The Dynamic Stall Module for FAST 8

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Abstract

The new modularization framework of FAST v.8 [**?**] required a complete overhaul of the aerodynamics routines. AeroDyn is an aerodynamics module that can then utilize either blade element momentum theory (BEMT) or generalized dynamic wake (GDW) to calculate aerodynamic forces on blade elements. Under asymmetric conditions, such as wind shear, yawed and tilted flow, the individual blade elements undergo variations in angle of attack (AOA) that lead to unsteady aerodynamics phenomena, which can no longer be captured through the static airfoil lift and drag look-up tables. This study lays out the main theory and the organization into the modularization framework of the dynamic stall (DS) module, which includes unsteady aerodynamics under attached flow conditions and DS. DS can be called by either BEMT or GDW.

Chapter 1

Overview

The main theory follows the work by **??????**. Dynamic stall is a well-known phenomenon that can affect wind turbine performance and loading especially during yawed operations, and that can result in large unsteady stresses on the structures.

Dynamic stall manifests as a delay in the onset of flow separation to higher AOAs that would otherwise occur under static (steady) conditions, followed by an abrupt flow separation from the leading edge (LE) of the airfoil **?**. The LE separation is the fundamental characteristic of the DS of an airfoil; in contrast, quasi-steady stall would start from the airfoil trailing edge (TE).

DS occurs for reduced frequencies above 0.02.

$$k = \frac{\omega c}{2U} \tag{1.1}$$

The five stages of DS are as follows and shown in Fig. 1 and Fig. 1:

1. Onset of flow reversal
2. Flow separation and vorticity accumulation at the LE
3. Shedding of the vortex and convection along the suction surface of the airfoil (lift increases)
4. Lift Stall: vortex is shed in the wake and lift abrupt drop-off
5. Re-attachment of the flow at AOAs considerably lower than static AOAs (hysteresis)

The model chosen to represent unsteady aerodynamics and dynamic stall is the Leishman-Beddoes model (LBM), because it is the most widely used and has the most available support throughout the community and it has shown

Figure 1.1: Conventional stages of DS, from **?**.

Figure 1.2: Conventional stages of DS and associated C_l, C_d, C_m as functions of AOA from **?**.

Figure 1.3: Main definitions of BE forces (denoted via their normalized coefficients) for the unsteady aerodynamics treatment, from **?**.

reasonable success when compared to experimental data. The LBM is a postdictive model, and as such it will not solve equations of motion, though the principles are fully rooted in the physics of unsteady flow.

In the LBM, the different processes are modeled as first-order subsystems with differential equations with pre-determined constants to match experimental results. Therefore, knowledge of the airfoil characteristics under unsteady aerodynamics is a prerogative of the LBM. The LBM may also be described as an indicial response (i.e.,response to a series of small disturbances) model for attached flow, extended to account for separated flow effects and vortex lift. Forces are computed as normal and tangential (to chord) and pitching moment about the 1/4-chord location, see also Fig. 1 and Eq. (1.2).

$$\begin{aligned} C_l &= C_n \cos\alpha + C_c \sin\alpha \\ C_d &= C_n \sin\alpha - C_c \cos\alpha + C_{d0} \end{aligned} \tag{1.2}$$

$$\begin{aligned} C_n &= C_l \cos\alpha + (C_d - C_{d0}) \sin\alpha \\ C_c &= C_l \sin\alpha - (C_d - C_{d0}) \cos\alpha \end{aligned} \tag{1.3}$$

The original model was developed for helicopters, but it has been successfully applied to wind turbines (see **??**). Yawed flowed conditions, Coriolis and centrifugal forces that lead to three-dimensional effects were not included in the original model.

Unsteady aerodynamics is mostly driven by 2D flow aspects, where the time scale is on the order of tenths of seconds, or $\sim c/\omega R$.

