 2: Velocity Tracking

relative degree, γ =1

$$\dot{\xi}(t) = -(\bar{f}_0 + \bar{f}_1(\xi(t) + v_d) + \bar{f}_2(\xi(t) + v_d)^2) + \bar{F}_{tr}u(t)$$

$$u(t) = \frac{1}{\bar{F}_{tr}}\{\bar{f}_0 + \bar{f}_1(\xi(t) + v_d) + \bar{f}_2(\xi(t) + v_d)^2 - k\xi(t)\}$$

Control Method - CLF

- Case 1: Distance Tracking

$$V = \frac{1}{2}\xi^T(t)\xi(t)$$

$$\dot{V} = \xi_1(t)\xi_2(t) + \xi_2(t)\{\dot{v}_f + \bar{f}_0 + \bar{f}_1(v_f(t) - \xi_2(t)) \\ + \bar{f}_2(v_f(t) - \xi_2(t))^2 - \bar{F}_{tr}u(t)\}$$

$$\phi_0 = L_f V + \alpha V = \xi_1(t)\xi_2(t) + \xi_2(t)\{\dot{v}_f(t) + \bar{f}_0 \\ + \bar{f}_1(v_f(t) - \xi_2(t)) + \\ \bar{f}_2(v_f(t) - \xi_2(t))^2\} + 2(\xi_1^2(t) + \xi_2^2(t))$$

$$\phi_1 = L_g V = -\bar{F}_{tr}\xi_2(t)$$

$$u(t) = \begin{cases} 0, & \phi_0 \leq 0 \\ -\frac{\phi_0}{\phi_1}, & \phi_0 > 0 \end{cases}$$

…Continued

Case 2: Velocity Tracking

$$V = \frac{1}{2}\xi^2(t)$$

$$\begin{aligned}\dot{V} = -\xi(t)(\bar{f}_0 + \bar{f}_1(\xi(t) + v_d)+ \\ \bar{f}_2(\xi(t) + v_d)^2) \\ +\xi(t)\bar{F}_{tr}u(t)\end{aligned}$$

$$\phi_0 = -\xi(t)(\bar{f}_0 + \bar{f}_1(\xi(t) + v_d) + \bar{f}_2(\xi(t) + v_d)^2) + 2\xi^2(t)$$

$$\phi_1 = \xi(t)\bar{F}_{tr}$$

$$u(t) = \begin{cases} 0, & \phi_0 \leq 0 \\ -\frac{\phi_0}{\phi_1}, & \phi_0 > 0 \end{cases}$$

Control Method – Sliding Mode

- Case 1: Distance Tracking

$$S(\xi) = \dot{\xi}_1 + \lambda\xi_1$$

$$\dot{S}(\xi) = \dot{v}_f - \bar{F}_{TR}u + \left\{\bar{f}_0 + \bar{f}_1(v_f - \xi_2) + \bar{f}_2(v_f - \xi_2)^2\right\} + \lambda\xi_2$$

$$u = \frac{1}{\bar{F}_{TR}}\left[\lambda\xi_2 + \dot{v}_f + \bar{f}_0 + \bar{f}_1(v_f - \xi_2) + \bar{f}_2(v_f - \xi_2)^2 + 1.1sat(\frac{s}{.01})\right]$$

Control Method – Sliding Mode

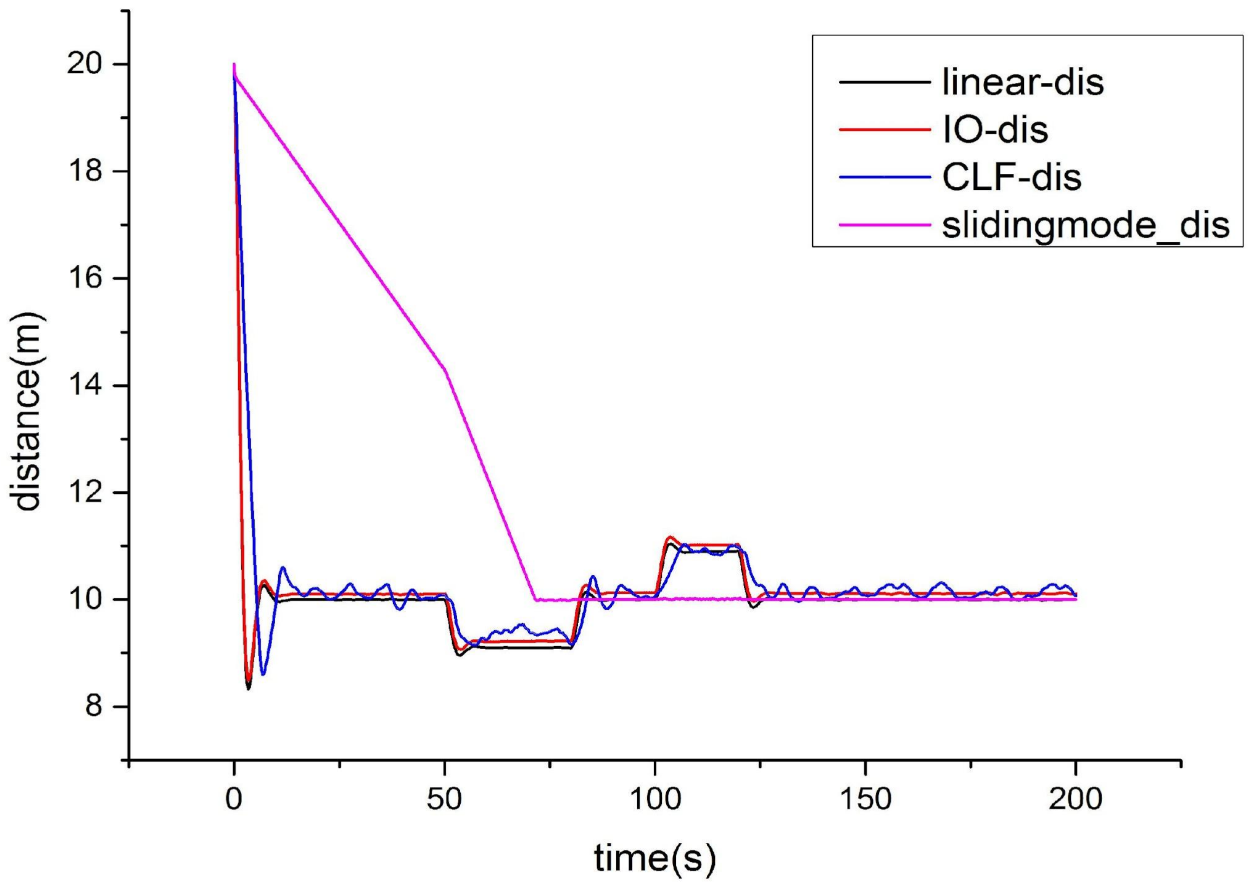
- Case 2: Velocity Tracking

$$S(\xi) = \lambda \xi_1$$

$$\dot{S}(\xi) = \lambda(\bar{F}_{TR}u - \{\bar{f}_0 + \bar{f}_1(\xi_1 + v_d) + \bar{f}_2(\xi_1 + v_d)^2\})$$

