Vector Subspaces

$(R^n, +, \times, \vec{\theta})$

If $S \subseteq R^n$ such that $(S, +, \times, \vec{\theta})$ is a vector space:
Then S is a subspace of R^n

Example

$(R^3, +, \times, \vec{\theta})$

S = $\{\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \mid x, y \in R\}$

- $S \subseteq R^3$
- $\begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ 0 \end{bmatrix} \in S$
- $\lambda \in R, \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} \in S$
    - $\lambda \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda_1 x_1 \\ \lambda_1 x_2 \\ 0 \end{bmatrix} \in S$
- $[0, 0, 0] \in S$
  Therefore, S is a subspace of R^3

Another Definition

C = $\{ \vec{e} \in R^n : A\vec{x} = \vec{\theta} \}$

$A\vec{x} = \vec{\theta}$: Homogenous system of linear equations

C is a subspace of R^n

$A_{mn}\vec{x} = \vec{\theta}$: solutions is a subspace R^n