The LBM considers a number of unsteady aerodynamics conditions, namely: attached flow conditions and TE separation before stall; delays and lags associated with the unsteady onset of dynamic stall and accompanying boundary layer development; advection of the LE vortex, shedding in the wake, and suppression of TE separation in favor of LE separation. The LBM can be subdivided into three main submodules:

1. Unsteady, Attached Flow Solution via Indicial Treatment (potential flow)

2. TE Flow Separation

3. DS and Vorticity Advection

1.1 Unsteady Attached Flow and Its Indicial Treatment

The advantage of the indicial treatment is that a response to an arbitrary forcing can be obtained through superposition of response-functions to a step variation in AOA, in pitch rate, or in heave (plunging) motion. The superposition is carried out via the so-called Duhamel Integral **?**, which for the generic response $F_R(t)$ to a generic disturbance $\epsilon(t)$ can be written as in Eq. (1.4):

$$\begin{aligned} F_R(t) &= \epsilon(0)\phi(t,M) + \int_0^t \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\sigma_t}(\sigma_t)\,\phi(t-\sigma_t,M)\mathrm{d}\sigma_t \\ \text{or} \\ F_R(s) &= \epsilon(0)\phi(s,M) + \int_0^s \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\sigma_s}(\sigma_s)\,\phi(s-\sigma_s,M)\mathrm{d}\sigma_s \end{aligned} \tag{1.4}$$

where the non-dimensional distance, s, is defined as:

$$s = \frac{2}{c}\int_0^t U(t)dt \tag{1.5}$$

where the airfoil half chord is considered as the non-dimensionalizing factor $(c/2)$.

The indicial functions are surmised into two components: the first is related to the non-circulatory loading (piston theory and acoustic wave theory); the second originates from the development of circulation about the airfoil.

The non-circulatory part depends on the instantaneous airfoil motion, but also on the time history of the prior motion. The circulatory response can be calculated via the 'lumped approach', where the effects of step changes in AOA, pitch rate, heave motion, etc., are combined into an effective AOA at the 3/4-chord station.

The normal force coefficient response to a step-change in non-dimensional pitch rate q and a step-change in AOA can be written as a function of the indicial functions as shown in Eq. (1.6):

$$\begin{aligned} C_{n_{\alpha,q}}(s,M) &= C_{n_\alpha}(s,M) + C_{n_q}(s,M) = C_{n\alpha}\alpha + C_{nq}(s,M)q \\ C_{n_\alpha}(s,M) &= \tfrac{4}{M}\phi_\alpha^{nc} + \tfrac{C_{n\alpha}}{\beta_M}\phi_\alpha^c \\ C_{n_q}(s,M) &= \tfrac{1}{M}\phi_q^{nc} + \tfrac{C_{n\alpha}}{2\beta_M}\phi_q^c \end{aligned} \tag{1.6}$$

Analogously the pitching moment coefficient about the 1/4-chord can be derived via indicial response as shown in Eq.(1.7):

$$\begin{aligned} C_{m_{\alpha,q}}(s,M) &= C_{m_\alpha}(s,M) + C_{m_q}(s,M) = C_{m\alpha}\alpha + C_{mq}(s,M)q \\ C_{m_\alpha}(s,M) &= -\tfrac{1}{M}\phi_{m,\alpha}^{nc} - \tfrac{C_{n\alpha}}{\beta_M}\phi_\alpha^c\,(\hat{x}_{AC} - 0.25) + C_{m0} \\ C_{m_q}(s,M) &= -\tfrac{7}{12M}\phi_{m,q}^{nc} - \tfrac{C_{n\alpha}}{16\beta_M}\phi_{m,q}^c \end{aligned} \tag{1.7}$$

where C_{m0} is positive if causes a pitch up of the airfoil, as seen in Fig. 1. Also note that the circulatory component of the pitching moment response to a step-change in α is a function of the $C_{n_\alpha}^c(s,M)$.