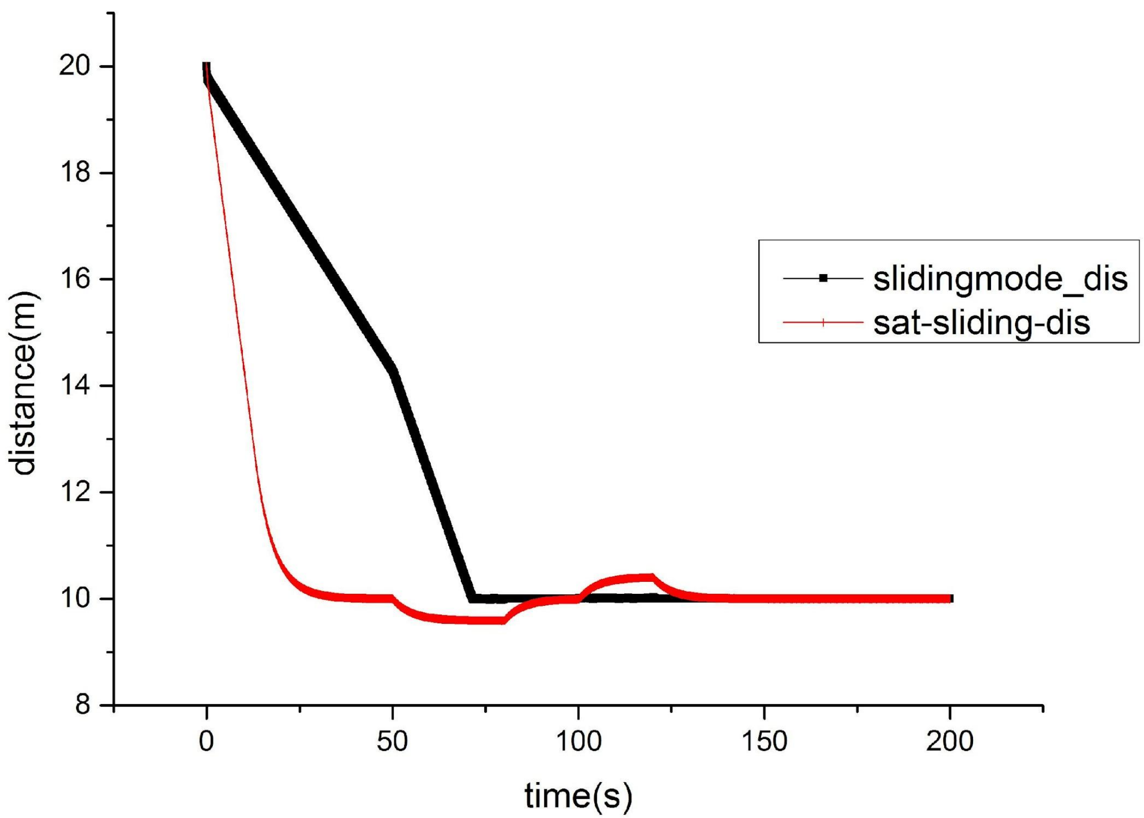
$$u = \frac{1}{\lambda \bar{F}_{TR}}\left[\lambda\left(\bar{f}_0 + \bar{f}_1(\xi_1 + v_d) + \bar{f}_2(\xi_1 + v_d)^2\right) - 1.1 sat(\frac{s}{.01})\right]$$

