Relations

A relation R between arbitrary sets A and B is a subset of the product $A \times B$

We say that R is of type $A \times B$

If $R \subseteq A^2$, we say that R is a binary relation on A.

We will use $a\ R\ b$ or $R(a, b)$ for $\langle a, b\rangle \in R$

For $A = \{a, b\}$, there are 16 binary relations on A:

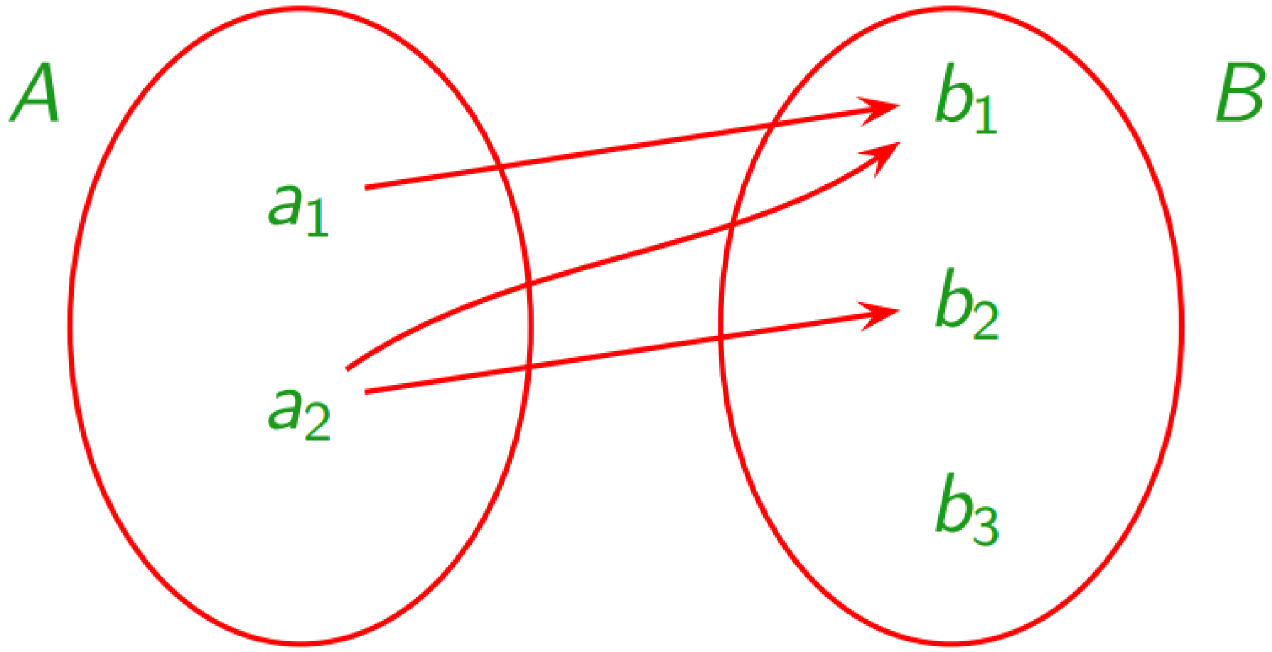
$$
\begin{array}{ll}
\emptyset & \{\langle a,b\rangle, \langle b,a\rangle\} \\
\{\langle a,a\rangle\} & \{\langle a,b\rangle, \langle b,b\rangle\} \\
\{\langle a,b\rangle\} & \{\langle b,a\rangle, \langle b,b\rangle\} \\
\{\langle b,a\rangle\} & \{\langle a,a\rangle, \langle a,b\rangle, \langle b,a\rangle\} \\
\{\langle b,b\rangle\} & \{\langle a,a\rangle, \langle a,b\rangle, \langle b,b\rangle\} \\
\{\langle a,a\rangle, \langle a,b\rangle\} & \{\langle a,a\rangle, \langle b,a\rangle, \langle b,b\rangle\} \\
\{\langle a,a\rangle, \langle b,a\rangle\} & \{\langle a,b\rangle, \langle b,a\rangle, \langle b,b\rangle\} \\
\{\langle a,a\rangle, \langle b,b\rangle\} & \{\langle a,a\rangle, \langle a,b\rangle, \langle b,a\rangle, \langle b,b\rangle\}
\end{array}
$$

Note that there are four elements (possible pairs) in A^2, and each relation is a subset of A^2, so there are $|\wp A^2| = 16$ possibilities.

