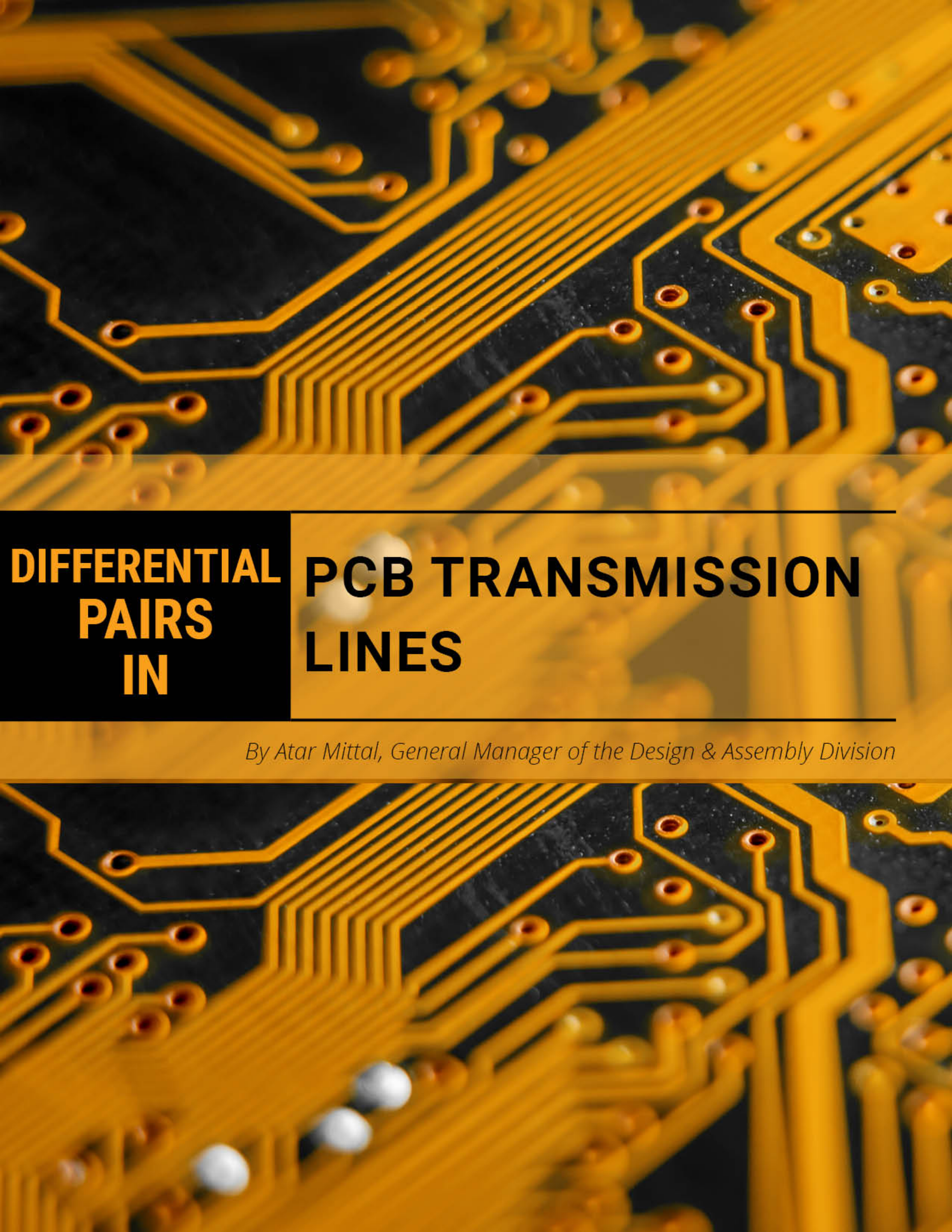

DIFFERENTIAL PAIRS IN PCB TRANSMISSION LINES

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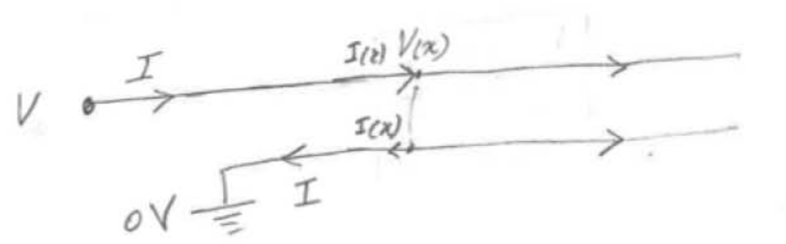
1. Differential and Common Mode Signals

