Rotation Matrices

In R^2 space

Rotate everything in space by θ degrees:

Rotation Matrix = R_θ = $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

R_ϕ = $\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}$

If rotating in same plane:

$R(\phi)R(\theta) = R(\theta + \phi) = R(\theta)R(\phi)$

Not commutative otherwise! Order Matters!

In R^n space

Rotation by θ in the $(i - j)^{th}$ Plane (all other axis are unchanged)

$$R_{ij}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \dots & \dots & 0 \\ 0 & \dots & \cos\theta & -\sin\theta & 0 \\ 0 & \dots & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Important bit in i, j columns

If $(j - i)^{th}$ Plane, swap columns:

$$R_{ji}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \dots & \dots & 0 \\ 0 & \dots & -\sin\theta & \cos\theta & 0 \\ 0 & \dots & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Determinant of Rotation Matrix = 1

Rotation Matrix = Square

$$R^T R = I$$

$$R R^T = I$$

$$R^T = R^{-1}$$