Simulation Results--Distance Tracking Distance

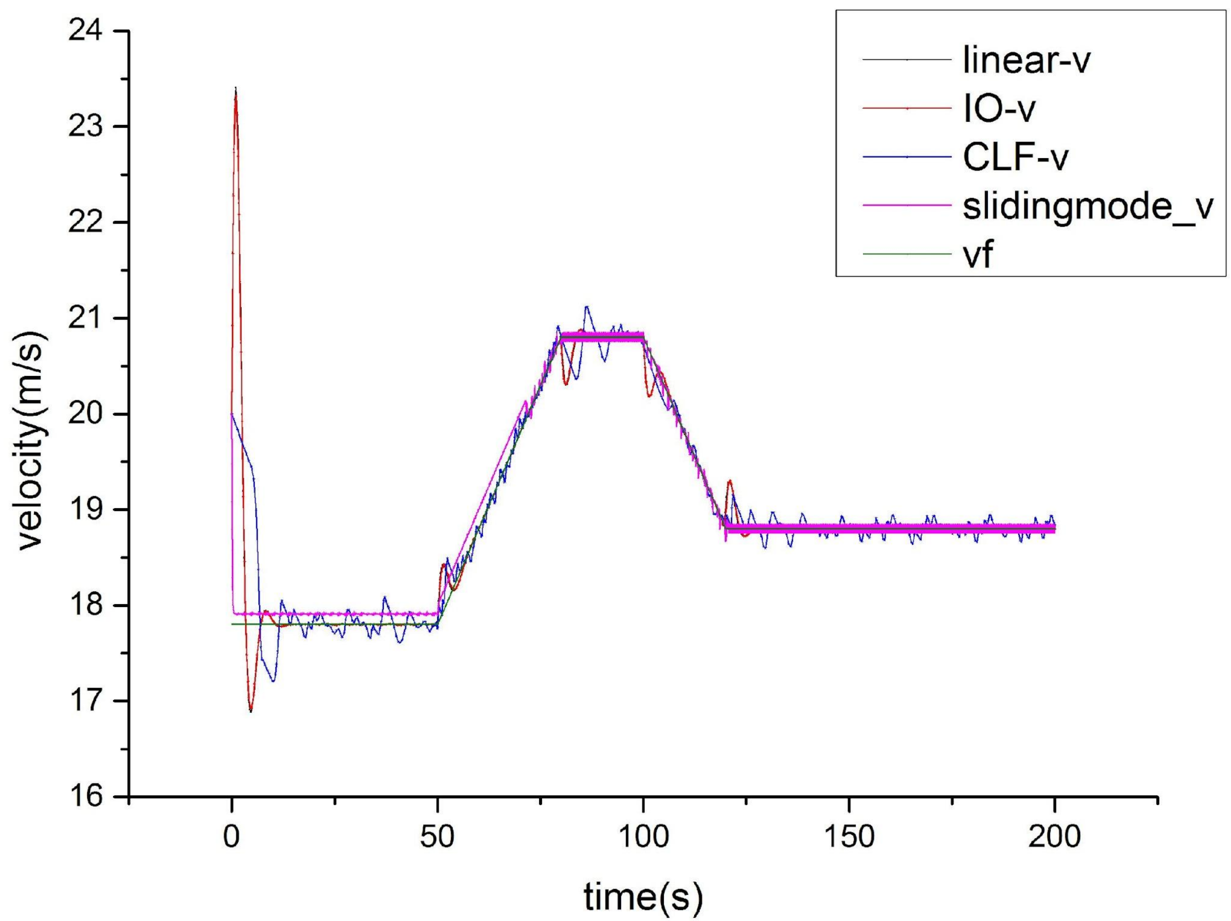


Simulation Results--Distance Tracking Distance

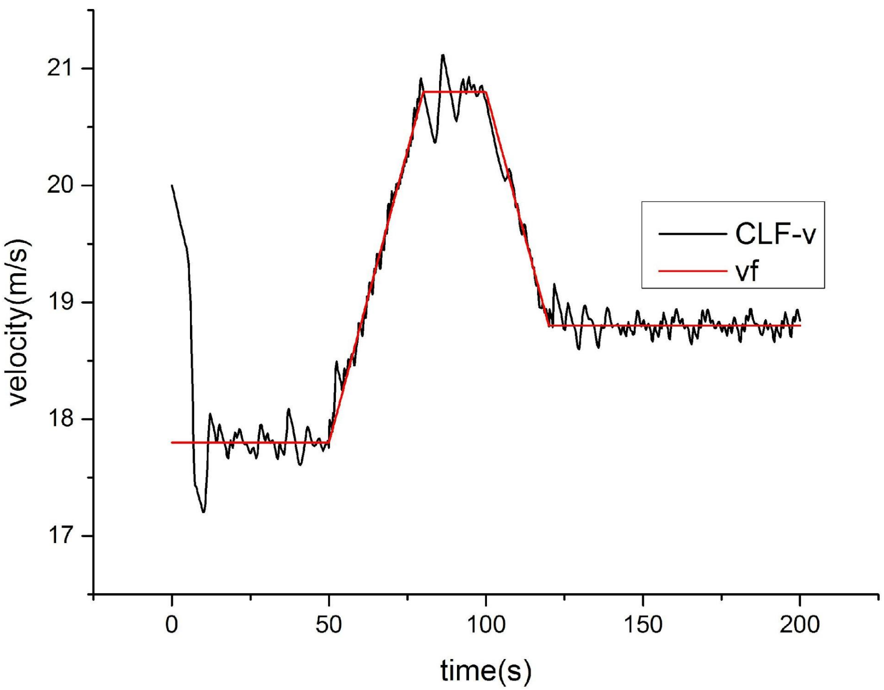
Saturated sliding mode & Sliding mode



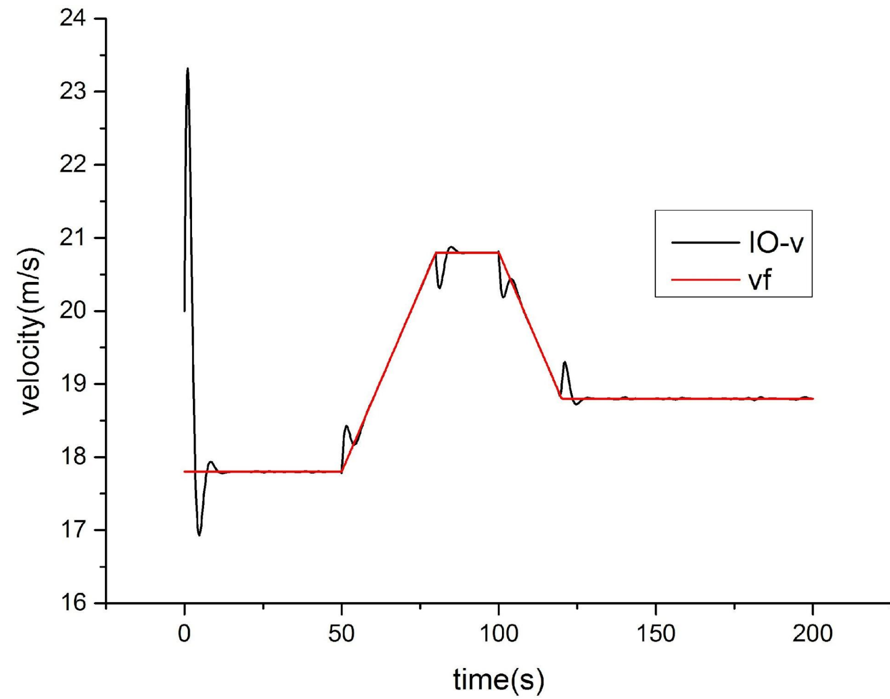
Simulation Results--Distance Tracking Velocity