Diagrams

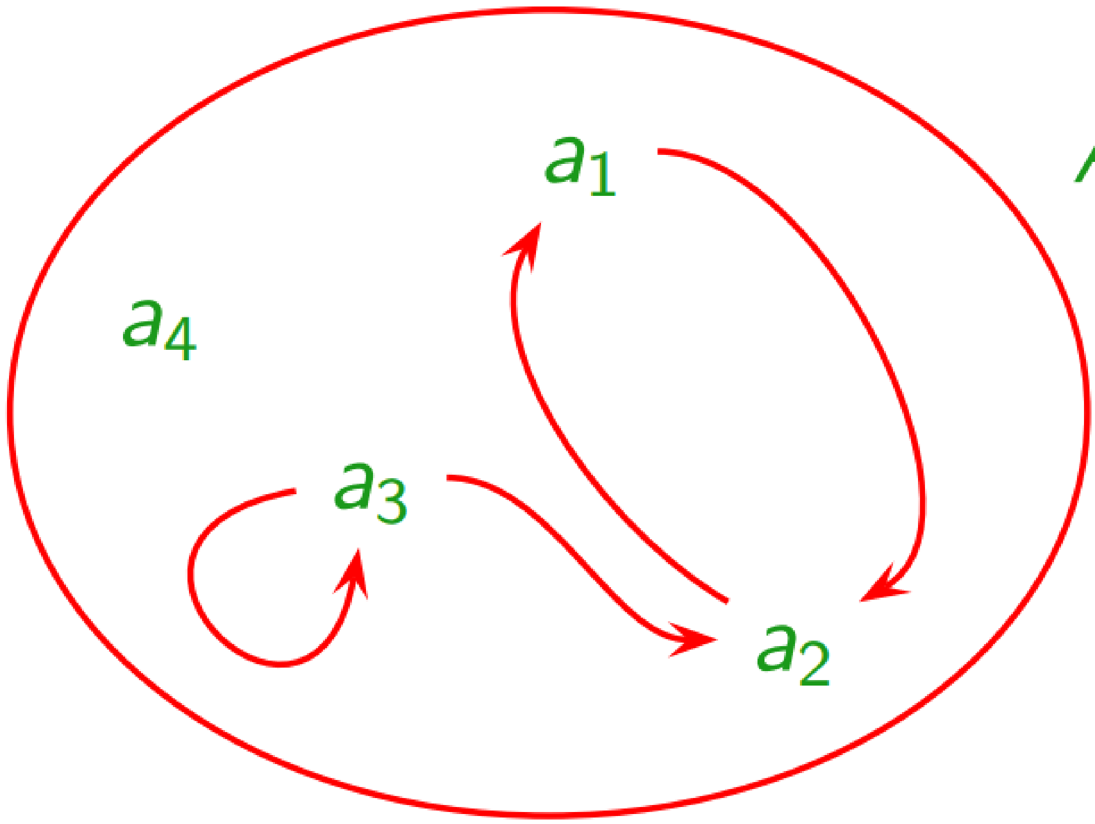
Let $A = \{a_1, a_2\}, B = \{b_1, b_2, b_3\}$ and $R = \{\langle a_1, b_1\rangle, \langle a_2, b_1\rangle, \langle a_2, b_2\rangle\}$

The relation R can be represented by this diagram:



