The Dynamic Stall Module for FAST 8

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Abstract

The new modularization framework of FAST v.8 [Jonkman, 2013] required a complete overhaul of the aerodynamics routines. AeroDyn is an aerodynamics module that can then utilize either blade element momentum theory (BEMT) or generalized dynamic wake (GDW) to calculate aerodynamic forces on blade elements. Under asymmetric conditions, such as wind shear, yawed and tilted flow, the individual blade elements undergo variations in angle of attack (AOA) that lead to unsteady aerodynamics phenomena, which can no longer be captured through the static airfoil lift and drag look-up tables. This study lays out the main theory and the organization into the modularization framework of the dynamic stall (DS) module, which includes unsteady aerodynamics under attached flow conditions and DS. DS can be called by either BEMT or GDW.

Acronyms

2D	two-dimensional
AOA	angle of attack
AOI	angle of incidence
BE	blade element
BEMT	blade element momentum theory
DS	dynamic stall
GDW	generalized dynamic wake
LBM	Leishman-Beddoes model
LE	leading edge
TE	trailing edge
UA	Unsteady-Aerodynamics module

List of Symbols

AFI_{Params}	airfoil static tables of C_l C_d C_m and DS parameters
$AFidx$	index pointing to the correct airfoil file/table/database
A_1	constant in the expression of ϕ^c_α andϕ^c_q; from Leishman [2011] experimental results set it to 0.3. This value is relatively insensitive for thin airfoils, but may be different for turbine airfoils. Generally speaking it should not be tuned by the user.
A_2	constant in the expression of ϕ^c_α andϕ^c_q; from Leishman [2011] experimental results set it to 0.7. This value is relatively insensitive for thin airfoils, but may be different for turbine airfoils. Generally speaking it should not be tuned by the user.
A_5	constant in the expression of K'''_q, $C^{nc}_{m_q}(s, M)$, and $k_{m,q}(M)$; from Leishman [2011] experimental results set it to 1
$DSMod$	switch to select handling of options and possible methods in dynamic stall

$F_R(s)$	Response Function to generic disturbance $\epsilon(s)$
$F_R(t)$	Response Function to generic disturbance $\epsilon(t)$
$FirstPass$	flag indicating first time step
$LESF$	leading edge separation flag
M	Mach Number
$NumOuts$	number of output channels
$OutFmt$	format for output data
$OutList$	list of output channels
$OutSFmt$	format for output data
R	Rotor Radius
S_1	constant in the f curve best-fit; by definition it depends on the airfoil.
S_2	constant in the f curve best-fit; by definition it depends on the airfoil.
St_{sh}	Strouhal's shedding frequency constant, commonly taken equal to 0.19
T'_α	Mach-dependent, non-dimensional time constant in the expression of ϕ^{nc}_α; it is equal to $2UT_\alpha(M)/c$
T'_q	Mach-dependent time constant in the expression of ϕ^{nc}_q
$TESF$	trailing edge separation flag
T_I	time constant in the expression of $\phi^{nc}_\alpha = c/a_s$
T_V	time constant associated with the vortex lift decay process; it is used in the expression of C^v_n. It depends on Re,M, and airfoil class.
$T_\alpha(M)$	Mach-dependent time constant in the expression of ϕ^{nc}_α
T_f	constant dependent on Mach, Re, and airfoil shape; it is used in the expression of D_f and f''
T_p	boundary-layer,leading edge (LE) pressure gradient time constant; in the expression of D_p. It should be tuned based on airfoil experimental data.
$T_q(M)$	Mach-dependent time constant in the expression of ϕ^{nc}_q
T_{V0}	initial value of T_V.
T_{VL}	time constant associated with the vortex advection process; it represents the non-dimensional time in semi-chords, needed for a vortex to travel from LE to trailing edge (TE); it is used in the expression of C^v_n. It depends on Re, M (weakly), and airfoil. Value's range = $[6; 13]$.
T_{f0}	initial value of T_f

$T'_{m,q}$	Mach-dependent time constant in the expression of $\phi^{nc}_{m,q}$
$T_{m,q}(M)$	Mach-dependent time constant in the expression of $\phi^{nc}_{m,q}$
T_{sh}	time constant associated with the vortex shedding. It allows multiple vortices to be shed at a Strouhal's frequency of 0.19.
U	relative air-speed
$VRTX$	vortex advection flag
X_1	deficiency function used in the development of $C^c_{n_\alpha}(s, M)$
X_2	deficiency function used in the development of $C^c_{n_\alpha}(s, M)$
Δt	time step
$\bar{\bar{x}}_{cp}$	constant in the expression of $\hat{x}^v_{cp}$, usually $= 0.2$
α_{-1}	previous time-step value of α
α_{-2}	two time-steps ago value of α
f''_{-1}	previous time-step value of f''
f'_{-1}	previous time-step value of f'
q_{-1}	previous time-step value of q
q_{-2}	two time-steps ago value of q
$\hat{k}_1$	constant in the C_c expression due to LE vortex effects
$\hat{k}_2$	constant in the C_c expression due to LE vortex effects, taken equal to $2(C'_n - C_{n1}) + (f'' - f)$
$\hat{x}_{AC}$	aerodynamic center distance from LE in percent chord
$\hat{x}^v_{cp}$	center-of-pressure distance from the 1/4-chord, in percent chord, during the LE vortex advection process
$\hat{x}_{cp}$	center-of-pressure distance from LE in percent chord
ν	kinematic viscosity
^{c}	circulatory component of the quantity at the base
^{nc}	non-circulatory component of the quantity at the base
$_\alpha$	relative to a step-change in α
$_n$	relative to the n-th time step
$_q$	relative to a step-change in q
$_t$	relative to the t-th time step
a_s	speed of sound

b_1	constant in the expression of ϕ_α^c andϕ_q^c; from Leishman [2011] experimental results set it to 0.14. This value is relatively insensitive for thin airfoils, but may be different for turbine airfoils. Generally speaking it should not be tuned by the user.
b_2	constant in the expression of ϕ_α^c andϕ_q^c; from Leishman [2011] experimental results set it to 0.53. This value is relatively insensitive for thin airfoils, but may be different for turbine airfoils. Generally speaking it should not be tuned by the user.
b_5	constant in the expression of K_q''', $C_{m_q}^{nc}(s, M)$, and $k_{m,q}(M)$; from Leishman [2011] experimental results set it to 5
k_0	constant in the $\hat{x}_{cp}$ curve best-fit; $= (\hat{x}_{AC} - 0.25)$
k_1	constant in the $\hat{x}_{cp}$ curve best-fit
k_2	constant in the $\hat{x}_{cp}$ curve best-fit
k_3	constant in the $\hat{x}_{cp}$ curve best-fit
$k_\alpha(M)$	Mach-dependent constant in the expression of $T_\alpha(M)$
$k_q(M)$	Mach-dependent constant in the expression of $T_q(M)$
$k_{m,q}(M)$	Mach-dependent constant in the expression of $T_{m,q}(M)$ and $C_{m_q}^{nc}(s, M)$
q	non-dimensional pitching Rate $= \dot{\alpha}c/U$
s	non-dimensional Distance
t	time
$D_{\alpha f,-1}$	previous time-step value of $D_{\alpha f}$
$D_{f,-1}$	previous time-step value of D_f
$D_{p,-1}$	previous time-step value of D_p
$K'_{\alpha,-1}$	previous time-step value of K'_α
$K'''_{q}{}_{,-1}$	previous time-step value of K'''_q
$K''_{q,-1}$	previous time-step value of K''_q
$K'_{q,-1}$	previous time-step value of K'_q
$X_{1,-1}$	previous time-step value of X_1
$X_{2,-2}$	previous time-step value of X_2
$C_n^{pot}{}_{,-1}$	previous time-step value of C_n^{pot}
$C^v_{n,-1}$	previous time-step value of C_n^v
$C_{V,-1}$	previous time-step value of C_V
c	chord length
C_c	2D tangential (along chord) force coefficient
C_c^{fs}	2D tangential (along chord) force coefficient under separated TE flow separation conditions.