The non-dimensional pitch-rate q is given by:

$$\begin{array}{rcl} q & = & {}^{\dot{\alpha}c}/_{U} \simeq {}^{K_{\alpha t}c}/_{U} \\ \text{with :}\quad K_{\alpha t} & = & \frac{\alpha_t - \alpha_{t-1}}{\Delta t} \end{array} \tag{1.8}$$

The indicial responses can then be approximated as in Eq. (1.9) [**??**]:

$$\begin{array}{rcl} \phi_\alpha^c & = & \phi_q^c = 1 - A_1 \exp\left(-b_1\beta_M^2 s\right) - A_2 \exp\left(-b_2\beta_M^2 s\right) \\ \phi_\alpha^{nc} & = & \exp\left(-\frac{s}{T'_\alpha}\right) \\ \phi_q^{nc} & = & \exp\left(-\frac{s}{T'_q}\right) \\ \phi_{m,q}^{nc} & = & \exp\left(-\frac{s}{T'_{m,q}}\right) \end{array} \tag{1.9}$$

One could find analogous expressions for $\phi_{m,q}^c$,$\phi_{m,\alpha}^{nc}$, but they are not shown here because further simplified expressions will be derived below.

By making use of exact results for short times $0 \leq s \leq 2M/(M+1)$ [**?**], **?** shows that:

$$\begin{array}{rcl} T_\alpha(M) & = & \frac{c}{2U}T'_\alpha = \dfrac{c}{2Ma_s}T'_\alpha = k_\alpha(M)T_I \\ T_q(M) & = & \frac{c}{2U}T'_q = \dfrac{c}{2Ma_s}T'_q = k_q(M)T_I \end{array} \tag{1.10}$$

where:

$$\begin{array}{rcl} k_\alpha(M) & = & \left[(1-M) + \frac{C_{n\alpha}}{2}M^2\beta_M\left(A_1b_1 + A_2b_2\right)\right]^{-1} = \left[(1-M) + \frac{C_{n\alpha}}{2}M^2\beta_M 0.413\right]^{-1} \\ k_q(M) & = & \left[(1-M) + C_{n\alpha}M^2\beta_M\left(A_1b_1 + A_2b_2\right)\right]^{-1} = \left[(1-M) + C_{n\alpha}M^2\beta_M 0.413\right]^{-1} \\ T_I & = & \frac{c}{a_s} \end{array} \tag{1.11}$$

Note that **?** recommends the use of $0.75T_\alpha(M)$ in place of $T_\alpha(M)$ to account for three-dimensional effects not included in piston theory.

The values of the A_1-b_2 constants are independent of M and are determined from experimental data on oscillating airfoils in wind tunnels.

For the circulatory component of the aerodynamic force response, the lumped approach can lead to a direct solution of $C_{n_{\alpha,q}}^c(s,M)$. considering the circulatory part $C_{n_\alpha}^c(s,M)$ of Eq. (1.6) for the response to the step in α, one can write:

$$C_{n_\alpha}^c(s,M) = \int_{s_0}^{s} \frac{C_{n\alpha}}{\beta_M}\phi_\alpha^c\alpha(s)\mathrm{d}s \simeq C_{n\alpha}^c(s,M)\Delta\alpha \tag{1.12}$$

By using Eq. (1.4) with $\phi(s,M)$ replaced by ϕ^c_α and $\epsilon(s)$ by α, Eq. (1.12) rewrites:

$$C^c_{n_{\alpha,q}}(s,M) = C^c_{n\alpha}(s,M)\left[\alpha(s_0)\phi^c_\alpha(s) + \int_{s_0}^{s} \frac{\mathrm{d}\alpha}{\mathrm{d}\sigma_s}(\sigma_s)\,\phi^c_\alpha(s-\sigma_s,M)\mathrm{d}\sigma_s\right] = C^c_{n\alpha}(s,M)\alpha_e \tag{1.13}$$

where α_e is an effective angle of attack at 3/4-chord accounting for a step variation in α, pitching rate, heave, and velocity (lumped approach). By applying the first of Eq. (1.9), and setting $s_0 = 0$, Eq. (1.13) can be simplified to arrive at an expression for α_e at the n-th time step, i.e.,α_{e_n}:

