Functions to Matrices

Gaussian Elimination Method

- No solution: Contradiction in matrix
- 1 solution: Identity matrix
- Infinite solutions: Free variables

Inverses

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots a_{1n}x_n = b_1 + 0b_2 + \ldots + 0b_n$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots a_{2n}x_n = 0b_1 + b_2 + \ldots + 0b_n$$
$$\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots a_{mn}x_n = 0b_1 + 0b_2 + \ldots + b_m$$

$A\vec{x} = I\vec{b}$
Apply EROs
$I\vec{x} = M\vec{b}$

Where M = Inverse of A

2x2 Inverse

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 2 & 0 & -5 & 3 \\ 0 & -1 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -5/2 & 3/2 \\ 0 & -1 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -5/2 & 3/2 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

Summary

Think of Matrices like functions.

Inverse matrix = inverse function.

Matrix Multiplication = Function Composition.

Vectors

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}$$ = Collection of Values or a point in an n-dimensional space

Linear Combination

$$\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ \dots \\ 0 \end{bmatrix} + \dots + x_n \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix}$$

$$\{ \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ \dots \\ 0 \end{bmatrix}, \dots \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix} \}$$ = **Standard Ordered Bases Of** R^n

Every Vector is a Linear Combination of Standard Ordered Bases

Every Vector is a Linear Combination of any Base

Matrix Interpretation

$$A = [\vec{a_1} \quad \vec{a_2} \quad \dots \quad \vec{a_n}]$$

$$A \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix} = \vec{a_1}$$ = First column of Matrix A = etc

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$f_A(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

$$f_A\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x f_A\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) + y f_A\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) + z f_A\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)$$