Initially, we're assuming

Total number of jobs = N
Total number of worker machine = M
probability of failing a job = p
Time taken to finish a job at machine i = t_i sec.
or, we can express this using machine capability.
Assuming ith machine has capability c_i
then we can express c_i as a function of t_i
when c_i increases, t_i decreases.

so, $$c_i \propto \frac{1}{t_i} \Rightarrow c_i = \frac{r}{t_i}$$

here r is the proportional constant.

so, $t_i = \frac{r}{c_i}$, we can call r as a base time taken to finish a job.

so, r = base time.

Let's start with the probability mass function of a random Variable X which expresses the number of attempt until success.

$$P(X=K) = p^{K-1}(1-p)$$

here K is a constant. This probability mass function represents the probability of getting sucess at K tries or attempts.

So, the expected number of attempt until success will be E[X]

$$E[X] = \sum_{K=1}^{\infty} K\, P(X=K)$$

$$= \sum_{K=1}^{\infty} K \times p^{K-1} \times (1-p)$$

$$E[X] = (1-p) \times \sum_{K=1}^{\infty} Kp^{K-1} \quad \text{.........} \quad (i)$$

we know the summation of Geometric series looks like this

$$\sum_{K=0}^{\infty} p^K = 1 + p + p^2 + p^3 + \cdots\cdots + \infty$$

$$= \frac{1}{1-p}, \text{ when } p < 1$$

$$\text{so, } \sum_{K=0}^{\infty} p^K = \frac{1}{1-p} \quad \text{..........} \quad (ii)$$

Now differentiating equation (ii) with respect to p

$$\sum_{K=0}^{\infty} K \times p^{K-1} = (-1) \times (-1) \times (1-p)^{-2}$$

$$\Rightarrow 0 \times p^{0-1} + \sum_{K=1}^{\infty} K p^{K-1} = \frac{1}{(1-p)^2}$$

$$\Rightarrow \sum_{K=1}^{\infty} K p^{K-1} = \frac{1}{(1-p)^2}$$

$$= \frac{1}{(1-p)^2} = \sum_{K=1}^{\infty} K p^{K-1} \quad \text{.......(iii)}$$

Now, equation (i) ÷ equation (iii)

$$\frac{E[X]}{\frac{1}{(1-p)^2}} = \frac{(1-p) \cancel{\sum_{K=1}^{\infty} K p^{K-1}}}{\cancel{\sum_{K=1}^{\infty} K p^{K-1}}}$$

$$\Rightarrow \frac{E[X]}{\frac{1}{(1-p)^2}} = (1-p)$$

$$\Rightarrow E[X] = \frac{1}{1-p}$$

$$E[X] = \frac{1}{1-p}$$

And this is the average number of attempts until success.

Now, from our assumption, we know that ith machine can finish a job at t_i sec

we can write the line in reverse way

In t_i sec ith machine can finish 1 Job(s)

$\therefore$ 1 " " " " " $\frac{1}{t_i}$ "

So, per unit time, if we have only one machine we can finish ~~job~~ $\frac{1}{t_i}$ many Job(s) [Job rate)

However, we have M machines. So, the job completion rate will be $= R_{total}$.

$$\text{so, } R_{total} = \sum_{i=1}^{M} \left(\frac{1}{t_i}\right)$$

$$= \sum_{i=1}^{M} \left(\frac{1}{\frac{r}{c_i}}\right) = \sum_{i=1}^{M} \left(\frac{c_i}{r}\right)$$

$$\text{So, } R_{total} = \sum_{i=1}^{M} \left(\frac{c_i}{r}\right)$$

Now, first we'll calculate the expected time to finish a job using all heterogeneous machine. Let's call it T_{job}.

$$T_{job} = (\text{Average number of attempts until success}) \times (\text{time taken to perform one attempt with all resources})$$

(attempts)

R_{total} is the rate of job completions using all resources which means with all heterogeneous machine we can attempt R_{total} Job(s) in unit time.

so, to attempt one job it will be $= \frac{1}{R_{total}}$.

$$\text{So, } T_{job} = E[X] \times \frac{1}{R_{total}}$$

Since, we have N jobs in the experiment. So average completion time of the experiment will be,

$$T_{average(exp)} = N \times T_{job}$$

$$= N \times E[X] \times \frac{1}{R_{total}}$$

$$= N \times \frac{1}{1-p} \times \frac{1}{\frac{1}{r} \sum_{i=1}^{M} (c_i)}$$

$$= N \times \frac{1}{1-p} \times \frac{r}{\sum_{i=1}^{M} (c_i)}$$

$$T_{total} = \frac{Nr}{1-p} \left[\sum_{i=1}^{M} (c_i) \right]^{-1}$$

Here T_{total} is a function of machines' capability.

or, we can express it as function of time taken to complete job in each heterogeneous machine.

$$T_{total} = \frac{N}{1-p} \left[\sum_{i=1}^{M} \left(\frac{1}{t_i} \right) \right]^{-1}$$