

Russell's Paradox

Russell's Paradox:

- Assume $R \triangleq \{ X \mid X \notin X \}$ is a set
- This is a hypothetical set (don't try to think of examples)
- Two cases (by Law of Excluded Middle):
 - $R \in R$
 - By definition of R : $R \notin R$
 - Contradiction!
 - $R \notin R$
 - By definition of R : $R \in R$
 - Contradiction!
 - Either way, there is a contradiction
- Therefore, our main assumption was wrong
- R is not a set

$R \triangleq \{ X \mid X \notin X \}$ is **not** well defined. When defining a set, you should only allow selection from a set $\{ x \in A \mid \dots \}$