Simulation Results--Distance Tracking Velocity



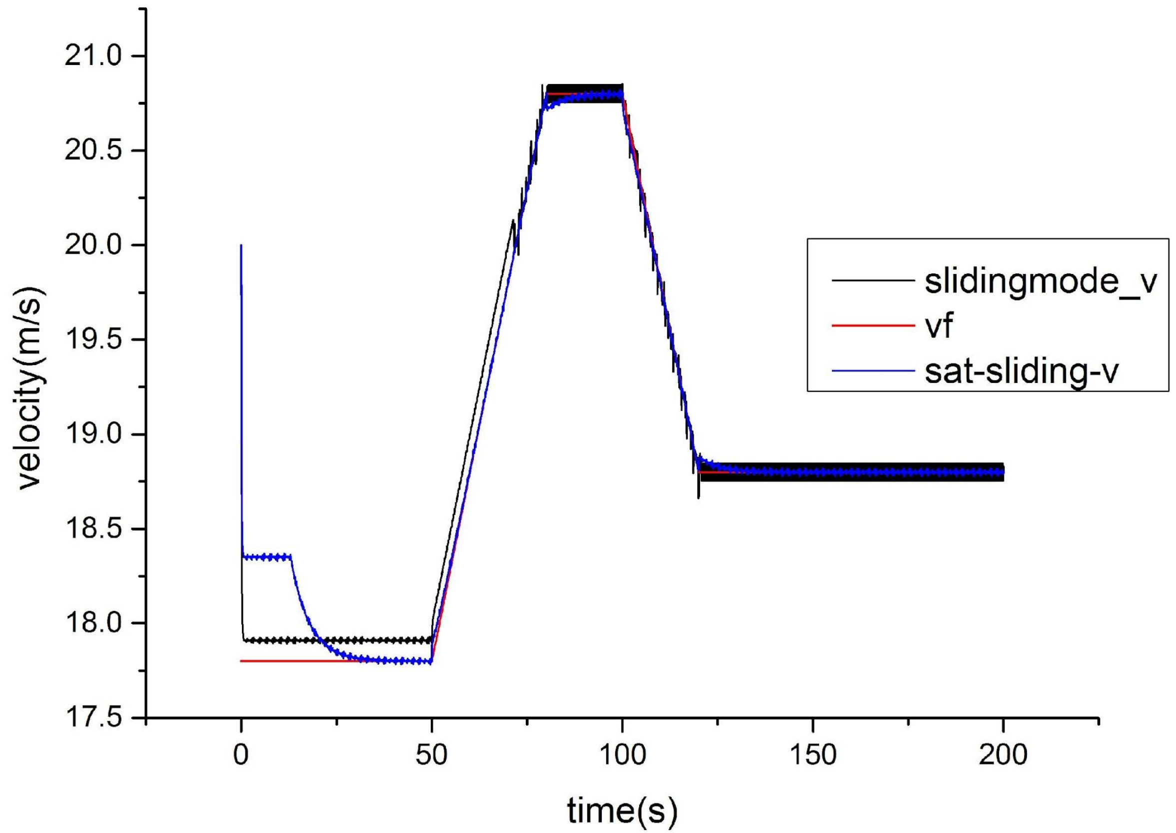
Control Lyapunov Function



Input-output linearization

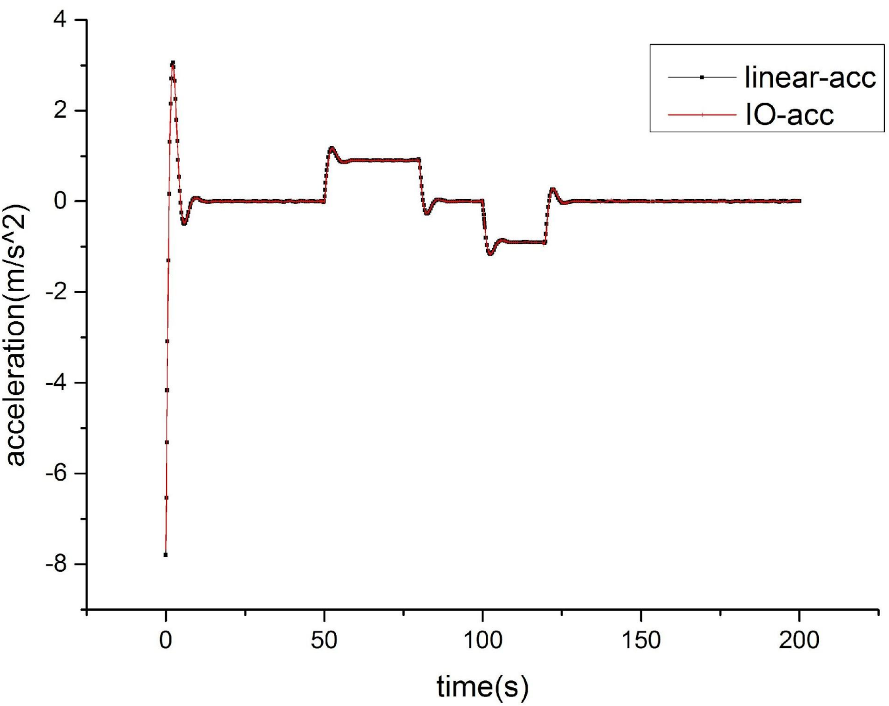
Simulation Results--Distance Tracking Velocity