C_c^{pot}	2D along-chord force coefficient under attached (potential) flow conditions
C_d	2D drag coefficient
C_{d0}	2D drag coefficient at 0-lift
C_l	2D lift coefficient
$C_{l\alpha}$	slope of the 2D lift coefficient curve
C_m	2D pitching moment coefficient about 1/4-chord, positive if nose up
C_{m0}	2D pitching moment coefficient at 0-lift, positive if nose up
$C_{m_\alpha}(s, M)$	pitching moment coefficient response to step change in α
$C_{m_\alpha}^c(s, M)$	circulatory component of the pitching moment coefficient response to step change in α
$C_{m_\alpha}^{nc}(s, M)$	non-circulatory component of the pitching moment coefficient response to step change in α
$C_{m_{\alpha,q}}(s, M)$	moment coefficient response to step change in α and q
$C_{m_{\alpha,q}}^c(s, M)$	circulatory component of $C_{m_{\alpha,q}}(s, M)$
$C_{m_{\alpha,q}}^{nc}(s, M)$	non-circulatory component of $C_{m_{\alpha,q}}(s, M)$
$C_{m_q}(s, M)$	pitching moment coefficient response to step change in q
$C_{m_q}^c(s, M)$	circulatory component of the pitching moment coefficient response to step change in q
$C_{m_q}^{nc}(s, M)$	non-circulatory component of the moment coefficient response to step change in q
$C_{m\alpha}$	slope of the 2D pitching moment coefficient curve
C_m^{fs}	2D tangential 1/4-chord pitching moment coefficient under separated TE flow separation conditions.
C_m^{pot}	2D moment coefficient under attached (potential) flow conditions about 1/4-chord location
$C_{mq}(s, M)$	slope of the pitching moment coefficient vs. q curve
C_m^v	pitching moment coefficient due to the presence of LE vortex
C_n	2D normal-to-chord force coefficient

C_{n1}	critical value of C'_n at LE separation. It should be extracted from airfoil data at a given Mach and Reynolds number. It can be calculated from the static value of C_n at either the break in the pitching moment or the loss of chord force at the onset of stall. It is close to the condition of maximum lift of the airfoil at low Mach numbers.
C_{n2}	critical value of C'_n at LE separation for negative AOAs; analogous to C_{n1}.
$C_{n_\alpha}(s, M)$	normal force coefficient response to step change in α
$C^c_{n_\alpha}(s, M)$	circulatory component of the normal force coefficient response to step change in α
$C^{nc}_{n_\alpha}(s, M)$	non-circulatory component of the normal force coefficient response to step change in α
$C_{n_{\alpha,q}}(s, M)$	normal force coefficient response to step change in α and q
$C^c_{n_{\alpha,q}}(s, M)$	circulatory component of $C_{n_{\alpha,q}}(s, M)$
$C^{nc}_{n_{\alpha,q}}(s, M)$	non-circulatory component of $C_{n_{\alpha,q}}(s, M)$
$C^c_n(s, M)$	circulatory component of $C_{n_{\alpha,q}}(s, M)$
$C_{n_q}(s, M)$	normal force coefficient response to step change in q
$C^c_{n_q}(s, M)$	circulatory component of the normal force coefficient response to step change in q
$C^{nc}_{n_q}(s, M)$	non-circulatory component of the normal force coefficient response to step change in q
$C_{n\alpha}$	slope of the 2D normal coefficient curve, should be similar to $C_{l\alpha}$
$C^c_{n\alpha}(s, M)$	slope of the circulatory normal force coefficient vs. α curve
C^{fs}_n	normal force coefficient under separated TE flow separation conditions.
C'_n	lagged component of C_n in the TE separated treatment
C^{pot}_n	2D normal-to-chord force coefficient under attached (potential) flow conditions
$C^{pot,c}_n$	circulatory part of 2D normal-to-chord force coefficient under attached (potential) flow conditions
$C_{nq}(s, M)$	slope of the normal force coefficient vs. q curve
C^v_n	normal force coefficient due to the presence of LE vortex
C_V	contribution to the normal force coefficient due to accumulated vorticity in the LE vortex

$D_{\alpha f}$	deficiency function for α_f
D_f	deficiency function for f'
D_p	deficiency function for C_n'
f	separation point distance from LE in percent chord
f_m	CENER's proposed version of f extracted from the C_m static tables, assuming $C_m = C_n f_m$
f_m'	version of f_m extracted from the airfoil C_m static tables with α_f as input parameter
f_m''	lagged version of f_m'
f'	separation point distance from LE in percent chord under unsteady conditions
f''	lagged version of f'accounting for unsteady boundary layer response
k	reduced frequency
K_α	backward finite difference of α at the t-th time step
K_α'	deficiency function for $C_{n_\alpha}^{nc}(s, M)$
K_q	backward finite difference of q at the t-th time step
K_q'	deficiency function for $C_{n_q}^{nc}(s, M)$
K_q''	deficiency function for $C_{m_q}^{nc}(s, M)$
K_q'''	deficiency function for $C_{m_q}^{c}(s, M)$
Re	airfoil-chord Reynolds Number
U	air velocity magnitude relative to the airfoil

Greek Symbols

$\Delta_{\alpha 0}$	α-α_0
$\epsilon(s)$	generic disturbance
$\epsilon(t)$	generic disturbance
η_e	recovery factor $\simeq [0.85 - 0.95]$ to account for viscous effects at limited or no separation on C_c
ω	generic frequency
$\phi(s, M)$	indicial response function
$\phi(t, M)$	indicial response function
ϕ_α^c	normal force coefficient, circulatory indicial response function to a step change in α
ϕ_α^{nc}	normal force coefficient, non-circulatory indicial response function to a step change in α

ϕ_q^c	normal force coefficient, circulatory indicial response function to a step change in q
ϕ_q^{nc}	normal force coefficient, non-circulatory indicial response Function to a step change in q
$\phi_{m,\alpha}^{nc}$	pitching moment coefficient, non-circulatory indicial response function to a step change in α
$\phi_{m,q}^c$	pitching moment coefficient, circulatory indicial response function to a step change in q
$\phi_{m,q}^{nc}$	pitching moment coefficient, non-circulatory indicial response function to a step change in q
σ_1	generic multiplier for T_f
σ_3	generic multiplier for T_V
σ_s	generic integrand coordinate
σ_t	generic integrand coordinate
τ_V	time variable, tracking the travel of the LE vortex over the airfoil suction surface. It is made dimensionless via the semi-chord: $\tau_V = t * 2U/c$. If less than $2T_{VL}$, it renders $VRTX$=True; if less than T_{VL} then the vortex is still on the airfoil.

α	angle of attack
α_0	0-lift angle of attack
α_1	angle of attack at f=0.7, (approximately the stall angle) for $\alpha \geq \alpha_0$
α_e	effective angle of attack at 3/4-chord
α_f	effective angle of incidence (AOI) which would give the same unsteady LE pressure gradient under static conditions; used to calculate f'
α_f'	lagged version of α_f; used in Minnema [1998] calculation of C_m under separated conditions

β_M	Prandtl-Glauert compressibility correction factor $\sqrt{1-M^2}$

Chapter 1

Overview

The main theory follows the work by Leishman and Beddoes [1986, 1989]; Pierce and Hansen [1995]; Pierce [1996]; Leishman [2011]; Damiani [2011]. Dynamic stall is a well-known phenomenon that can affect wind turbine performance and loading especially during yawed operations, and that can result in large unsteady stresses on the structures.

Dynamic stall manifests as a delay in the onset of flow separation to higher AOAs that would otherwise occur under static (steady) conditions, followed by an abrupt flow separation from the LE of the airfoil Leishman [2011]. The LE separation is the fundamental characteristic of the DS of an airfoil; in contrast, quasi-steady stall would start from the airfoil TE.