1.1 Single-Ended Line

In our PCB Transmission Lines eBook, we established that a single-ended transmission line can be modeled as follows:



The relation between the voltage and current at any point on the line is given by:

$$V=Z_0 I \qquad (1)$$

Where Z_0 is the characteristic (or instantaneous) impedance of the line. For a lossless or almost lossless line, we saw that Z_0 is given by:

$$Z_0=\sqrt{(L_0)}/C_0 \quad \text{ohms} \qquad (2)$$

Where L_0 and C_0 are respectively the inductance and capacitance per unit length (pul) of the line.

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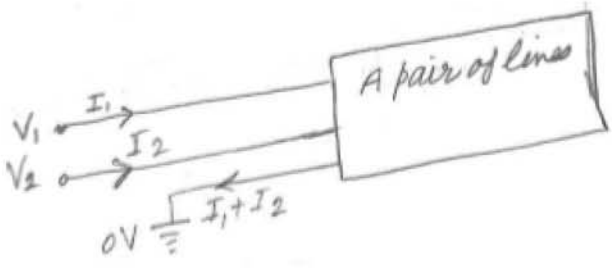
1.2 Differential Pair Line

A pair of lines can be modeled as follows:



We will assume here that both of the lines of the pair are identical and uniform. And that they have the same separation from each other along the entire length of the line. These are precisely the characteristics of a pair of lines to be designated as a differential pair.

When we have a pair of lines close to each other, it is fair to say that the presence of a current in line 2 will induce some voltage in line 1 and a current in line 1 will induce some voltage in line 2. Thus, the voltage V_1 at line 1 will not only depend on current I_1 in line 1 (through impedance Z_0 of line 1). It will also depend on current I_2 in line 2 through coupling or mutual impedance Z_m between lines 1 and 2. This situation can be expressed by the following equation:

$$V_1 = Z_{se} I_1 + Z_m I_2 \qquad (3)$$

Where Z_{se} is the characteristic impedance of line 1 and Z_m is the mutual or coupling impedance between line 1 and line 2.

Similarly for line 2 (of the differential pair), being identical to line 1, we can write the following equation:

$$V_2 = Z_{se} I_2 + Z_m I_1 \qquad (4)$$

The mutual impedance Z_m arises because of coupling between the two lines. And the most important coupling agents are L_m, the mutual inductance pul and C_m, the mutal capacitance pul between lines 1 and 2.

Closer are the two lines to each other, greater is the coupling between them. In fact, if the separation 'S' between the lines is reduced, the values of all three parameters – L_m, C_m and Z_m – increase.

The equations (3) and (4) are true for any point on the line 1 and the corresponding point on the line 2. And for a uniform differential pair, the Z_{se} and Z_m have the same value at every location along the differential pair.

1.3 Coupling Coefficient

Since 'Z_m' provides the magnitude of the signal voltage coupled from one line to another, as compared to the signal contributed through its own 'Z_{se}', we may define the ratio 'Z_m/Z_{se}' as the coupling coefficient between the two lines of a differential pair:

$$K = Z_m / Z_{se} \qquad (5)$$

1.4 Differential and Common Mode

1.4.1 Odd and Even Modes

Let V_1 and V_2 be the signal voltages and I_1 and I_2 be the signal currents in the two lines of a differential pair characterized by impedances Z_{se} and Z_m. We know these six quantities are related through equations (3) and (4).

