Axis of Rotation

Basic Rotation: $R_{ij}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \dots & \dots & 0 \\ 0 & \dots & \cos\theta & -\sin\theta & 0 \\ 0 & \dots & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$\cos(\theta)$ in i row, i column
$-\sin(\theta)$ in i row, j column
$\sin(\theta)$ in j row, i column
$\cos(\theta)$ in j row, j column

Composition of Rotations = $R = R_{nm} \dots R_{13} \dots R_{12}$

Axis of Rotation In R^3 space

$R\vec{x} = \vec{x}$
$(R - I)\vec{x} = \vec{0}$

Solutions = $Ker(R - I)$ = Axis of Rotation

$\dim(Ker(R - I)) = 1$

Axis of Rotation In R^4 space

$\dim(Ker(R - I)) = 2$

Axis of Rotation In R^n space

$Ker(R - I)$ = Axis of Rotation
$\dim(Ker(R - I)) = n - 2$

Example 1

In R^4 space

$$R_{12}(90^o) = \begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) & 0 & 0 \\ \sin(\pi/2) & \cos(\pi/2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{23}(\pi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\pi) & -\sin(\pi) & 0 \\ 0 & \sin(\pi) & \cos(\pi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = R_{23}(\pi)R_{12}(\pi/2) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R\vec{x} = \vec{x}$$

$$(R - I)\vec{x} = \vec{0}$$

$$R - I = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

EROs to RREF

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = -x_2$$

$$x_2 = x_2$$

$$x_3 = 0$$

$$x_4 = x_4$$

$$\vec{x} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathrm{Ker}(R - I) = \text{span}\left\{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}$$