Examples

Prove that If ‘P implies Q’, then ‘(not Q) implies (not P)’

- Assume P holds
- Assume ‘P implies Q’ holds.
- Assume ‘not Q’ holds.
- By step ‘Assumption’, P is True.
- By step ‘Assumption’, ‘P implies Q’ is True.
- By step ‘Using implies’, we have that Q is True.
- By step ‘Assumption’, we have that ‘not Q’ is True.
- We have a contradiction.
- Therefore, ‘not P’ is True.
- By assuming that ‘P implies Q’ holds, and that ‘not Q’ holds, we have that ‘not P’ holds.
- Therefore, by the ‘Showing Implies’ step, we have that If ‘P implies Q’, then ‘(not Q) implies (not P)’

Example 2

If 'for every x in A we have that $P(x)$ implies $Q(x)$', then '(there exists y in A such that $P(y)$) implies (there exists z in A such that $Q(z)$)' holds.

Proof: Our aim is to prove that '(there exists y in A such that $P(y)$) implies (there exists z in A such that $Q(z)$)'. We can use the assumption that 'for every x in A, $P(x)$ implies $Q(x)$'.

To prove the implication, we assume 'there exists y in A such that $P(y)$'; we call o in A the object that satisfies P, so $P(o)$ holds.

Then taking o for x, we have that '$P(o)$ implies $Q(o)$'. So $Q(o)$ holds. So there exists an object in A that satisfies Q (namely o), so 'there exists z in A such that $Q(z)$'.

Then we have that '(there exists y in A such that $P(y)$)' implies (there exists z in A such that $Q(z)$)' holds. $\square$