Saturated sliding mode & sliding mode

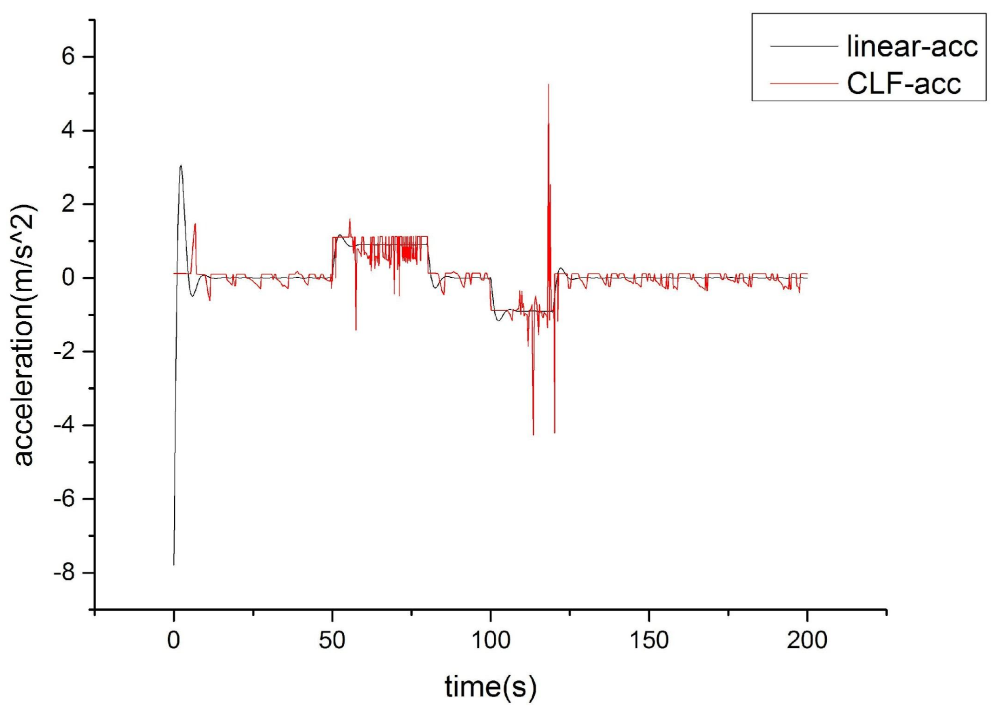


Simulation Results--Distance Tracking acceleration

Linear & IO linearization

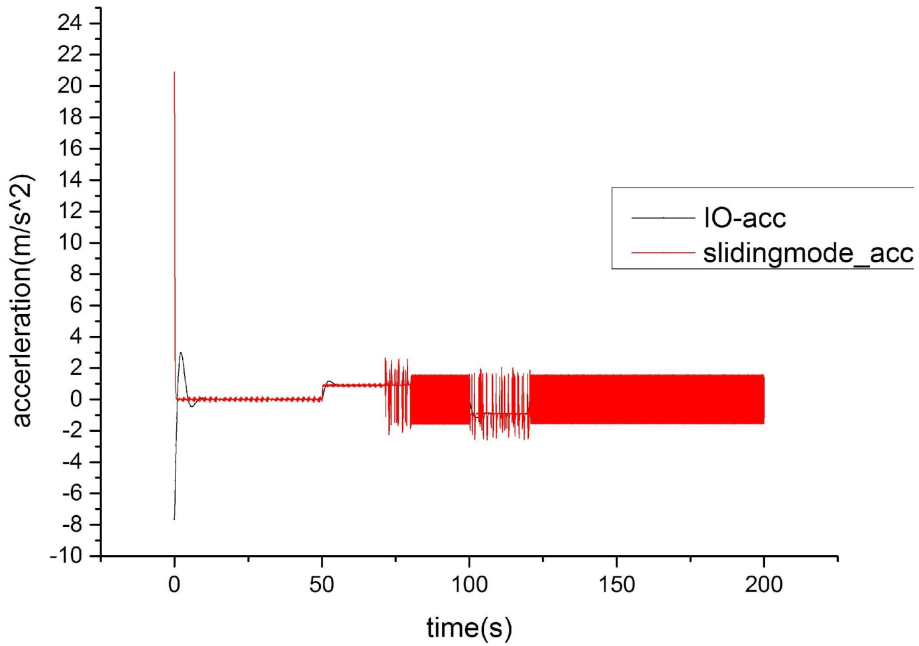


Linear & Control Lyapunov Function

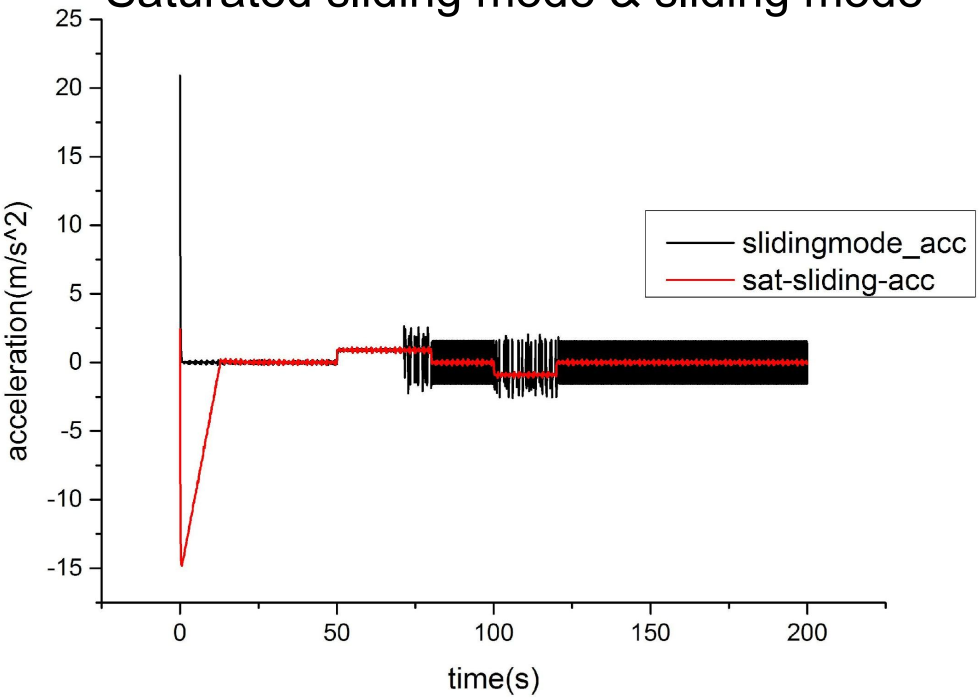