The difference in signal voltage V_1 and V_2 is called the differential signal V_{diff}. Half of it is also called the odd mode signal:

$$V_{diff} = V_1 - V_2$$

$$V_{odd} = \frac{V_{diff}}{2} = \frac{V_1 - V_2}{2} \qquad (5)$$

The average value of V_1 and V_2 is called the common mode signal V_{com}. It is also called the even mode signal:

$$V_{com} \equiv V_{even} = \frac{V_1 - V_2}{2} \qquad (6)$$

From 5 and 6, we can express V_1 and V_2 in terms of V_{diff} and V_{com} as follows:

$$V_1 = V_{com} + \frac{V_{diff}}{2} \equiv V_{even} + V_{odd} \qquad (7a)$$

$$V_2 = V_{com} - \frac{V_{diff}}{2} \equiv V_{even} - V_{odd} \qquad (7b)$$

Furthermore, the equations (7a) and (7b) also allow us to think that V_{com} or V_{even} part of the signals in V_1 and V_2 are a kind of bias on top of which the differential mode (or odd mode) signals $+V_{odd}$' and $-V_{odd}$ ride to result in V_1 and V_2. This viewpoint is the most important aspect of differential signaling analysis.

At this point, before we proceed further in our conceptual analysis, let's keep in mind that the use of transmission lines – single-ended or differential – is to propagate time varying signals – usually high-speed digital signals or high-frequency analog signals – from one place to another. It is time varying signals that constitute information. Static voltages and currents do not have any information.

Therefore, looking at equations (7a) and (7b) above, we need to emphasize that V_{com} (or V_{even}) is only a de bias voltage on the two lines of a differential pair. The main signal is the differential signal (V_1 – V_2). Half of which is added to line 1, usually called the positive line. It is identified by the suffix + or P in the name of the signal on it. The other half is subtracted from line 2, usually called the negative line and identified by the suffix '-' or 'N' in the name of the signal on it, to constitute 'V_1' and 'V_2' at the signal transmitter end.

At the destination, the two lines go to the inputs of a differential receiver which detects the difference ('V_1 – V_2') in the amplitude of the signals on the two lines as the true signal. Thereby, in the process rejecting any common mode signal – either deliberate AC bias and/or common mode noise.

This ability of rejecting the common mode signal and thus any common mode noise in differential signaling makes it far superior to single-ended signaling, where there is no way to separate the noise from the actual signal.

Having stated this, we may be tempted to think that we need to analyze deeper only the differential or odd mode and ignore the common mode. But let's not forget that as signals propagate on the lines, they are suspect to all kinds of noise that gets superimposed on them. This may affect signal integrity adversely. Therefore, while differential pair lines' response to the differential (ie. odd) part of the signal is our main concern, we must also analyze the differential pair's response to the common (or even) mode signal.

1.5 Differential and Odd Mode Signals

We will now analyze the differential pair when we have sent only odd mode signals on it – without any common mode part.

In this case, since '$V_{com} = V_{even} = 0$', we have from (7a) and (7b):

$$V_1 = \frac{V_{diff}}{2} = V_{odd}$$

$$And \qquad V_2 = \frac{-V_{diff}}{2} = -V_{odd}$$

$$\therefore \qquad V_2 = -V_1$$

And since the lines are identical, we will have '$I_2 = -I$' so that '$I_1 + I_2 = 0$'. Thus, there will be zero current in the return path.

Equation (3) or (4) now gives us:

$$V_1 \equiv V_{odd} = Z_{se}I_1 + Z_mI_2 = Z_{se}I_1 - Z_mI_1 = (Z_{se} - Z_m)I_1 = (Z_{se} - Z_m)I_{odd}$$

We, as usual, define the ratio of 'Vodd/Iodd' as the odd mode impedance of the line:

$$Z_{odd}=V_{odd}/I_{odd}=Z_{se}-Z_m \qquad (8)$$

Now 'Zm = K.Zse' where 'K' is the coefficient of coupling.

$$Z_{odd}=Z_{se}(1\text{-}K) \qquad (8a)$$

From this, it is clear that odd mode impedance is less than the single-ended impedance 'Z_{se}' of the single line. And greater the 'Z_m' (or coupling between the two lines of the pair) is, lesser will 'Z_{odd}' be from 'Z_0'.

In our next article, we will discuss differential impedance and even or common mode.