$$\alpha_{e_n}(s,M) = (\alpha_n - \alpha_0) - X_{1_n}(\Delta s) - X_{2_n}(\Delta s) \tag{1.14}$$

where the $\int_{s_0}^{s}[...]\mathrm{d}\sigma_s$ was divided into two steps considering a distance interval Δs, i.e.,$\int_0^s[...]\mathrm{d}\sigma_s$ and $\int_s^{s+\Delta s}[...]\mathrm{d}\sigma_s$, by carrying out the algebra a recursive expression for X_1 and X_2 can be found:

$$\begin{aligned} X_{1_n} &= X_{1_{n-1}}\exp\left(-b_1\beta_M^2\Delta s\right) + A_1\exp\left(-b_1\beta_M^2\tfrac{\Delta s}{2}\right)\Delta\alpha_n \\ X_{2_n} &= X_{2_{n-1}}\exp\left(-b_2\beta_M^2\Delta s\right) + A_2\exp\left(-b_2\beta_M^2\tfrac{\Delta s}{2}\right)\Delta\alpha_n \end{aligned} \tag{1.15}$$

Note that α_0 was introduced into Eq.(1.14), because α_e is an effective angle of incidence (AOI) and not AOA.

Similarly to the above development, the circulatory contribution to $C^c_{n_{\alpha,q}}(s,M)$ from a step change in q can be derived as:

$$C^c_{n_q}(s,M) = \frac{C^c_{n\alpha}(s,M)}{2}(q_n - q_0) - X_{3_n}(\Delta s) - X_{4_n}(\Delta s) \tag{1.16}$$

However, the lumped approach can account for any effect to α, including step changes in q, so Eq. (1.16) is not necessary, and it is virtually included via Eq. (1.13) and (1.14).

The non-circulatory part cannot be handled via the superposition (lumped approach), therefore the contribution from step changes in α and q need to be kept separate:

$$C^{nc}_{n_{\alpha,q}}(s,M) = C^{nc}_{n_\alpha}(s,M) + C^{nc}_{n_q}(s,M) \tag{1.17}$$

Now, using Duhamel's integral (1.4) on the non-circulatory component $C^{nc}_{n_\alpha}(s,M)$ (see Eq.(1.17)) with the ϕ^{nc}_α from Eq.(1.9), one can arrive at:

$$\begin{aligned} C^{nc}_{n_\alpha}(s,M) &= \tfrac{4T_\alpha(M)}{M}\left(K_{\alpha t} - K'_{\alpha t}\right) \\ K_{\alpha t} &= \tfrac{\alpha_t - \alpha_{t-1}}{\Delta t} \\ K'_{\alpha t} &= K'_{\alpha t-1}\exp\left(-\tfrac{\Delta t}{T_\alpha(M)}\right) + (K_{\alpha t} - K_{\alpha t-1})\exp\left(-\tfrac{\Delta t}{2T_\alpha(M)}\right) \end{aligned} \tag{1.18}$$

Note that in Eq. (1.18), K'_α is the deficiency function for $C^{nc}_{n_\alpha}(s,M)$.

For $C^{nc}_{n_q}(s,M)$, an analogous procedure leads to:

$$\begin{aligned}
C^{nc}_{n_q}(s,M) &= \tfrac{T_q(M)}{M}\left(K_{q_t} - K'_{q_t}\right)\\
K_{q_t} &= \tfrac{q_t - q_{t-1}}{\Delta t}\\
K'_{q_t} &= K'_{q_{t-1}}\exp\left(-\tfrac{\Delta t}{T_q(M)}\right) + \left(K_{q_t} - K_{q_{t-1}}\right)\exp\left(-\tfrac{\Delta t}{2T_q(M)}\right)
\end{aligned} \tag{1.19}$$