DS occurs for reduced frequencies above 0.02.

$$k = \frac{\omega c}{2U} \tag{1.1}$$

The five stages of DS are as follows and shown in Fig. 1 and Fig. 1:

1. Onset of flow reversal
2. Flow separation and vorticity accumulation at the LE
3. Shedding of the vortex and convection along the suction surface of the airfoil (lift increases)
4. Lift Stall: vortex is shed in the wake and lift abrupt drop-off
5. Re-attachment of the flow at AOAs considerably lower than static AOAs (hysteresis)

The model chosen to represent unsteady aerodynamics and dynamic stall is the Leishman-Beddoes model (LBM), because it is the most widely used and has the most available support throughout the community and it has shown reasonable success when compared to experimental data. The LBM is a post-dictive model, and as such it will not solve equations of motion, though the principles are fully rooted in the physics of unsteady flow.

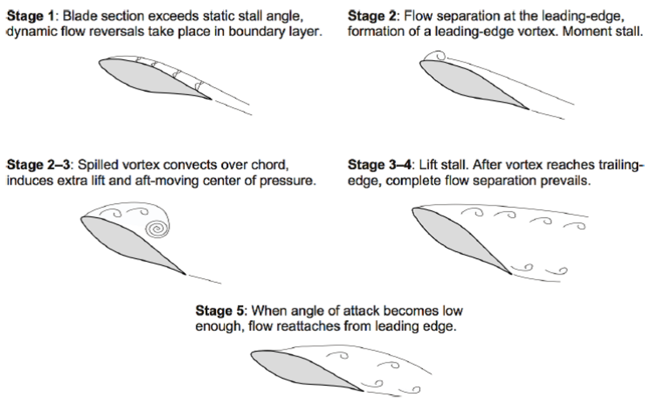

Figure 1.1: Conventional stages of DS, from Leishman [2006].

In the LBM, the different processes are modeled as first-order subsystems with differential equations with pre-determined constants to match experimental results. Therefore, knowledge of the airfoil characteristics under unsteady aerodynamics is a prerogative of the LBM. The LBM may also be described as an indicial response (i.e.,response to a series of small disturbances) model for attached flow, extended to account for separated flow effects and vortex lift. Forces are computed as normal and tangential (to chord) and pitching moment about the 1/4-chord location, see also Fig. 1 and Eq. (1.2).

$$\begin{aligned} C_l &= C_n \cos\alpha + C_c \sin\alpha \\ C_d &= C_n \sin\alpha - C_c \cos\alpha + C_{d0} \end{aligned} \tag{1.2}$$

$$\begin{aligned} C_n &= C_l \cos\alpha + (C_d - C_{d0}) \sin\alpha \\ C_c &= C_l \sin\alpha - (C_d - C_{d0}) \cos\alpha \end{aligned} \tag{1.3}$$

The original model was developed for helicopters, but it has been successfully applied to wind turbines (see Pierce [1996]; Gupta and Leishman [2006]). Yawed flowed conditions, Coriolis and centrifugal forces that lead to three-dimensional effects were not included in the original model.

Unsteady aerodynamics is mostly driven by 2D flow aspects, where the time scale is on the order of tenths of seconds, or $\sim c/\omega R$.

The LBM considers a number of unsteady aerodynamics conditions, namely: attached flow conditions and TE separation before stall; delays and lags associated with the unsteady onset of dynamic stall and accompanying boundary layer development; advection of the LE vortex, shedding in the wake, and suppression of TE separation in favor of LE separation. The LBM can be subdivided into three main submodules:

1. Unsteady, Attached Flow Solution via Indicial Treatment (potential flow)

2. TE Flow Separation

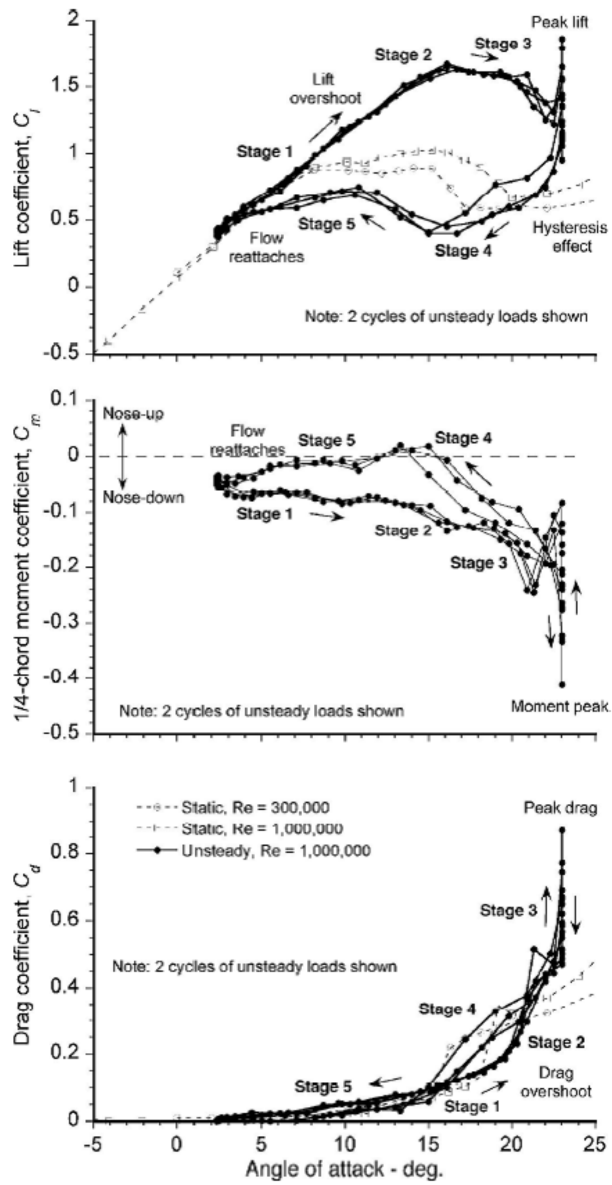

Figure 1.2: Conventional stages of DS and associated C_l, C_d, C_m as functions of AOA from Leishman [2006].

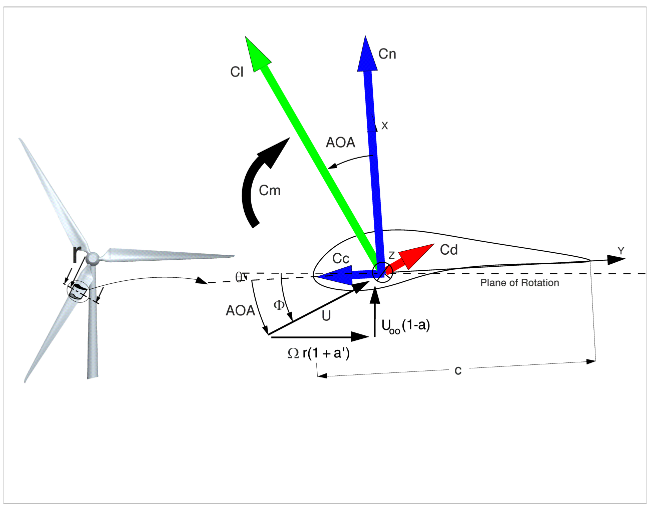

Figure 1.3: Main definitions of BE forces (denoted via their normalized coefficients) for the unsteady aerodynamics treatment, from Damiani [2011].

1.1 Unsteady Attached Flow and Its Indicial Treatment

The advantage of the indicial treatment is that a response to an arbitrary forcing can be obtained through superposition of response-functions to a step variation in AOA, in pitch rate, or in heave (plunging) motion. The superposition is carried out via the so-called Duhamel Integral Leishman [2006], which for the generic response $F_R(t)$ to a generic disturbance $\epsilon(t)$ can be written as in Eq. (1.4):

$$\begin{array}{lrcl} & F_R(t) & = & \epsilon(0)\phi(t,M) + \int_0^t \frac{\mathrm{d}\epsilon}{\mathrm{d}\sigma_t}(\sigma_t)\,\phi(t-\sigma_t,M)\mathrm{d}\sigma_t \\ \text{or} & & & \\ & F_R(s) & = & \epsilon(0)\phi(s,M) + \int_0^s \frac{\mathrm{d}\epsilon}{\mathrm{d}\sigma_s}(\sigma_s)\,\phi(s-\sigma_s,M)\mathrm{d}\sigma_s \end{array} \tag{1.4}$$

where the non-dimensional distance, s, is defined as:

$$s = \frac{2}{c}\int_0^t U(t)dt \tag{1.5}$$

where the airfoil half chord is considered as the non-dimensionalizing factor $(c/2)$.

