Using Statements

AND

- Assume ‘P and Q’ holds.
- Therefore, ‘P and Q’ is True.
- Therefore, P is True and Q is True.

OR

- Assume ‘P or Q’ holds.
- Therefore, ‘P or Q’ is True.
- Therefore, there are two cases:
  - P is True
  - Q is True

Implies

- Assume P holds
- Therefore, P is True.
- Assume ‘P implies Q’ holds
- Therefore, ‘P implies Q’ is True.
- Therefore, Q is True.

Law of Excluded Middle

‘P or (not P)’ is True, for all P.

Contradictions

From a contradiction, anything follows. We have a contradiction, then P is True for any P.

For all x

Let o be any object that can be used for x
Then P(o) holds

There exists an x

- Let o be that object
- Then P(o) holds

Proof by Contradiction

This shows that P holds because it cannot be true that it does not hold.

A proof using this step would have the structure:

- Assume that $\mathrm{not\ P}$ holds
- . . . .
- We have a contradiction
- Then P is true
- $\mathrm{not\ P}$ ceases to be an assumption