Diagonalizability

A is diagonalizable if $\exists$ a 'Basis change' matrix B such that $B^{-1}AB$ is a diagonal matrix

Example

A = $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

$spectrum(A) = \{5, -2\}$

$E_5 = span\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}\}$

$E_{-2} = span\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\}$

$A_{EE} : R^n_E \to R^n_E$
$A_{VV} : R^n_V \to R^n_V$

Need:
I_{EV}, I_{VE}

$V = span\{eigenvectors\ of\ A\} =$ will span whole of R^n

$A_{VV} = I_{VE}A_{EE}I_{EV}$ = $I_{EV}^{-1}A_{EE}I_{EV}$
A_{VV} is a diagonal matrix

$SOB = E = (\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix})$
$V = (\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix})$

I_{EV} = $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$

$I_{VE} = I_{EV}^{-1} = (1/7)\begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}$

$A_{VV} = I_{VE}A_{EE}I_{EV} = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$ = Entries are the eigenvalues!

Change order of Eigenvectors $\implies$ change order of eigenvalues in A_{VV}

Summary

1. Start with matrix A
2. Compute $Spectrum(A) = \{\lambda_1, \lambda_2 ..., \lambda_k\}$
3. Compute Bases of $\{E_{\lambda_1}, E_{\lambda_2} ... E_{\lambda_k}\}$
4. $\sum_{i=1}^{k}$ $(dim(E_{\lambda_i}))$ = n $\implies$ A is diagonalizable
    1. B = [$\vec{v_1}, \vec{v_2}, \dots, \vec{v_n}$]