Directed Graphs

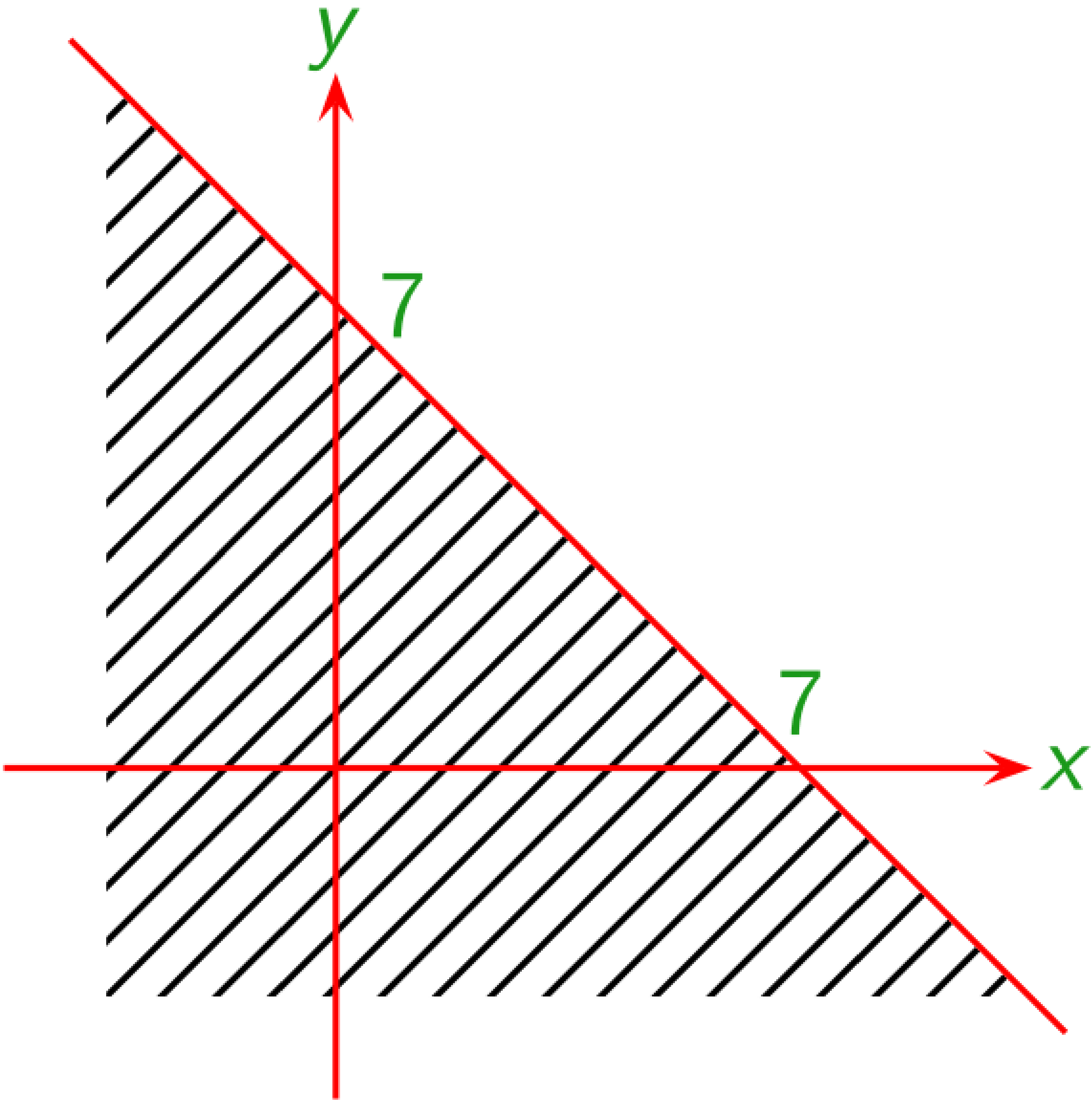
Let R be a binary relation on $A = \{a_1, a_2, a_3, a_4\}$ with
$R = \{\langle a_1, a_2\rangle, \langle a_2, a_1\rangle, \langle a_3, a_2\rangle, \langle a_3, a_3\rangle\}$ The directed graph of this relation is:



Notice that **the direction of the arrows matters**

Special representation

The relation R defined by $R = \{\langle x, y\rangle \in R^2 \mid x + y \leq 7\}$ can be represented by:



Matrix Representation

Let $A = \{a_1, a_2\}, B = \{b_1, b_2, b_3\}$ and $R = \{\langle a_1, b_1\rangle, \langle a_2, b_1\rangle, \langle a_2, b_2\rangle\}$ as before. The matrix representation of R is:

$$\begin{pmatrix} \text{True} & \text{False} & \text{False} \\ \text{True} & \text{True} & \text{False} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} T & F & F \\ T & T & F \end{pmatrix}$$

This representation can be generalised to arbitrary finite sets A and B.