2. Differential Impedance, Even or Common Mode

2.1 Differential Impedance

The differential impedance is the impedance seen by a purely differential (ie. odd mode) signal over a differential pair. As:

$$V_{diff} = 2V_{odd}$$

$$And \qquad I_2 = -I_1, \qquad \therefore \qquad I_{diff} = I_1 = I_{odd}$$

$$\therefore \qquad Z_{diff} = \frac{V_{diff}}{I_{diff}} = \frac{2V_{odd}}{I_{odd}} = 2Z_{odd} = 2Z_{se}(1-K) \qquad (9)$$

Thus, the differential impedance is twice the odd mode impedance. Or the odd mode impedance is half of the differential impedance.

Most often, the only specified requirement of a differential pair is its differential impedance. Typical values for most common differential signal types are 90 ohms differential, 100 ohms differential or 120 ohms differential. In some cases, 75-ohm differential impedance is also used.

2.2 Even or Common Mode

Let's now take the case when both the lines of the differential pair are excited by a common signal:

$$V_{com} \equiv V_{even} = V_1 = V_2$$

Let's now take the case when both the lines of the differential pair are excited by a common signal:

$$Z_{even} = \frac{V_{even}}{I_1} = (Z_{se} + Z_m) \qquad (10)$$

As:

$$Z_m = K.Z_{se} \qquad \therefore \qquad Z_{even} = Z_{se}(1+K) \qquad (10a)$$

And the common mode signal current in the differential pair is '$I_1 + I_2 = 2I_1$' so that the common mode impedance of the differential pair is:

$$Z_{com} = \frac{V_{com}}{I_{com}} = \frac{V_{even}}{2I_1} = \frac{1}{2}Z_{even} = \frac{Z_{se}}{2}(1+K) \qquad (11)$$

Thus, the common mode impedance of the differential pair is half of the even mode impedance of one line. It basically equals two even mode impedances in parallel.

From the equation (10), it is clear that 'Z_{even}' is greater than 'Z_{se}'. Greater the coupling between the two lines of the pair is, larger will 'Z_{even}' be compared to 'Z_{se}'.

2.3 Recap: Odd and Even Mode

Odd and even mode impedances of a differential pair lines are fundamental characteristics of the pair. All other impedances can be evaluated from their values, as illustrated below:

Since:

$$Z_{even} = Z_{se} + Z_m = Z_{se}(1 + K)$$

$$And \qquad Z_{odd} = Z_{se} - Z_m = Z_{se}(1 - K)$$

$$\therefore \qquad Z_{se} = \frac{Z_{even} + Z_{odd}}{2} \qquad (12a)$$

$$And \qquad Z_m = \frac{Z_{even} - Z_{odd}}{2} \qquad (12b)$$

$$And \qquad K = \frac{Z_m}{Z_{se}} = \frac{Z_{even} - Z_{odd}}{Z_{even} + Z_{odd}} = \left(1 - \frac{Z_{odd}}{Z_{se}}\right) \qquad (12c)$$

To these, we can add:

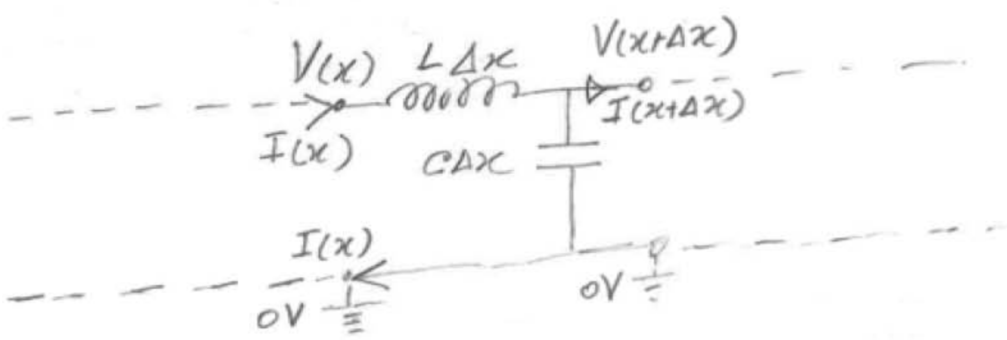
$$Z_{diff} = 2Z_{odd} = 2Z_{se}(1 - K) \qquad (9)$$

$$Z_{com} = \frac{Z_{even}}{2} = \frac{Z_{se}}{2}(1 + K) \qquad (11)$$

3. The Physical Parameters

3.1 Detailed Analysis of a Differential Pair in Terms of Line Inductances and Capacitances

The analysis of a lossless single-ended transmission line was done using the following circuit model for an infinitesimally small line of length 'delta x':



