Cayley-Hamilton Theorem

A

Characteristic Polynomial of A = $det(A - \lambda I)$ = $a_0 + a_1\lambda + a_2\lambda^2 + ... + a_n\lambda^n$

$a_0 I + a_1 A + a_2 A^2 + ... + a_n A^n = [0]$

$A^n = -(1/a_n)(a_0 I + a_1 A + a_2 A^2 + ... + a_{n-1}A^{n-1})$ = Equation 1

$A^{n+1} = AA^n = A(-(1/a_n)(a_0 I + a_1 A + a_2 A^2 + ... + a_{n-1}A^{n-1})) = -(1/a_n)(a_0 A + a_1 A^2$-

$A^{n+1} = f(I, A, A^2, ..., A^{n-1})$

$I = -(1/a_0)(a_1 A + a_2 A^2 + ... a_n A^n)$

Assuming A has an inverse:

$A^{-1} = -(1/a_0)(a_1 I + a_2 A + ... a_n A^{n-1})$

$A^k =$ Linear combination of $(I, A, A^2, ..., A^{n-1})$

Example

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

Characteristic Polynomial = $(\lambda - 5)(\lambda + 2)$

$(A - 5I)(A + 2I) = 0$

$A^2 - 3A - 10I = 0$

$A^{-1} = (1/10)(A - 3I)$