The indicial functions are surmised into two components: the first is related to the non-circulatory loading (piston theory and acoustic wave theory); the second originates from the development of circulation about the airfoil.

The non-circulatory part depends on the instantaneous airfoil motion, but also on the time history of the prior motion. The circulatory response can be calculated via the 'lumped approach', where the effects of step changes in AOA, pitch rate, heave motion, etc., are combined into an effective AOA at the 3/4-chord station.

The normal force coefficient response to a step-change in non-dimensional pitch rate q and a step-change in AOA can be written as a function of the indicial functions as shown in Eq. (1.6):

$$\begin{array}{lcr} C_{n_{\alpha,q}}(s,M) & = & C_{n_\alpha}(s,M) + C_{n_q}(s,M) = C_{n\alpha}\alpha + C_{nq}(s,M)q \\ C_{n_\alpha}(s,M) & = & \frac{4}{M}\phi_\alpha^{nc} + \frac{C_{n\alpha}}{\beta_M}\phi_\alpha^c \\ C_{n_q}(s,M) & = & \frac{1}{M}\phi_q^{nc} + \frac{C_{n\alpha}}{2\beta_M}\phi_q^c \end{array} \tag{1.6}$$

Analogously the pitching moment coefficient about the 1/4-chord can be derived via indicial response as shown in Eq.(1.7):

$$\begin{array}{lcr} C_{m_{\alpha,q}}(s,M) & = & C_{m_\alpha}(s,M) + C_{m_q}(s,M) = C_{m\alpha}\alpha + C_{mq}(s,M)q \\ C_{m_\alpha}(s,M) & = & -\frac{1}{M}\phi_{m,\alpha}^{nc} - \frac{C_{n\alpha}}{\beta_M}\phi_\alpha^c\,(\hat{x}_{AC} - 0.25) + C_{m0} \\ C_{m_q}(s,M) & = & -\frac{7}{12M}\phi_{m,q}^{nc} - \frac{C_{n\alpha}}{16\beta_M}\phi_{m,q}^c \end{array} \tag{1.7}$$

where C_{m0} is positive if causes a pitch up of the airfoil, as seen in Fig. 1. Also note that the circulatory component of the pitching moment response to a step-change in α is a function of the $C^c_{n_\alpha}(s, M)$.

The non-dimensional pitch-rate q is given by:

$$\begin{aligned} q &= \dot{\alpha}c/U \simeq K_{\alpha t}c/U \\ \text{with}: \quad K_{\alpha t} &= \frac{\alpha_t - \alpha_{t-1}}{\Delta t} \end{aligned} \tag{1.8}$$

The indicial responses can then be approximated as in Eq. (1.9) [Leishman and Beddoes, 1989; Johansen, 1999]:

$$\begin{aligned} \phi^c_\alpha &= \phi^c_q = 1 - A_1 \exp\left(-b_1\beta^2_M s\right) - A_2 \exp\left(-b_2\beta^2_M s\right) \\ \phi^{nc}_\alpha &= \exp\left(-\frac{s}{T'_\alpha}\right) \\ \phi^{nc}_q &= \exp\left(-\frac{s}{T'_q}\right) \\ \phi^{nc}_{m,q} &= \exp\left(-\frac{s}{T'_{m,q}}\right) \end{aligned} \tag{1.9}$$

One could find analogous expressions for $\phi^c_{m,q}$,$\phi^{nc}_{m,\alpha}$, but they are not shown here because further simplified expressions will be derived below.

By making use of exact results for short times $0 \leq s \leq 2M/(M+1)$ [Lomax et al., 1952], Leishman [2011] shows that:

$$\begin{aligned} T_\alpha(M) &= \frac{c}{2U}T'_\alpha = \frac{c}{2Ma_s}T'_\alpha = k_\alpha(M)T_I \\ T_q(M) &= \frac{c}{2U}T'_q = \frac{c}{2Ma_s}T'_q = k_q(M)T_I \end{aligned} \tag{1.10}$$

where:

$$\begin{aligned} k_\alpha(M) &= \left[(1-M) + \frac{C_{n\alpha}}{2}M^2\beta_M\left(A_1b_1 + A_2b_2\right)\right]^{-1} = \left[(1-M) + \frac{C_{n\alpha}}{2}M^2\beta_M 0.413\right]^{-1} \\ k_q(M) &= \left[(1-M) + C_{n\alpha}M^2\beta_M\left(A_1b_1 + A_2b_2\right)\right]^{-1} = \left[(1-M) + C_{n\alpha}M^2\beta_M 0.413\right]^{-1} \\ T_I &= \frac{c}{a_s} \end{aligned} \tag{1.11}$$

Note that Leishman [2011] recommends the use of $0.75T_\alpha(M)$ in place of $T_\alpha(M)$ to account for three-dimensional effects not included in piston theory.

The values of the A_1-b_2 constants are independent of M and are determined from experimental data on oscillating airfoils in wind tunnels.

For the circulatory component of the aerodynamic force response, the lumped approach can lead to a direct solution of $C^c_{n_{\alpha,q}}(s, M)$. considering the circulatory part $C^c_{n_\alpha}(s, M)$ of Eq. (1.6) for the response to the step in α, one can write:

$$C^c_{n_\alpha}(s,M) = \int_{s_0}^{s} \frac{C_{n\alpha}}{\beta_M}\phi^c_\alpha \alpha(s)\mathrm{d}s \simeq C^c_{n\alpha}(s,M)\Delta\alpha \tag{1.12}$$

By using Eq. (1.4) with $\phi(s,M)$ replaced by ϕ^c_α and $\epsilon(s)$ by α, Eq. (1.12) rewrites:

$$C^c_{n_{\alpha,q}}(s,M) = C^c_{n\alpha}(s,M)\left[\alpha(s_0)\phi^c_\alpha(s) + \int_{s_0}^{s}\frac{\mathrm{d}\alpha}{\mathrm{d}\sigma_s}(\sigma_s)\,\phi^c_\alpha(s-\sigma_s,M)\mathrm{d}\sigma_s\right] = C^c_{n\alpha}(s,M)\alpha_e \tag{1.13}$$

where α_e is an effective angle of attack at 3/4-chord accounting for a step variation in α, pitching rate, heave, and velocity (lumped approach). By applying the first of Eq. (1.9), and setting $s_0 = 0$, Eq. (1.13) can be simplified to arrive at an expression for α_e at the n-th time step, i.e.,α_{e_n}:

$$\alpha_{e_n}(s,M) = (\alpha_n - \alpha_0) - X_{1_n}(\Delta s) - X_{2_n}(\Delta s) \tag{1.14}$$

where the $\int_{s_0}^{s}[...]\mathrm{d}\sigma_s$ was divided into two steps considering a distance interval Δs, i.e.,$\int_0^s[...]\mathrm{d}\sigma_s$ and $\int_s^{s+\Delta s}[...]\mathrm{d}\sigma_s$, by carrying out the algebra a recursive expression for X_1 and X_2 can be found:

$$\begin{aligned} X_{1_n} &= X_{1_{n-1}}\exp\left(-b_1\beta_M^2\Delta s\right) + A_1\exp\left(-b_1\beta_M^2\tfrac{\Delta s}{2}\right)\Delta\alpha_n \\ X_{2_n} &= X_{2_{n-1}}\exp\left(-b_2\beta_M^2\Delta s\right) + A_2\exp\left(-b_2\beta_M^2\tfrac{\Delta s}{2}\right)\Delta\alpha_n \end{aligned} \tag{1.15}$$

Note that α_0 was introduced into Eq.(1.14), because α_e is an effective AOI and not AOA.