Here, 'L' and 'C' are respectively the inductance and capacitance per unit length of the line. After analysis, we had derived that the characteristic (or instantaneous) impedance of the line at a point was given by:

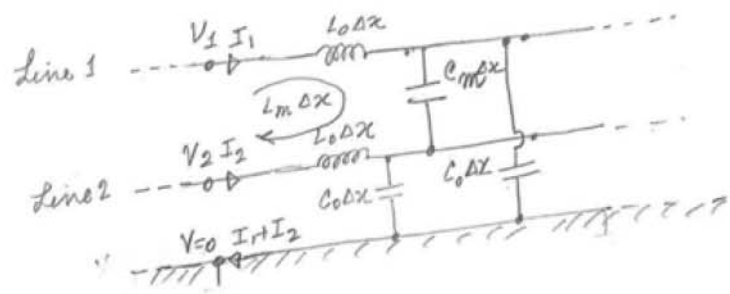
$$Z_0=\sqrt{(L/C)} \qquad \text{;ohms}$$

And the propagation delay 'P_d' was given by:

$$P_d=\sqrt{LC} \quad \text{in time pul}$$

We can now apply the above model and results to the analysis of the lines of a differential pair in terms of inductances and capacitances involved. And we are assuming that lines as such that the conductor resistances ('R's') and dielectric conductances ('G's') can be neglected for the impedance and propagation delay purposes. This will be the case at practical frequencies of interest.

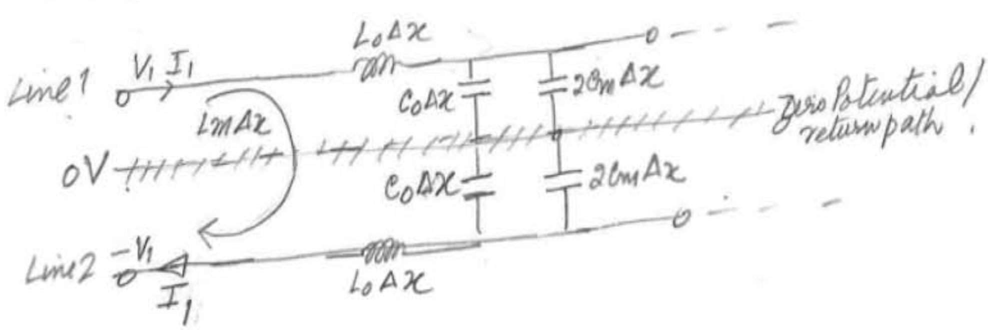
The following diagram gives the circuit model of an infinitesimally small length of a differential pair line:



Here ‘L_0’ and ‘C_0’ are respective by the inductance and capacitance of each line per unit length. ‘Lm’ is the mutual inductance per unit length between line 1 and line 2. ‘Cm’ is the capacitance per unit length between line 1 and line 2.

3.2 Case 1: Odd Mode (Purely Differential Signal)Impedances

Here ‘V_2 = -V_1’ and ‘I_2 = -I_1’. Thus, no current flows in the return path. ‘Cm delta x’ can be considered as two capacitors, each of value ‘2_{Cm} delta x’, in series whose center point is at zero potential. (That is because of the potential division between two equal capacitors.) Thus, the equivalent circuit of above in odd mode becomes:



Let’s look at line 1 (the line 2 case is exactly similar).

