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Conference Paper · November 2011

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ISSN 1726-9679

DANUBE ADRIA ASSOCIATION FOR AUTOMATION & MANUFACTURING

DAAAM INTERNATIONAL VIENNA

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ANNALS OF DAAAM FOR 2011 & PROCEEDINGS

OF THE 22ND INTERNATIONAL DAAAM SYMPOSIUM
"INTELLIGENT MANUFACTURING & AUTOMATION:
POWER OF KNOWLEDGE AND CREATIVITY"
23-26TH NOVEMBER 2011, VIENNA, AUSTRIA

ORGANIZED BY:
DAAAM INTERNATIONAL VIENNA
INTERNATIONAL ACADEMY OF ENGINEERING
VIENNA UNIVERSITY OF TECHNOLOGY, UNIVERSITY OF APPLIED SCIENCES
TECHNIKUM VIENNA AND AUSTRIAN SOCIETY OF ENGINEERS AND ARCHITECTS - ÖIAV 1848

UNDER THE AUSPICES OF: THE DANUBE RECTORS' CONFERENCE &
RECTORS' HONOR COMMITTEE OF DAAAM INTERNATIONAL

EDITOR: B.[RANKO] KATALINIC

DANUBE ADRIA ASSOCIATION FOR AUTOMATION & MANUFACTURING

DAAAM INTERNATIONAL VIENNA

PDF OFF-PRINTS

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This Publication has to be referred as:

Koleda, P. & Nascak, L.: Circular Interpolation for Circular Trajectory, *Annals of DAAAM for 2011 & Proceedings of the 22nd International DAAAM Symposium*, ISBN 978-3-901509-83-4, ISSN 1726-9679, pp 1017-1018, Editor B[ranko] Katalinic, Published by DAAAM International, Vienna, Austria 2011

www.daaam.com

Annals of DAAAM for 2011 & Proceedings of the 22nd International DAAAM Symposium, Volume 22, No. 1, ISSN 1726-9679
ISBN 978-3-901509-83-4, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2011
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Annals & Proceedings of DAAAM International 2011

CIRCULAR INTERPOLATION FOR CIRCULAR TRAJECTORY

KOLEDA, P[eter] & NASCAK, L[ubomir]

Abstract: *There are many methods of trajectory interpolation used in manufacturing and manipulating industrial devices. This article deals with possibilities of circular interpolation by substitution of trajectory of circular shape. Trajectory deviations and courses of passed trajectories are derived, wherein emphasis is placed on aplication during control of positioning system with two degrees of freedom. The criterias for selection of specific interpolation method are described in last part of this article.*
Key words: *interpolation, circular interpolation, parametrisation of actuating variables, optimal trajectory*

1. INTRODUCTION

During the movement control of robot, eventually manipulator, or some other positioning system, the need of description of its trajectory arises. Linear and circular sections, which are in the plain, create the simplest variant of this trajectory. The given description has to be suitable in terms of possible enforceable movements, control algorithms and difficulty of their processing, mainly in computing operations with relevant data. In (Jaklic et al., 2007) and (du Plessis & Snyman, 2003) the geometric interpolation of circle-like curves is studied as well as path interpolation methodology.

There are different methods of linear and circular interpolation in case of biaxial planar mechanism if simultaneous speed control is possible on both axes (Koleda & Nascak, 2010). The circle is substituted by linear sections, which the biaxial system is able to carry out by means of speed control of movement (on the corresponding axes) according to steepness of this section. The principle of linear trajectory interpolation, which is necessary for interpolation of linear parts of required trajectory, results from this argument. Indeed, an optimalization and design of controller of controlled process can not be forgotten (Pontriagin et al., 1962), (Nascak & Suriansky, 2005).

