Identity and Inverse

Let A be a set. We define the **identity** function on A, denoted $Id_A : A \to A$, by $Id_A(a) = a$ for all $a \in A$

Let $f : A \to B$ be an arbitrary function. The function $g : B \to A$ is:

A left inverse of f: $\forall a \in A \; g \circ f = Id_A$
A right inverse of f: $\forall b \in B \; f \circ g = Id_B$

The function $g : B \to A$ is an inverse of f (written f^{-1}) when **it is both a left and right inverse**

Proposition

$f : A \to B$ has an inverse $\iff$ f is a bijection
The Inverse is unique

Example

Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by

$$f(x) = \begin{cases} x+1, & x \text{ even} \\ x-1, & x \text{ odd} \end{cases}$$

It is easy to check that $(f \circ f)(x) = x$, considering the cases when x is odd and even separately.

Therefore f is its own inverse, and we can deduce that it is a *bijection*.