So finally, the expression for the total normal force under attached conditions can be expressed as:

$$C_n^{pot} = C^c_{n_{\alpha,q}}(s,M) + C^{nc}_{n_{\alpha,q}}(s,M) = C^c_{n\alpha}(s,M)\alpha_e + \frac{4T_\alpha(M)}{M}\left(K_{\alpha t} - K'_{\alpha t}\right) + \frac{T_q(M)}{M}\left(K_{q_t} - K'_{q_t}\right) \tag{1.20}$$

Now turning back to the pitching moment, following **?** the circulatory component $C^c_{m_q}(s,M)$ can be written as:

$$C^c_{m_q}(s,M) = -\frac{C_{n\alpha}}{16\beta_M}\left(q - K'''_q\right)\frac{c}{U} \tag{1.21}$$

where

$$K'''_{q\ t} = K'''_{q\ t-1}\exp\left(-b_5\beta_M^2\Delta s\right) + A_5\Delta q_t\exp\left(-b_5\beta_M^2\frac{\Delta s}{2}\right) \tag{1.22}$$

The non-circulatory component of the pitching moment response to step-change in α, $C^{nc}_{m_\alpha}(s,M)$, writes [**??**]:

$$C^{nc}_{m_\alpha}(s,M) = -\frac{1}{M}\phi^{nc}_{m,\alpha} = -\frac{C^{nc}_{n_\alpha}(s,M)}{4} \tag{1.23}$$

which implies

$$\phi^{nc}_{m,\alpha} = \phi^{nc}_\alpha \tag{1.24}$$

The other component $C^{nc}_{m_q}(s,M)$ writes [**?**]:

$$
\begin{aligned}
C^{nc}_{m_q}(s,M) &= -\tfrac{7}{12M}\phi^{nc}_{m,q} = -\tfrac{7k_{m,q}(M)^2T_I}{12M}\left(K_q - K''_q\right)
\end{aligned}
$$

with:

$$
\begin{aligned}
k_{m,q}(M) &= \tfrac{7}{15(1-M)+1.5C_{n\alpha}A_5b_5\beta_M M^2} \\
T_{m,q}(M) &= k_{m,q}(M)T_I \\
K''_{q\,t} &= K''_{q\,t-1}\exp\left(-\tfrac{\Delta t}{k_{m,q}(M)^2T_I}\right) + \left(K_{q_t} - K_{q_{t-1}}\right)\exp\left(-\tfrac{\Delta t}{2k_{m,q}(M)^2T_I}\right)
\end{aligned}
\tag{1.25}
$$

where the same procedure as before was used to arrive at a deficiency function using Duhamel's integral (Eq. (1.4)) and equating the expressions of the $C_{m_{\alpha,q}}(s,M)$ at $s \to 0$ (see **?**).

Finally, the expression for the total pitching moment at 1/4-chord under attached conditions can be expressed as:

$$
\begin{aligned}
&C^{pot}_m = C^c_{m_{\alpha,q}}(s,M) + C^{nc}_{m_{\alpha,q}}(s,M) = \\
&C_{m0} - \tfrac{C_{n\alpha}}{\beta_M}\phi^c_\alpha\left(\hat{x}_{AC} - 0.25\right) + \\
&-\tfrac{C_{n\alpha}}{16\beta_M}\left(q - K'''_q\right)\tfrac{c}{U} + \\
&-\tfrac{T_\alpha(M)}{M}\left(K_{\alpha t} - K'_{\alpha t}\right) + \\
&-\tfrac{7k_{m,q}(M)^2T_I}{12M}\left(K_q - K''_q\right)
\end{aligned}
\tag{1.26}
$$

Note that the pitching moment treatment is slightly different from what is in AeroDyn v13 and **?**, and it is more in line with **?** and **?**. If **?** suggestions is used, then the $C^{nc}_{m_q}(s,M)$ (last term in Eq. (1.26)) is to be replaced by:

$$
C^{nc}_{m_q}(s,M) = -\frac{7}{12M}\phi^{nc}_{m,q} = -\frac{C^{nc}_{n_q}(s,M)}{4} - \frac{k_\alpha(M)^2T_I}{3M}\left(K_q - K''_q\right) \tag{1.27}
$$

1.1.1 Tangential Force

The tangential force along the chord can be written as in Eq. (1.28) from **?**:

$$
C^{pot}_c = C^{pot}_n \tan\left(\alpha_e + \alpha_0\right) \tag{1.28}
$$

In potential flow, D'Alambert's paradox leads to the absence of drag; therefore, from Eq. (1.3), $C_l\cos\alpha = C_n$, and $C_c = C_l\sin\alpha$ which bring forth Eq.(1.28). Since α_e is a virtual AOI at 3/4-chord, we needed to add the α_0.