Similarly to the above development, the circulatory contribution to $C^c_{n_{\alpha,q}}(s,M)$ from a step change in q can be derived as:

$$C^c_{n_q}(s,M) = \frac{C^c_{n\alpha}(s,M)}{2}(q_n - q_0) - X_{3_n}(\Delta s) - X_{4_n}(\Delta s) \tag{1.16}$$

However, the lumped approach can account for any effect to α, including step changes in q, so Eq. (1.16) is not necessary, and it is virtually included via Eq. (1.13) and (1.14).

The non-circulatory part cannot be handled via the superposition (lumped approach), therefore the contribution from step changes in α and q need to be kept separate:

$$C^{nc}_{n_{\alpha,q}}(s,M) = C^{nc}_{n_\alpha}(s,M) + C^{nc}_{n_q}(s,M) \tag{1.17}$$

Now, using Duhamel's integral (1.4) on the non-circulatory component $C^{nc}_{n_\alpha}(s,M)$ (see Eq.(1.17)) with the ϕ^{nc}_α from Eq.(1.9), one can arrive at:

$$\begin{aligned}
C^{nc}_{n_\alpha}(s,M) &= \tfrac{4T_\alpha(M)}{M}\left(K_{\alpha t}-K'_{\alpha t}\right)\\
K_{\alpha t} &= \tfrac{\alpha_t-\alpha_{t-1}}{\Delta t}\\
K'_{\alpha t} &= K'_{\alpha t-1}\exp\left(-\tfrac{\Delta t}{T_\alpha(M)}\right)+\left(K_{\alpha t}-K_{\alpha t-1}\right)\exp\left(-\tfrac{\Delta t}{2T_\alpha(M)}\right)
\end{aligned} \tag{1.18}$$

Note that in Eq. (1.18), K'_α is the deficiency function for $C^{nc}_{n_\alpha}(s,M)$.
For $C^{nc}_{n_q}(s,M)$, an analogous procedure leads to:

$$\begin{aligned}
C^{nc}_{n_q}(s,M) &= \tfrac{T_q(M)}{M}\left(K_{q_t}-K'_{q_t}\right)\\
K_{q_t} &= \tfrac{q_t-q_{t-1}}{\Delta t}\\
K'_{q_t} &= K'_{q_{t-1}}\exp\left(-\tfrac{\Delta t}{T_q(M)}\right)+\left(K_{q_t}-K_{q_{t-1}}\right)\exp\left(-\tfrac{\Delta t}{2T_q(M)}\right)
\end{aligned} \tag{1.19}$$

So finally, the expression for the total normal force under attached conditions can be expressed as:

$$C^{pot}_n = C^c_{n_{\alpha,q}}(s,M)+C^{nc}_{n_{\alpha,q}}(s,M) = C^c_{n\alpha}(s,M)\alpha_e+\frac{4T_\alpha(M)}{M}\left(K_{\alpha t}-K'_{\alpha t}\right)+\frac{T_q(M)}{M}\left(K_{q_t}-K'_{q_t}\right) \tag{1.20}$$

Now turning back to the pitching moment, following Johansen [1999] the circulatory component $C^c_{m_q}(s,M)$ can be written as:

$$C^c_{m_q}(s,M) = -\frac{C_{n\alpha}}{16\beta_M}\left(q-K'''_q\right)\frac{c}{U} \tag{1.21}$$

where

$$K'''_{q\ t} = K'''_{q\ t-1}\exp\left(-b_5\beta_M^2\Delta s\right)+A_5\Delta q_t\exp\left(-b_5\beta_M^2\frac{\Delta s}{2}\right) \tag{1.22}$$

The non-circulatory component of the pitching moment response to step-change in α, $C^{nc}_{m_\alpha}(s,M)$, writes [Leishman and Beddoes, 1986; Johansen, 1999]:

$$C^{nc}_{m_\alpha}(s,M) = -\frac{1}{M}\phi^{nc}_{m,\alpha} = -\frac{C^{nc}_{n_\alpha}(s,M)}{4} \tag{1.23}$$

which implies

$$\phi^{nc}_{m,\alpha} = \phi^{nc}_\alpha \tag{1.24}$$

The other component $C^{nc}_{m_q}(s,M)$ writes [Leishman, 2006]:

$$
\begin{aligned}
C^{nc}_{m_q}(s,M) &= -\tfrac{7}{12M}\phi^{nc}_{m,q} = -\tfrac{7k_{m,q}(M)^2T_I}{12M}\left(K_q - K''_q\right)
\end{aligned}
$$

with:

$$
\begin{aligned}
k_{m,q}(M) &= \tfrac{7}{15(1-M)+1.5C_{n\alpha}A_5b_5\beta_M M^2} \\
T_{m,q}(M) &= k_{m,q}(M)T_I \\
K''_{q\,t} &= K''_{q\,t-1}\exp\left(-\tfrac{\Delta t}{k_{m,q}(M)^2T_I}\right) + \left(K_{q_t} - K_{q_{t-1}}\right)\exp\left(-\tfrac{\Delta t}{2k_{m,q}(M)^2T_I}\right)
\end{aligned}
\tag{1.25}
$$

where the same procedure as before was used to arrive at a deficiency function using Duhamel's integral (Eq. (1.4)) and equating the expressions of the $C_{m_{\alpha,q}}(s,M)$ at $s \to 0$ (see Leishman [2006]).

Finally, the expression for the total pitching moment at 1/4-chord under attached conditions can be expressed as:

$$
\begin{aligned}
& C^{pot}_m = C^c_{m_{\alpha,q}}(s,M) + C^{nc}_{m_{\alpha,q}}(s,M) = \\
& C_{m0} - \tfrac{C_{n\alpha}}{\beta_M}\phi^c_\alpha\left(\hat{x}_{AC} - 0.25\right) + \\
& -\tfrac{C_{n\alpha}}{16\beta_M}\left(q - K'''_q\right)\tfrac{c}{U} + \\
& -\tfrac{T_\alpha(M)}{M}\left(K_{\alpha t} - K'_{\alpha t}\right) + \\
& -\tfrac{7k_{m,q}(M)^2T_I}{12M}\left(K_q - K''_q\right)
\end{aligned}
\tag{1.26}
$$

Note that the pitching moment treatment is slightly different from what is in AeroDyn v13 and Damiani [2011], and it is more in line with Leishman [2006] and Johansen [1999]. If Minnema [1998] suggestions is used, then the $C^{nc}_{m_q}(s,M)$ (last term in Eq. (1.26)) is to be replaced by:

$$
C^{nc}_{m_q}(s,M) = -\frac{7}{12M}\phi^{nc}_{m,q} = -\frac{C^{nc}_{n_q}(s,M)}{4} - \frac{k_\alpha(M)^2T_I}{3M}\left(K_q - K''_q\right) \tag{1.27}
$$

1.1.1 Tangential Force

The tangential force along the chord can be written as in Eq. (1.28) from Leishman [2011]:

$$
C^{pot}_c = C^{pot,c}_n \tan\left(\alpha_e + \alpha_0\right) \tag{1.28}
$$

In potential flow, D'Alambert's paradox leads to the absence of drag; therefore, from Eq. (1.3), $C_l\cos\alpha = C_n$, and $C_c = C_l\sin\alpha$ which bring forth Eq.(1.28). Since α_e is a virtual AOI at 3/4-chord, we needed to add the α_0.

Since this drag treatment has roots only in the circulatory lift derivation, the non-circulatory part is dropped as seen in Eq. (1.28).

