L4 - An Introduction to Boolean Algebra

Propositional Logic

- Propositions may be True or False
- They are stated as functions containing:
    - Other propositions
    - The three basic logical connectives: AND, OR, and NOT
- For example, the statement:
    - *"I will take an umbrella with me if it is raining or the weather forecast is bad"*
    - This connects the proposition *'I will take an umbrella with me'* to the two propositions '*it is raining*' and '*the weather forecast is bad*'.

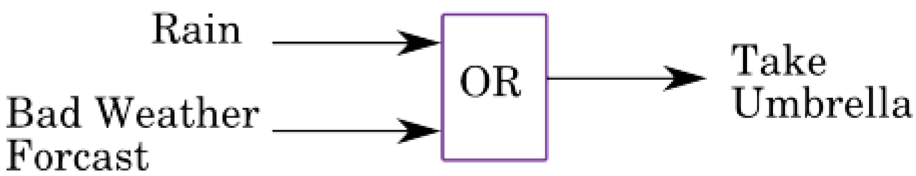

Figure 1: A simple Proposition

Another Example

- *"If I do not take the car then I will take the umbrella if it is raining or the weather forecast is bad"*

    (*Take Umbrella*) = (NOT (*Take Car*)) AND ((*Bad Forecast*) OR (*Raining*))

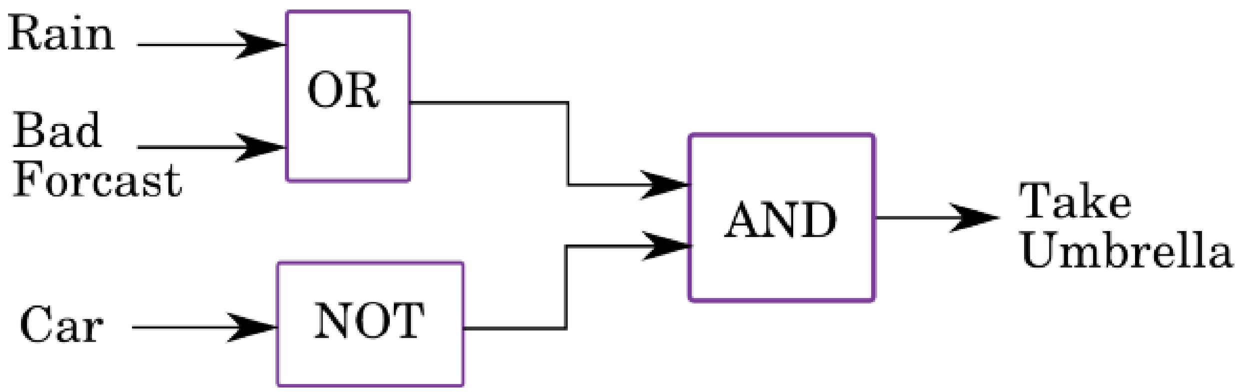

Figure 2: Another Proposition

Boolean Algebra

- 1 for True
- 0 for False
- · for And
- - for Or
- A' for (Not A)

AND ·

A	B	R
0	0	0
0	1	0
1	0	0
1	1	1

OR +

A	B	R
0	0	0
0	1	1
1	0	1
1	1	1

NOT ′

A	R
0	1
1	0

Precedence of Boolean Operations

Operator	Symbol	Precedence
NOT	$'$	Highest
AND	$\cdot$	Middle
OR	$+$	Lowest

Expressions inside brackets are always evaluated first, overriding the precedence order.

Fully bracketed form of Figure 2:

$$U = ((C') \cdot ((W) + (R)))$$

Using Precedence rules, this can be simplified to:

$$U = C' \cdot (W + R)$$

Truth Table Method

We can use the truth tables for And, Or and Not to evaluate the overall truth table of a more complex expression.

$$U = C' \cdot (W + R)$$

We can evaluate the overall truth of the proposition given in the above equation for every possible input combination. Here, there are eight possible sets of input values since there are three inputs and $8 = 2^3$.

R	W	C	$X_1 = R + W$	$X_2 = C'$	$U = X_1 \cdot X_2$
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	0

Boolean Expression Rules

$$(A')' = A$$
$$A \cdot A' = 0$$
$$A + A' = 1$$

Associative	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
	$(A + B) + C = A + (B + C)$
Commutative	$A \cdot B = B \cdot A$
	$A + B = B + A$
Distributive	$A \cdot (B + C) = A \cdot B + A \cdot C$
	$A + (B \cdot C) = (A + B) \cdot (A + C)$ (the weird one!)

$$A \cdot A = A$$
$$A + A = A$$

$$A \cdot 0 = 0$$
$$A \cdot 1 = A$$
$$A + 0 = A$$
$$A + 1 = 1$$

$A + A \cdot (B) = A$ (proof: $A + A \cdot B = A \cdot (1 + B) = A \cdot 1 = A$)

In these expressions, A and B may stand for any complex Boolean expression.

De Morgan's Theorem

$$(A + B)' = A' \cdot B'$$
$$(A \cdot B)' = A' + B'$$

This theorem holds for any number of terms, so:

$$(A \cdot B \cdot C \cdot .... \cdot X)' = A' + B' + C' + ... + X'$$

Duality

Every equation has its dual.

This can be generated by:

- Replacing the AND operators with ORs (and vice versa)
- Replacing the constants 0 with 1s (and vice versa)
- Add brackets to maintain evaluation order