1.2 TE Flow Separation

The base of this dynamic system is Kirchoff's theory, which can be expressed as follows [?]:

$$\begin{aligned} C_n(\alpha, f) &= C_{n\alpha}(\alpha - \alpha_0)\left(\tfrac{1+\sqrt{f}}{2}\right)^2 \\ C_c(\alpha, f) &= \eta_e C_{n\alpha}(\alpha - \alpha_0)\sqrt{f}\tan(\alpha) \end{aligned} \tag{1.29}$$

with f the separation point distance from LE in percent chord, and η_e the recovery factor $\simeq [0.85 - 0.95]$ to account for viscous effects at limited or no separation on C_c.

If the airfoil's C_l,C_d, and C_m characteristics are known, then Eq.1.29 may be solved for f. **?** suggests the use of best-fit curves obtained from static measurements on airfoils, of the type:

$$f = \begin{cases} 1 - 0.3\exp\left(\tfrac{|\alpha|-\alpha_1}{S_1}\right), \text{if —}\alpha\text{—} < \alpha_1 \\ 0.04 + 0.66\exp\left(\tfrac{\alpha_1-|\alpha|}{S_2}\right), \text{if —}\alpha\text{—} \geq \alpha_1 \end{cases} \tag{1.30}$$

S_1 and S_2 are best-fit constants that define the abruptness of the static stall. α_1 is the angle of attack at f=0.7, approximately the stall angle.

Now accounting for unsteady conditions, the TE separation point gets modified due to temporal effects on airfoil pressure distribution and boundary layer response. LE separation occurs when a critical pressure at the LE, corresponding to a critical value of the normal force C_{n1}, is reached. The circulatory normal force needs to be modified to account for the lagged boundary layer response. In order to arrive at a new expression for C_n, we start by accounting for the separation point location under unsteady conditions, which can be calculated starting from an effective AOI, α_f:

$$\alpha_f = \frac{C_n'}{C_{n\alpha}} + \alpha_0 \tag{1.31}$$

where an effective C_n' is used, calculated as in Eq. (1.32):

$$\begin{aligned} C_n' &= C_n^{pot} - D_p \\ D_{p_t} &= D_{p_{t-1}}\exp\left(-\tfrac{\Delta s}{T_p}\right) + \left(C_n^{pot}{}_t - C_n^{pot}{}_{t-1}\right)\exp\left(-\tfrac{\Delta s}{2T_p}\right) \end{aligned} \tag{1.32}$$

Note T_p is boundary-layer,LE pressure gradient time constant; in the expression of D_p. It should be tuned based on airfoil experimental data. and is an empirically based quantity that should be tuned from experimental data. **?** employs two time constants $T_{p\alpha}$ and T_{pq} as the C_n' is separated into two contributions, one from α and from q.

Given the new C_n', one can attain a new formulation for f, i.e.,f'', which accounts for delays in the boundary layer, and that will be used via Kirchoff's treatment to arrive at the new C_n:

$$\begin{aligned} f'' &= f' - D_f \\ D_{f\,t} &= D_{f\,t-1} \exp\left(-\tfrac{\Delta s}{T_f}\right) + \left(f'_t - f'_{t-1}\right) \exp\left(-\tfrac{\Delta s}{2T_f}\right) \end{aligned} \tag{1.33}$$

where f' is the separation point distance from LE in percent chord under unsteady conditions that can be obtained from the best-fit in Eq. (1.30) replacing α with α_f. Alternatively, f' can be derived from a direct lookup table of static airfoil data reversing Eq. (1.29). In fact, two values of f' could be calculated: one for C_n and one for C_c.

