Rank and Nullity

$\mathrm{Rank}(A) = \mathrm{rk}(A) = \dim(\mathrm{Im}(A))$ = Number of pivots = Number of linearly independent columns of A
$\mathrm{Null}(A) = \dim(\mathrm{Ker}(A))$ = Number of Free variables

$\mathrm{Rank}(A) + \mathrm{Null}(A)$ **= Number of columns in A** = n

EROs do not change the linear (in)dependence of rows or columns

$\mathrm{Rank}(A) = \mathrm{Rank}(A^T)$

For A_{mxn}

$\dim(\mathrm{Im}(A)) = \dim(\mathrm{Im}(A^T))$
$\dim(\mathrm{Ker}(A)) + \dim(\mathrm{Im}(A)) = n$
$\dim(\mathrm{Ker}(A^T)) + \dim(\mathrm{Im}(A^T)) = m$
$\dim(\mathrm{Ker}(A)) - \dim(\mathrm{Ker}(A^T)) = n - m$