Transformation

Example 1

$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix} : R^3 \to R^2$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \to \begin{bmatrix} 3x_1 \\ 4x_2 \end{bmatrix}$

Stretching, Squishing, Rotating, Projecting

Rules

Given: $R^n_{B_1} \to R^m_{D_1}$ through Φ
Compute: $R^n_{B_2} \to R^m_{D_2}$

$I_{B_1 B_2} : R^n_{B_2} \to R^n_{B_1}$
$I_{D_2 D_1} : R^m_{D_1} \to R^m_{D_2}$

$\Phi = A_{AB}$
Find $A_{D_2 B_2}$ = Answer

Example

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} : R^3_{SOB} \to R^2_{SOB}$

$B = \left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right)$ = Basis for R^3

$D = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$

$\vec{x}$ w.r.t B = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ w.r.t B

$\vec{y} = A\vec{x}$

What is $\vec{y}$ w.r.t D?

S = Standard Ordered Basis = SOB

I_{DS} (A (I_{SB} ($\vec{x}$ w.r.t B))) = $\vec{y}$ w.r.t D

(I_{DS} A I_{SB}) ($\vec{x}$ w.r.t B) = $\vec{y}$ w.r.t D

M = I_{DS} A I_{SB}

$$I_{SB} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$I_{DS} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

Sub in to find y

Example 2

$$A = \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right)$$

$$B = \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix}\right)$$

Start at $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ WRT A

End at $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ WRT B through transformation T

E = SOB

Determine a possible T

T I_{EA} ($\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ WRT A) = I_{EB} ($\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ WRT B)

$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} =$ *One possible T*