Simulation Results--Distance Tracking acceleration

Sliding mode & IO linearization

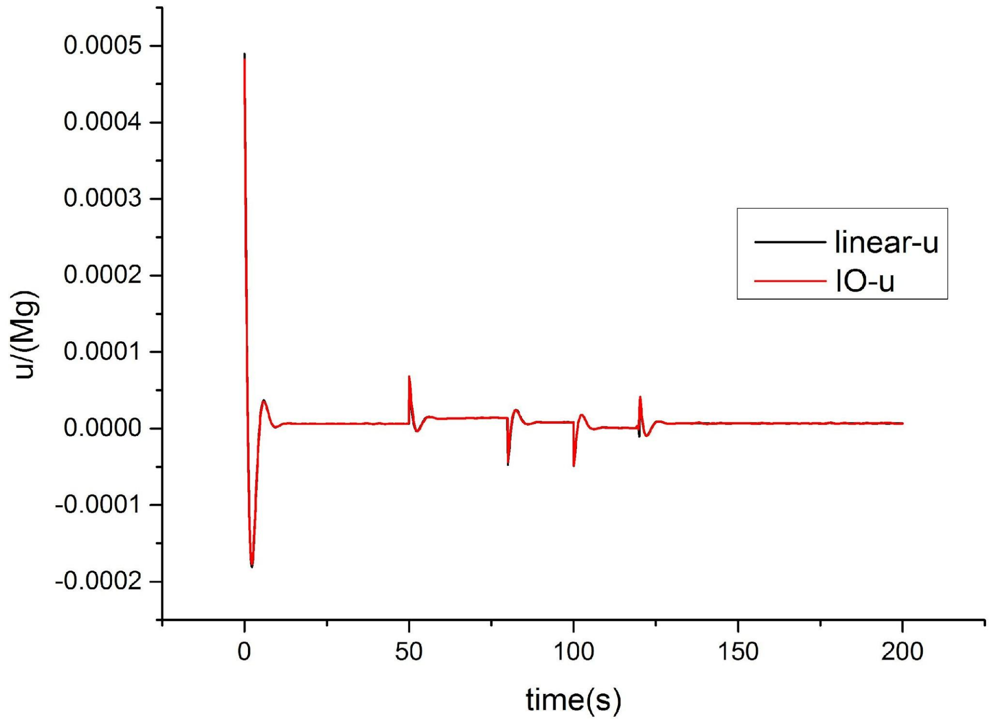


Saturated sliding mode & sliding mode

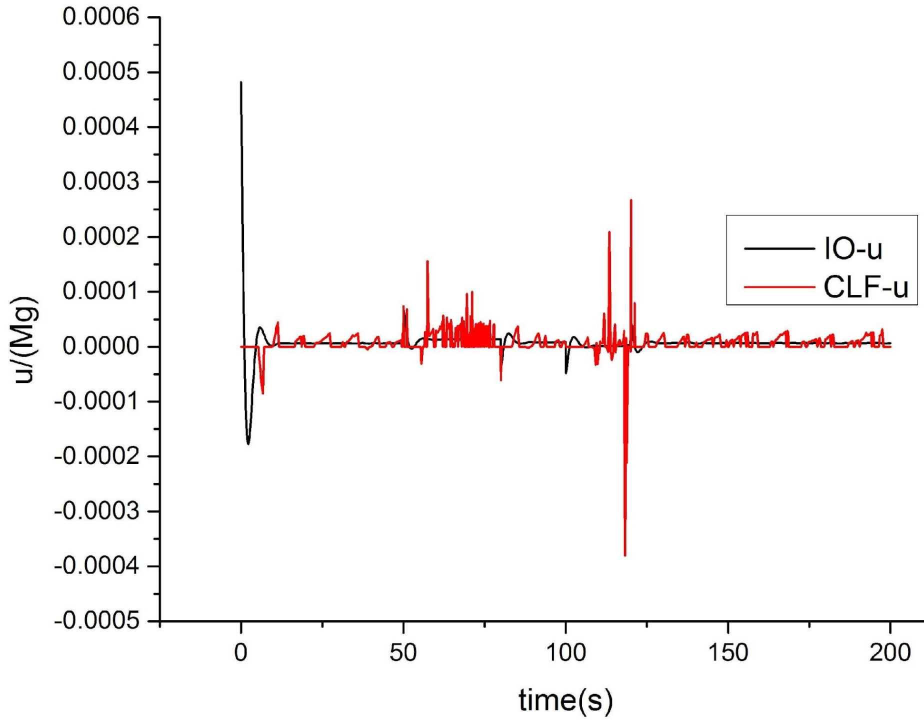


Simulation Results--Distance Tracking Control input u/(Mg)

