

# Cardinality of function spaces

Let  $B^A$  denote the set of all functions from  $A$  to  $B$ , where  $A$  and  $B$  are **finite** sets. If  $|A| = m$  and  $|B| = n$ , then  $|B^A| = n^m$

**Proof:** By counting. For each element of  $A$ , there are  $n$  independent ways of mapping it to an element of  $B$ . We can view this as establishing how many numbers we can represent with  $m$  positions in base  $n$ : the answer to this is  $n^m$ .  $\square$