Set Definitions

Constructing Sets

We define a set by listing its elements inside curly brackets:
$V = \{a, e, i, o, u\}$

Alternatively, we can define a set by stating the property that its elements satisfy:

$P \triangleq \{ p \in N \mid 'p \text{ is a prime number}' \}$

Set Equality

Let A, B be any two sets. Then A is the same set as B , written $A = B$, is defined as:

$$A = B \equiv A \subseteq B \text{ and } B \subseteq A$$

Subsets

Let A, B be any two sets.
Then A is a subset of B , written $A \subseteq B$, when all the elements of A are also elements of B :

$$A \subseteq B \equiv \forall x \in A \, (x \in B)$$

Union

Let A and B be any sets:

$$A \cup B \triangleq \{ x \mid 'x \in A \text{ or } x \in B' \}$$

This is not well defined, however, this is assumed to create a set since we have no choice.

Intersection

Let A and B be any sets:

$$A \cap B \triangleq \{x \in A \cup B \mid {}'x \in A \text{ and } x \in B' \}$$

$$A \cap B \triangleq \{x \in A \mid x \in B\}$$

Well defined = Now that we have defined $A \cup B$, we must use this.

Difference

Let A and B be any sets:

$$A \setminus B \triangleq \{ x \in A \cup B \mid {}'x \in A \text{ and } x \notin B' \}$$

$$A \setminus B \triangleq \{x \in A \mid x \notin B\}$$

Symmetric Difference

Let A and B be any sets:

$$A \Delta B \triangleq (A \setminus B) \cup (B \setminus A)$$

We can use this since all operators involved have been well defined.

Disjoint Sets

Let A and B be any sets:

$$A \text{ and } B \text{ are disjoint if } A \cap B = \emptyset$$