The inductive voltage in line 1 will comprise of two parts. One due to ‘I’, flowing through ‘L_0 delta x’. The other one due to ‘I_2 = -I_1’, flowing through ‘L_m delta x’. These can be equivalently stated as due to ‘I’, flowing through ‘(L_0 – L_m) delta x’. Thus, the effective inductance per unit length of line 1 in odd mode, denoted by ‘L_{odd}’, will be given by:

$$L_{odd}=L_0-L_m \qquad (13a)$$

And the effective capacitance between line 1 and zero potential line is '(C0 + 2Cm) delta x'. Thus, the odd mode line capacitance per unit length is:

$$C_{odd}=C_0+2C_m \qquad (13b)$$

By definition and similar to results derived in the case of the single-ended line, the odd mode characteristic impedance is given by:

$$Z_{odd}=\sqrt{(L_{odd}/C_{odd})}=\sqrt{((L_0-L_m)/(C_0+2C_m))} \qquad (13c)$$

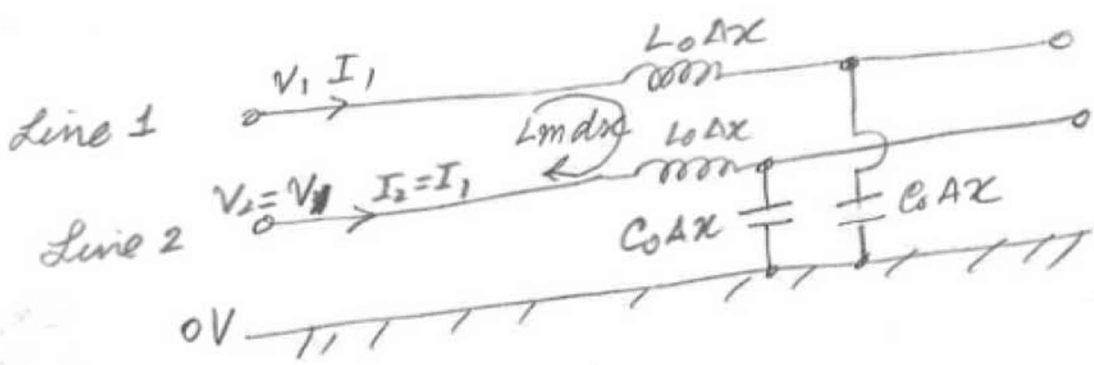
And the propagation delay per unit length for odd mode signals is given by:

$$P_{dood}=\sqrt{(L_{odd}C_{odd})}=\sqrt{((L_0-L_m)(C_0+2C_m))} \qquad (13d)$$

This is the propagation delay per unit length that will take place for purely the differential part of the signal. The point to also remember here is that the odd mode or purely differential signal's electromagnetic waves exist largely in and around the space between the two lines. And they are relatively less affected by the reference or ground planes.

3.3 Case 2: Even Mode (Purely Common Mode Signal)

In this case, '$V_2 = V_1$' and as the two lines of the pair are identical, '$I_1 = I_2$'. Since '$V_1 = V_2$', the capacitance 'Cm' between the two lines will have no effect on the currents in the two lines. It can therefore be ignored, leading to the following equivalent circuit:



Let's look at line 1 (the line 2 case is identical).

The inductive voltage in line 1 will be contributed by 'I_1', flowing in 'L_0 delta x'. And by '$I_2 = I_1$", flowing in 'Lm delta x'. This is equivalent to saying 'I_1' flowing through '($L_0 + L_m$) delta x'. Therefore, the effective inductance per unit length of either line in even mode will be:

$$L_{even}=L_0+L_m \qquad (14a)$$

And the effective capacitance per unit length of either line in even mode will be:

$$C_{even}=C_0 \qquad (14b)$$

Therefore, the even mode impedance of either line will be given by:

$$Z_{even}=\sqrt{(L_{even}/C_{even})}=\sqrt{((L_0+L_m)/C_0)} \qquad (14c)$$

And the propagation delay per unit length for even mode signals is given by:

$$P_{deven}=\sqrt{(L_{even}\,C_{even})}=\sqrt{((L_0+L_m)\,C_0)} \qquad (14d)$$

Equations (13c) and (14c) clearly indicate that 'Zeven' is greater than 'Z_{odd}'. From theory, it is also known that 'L_m' is lesser than 'L_0' and 'C_m' is lesser than 'C_0'.

