Basis Change

Let B = $(b_1, b_2, ..., b_n)$ are ordered Basis of R^n
Let D = $(d_1, d_2, ..., d_n)$ are ordered Basis of R^n

$\vec{x} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdots \\ \beta_n \end{bmatrix}$ WRT B = $\begin{bmatrix} \delta_1 \\ \delta_2 \\ \cdots \\ \delta_n \end{bmatrix}$ WRT D

I_{DB} $\vec{x}$ WRT B = $\vec{x}$ WRT D
$I_{DB} : \Phi : R^n \to R^n$

$\vec{b_i} \to i^{th}$ column of I_{DB}

Example 1

$$B = (\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix})$$

$$D = (\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix})$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \gamma_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \gamma_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

I_{DB} **=** $\begin{bmatrix} \alpha_1 & \gamma_1 \\ \alpha_2 & \gamma_2 \end{bmatrix}$ **= Basis Change Matrix**

Basis Change Matrix = Square

E = Standard Ordered Bases
$B = \{b_1, b_2, ..., b_n\}$

$$I_{EB} = [b_1 \quad b_2 \quad \ldots \quad b_n]$$

In general

$$I_{AB} = I_{BA}^{-1}$$

$$I_{AB} = I_{AE} . I_{EB} = I_{EA}^{-1} . I_{EB}$$