1.2 TE Flow Separation

The base of this dynamic system is Kirchoff's theory, which can be expressed as follows [Leishman, 2006]:

$$
\begin{aligned}
C_n(\alpha, f) &= C_{n\alpha}(\alpha - \alpha_0)\left(\tfrac{1+\sqrt{f}}{2}\right)^2 \\
C_c(\alpha, f) &= \eta_e C_{n\alpha}(\alpha - \alpha_0)\sqrt{f}\tan(\alpha)
\end{aligned}
\tag{1.29}
$$

with f the separation point distance from LE in percent chord, and η_e the recovery factor $\simeq [0.85 - 0.95]$ to account for viscous effects at limited or no separation on C_c.

If the airfoil's C_l,C_d, and C_m characteristics are known, then Eq.1.29 may be solved for f. Leishman [2011] suggests the use of best-fit curves obtained from static measurements on airfoils, of the type:

$$
f = \begin{cases}
1 - 0.3\exp\left(\frac{\alpha-\alpha_1}{S_1}\right), \text{if } \alpha_0 \leq \alpha \leq \alpha_1 \\
1 - 0.3\exp\left(\frac{\alpha_2-\alpha}{S_1}\right), \text{if } \alpha_2 \leq \alpha < \alpha_0 \\
0.04 + 0.66\exp\left(\frac{\alpha_1-\alpha}{S_2}\right), \text{if } \alpha > \alpha_1 \\
0.04 + 0.66\exp\left(\frac{\alpha-\alpha_2}{S_2}\right), \text{if } \alpha < \alpha_2
\end{cases}
\tag{1.30}
$$

S_1 and S_2 are best-fit constants that define the abruptness of the static stall. α_1 is the angle of attack at f=0.7, (approximately the stall angle) for $\alpha \geq \alpha_0$, whereas α_2 is the angle of attack at f=0.7, for $\alpha < \alpha_0$.

Now accounting for unsteady conditions, the TE separation point gets modified due to temporal effects on airfoil pressure distribution and boundary layer response. LE separation occurs when a critical pressure at the LE, corresponding to a critical value of the normal force C_{n1}, is reached. The circulatory normal force needs to be modified to account for the lagged boundary layer response. In order to arrive at a new expression for C_n, we start by accounting for the separation point location under unsteady conditions, which can be calculated starting from an effective AOI, α_f:

$$
\alpha_f = \frac{C'_n}{C_{n\alpha}} + \alpha_0 \tag{1.31}
$$

where an effective C'_n is used, calculated as in Eq. (1.32):

$$\begin{aligned} C'_n &= C_n^{pot} - D_p \\ D_{p_t} &= D_{p_{t-1}} \exp\left(-\tfrac{\Delta s}{T_p}\right) + \left(C_n^{pot}{}_t - C_n^{pot}{}_{t-1}\right) \exp\left(-\tfrac{\Delta s}{2T_p}\right) \end{aligned} \tag{1.32}$$

Note T_p is boundary-layer,LE pressure gradient time constant; in the expression of D_p. It should be tuned based on airfoil experimental data. and is an empirically based quantity that should be tuned from experimental data. Johansen [1999] employs two time constants $T_{p\alpha}$ and T_{pq} as the C'_n is separated into two contributions, one from α and from q.

Given the new C'_n, one can attain a new formulation for f, i.e.,f'', which accounts for delays in the boundary layer, and that will be used via Kirchoff's treatment to arrive at the new C_n:

$$\begin{aligned} f'' &= f' - D_f \\ D_{f_t} &= D_{f_{t-1}} \exp\left(-\tfrac{\Delta s}{T_f}\right) + \left(f'_t - f'_{t-1}\right) \exp\left(-\tfrac{\Delta s}{2T_f}\right) \end{aligned} \tag{1.33}$$

where f' is the separation point distance from LE in percent chord under unsteady conditions that can be obtained from the best-fit in Eq. (1.30) replacing α with α_f.

Alternatively, f' can be derived from a direct lookup table of static airfoil data reversing Eq. (1.29). In fact, two values of f' could be calculated: one for C_n and one for C_c.

Also note that T_f is a Mach, Re nad airfoil dependent time constant associated with the motion of the separation point along the suction surface of the airfoil. constant dependent on Mach, Re, and airfoil shape; it is used in the expression of D_f and f'' gets modified via multipliers (σ_1) depending on the phase of the separation or reattachment as discussed later; here it suffices noting that T_f can be written as a modified version of the initial value T_{f0}:

$$T_f = T_{f0}/\sigma_1 \tag{1.34}$$

Finally, the normal force coefficient C_n^{fs}, after accounting for separated flow from the TE becomes:

$$C_n^{fs} = C_{n_{\alpha,q}}^{nc}(s, M) + C_{n_{\alpha,q}}^{c}(s, M)\left(\frac{1+\sqrt{f''}}{2}\right)^2 = C_{n_{\alpha,q}}^{nc}(s, M) + C_{n\alpha}^{c}(s, M)\alpha_e\left(\frac{1+\sqrt{f''}}{2}\right)^2 \tag{1.35}$$

Note that González [2014]; Sheng [2007] propose the corrective factor to be:

$$C_n^{fs} = C_{n_{\alpha,q}}^{nc}(s, M) + C_{n\alpha}^{c}(s, M)\alpha_e\left(\frac{1+2\sqrt{f''}}{3}\right)^2 \tag{1.36}$$

to account for lower values of C_n when f=0 that were seen from experimental data.

1.2.1 Tangential Force

The along-chord force coefficient analogously becomes:

$$C_c^{fs} = C_n^{pot,c} \tan(\alpha_e + \alpha_0)\eta_e \sqrt{f''} = C_{n\alpha}^c(s,M)\alpha_e \tan(\alpha_e + \alpha_0)\eta_e \sqrt{f''} \quad (1.37)$$

González [2014] proposes a slightly different formulation:

$$C_c^{fs} = C_{n\alpha}^c(s,M)\alpha_e \tan(\alpha_e + \alpha_0)\eta_e \left(\sqrt{f''} - 0.2\right) \quad (1.38)$$

This modification is to account for negative values seen at f=0.

1.2.2 Pitching Moment

Now turning to the pitching moment, the contribution due to unsteady separated flow is on the circulatory component alone. Leishman [2011] suggests using this formulation that modified C_m^{pot} for the $C_{m_\alpha}^c(s,M)$ component:

$$C_m^{fs} = C_{m0} - C_{n_{\alpha,q}}^c(s,M)(\hat{x}_{cp} - 0.25) + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \quad (1.39)$$

where $\hat{x}_{cp}$ is center-of-pressure distance from LE in percent chord and can be approximated by [Leishman, 2011]:

$$\hat{x}_{cp} = k_0 + k_1(1 - f'') + k_2 \sin\left(\pi f''^{k_3}\right) \quad (1.40)$$

where k_0=0.25-$\hat{x}_{AC}$, and the k_1-k_3 constants are calculated via best-fits of experimental data. Other expressions could be used to perform the best fit of $\hat{x}_{cp}$ vs. f from static C_m airfoil data.

Minnema [1998] suggests a different approach where an effective lagged AOI is calculated as follows:

$$\begin{aligned} \alpha'_f &= \alpha_f - D_{\alpha f} \\ D_{\alpha f_t} &= D_{\alpha f_{t-1}} \exp\left(-\tfrac{\Delta s}{T_f}\right) + \left(f'_t - f'_{t-1}\right) \exp\left(-\tfrac{\Delta s}{2T_f}\right) \end{aligned} \quad (1.41)$$

then the new AOI is used to derive the contribution to the circulatory component of the pitching moment, which is extracted from a look-up table of static coefficients C_m vs. α.

$$C_m^{fs} = C_m\left(\alpha'_f\right) + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \quad (1.42)$$

The method proposed by González [2014] uses a value of f' (=f'_m) extracted from a static data table where it is assumed that $C_m = f_m C_n$ (loosely correlating f to $\hat{x}_{cp}$). The angle used for interpolation of the lookup table is α_f. In order to resolve the contribution to C_m from the unsteady TE separation, this method then uses the full C_n^{fs} from Eq. (1.36) and the lagged = f''_m calculated from f'_m and Eq. (1.33), replacing f' with f'_m.

$$C_m^{fs} = C_n^{fs} f''_m + C^c_{m_q}(s,M) + C^{nc}_{m_\alpha}(s,M) + C^{nc}_{m_q}(s,M) \tag{1.43}$$

In this case, the two treatments [Minnema, 1998; González, 2014] seem somewhat equivalent, with the exception that González [2014] proposes 21 (7 for each f'' related to C_n, C_c, and C_m)different multipliers for T_f depending on the state of the airfoil aerodynamics (e.g.,increasing AOA and above a critical C_{n1}, increasing AOA and below a critical C_{n1}).