'Z_{se}', 'Z_m' and 'K' in terms of 'L_0', 'C_0', 'L_m', 'C_m'

Using equations (12), (13) and (14), we have:

$$Z_{se}=\frac{Z_{even}+Z_{odd}}{2}=\frac{1}{2}\left(\sqrt{\frac{L_0+L_m}{C_0}}+\sqrt{\frac{L_0-L_m}{C_0+2C_m}}\right) \qquad (15a)$$

$$Z_{m}=\frac{Z_{even}-Z_{odd}}{2}=\frac{1}{2}\left(\sqrt{\frac{L_0+L_m}{C_0}}-\sqrt{\frac{L_0-L_m}{C_0+2C_m}}\right) \qquad (15b)$$

$$K=\frac{Z_m}{Z_{se}}=\frac{\sqrt{\frac{L_0+L_m}{C_0}}-\sqrt{\frac{L_0-L_m}{C_0+2C_m}}}{\sqrt{\frac{L_0+L_m}{C_0}}+\sqrt{\frac{L_0-L_m}{C_0+2C_m}}} \qquad (15c)$$

3.4 Single-Ended Impedance

Some words about 'Zse'

'Z_{se}' is the single-ended impedance of either of the two lines in the presence of the other line. It is not exactly the same as the single-ended impedance of a single line will be when there is no second line. The presence of the second line somewhat reduces the impedance. The more coupled (or closer) the two lines are, 'Z_{se}' will become a little more less.

It is important to note that 'Z_{se}' does not change much if the coupling between the two lines is not high. Or if we can make the separation between the two lines greater than the maximum of the conductor width or dielectric height between the signal layer and the closest ground/ reference plane.

i.e. 'Z_{se}' is relatively the same if the separation is greater than the maximum signal trace width, dielectric height between the signal layer and the nearest reference plane.

Further, 'Z_{se}' is not approximate to $\sqrt{}$'L_0/C_0' but is nearer to $\sqrt{}$'$L_0/(C_0+C_m)$'. This may appear surprising at first glance but that is the case.

It is not out of place to define a few new terms:

$$Z_0 = \sqrt{\frac{L_0}{C_0 + C_m}} \qquad (16a)$$

$$PD_0 = \sqrt{L_0(C_0 + C_m)} \qquad (16b)$$

$$K_L = \frac{L_m}{L_0} \qquad (16c)$$

$$K_C = \frac{C_m}{C_0 + C_m} \qquad (16d)$$

'KL' and 'KC' can be called the inductive and capacitive coupling coefficients respectively. Using these, we can write equation (15) as:

$$Z_{odd}=Z_0 \sqrt{((1-K_L)/(1+K_C))}=Z_0 (1-K_L)^{(1/2)} (1+K_C)^{(-1/2)} \qquad (17a)$$

$$Z_{even}=Z_0 \sqrt{((1+K_L)/(1-K_C))}=Z_0 (1+K_L)^{(1/2)} (1-K_C)^{(-1/2)} \qquad (17b)$$

Since 'KL' and 'KC' are lesser than 1, and for most practical cases will be significantly lesser than 1, we can retain only the first order terms so that:

$$Z_{odd} \approx Z_0 (1-K_L/2)(1\ K_C/2) \approx Z_0 (1-(K_L+K_C)/2) \qquad (18a)$$

$$Z_{even} \approx Z_0 (1+K_L/2)(K_C/2) \approx Z_0 (1+(K_L+K_C)/2) \qquad (18b)$$

If we compare equation 18a with 8a and 18b with 10a, it is easy to conclude that:

$$Z_{se} \approx Z_0 \qquad (18c)$$

And:

$$K \approx (K_L+K_C)/2 \qquad (18d)$$

It is worth mentioning here that in most practical cases of stripline differential pairs, the inductive coupling coefficient 'KL' and the capacitive coupling coefficient 'KC' are nearly equal if the dielectric constants of the PCB materials above and below the signal layer are nearly equal.

