The Pigeonhole Principle

If a set of n distinct objects is partitioned into k subsets, where $0 < k < n$, then at least one subset contains at least two elements.

Proof: Let $|A| = n$, and $A_1, \ldots, A_k$ be a partition of A; in particular, each A_i is not empty.

Assume that each A_i has only one element. Since $A = \cup_{i=1}^{k} A_i$ is a partition of A, we know that $|A| = |\cup_{i=1}^{k} A_i|$, with the A_i pairwise disjoint, so $|\cup_{i=1}^{k} A_i| = \Sigma_{i=1}^{k} |A_i|$.

Notice that then $n = |A| = |\cup_{i=1}^{k} A_i| = \Sigma_{i=1}^{k} |A_i| = k$.

It is impossible for $k < n$ and $n = k$, so we have a contradiction. So there is at least one A_i that has at least two elements.