Equivalence classes

Let R be an equivalence relation on A.

For any $a \in A$, the **equivalence class** of a with respect to R, denoted $[a]_R$, is defined as:

$[a]_R \triangleq \{ b \in A \mid a \sim_R b \}$

We write $[a]$ instead of $[a]_R$ when it is clear which R is meant.

The set of equivalence classes is the **quotient set** A/R

A Proposition...

The set $\{ V \in \wp A \mid \exists a \in A \ (V = [a]_R) \}$ of equivalence classes forms a partition of A.

Proof: Given $a \in A$, then $a \sim a$ by reflexivity and so $a \in [a]$.
For every $a \in A$, $a \in \bigcup_{a \in A}[a]$, and hence the classes cover A.
Suppose $[a] \cap [b] \neq \emptyset$, and let $w \in [a] \cap [b]$. This means that $a \sim w$ and $b \sim w$, and hence $w \sim b$ by symmetry, and $a \sim b$ by transitivity. To show that $[b] \subseteq [a]$, take $v \in [b]$, then $b \sim v$, and $a \sim v$ by transitivity. Hence $v \in [a]$.
Likewise we can show that $[a] \subseteq [b]$, so $[a] = [b]$. $\square$