# **Proof Styles**

Stylised has been renamed to semi-structured!

## What is a proof?

Assume the following facts are true:

- (1) A person is happy if all of their children are rich.
- (2) Someone is a supervillain if at least one of their parents is a supervillain.
- (3) All supervillains are rich.

#### Show that:

All supervillains are happy.

(state any additional assumptions that you make)

Need an extra assumption:

(4) A supervillain is also a person.

#### Given:

(1) 
$$\forall x. [person(x) \land \forall y. [childof(y, x) \longrightarrow rich(y)] \longrightarrow happy(x)]$$

(2) 
$$\forall x. [\exists y. [childof(x, y) \land supervillain(y)] \longrightarrow supervillain(x)]$$

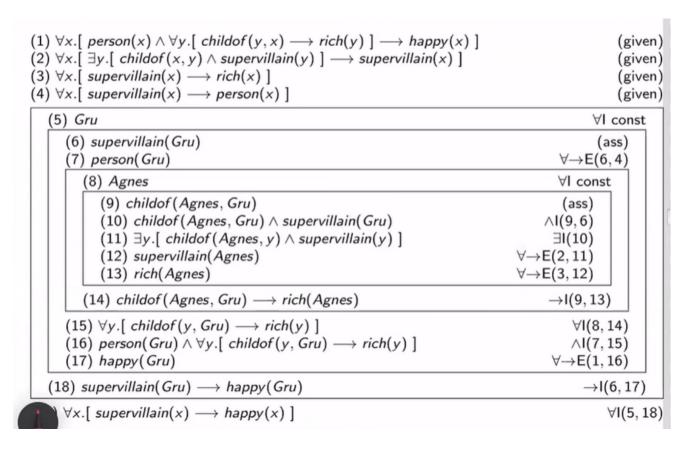
(3) 
$$\forall x.[$$
 supervillain( $x$ )  $\longrightarrow$  rich( $x$ ) ]

(4) 
$$\forall x.[$$
 supervillain( $x) \longrightarrow person(x) ]$ 

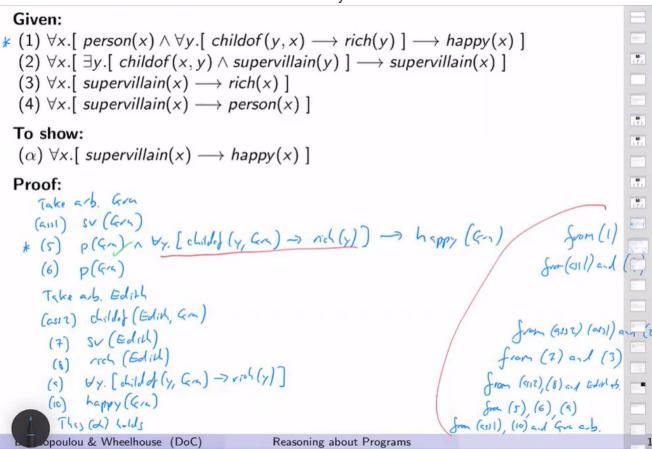
#### To show:

$$(\alpha) \ \forall x. [ supervillain(x) \longrightarrow happy(x) ]$$

Now, this can be proven.



#### **Semi-Structured Proofs**



# **Comparing the Proof Styles**

## (1) Free-Form Proofs:

- (+) short to develop
- (+) might highlight the intuition
- (-) error prone

## (2) Natural Deduction Proofs:

- (+) total confidence in the proof
- (-) very lengthy
- (-) layout sometimes (often?) difficult
- (-) intuition may be lost in the detail

# (3) Stylised Proofs:

- (+) structure of argument made explicit
- (+) few errors
- (-) errors are still possible

#### Aims of a Proof

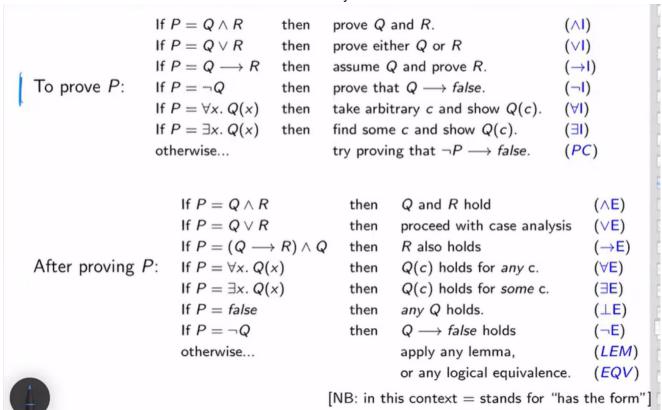
Aim	Free-Form	Natural Deduction	Stylised
prove only valid	X	✓	$\approx$
easily read at time	Х	~	✓
easily read later	X	≈	<b>√</b>
others can check	X	<b>√</b>	<b>✓</b>
highlight intuitions	<b>√</b>	X	$\approx$

### **Semi-Structured Proofs: Rules**

- Rule 1. write out and name each given formula.
- Rule 2. write out and name each formula to be shown.
- Rule 3. plan out the proof and name intermediate results.
- Rule 4. justify each step of the proof.
- Rule 5. vary the size of each step as appropriate.

## Making the plan and the Justifications

**Proof Styles** 



## **Examples**

A  $\rightarrow$  C Given: (A)  $\forall x \exists y . [friend(x,y) \rightarrow happ(x)]$ To show: (C)  $\forall x . [\forall y . friend(x,y) \rightarrow happ(x)]$ 

Proof take x, arb.

The front (x, x)

need to show happe(x,)

for some choice of x, for x

(2) friend (x, x, ) => happe(x,)

fron (1)

Therefore

(4) happey (x,)

This were shown

(5) ty, friend (x, x) => happey(x,)

Since x, ves att. Her (c) follows from (5)

