

Expressiveness

$$\text{Add}(0, n) = n$$

$$\text{Add}(\text{Succ}(m), n) = \text{Succ}(\text{Add}(m, n))$$

$$\text{Mult}(0, n) = 0$$

$$\text{Mult}(\text{Succ}(m), n) = \text{Add}(\text{Mult}(m, n), n)$$

We can show that $3 + 4 = 7$, as expected

$$\begin{aligned} \text{Add}(3, 4) &\triangleq \text{Add}(S(S(S(0))), S(S(S(S(0))))) \\ &= S(\text{Add}(S(S(0)), S(S(S(S(0))))) \\ &= S(S(\text{Add}(S(0), S(S(S(S(0))))) \\ &= S(S(S(\text{Add}(0, S(S(S(S(0))))) \\ &= S(S(S(S(S(S(S(0))))) \triangleq 7 \end{aligned}$$