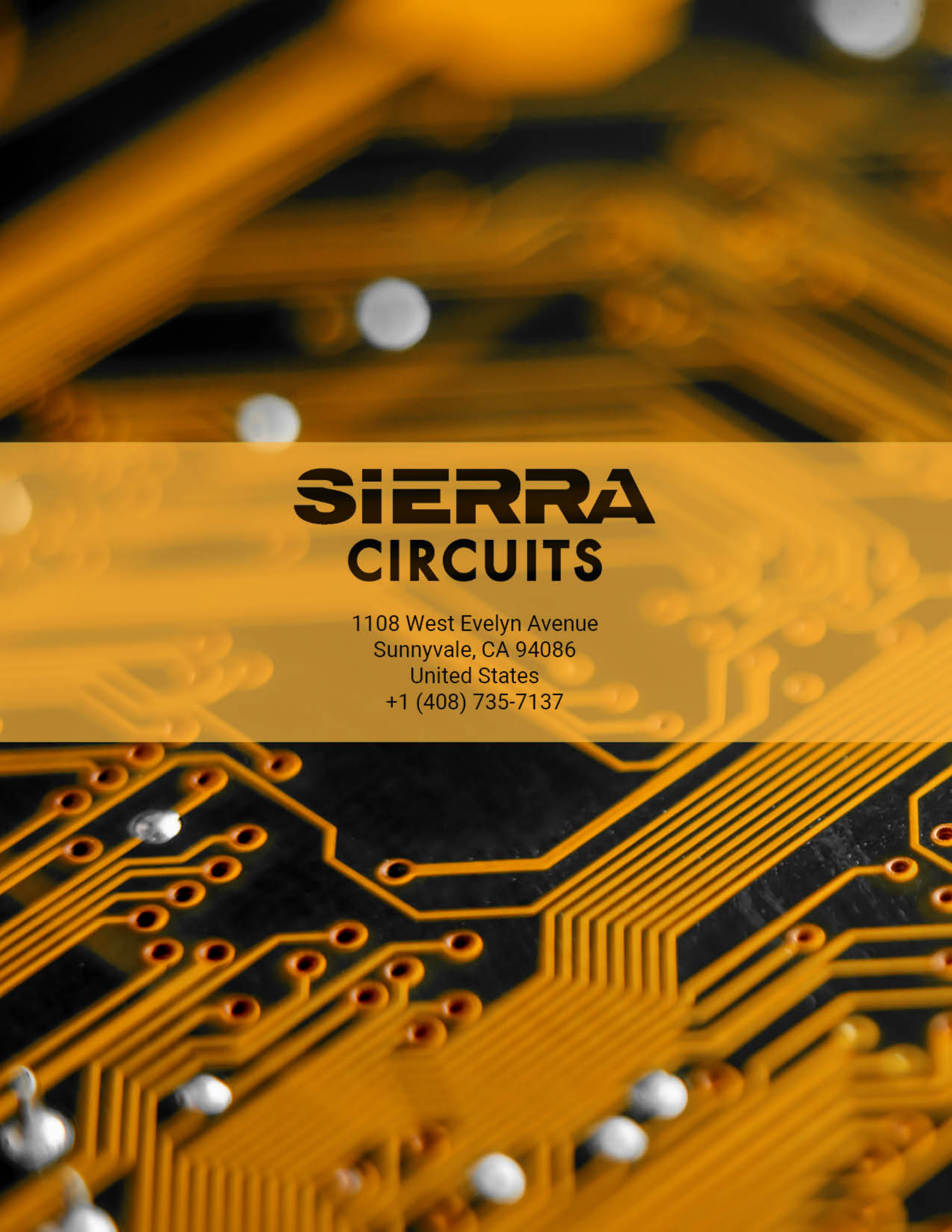
3.5 Two Crosstalk Related Parameters

Here, we would also like to define two more parameters, NEXT and FEXT:

$$NEXT=(Z_{even}-Z_{odd})/2(Z_{even}+Z_{odd}) \approx (K_L+K_C)/4 \approx K/2 \qquad (19a)$$

$$FEXT=(P_{dodd}-P_{-deven})/2 \approx ((K_C-K_{-L})/2).PD_0 \quad ; \quad \textit{in time pul} \qquad (19b)$$

NEXT is called the nearest crosstalk coefficient. FEXT is called the far end crosstalk coefficient. These parameters are important in crosstalk analysis when of the two nearby lines, one is the main signal line and the other line is the quiet line on which we wish to determine the crosstalk voltage induced due to signal voltage on the main line. We will develop this point in a next article about crosstalk analysis in detail.



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