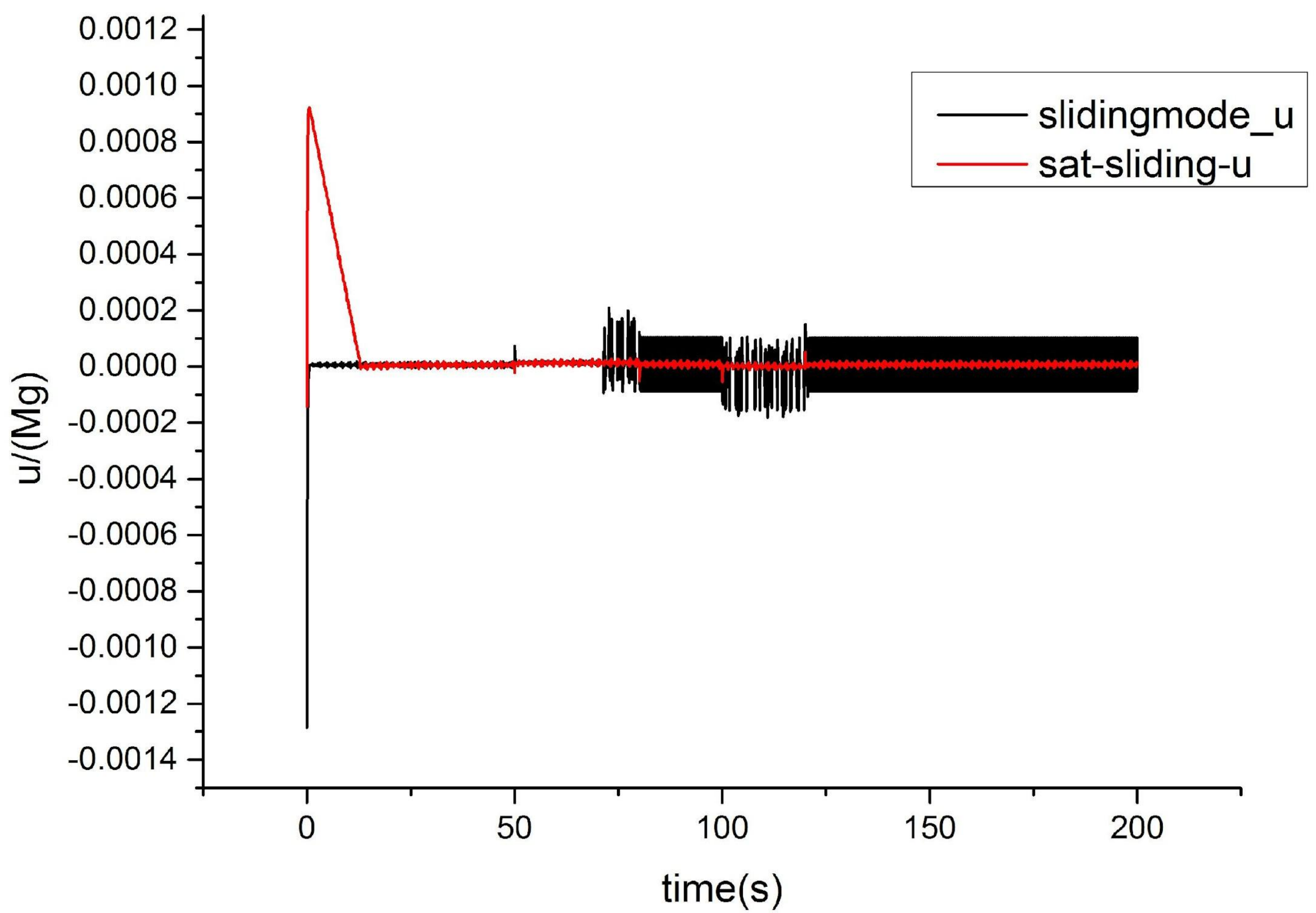
Linear & IO linearization



IO & Control Lyapunov Function



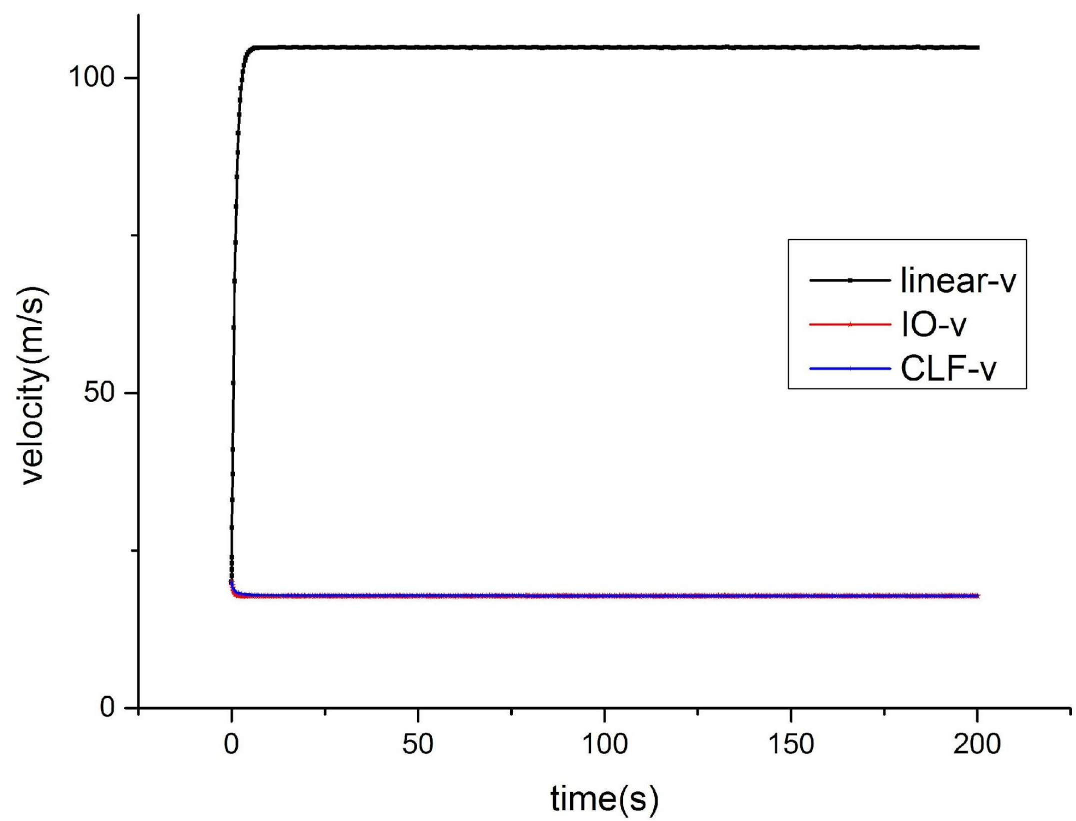
Simulation Results--Distance Tracking
Control input u/(Mg)



Conclusion for the distance tracking

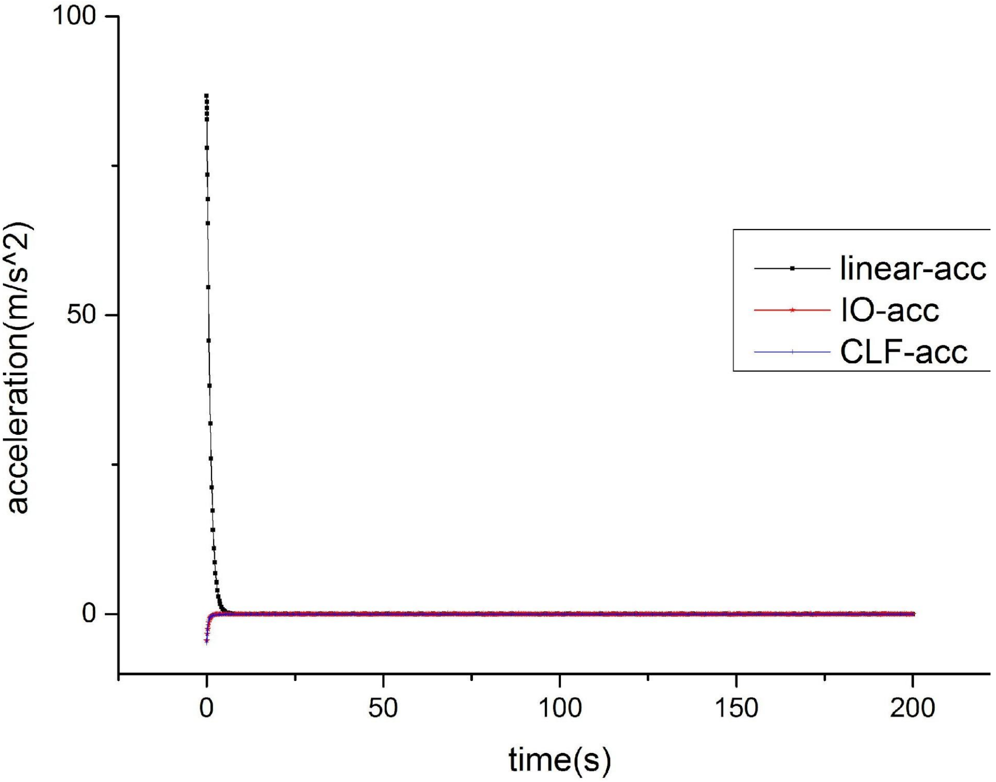
- 1. The controller for the linear cruise control model works as well as the input-output linearization controller, but both are hard to balance between large overshoots and slow settling time
- 2. The controller by Control Lyapunov Function does not work well for this model and has many jitters in steady state
- 3. The sliding mode controller has a small overshoot but the high-frequency jitters in the steady state can not be eliminated. After adding a saturation function, we can bound these jitters in a very small range.

