# Determinants

A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
$\det(A) = ad - bc$

$|\det(A)|$ = Area scale factor in $R^2$ = Volume scale factor in $R^3$

Standard Ordered Basis forms the edges of a unit square in $R^2$ (or a unit cube in $R^3$ or a unit n-dimensional cube in $R^n$

A square matrix $A_{n \times n}$ transforms the unit n-dimensional cube in $R^n$ to some n-dimensional parallelopiped in $R^n$

The columns of A are the edges of the n-dimensional parallelopiped.

Determinant of A = $|\det(A)|$ = n-dimensional 'volume' (area in case of $R^2$) of the parallelopiped

# The Rules

$R^2$: Unit Square transformed through 2x2 matrix becomes a parallelogram
$R^3$: Unit Cube transformed through 3x3 matrix becomes a parallelepiped

# Find Determinant of any Matrix using Gaussian Elimination

1. Swapping Rows
    1. Determinant swaps sign
2. Scale a row
    1. Determinant is also scaled
3. Adding Rows
    1. Determinant is unchanged
4. For Upper Triangular Matrix (square)
    1. All values below the diagonal are 0
    2. Determinant is product of the values in the diagonal

# Example

$T_0 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$T_1 = \begin{bmatrix} 3 & 6 \\ 3 & 4 \end{bmatrix} \implies \det(T_1) = 3\det(T_0)$

$T_2 = \begin{bmatrix} 3 & 6 \\ 0 & -2 \end{bmatrix} \implies \det(T_2) = \det(T_1)$

$T_3 = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \implies \det(T_3) = (1/3)\det(T_2)$

$\det(T_3) = \det(T_0)$

$R_i \to R_i + \lambda R_j$
*Determinant is unchanged.*

# Determinant = 0

$\det(A) = 0 \iff$ columns of A are **linearly dependent**
$\det(A) = 0 \iff \dim(Im(A)) < n$
$\dim(Im(A)) < n \iff \dim(Ker(A)) > 0$

A has a 0 row $\implies \det(A) = 0$