

Directed Graphs

In this course, a graph is undirected unless otherwise stated.

A directed graph is a set N of nodes and a set A of arcs such that each $a \in A$ is associated with an ordered pair of nodes (the endpoints of a).

- In diagrams, the arcs are shown with arrows from source node to target node
- In a path a_1, \dots, a_n in a directed graph the source of a_{i+1} must match the target of a_i (for $i = 1, \dots, n - 1$)
- For any pair of nodes x, y , if there is at most one arc from x to y then we can refer to this arc as (x, y)

The indegree of a node x is the number of arcs entering x .

The outdegree of a node x is the number of arcs leaving x .

For any directed graph, the sum of the indegrees of all nodes = the sum of the outdegrees of all nodes = the number of arcs.

A directed graph is strongly connected if for any $x, y \in \text{nodes}(G)$ there is a path from x to y . So, for any pair of nodes x, y we need paths both from x to y and from y to x .