

# Associativity

Let  $f : A \rightarrow B$ ,  $g : B \rightarrow C$ , and  $h : C \rightarrow D$  be arbitrary functions. Then  
 $h \circ (g \circ f) = (h \circ g) \circ f$

**Proof** Let  $a \in A$  be arbitrary. Then

$$\begin{aligned}(h \circ (g \circ f))(a) &\triangleq h((g \circ f)(a)) \\ &\triangleq h(g(f(a))) \\ &\triangleq (h \circ g)(f(a)) \\ &\triangleq ((h \circ g) \circ f)(a)\end{aligned}$$

We have shown this for arbitrary  $a \in A$ , so

$$h \circ (g \circ f) = (h \circ g) \circ f$$