N-ary Product

For any $n \geq 1$, an **n-tuple** is a sequence $\langle a_1, \ldots, a_n \rangle$ of n objects where the order of the a_i matter.

Let $A_1, \ldots, A_n$ be arbitrary sets. The n-ary product of the A_i, written $A_1 \times \ldots \times A_n$ or $\Pi_{i=1}^{n} A_i$, is defined by:

$$A_1 \times \ldots \times A_n \triangleq \{ \langle a_1, \ldots, a_n \rangle \mid \forall\, 1 \leq i \leq n \; (a_i \in A_i) \}$$

The n-ary product of As is written A^n, with A^2 corresponding to the Cartesian product.

Let A_i be finite sets for each $1 \leq i \leq n$
Then:

$$|A_1 \times \ldots \times A_n| = |A_1| \times \ldots \times |A_n|$$

This can be proved by **induction**.

We can form the product of three sets in three different ways:

- $A \times B \times C$
- $(A \times B) \times C$
- $A \times (B \times C)$

There is a natural correspondence between these three sets.