2. INTERPOLATION OF TRAJECTORY WITH CIRCLE SHAPE

The trajectory that has a shape of full circle eventually of its part can be substituted by different methods. A shape of substitute trajectory convergences to ideal circle up to the extent, that we can consider this trajectory for an ideal circle observing established criterias. The most used methods of circular interpolation, which are based on possibilities of simple analytic description of circle, are:

- rectangular interpolation – the circle trajectory is composed of linear rectangular sections,
- interpolation with constant step length on one of axes – circle trajectory is composed of linear sections, their projection into one of axes is constant,
- interpolation with maximum deviation δ – trajectory is composed of linear sections; the determining condition for its calculation is compliance of maximum deviation.

Algorithms for above mentioned methods of circular interpolation include calculation of corresponding data (coordinates of interpolation points).

Fig. 1 and Fig. 2 are illustrating described types of circular interpolation for trajectory which has a shape of circular arc with central angle α = 90°. Optional parameters are set in order to the differences of individual substitute trajectories were visible.

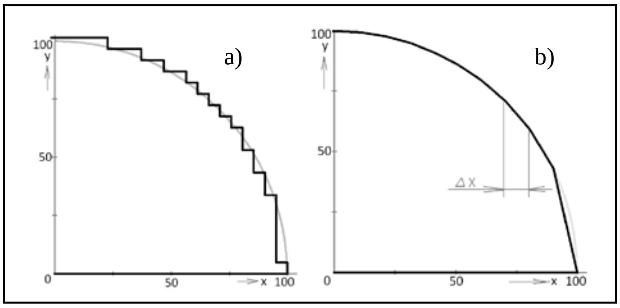

Fig. 1.a) Rectangular circular interpolation: step length = 5 mm, radius = 100 mm; b) Circular interpolation with constant step length: step length Δx = 10 mm, radius = 100 mm

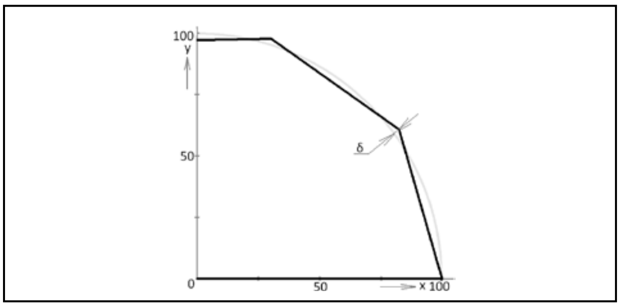

Fig. 2.Circular interpolation, maximum deviation δ = 5 mm, radius = 100 mm

There is evident from Fig. 1 and Fig. 2 that every algorithm of calculation of actuating variables, that represent a movement speeds on the corresponding axes, has different properties characterizing method of interpolation. These properties are mainly deviations from required trajectories, the actual total passed distances and speeds of translation movements on the x and y axes. Rate of similarity to ideal circle is expressed with deviation of actual trajectory from required trajectory and in rectangular coordinate system it can be calculated for example by means of quadratic distance

$$q_i = \sqrt{(X_{REALi} - X_{COMPi})^2 + (Y_{REALi} - Y_{COMPi})^2} \qquad (1)$$

wherein X_{REALi} and Y_{REALi} are coordinates of found point on trajectory which was really passed by mechanism; index i defines order of steps; X_{COMPi} and Y_{COMPi} are coordinates of point which lies on required trajectory and is the closest to point with X_{REALi} and Y_{REALi} coordinates.

