Relation Operators

Let $R, S \subseteq A \times B$

We define the relations $R \cup S$, $R \cap S$, and $\overline{R}$ all with type $A \times B$, by:

Relation Union

$R \cup S \triangleq \{\langle a, b\rangle \in A \times B \mid \langle a, b\rangle \in R \vee \langle a, b\rangle \in S\}$

Relation Intersection:

$R \cap S \triangleq \{ \langle a, b\rangle \in A \times B \mid \langle a, b\rangle \in R \wedge \langle a, b\rangle \in S \}$

Relation Complement:

$\overline{R} \triangleq \{ \langle a, b\rangle \in A \times B \mid \langle a, b\rangle \notin R \}$

Notice the difference between the relation and set operators.

Identity Relation

Given any set A, the **identity** on A, written Id_A, is the binary relation on A defined by:

$Id_A \triangleq \{ \langle x, y\rangle \in A^2 \mid x = y \}$

Inverse relation

The inverse of R, written R^{-1}, is defined by:

$R^{-1} \triangleq \{ \langle b, a\rangle \in B \times A \mid a \; R \; b \}$

Notice that $(R^{-1})^{-1} = R$

Notice also the difference between R^{-1} and $\overline{R}$