Linear Dependence

Quick Example

$[2, 3, 4] = [1, 1, 1] + [1, 2, 3]$

$\vec{v_1} = \lambda_2 \vec{v_2} + \lambda_3 \vec{v_3}$

Then $\vec{v_1}$ is linearly dependent on $\vec{v_2}$ and $\vec{v_3}$

Definition of Linear Independence

$[\vec{v_1}, \ldots \vec{v_k}]$

If $a_1 \vec{v_1} + \ldots + a_k \vec{v_k} = \vec{0} \implies a_1 = \ldots = a_k = 0$

Then $[\vec{v_1}, \ldots \vec{v_k}]$ is linearly independent

Quickly Finding Linear Independence

Vectors = $\vec{v_1}, \vec{v_2}, \ldots, \vec{v_k}$

$$V = \begin{bmatrix} \vec{v_1}^T \\ \vec{v_2}^T \\ \ldots \\ \vec{v_k}^T \end{bmatrix}$$

EROs to REF

Every column has a pivot $\implies$ linearly independent
Otherwise $\implies$ linearly dependent

Example

$$\mathrm{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \end{bmatrix}\right\} = \mathrm{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\}$$

Since the last two can be expressed as a combination of the first two

$\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\}$ can't be reduced further since they are linearly independent