

# Vector Spaces

## Definition of Vector Spaces

$$(S, \oplus, \otimes, \vec{0}_S)$$

Let  $\vec{S}_1, \vec{S}_2 \in S$

If:

- Closed under  $\oplus$  ( $\vec{S}_1 + \vec{S}_2 \in S$ )
- Closed under  $\otimes$  ( $\lambda \vec{S}_1 \in S$ )
- $\vec{0}_S + \vec{x} = \vec{x}$

Then  $(S, \oplus, \otimes, \vec{0}_S)$  is a vector space.

## Another Example

$(\mathbb{R}^+, 'x'$  (usual multiplication, used as vector addition),  $\exp$  (scalar multiplication), 1)

Take  $x, y \in \mathbb{R}^+$

- $x \oplus y = xy \in \mathbb{R}^+$
- $\lambda \otimes x = x^\lambda \in \mathbb{R}^+$
- $1 \oplus x = x \in \mathbb{R}^+$

## Another example 2

$$a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

$$(P, +, \times, 0)$$