1.3 Dynamic Stall

During DS there is shear layer roll-up at the LE, vortex formation, and vortex travel over the upper surface of the airfoil to be subsequently shed in the wake. The main condition to be met for the shear layer roll up is:

$$\begin{array}{lll} C'_n > C_{n1} & \text{for} & \alpha \geq \alpha_0 \\ C'_n < C_{n2} & \text{for} & \alpha < \alpha_0 \end{array} \tag{1.44}$$

The normal force coefficient contribution from the additional lift associated with the low pressure LE vortex can be written as Leishman [2011]:

$$C^v_{n\,t} = C^v_{n\,t-1} \exp\left(-\frac{\Delta s}{T_V}\right) + (C_{V\,t} - C_{V\,t-1}) \exp\left(-\frac{\Delta s}{2T_V}\right) \tag{1.45}$$

T_V is the time constant associated with the vortex lift decay process; it is used in the expression of C^v_n. It depends on Re,M, and airfoil class.. T_V gets modified via a multiplier σ_3 to account for various stages of the process as discussed later, here suffice to say that:

$$T_V = T_{V0}/\sigma_3 \tag{1.46}$$

C_V represents the contribution to the normal force coefficient due to accumulated vorticity in the LE vortex. C_V is modeled proportionally to the difference between the attached and separated circulatory contributions to C_n:

$$C_V = C^c_{n_{\alpha,q}}(s,M) - C^c_{n_{\alpha,q}}(s,M)\left(\frac{1+\sqrt{f''}}{2}\right)^2 = C^c_{n\alpha}(s,M)\alpha_e\left(1 - \frac{1+\sqrt{f''}}{2}\right)^2 \tag{1.47}$$

If González [2014] is used, then C_V writes as:

$$C_V = C^c_{n\alpha}(s,M)\alpha_e\left(1 - \frac{1+2\sqrt{f''}}{3}\right)^2 \tag{1.48}$$

If $\tau_V > T_{VL}$ and if α_f is not moving away from stall (i.e.,$[(\alpha_f - \alpha_0) * (\alpha_{f\,t} - \alpha_{f\,t-1})] > 0$) , then the vorticity is no longer allowed to accumulate, in which cases Eq.(1.45) rewrites as:

$$\begin{array}{lll} & C^v_{n\,t} & = C^v_{n\,t-1} \exp\left(-\frac{\Delta s}{T_{V0}/\sigma_3}\right) \\ \text{with} & \sigma_3 & = 2 \end{array} \tag{1.49}$$

where the decay of the normal force (due to vorticity at the LE) is accelerated at twice the original rate and no further accretion of vorticity is allowed. Eq. (1.49) should also be used when conditions in Eq. (1.44) are not met. Note that T_{VL} represents the time constant associated with the vortex advection process; it represents the non-dimensional time in semi-chords, needed for a vortex to travel from LE to TE; it is used in the expression of C_n^v. It depends on Re, M (weakly), and airfoil. Value's range = $[6; 13]$.

Finally the total normal force can be written as:

$$C_n = C_n^{fs} + C_n^v = C_{n\alpha}^c(s,M)\alpha_e \left(\frac{1+\sqrt{f''}}{2}\right)^2 + C_{n_{\alpha,q}}^{nc}(s,M) + C_n^v \tag{1.50}$$

Again, if González [2014] is used, then the correction factor for the separated flow treatment is slightly modified as in Eq.(1.48), i.e.:

$$C_n = C_n^{fs} + C_n^v = C_{n\alpha}^c(s,M)\alpha_e \left(\frac{1+2\sqrt{f''}}{3}\right)^2 + C_{n_{\alpha,q}}^{nc}(s,M) + C_n^v \tag{1.50b}$$

Note that multiple vortices can be shed at a given shedding frequency corresponding to:

$$T_{sh} = 2\frac{1-f''}{St_{sh}} \tag{1.51}$$

Therefore τ_V is reset to 0 if $\tau_V = 1 + \dfrac{T_{sh}}{T_{VL}}$.

1.3.1 Tangential Force

The along-chord force coefficient gets modified by the presence of the LE vortex as [Pierce, 1996]:

$$C_c = \eta_e C_c^{pot}\sqrt{f''} + C_n^v \tan(\alpha_e)(1-\tau_V) \tag{1.52}$$

González [2014] does not contain the vortex contribution to C_c based on experimental validation:

$$C_c = \eta_e C_c^{pot}\sqrt{f''} \tag{1.52b}$$

The original [Leishman and Beddoes, 1989] model had C_c written as:

$$C_c = \begin{cases} \eta_e C_c^{pot}\sqrt{f''}\sin(\alpha_e+\alpha_0) & , \quad C_n' \leq C_{n1} \\ \hat{k}_1 + C_c^{pot}\sqrt{f''}f''^{\hat{k}_2}\sin(\alpha_e+\alpha_0) & , \quad C_n' > C_{n1} \end{cases} \tag{1.53a}$$

$$\text{with :} \hat{k}_2 = 2(C_n' - C_{n1}) + f'' - f \tag{1.53b}$$

where $\hat{k}_1$ is a constant required to fit the C_c curve under static conditions for two-dimensional (2D) airfoils.

1.3.2 Pitching Moment

Leishman [2011] offers a form for the $\hat{x}^v_{cp}$, which is the center-of-pressure distance from the 1/4-chord, in percent chord, during the LE vortex advection process:

$$\begin{aligned} C^v_m &= -\hat{x}^v_{cp} C^v_n \\ \hat{x}^v_{cp}(\tau_V) &= \bar{\bar{x}}_{cp} \left(1 - \cos\left(\frac{\pi \tau_V}{T_{VL}}\right)\right) \end{aligned} \tag{1.54}$$

where $\bar{\bar{x}}_{cp}$ is a constant in the expression of $\hat{x}^v_{cp}$, usually $= 0.2$.

Finally, the expression for the total pitching moment can ve written as:

$$C_m = C_{m0} - C^c_{n_{\alpha,q}}(s,M)(\hat{x}_{cp} - 0.25) + C^c_{m_q}(s,M) + C^{nc}_{m_\alpha}(s,M) + C^{nc}_{m_q}(s,M) + C^v_m \tag{1.55}$$

If Minnema [1998]'s approach is used then Eq.(1.55) rewrites as:

$$C_m = C_m\left(\alpha_f\right) + C^c_{m_q}(s,M) + C^{nc}_{m_\alpha}(s,M) + C^{nc}_{m_q}(s,M) + C^v_m \tag{1.56}$$

and if González [2014]'s treatment is used then the total moment becomes:

$$C_m = C_n f''_m + C^c_{m_q}(s,M) + C^{nc}_{m_\alpha}(s,M) + C^{nc}_{m_q}(s,M) + C^v_m \tag{1.57}$$

Chapter 2

Inputs, Outputs, Parameters, and States for the Unsteady-Aerodynamics module (UA)

2.1 Init_Inputs

Beside the standard variables common to all modules ($OutFmt$, $OutSFmt$, $NumOuts$, $OutList$), the Init_Inputs to the UA are:

- Δt -time step
- a_s -speed of sound
- ν -kinematic viscosity
- $DSMod$ - switch to select handling of options and possible methods in dynamic stall
- AFI_{Params} -airfoil static tables of C_l C_d C_m and DS parameters
- $AFidx$ -index pointing to the correct airfoil file/table/database
- c - airfoil chord at each blade station for each blade.