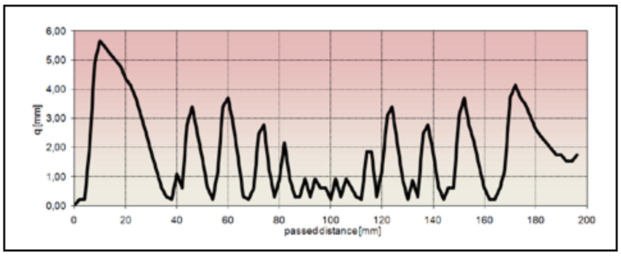

Fig. 3. Deviation during rectangular interpolation

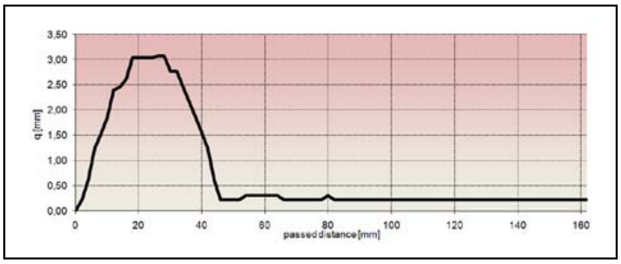

Fig. 4. Deviation during interpolation with constant step length

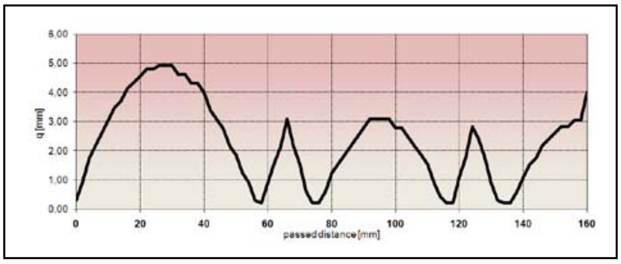

Fig. 5. Deviation during interpolation with maximum selected deviation

Found maximum quadratic deviations and length of interpolated trajectory are mentioned in Tab. 1. The trajectory computed by algorithm with constant step length achieves minimum deviation that is created at the beginning of trajectory. It is given by slope of linear section that could be expressed with ratio $\Delta y/\Delta x$, wherein Δx is constant. In sections where this ratio is large, the deviations from required trajectory will be the largest, too. The shortest passed distance account the trajectory calculated through algorithm with constant step length and maximum deviation. Such computed trajectories are similar by their shape to circular section.

Another parameter in terms of exact tracking of selected trajectory is sliding speed on corresponding axes. If initial point of trajectory lies on circle and its centre is simultaneously the centre of coordinate system, so in the first quadrant the speed v_x will be negative and speed v_y will be positive for all methods of interpolation. Courses of speeds for individual interpolation algorithms can be derived from derivations of distance according to time. Exact determination of speed depends on concrete behaviour of kinetic system, mainly on its dynamics and inertia.

In method of rectangular interpolation the problems can arise relating to the dynamics of system for the reasons of often increasing and decreasing of speed and of substantial accelerations. For an achievement of the largest accuracy we must anticipate the large number of these cycles. Smoother

Method of interpolation	q_{max} [mm]	Passed distance [mm]
Rectangular interpolation, step = 5 mm	5,65	196
Constant step length = 10 mm	3,07	162
Maximum deviation δ = 5 mm	4,92	160

Tab. 1.Found qualitative parameters of interpolation algorithms

course of speeds is achievable by other two methods, where required location is reached through gradual change of speed and deviation from ideal state is minimized.

Criterias for selection of trajectory can be described as follows:

1) **Minimum deviation from required trajectory** – the most important criteria for achievement of most exact tracking of selected curve. Limiting factor is resolution of sensor used in feedback.
2) **The shortest passed distance** – it depends on selected interpolation algorithm which substitutes required trajectory.
3) **The shortest running time** – time necessary to pass the whole trajectory; it depends on speed of movement and on length of trajectory.
4) **The lowest power consumption** necessary for relocation – preliminary criteria can be suitable for this condition. The shortest trajectory, eventually trajectory that takes the shortest running time, can have also the lowest power consumption. For its completion effective method for measuring of power consumption must be suggested.

Optimal trajectory should take into account all mentioned criteria in balance. If we select the shortest running time as the most significant, it can significantly influence and exacerbate accuracy of position, when the dynamics and inertia of system would be reflected.

3. CONCLUSION

Described interpolation methods and algorithm of their calculation by substitution of circle parts of trajectory are designed in order to use them in the proposed positioning device with two degrees of freedom created by compound table. Comparison of deviations illustrates and designs possibilities of precision improvement of corresponding methods, it is necessary to practically try out their real forms in further research. The options and methods of action values parameterization from these courses are also obvious.

The contribution is aimed at increasing the presicion of interpolation algorithms. Based on described interpolation methods, effective and exact interpolation procedures can be proposed, which will substitute trajectory of various form

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