Translating to English

Variables must be eliminated: English doesn't use them.

$\forall x(\exists y \exists z(\texttt{Bought}(x,y) \wedge \texttt{Bought}(x,z) \wedge \neg(y=z)) \to x = tony)$
'For all x, if x bought two different things then x is equal to Tony.'
'Anything that bought two different things is Tony.'
Care: it doesn't say Tony did buy 2 things, just that no one else did.

Express the sub-concepts in logic. Then build these pieces into a whole logical sentence.

- Sub-concept 'x is bought'/'x has a buyer': $\exists y\, \texttt{Bought}(y,x)$.
- Any bought thing isn't human:
  $\forall x(\exists y\, \texttt{Bought}(y,x) \to \neg\, \texttt{Human}(x))$.
  Important: $\forall x \exists y(\texttt{Bought}(y,x) \to \neg\, \texttt{Human}(x))$ would not do.
- Every PC is bought: $\forall x(\texttt{PC}(x) \to \exists y\, \texttt{Bought}(y,x))$.
- Some PC has a buyer: $\exists x(\texttt{PC}(x) \wedge \exists y\, \texttt{Bought}(y,x))$.
- No lecturer bought a PC:
  $\neg \exists x(\texttt{Lecturer}(x) \wedge \underbrace{\exists y(\texttt{Bought}(x,y) \wedge \texttt{PC}(y))}_{x \text{ bought a PC}})$.

You often need to say things like:

- 'All lecturers are human': $\forall x(\texttt{Lecturer}(x) \to \texttt{Human}(x))$.
  NOT $\forall x(\texttt{Lecturer}(x) \wedge \texttt{Human}(x))$.
  NOT $\forall x\, \texttt{Lecturer}(x) \to \forall x\, \texttt{Human}(x)$.
- 'Some lecturer is human': $\exists x(\texttt{Lecturer}(x) \wedge \texttt{Human}(x))$.
  NOT $\exists x(\texttt{Lecturer}(x) \to \texttt{Human}(x))$.
- Frank bought a PC: $\exists x(\texttt{PC}(x) \wedge \texttt{Bought}(\texttt{frank}, x))$

The patterns $\forall x(A \to B)$ and $\exists x(A \wedge B)$, are therefore very common.

$\forall x(A \wedge B)$, $\forall x(A \vee B)$, $\exists x(A \vee B)$ also crop up: they say everything/something is A and/or B.

But $\exists x(A \to B)$, especially if x occurs free in A, is *extremely rare.* If you write it, check to see if you've made a mistake.

Counting

- **There is at least one PC:**
  $\exists x\ \mathsf{PC}(x)$.

- **There are at least two PCs:**
  $\exists x \exists y(\mathsf{PC}(x) \wedge \mathsf{PC}(y) \wedge x \neq y)$,
  or (more deviously) $\forall x \exists y(\mathsf{PC}(y) \wedge y \neq x)$.

- **There are at least three PCs:**
  $\exists x \exists y \exists z(\mathsf{PC}(x) \wedge \mathsf{PC}(y) \wedge \mathsf{PC}(z) \wedge x \neq y \wedge y \neq z \wedge x \neq z)$,
  or $\forall x \forall y \exists z(\mathsf{PC}(z) \wedge z \neq x \wedge z \neq y)$.

- **There are no PCs:**
  $\neg \exists x\ \mathsf{PC}(x)$

- **There is at most one PC**: 3 ways:
  1. $\neg\exists x \exists y(\texttt{PC}(x) \wedge \texttt{PC}(y) \wedge x \neq y)$
     This says 'not(there are at least two PCs)' — see above.
  2. $\forall x \forall y(\texttt{PC}(x) \wedge \texttt{PC}(y) \to x = y)$
  3. $\exists x \forall y(\texttt{PC}(y) \to y = x)$
- **There's exactly one PC**: 2 ways:
  1. 'There's at least one PC' $\wedge$ 'there's at most one PC'
     $\exists x(\texttt{PC}(x) \wedge \forall y(\texttt{PC}(y) \to y = x))$
  2. $\exists x \forall y(\texttt{PC}(y) \leftrightarrow y = x)$