Basis Vectors

U = span$\{\vec{v_1}, \ldots, \vec{v_k}\}$ is a subspace of $R^n \wedge \{\vec{v_1}, \ldots, \vec{v_k}\}$ are linearly independent
$\implies \{\vec{v_1}, \ldots, \vec{v_k}\}$ is a basis of U

$\{\vec{v_1}, \ldots, \vec{v_k}\}$ = ordered basis

V = $\{\vec{v_1}, \ldots, \vec{v_k}\}$

$|V|$ = Dimension of U

Example 1

$$U = \text{span}\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}7\\8\\9\end{bmatrix}\}$$

U = subspace of R^3

$$\text{Let} A = \begin{bmatrix}1 & 4 & 7\\2 & 5 & 8\\3 & 6 & 9\end{bmatrix}$$

EROs

$$\text{Let } A' = \begin{bmatrix}1 & 0 & 0\\4 & -3 & 0\\7 & -6 & 0\end{bmatrix}$$

$$\text{span}\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}\} = \text{basis of U}$$

Dimension of U = 2

Example 2

X-Y plane = $\text{span}\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\}$

X-Y plane = $\text{span}\{\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}\}$ = **Also linearly independent**

But the first one is better, saving lots of computation, since there are lots of 0s

In general, if there are more 0s in basis then "simple" basis

Convert $\text{span}\{\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}\}$ to Simple Basis

Turn column vectors into rows

Apply EROs to RREF

Convert rows back into column vectors
This is your new basis