Simulation Results--Velocity Tracking
Velocity

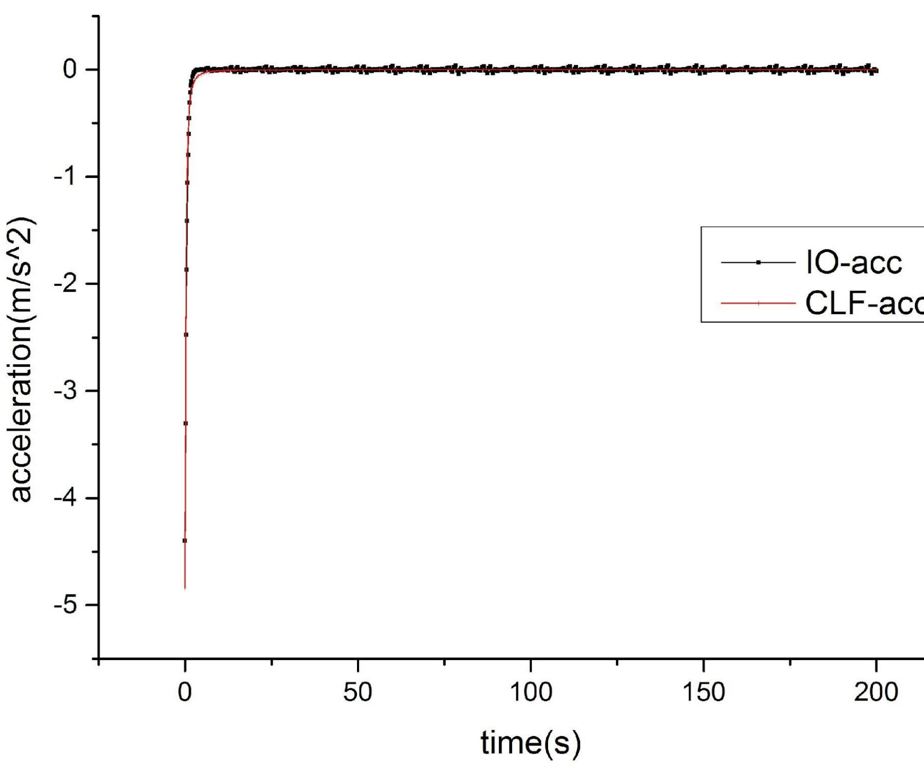


Simulation Results--Velocity Tracking Acceleration

Linear, IO & CLF



IO & CLF



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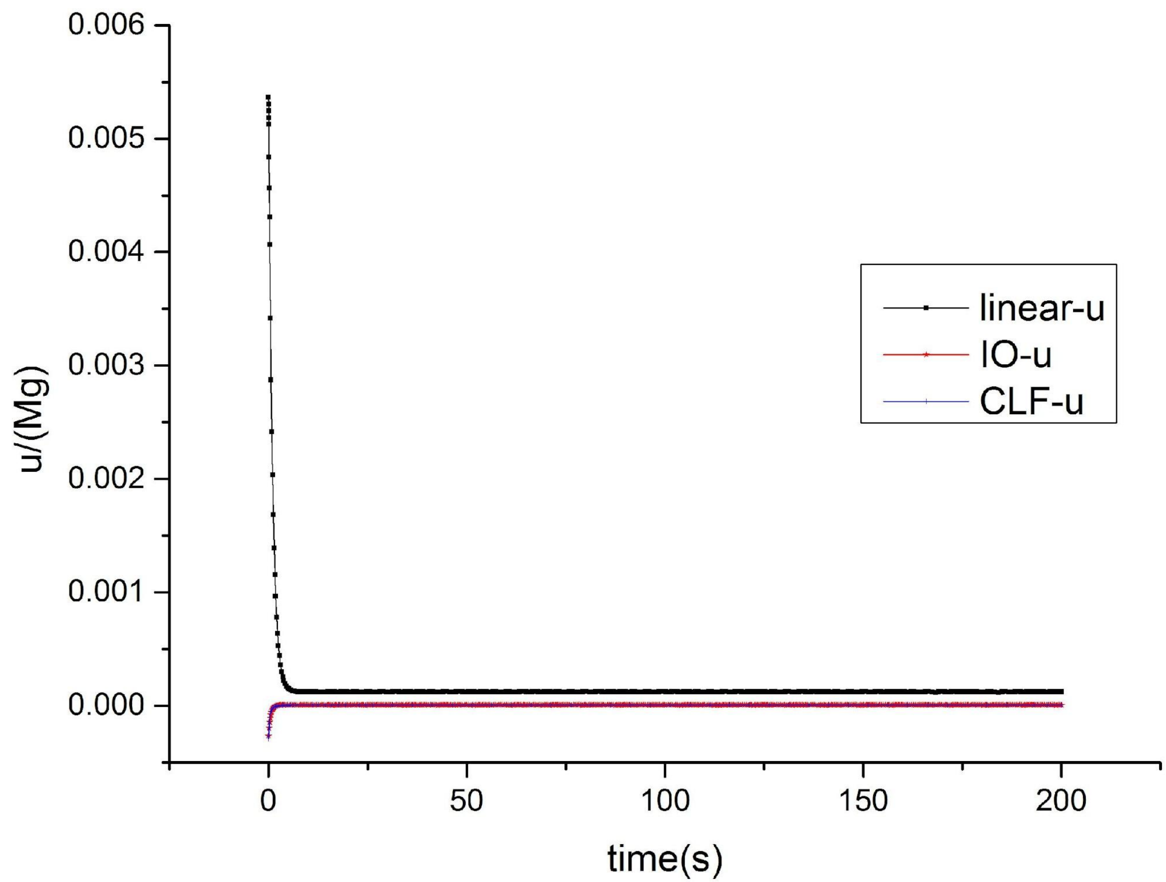
Simulation Results--Velocity Tracking
Control input u/(Mg)



Conclusion for velocity tracking

- 1. The linear controller does not work well for the nonlinear model. Although it can stabilize the system, it brings a very big steady state error(400%) and large initial acceleration which is not allowed
- 2. The controllers designed by input-output linearization and Control Lyapunov Function perform very similarly.

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