2.2 Inputs u

The Inputs to the UA are:

- α
- U

Note these are for a given node (within a given blade) and chord.

2.3 Outputs $\mathbf{y}$

The Outputs from the UA are:

- C_n
- C_c
- C_m
- C_l
- C_d

Note these are for a given node (within a given blade) and chord.

2.4 States $\mathbf{x}$

The States for the UA are:

Discrete States:

- α_{-1} -previous time-step value of α
- α_{-2} -previous time-step value of α
- q_{-1} -previous time-step value of q
- q_{-2} -two time-steps ago value of q
- $X_{1,-1}$ -previous time-step value of X_1
- $X_{2,-2}$ -previous time-step value of X_2
- $K'_{\alpha,-1}$ -previous time-step value of K'_α
- $K'_{q,-1}$ -previous time-step value of K'_q
- $K''_{q\ ,-1}$ -previous time-step value of K''_q
- $K'''_{q\ ,-1}$ -previous time-step value of K'''_q
- $D_{p,-1}$ -previous time-step value of D_p
- $D_{f\ ,-1}$ -previous time-step value of D_f

- $C_{n}^{pot}{}_{,-1}$ -previous time-step value of C_n^{pot}
- T_f -constant dependent on Mach, Re, and airfoil shape; it is used in the expression of D_f and f''
- f'_{-1} -previous time-step value of f'
- f''_{-1} -previous time-step value of f''
- τ_V -time variable, tracking the travel of the LE vortex over the airfoil suction surface. It is made dimensionless via the semi-chord: $\tau_V = t{*}2U/c$. If less than $2T_{VL}$, it renders $VRTX$=True; if less than T_{VL} then the vortex is still on the airfoil.
- $C^v_{n,-1}$ -previous time-step value of C^v_n
- $C_{V,-1}$ -previous time-step value of C_V
- $D_{\alpha f,-1}$ -previous time-step value of $D_{\alpha f}$

The FAST 8 framework does not allow logicals or discontinuous variables within states. For this reason, the following ones are declared as Other States.

Other States:

- σ_1 -generic multiplier for T_f
- σ_3 -generic multiplier for T_V
- $TESF$ -trailing edge separation flag
- $LESF$ -leading edge separation flag
- $VRTX$ -vortex advection flag
- $FirstPass$ -flag indicating first time step

Note that, in contrast to Inputs and Outputs, the States must be tracked by the UA module, therefore they are two-dimensional array (per blade, per node).

2.5 Parameters p

The Parameters for the UA are :

- Δt
- c
- $DSMod$
- a_s

- ν
- AFI_{Params}
- $AFidx$

The airfoil AFI_{Params} structure contains airfoil specific quantities, i.e. parameters and constants for the UA:

- α_0, α_1, $C_{n\alpha}$, C_{n1}, C_{n2}, η_e, C_{d0}, C_{m0}, $\bar{\bar{x}}_{cp}$, St_{sh}
- A_1,b_1,A_2,b_2,A_5,b_5
- S_1, S_2
- T_p, fairly independent of airfoil type
- T_{f0}, T_{V0}, T_{VL}
- k_0, k_1, k_2, k_3
- $\hat{k}_1$

2.6 Unsteady-Aerodynamics module Implementation

2.6.1 UA_Init

This routine initializes the States and sets the parameters (copies from the Init_Input).

2.6.2 UA_UpdateStates

The model is of the parsimonius, open loop, Kelvin-chain kind. Outputs of one subsystems go into inputs of the next subsystems. There are no differential equations to solve. There is no solver per se, for this reason states are discrete states only (and Other States see Ection 2.4).

The typical list of arguments to UA_UpdateStates gets augmented to pass indices to the blade and blade-node of interest. The indices point into the array of structures AFI_{Params} and into the array of chords.

Main Logical Flags

- IF $C'_n > C_{n1}$ ($C'_n < C_{n2}$ for $\alpha < \alpha_0$) THEN
  $TESF$=TRUE: this means LE separation can occur.
  ELSE $TESF$=False: this means reattachment can occur.
- IF $f''_t < f''_{t-1}$ THEN
  $TESF$=True: this means TE separation is in progress.
  ELSE $TESF$=False: this means TE reattachment is in progress.

- IF $0 < \tau_V \leq 2T_{VL}$ THEN
  $VRTX$=True: this means vortex advection is in progress.
  ELSE $VRTX$=False: this means vortex is in wake.
- IF $\tau_V \geq 1 + \dfrac{T_{sh}}{T_{VL}}$ THEN
  τ_V is reset to 0.

T_f modifications

The following conditional statements operate on a multiplier σ_1 that affects T_f, i.e.,the actual T_f is given by Eq.(1.34).

$$T_f = T_{f0}/\sigma_1 \qquad \text{(1.34 revisited)}$$

where T_{f0} is the initial value of T_f.

$\sigma_1 = 1$ (initialization default value)

$\Delta_{\alpha 0} = \alpha\text{-}\alpha_0$

IF $TESF$=True THEN: (separation)

IF $K_\alpha \Delta_{\alpha 0} < 0$ THEN $\sigma_1 = 2$ (accelerate separation point movement)

ELSEIF $TESF$=False THEN $\sigma_1 = 1$ (default value, LE separation can occur)

ELSEIF $f''_{n-1} \leq 0.7$ THEN $\sigma_1 = 2$ (accelerate separation point movement if separation is occurring)

ELSE σ_1 =1.75 (accelerate separation point movement)

ELSE: (reattachment, this means $TESF$=False)

IF $TESF$ = False THEN $\sigma_1 = 0.5$ (default: slow down reattachment)

IF $VRTX$=True AND $0 \leq \tau_V \leq T_{VL}$ THEN
$\sigma_1 = 0.25$ - No flow reattachment if vortex shedding in progress

IF $K_\alpha \Delta_{\alpha 0} > 0$ THEN $\sigma_1 = 0.75$

Note the last three conditional statements are separate IFs.

T_V modifications

For T_V, an analogous set of conditions is used to set the proper value of the time constant depending on subsystem stages:

σ_3=1 (initialization default value)

IF $T_{VL} \leq \tau_V \leq 2T_{VL}$ THEN
  σ_3=3 (post-shedding)
  IF $TESF$=False THEN
    σ_3=4 (accelerate vortex lift decay)
    IF $VRTX$=True AND $0 \leq \tau_V \leq T_{VL}$ THEN:
      IF $K_\alpha \Delta_{\alpha 0} < 0$ THEN σ_3=2 (accelerate vortex lift decay)
      ELSE σ_3=1 (default)

ELSEIF $K_\alpha \Delta_{\alpha 0} < 0$ THEN σ_3=4 (vortex lift must decay fast)

IF $TESF$=False AND $K_q \Delta_{\alpha 0} < 0$ THEN σ_3=1 (default)

DSMod Logical Flags

The options implemented in the code are as follows:
IF Gonzalez THEN
  Replace Eq. (1.55) with Eq. (1.57)
  Replace Eq. (1.52) with Eq. (1.52b)
  Replace Eq. (1.50) with Eq. (1.50b)
  Replace Eq. (1.47) with Eq. (1.48)
  Replace Eq. (1.39) with Eq. (1.43)
  Replace Eq. (1.37) with Eq. (1.38)
  Replace Eq. (1.35) with Eq. (1.36)
  Add Eq. (1.16) to $C^c_{n_{\alpha,q}}(s, M)$ Eq. (1.13)

IF Minemma THEN
  Replace Eq. (1.55) with Eq. (1.42)
  Replace Eq. (1.39) with Eq. (1.41)-(1.42)
  Replace Eq. (1.26) with Eq. (1.27)

IF flookup THEN
  Replace Eq. (1.30) with expressions for f'_n f'_n
  IF Gonzalez THEN calculate f''_m

IF CCLBMswitch THEN
  Replace Eq. (1.52) with Eq. (1.53)

2.6.3 UA_CalcOutput

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