Also note that T_f is a Mach, Re nad airfoil dependent time constant associated with the motion of the separation point along the suction surface of the airfoil. constant dependent on Mach, Re, and airfoil shape; it is used in the expression of D_f and f'' gets modified via multipliers (σ_1) depending on the phase of the separation or reattachment as discussed later; here it suffices noting that T_f can be written as a modified version of the initial value T_{f0}:

$$T_f = T_{f0}/\sigma_1 \tag{1.34}$$

Finally, the normal force coefficient C_n^{fs}, after accounting for separated flow from the TE becomes:

$$C_n^{fs} = C^{nc}_{n_{\alpha,q}}(s,M) + C^{c}_{n_{\alpha,q}}(s,M)\left(\frac{1+\sqrt{f''}}{2}\right)^2 = C^{nc}_{n_{\alpha,q}}(s,M) + C^{c}_{n\alpha}(s,M)\alpha_e\left(\frac{1+\sqrt{f''}}{2}\right)^2 \tag{1.35}$$

Note that **??** propose the corrective factor to be:

$$C_n^{fs} = C^{nc}_{n_{\alpha,q}}(s,M) + C^{c}_{n\alpha}(s,M)\alpha_e\left(\frac{1+2\sqrt{f''}}{3}\right)^2 \tag{1.36}$$

to account for lower values of C_n when f=0 that were seen from experimental data.

1.2.1 Tangential Force

The along-chord force coefficient analogously becomes:

$$C_c = C_n^{pot}\tan\left(\alpha_e + \alpha_0\right)\eta_e\sqrt{f''} = C^{c}_{n\alpha}(s,M)\alpha_e\tan\left(\alpha_e + \alpha_0\right)\eta_e\sqrt{f''} \tag{1.37}$$

? proposes a slightly different formulation:

$$C_c = C^{c}_{n\alpha}(s,M)\alpha_e\tan\left(\alpha_e + \alpha_0\right)\eta_e\left(\sqrt{f''} - 0.2\right) \tag{1.38}$$

This modification is to account for negative values seen at f=0.

1.2.2 Pitching Moment

Now turning to the pitching moment, the contribution due to unsteady separated flow is on the circulatory component alone. **?** suggests using this formulation that modified C_m^{pot} for the $C_{m_\alpha}^c(s,M)$ component:

$$C_m = C_{m0} - C_{n_{\alpha,q}}^c(s,M)(\hat{x}_{cp} - 0.25) + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \tag{1.39}$$

where $\hat{x}_{cp}$ is center-of-pressure distance from LE in percent chord and can be approximated by [**?**]:

$$\hat{x}_{cp} = k_0 + k_1\left(1 - f''\right) + k_2 \sin\left(\pi f''^{k_3}\right) \tag{1.40}$$

where k_0=0.25-$\hat{x}_{AC}$, and the k_1-k_3 constants are calculated via best-fits of experimental data. Other expressions could be used to perform the best fit of $\hat{x}_{cp}$ vs. f from static C_m airfoil data.

? suggests a different approach where an effective lagged AOI is calculated as follows:

$$\begin{aligned} \alpha'_f &= \alpha_f - D_{\alpha f} \\ D_{\alpha f_t} &= D_{\alpha f_{t-1}} \exp\left(-\tfrac{\Delta s}{T_f}\right) + \left(f'_t - f'_{t-1}\right) \exp\left(-\tfrac{\Delta s}{2T_f}\right) \end{aligned} \tag{1.41}$$

then the new AOI is used to derive the contribution to the circulatory component of the pitching moment, which is extracted from a look-up table of static coefficients C_m vs. α.

$$C_m = C_m\left(\alpha_f\right) + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \tag{1.42}$$

The method proposed by **?** uses a third value of f'' extracted from a static data table where it is assumed that $C_m = f_m C_n$ (loosely correlating f to $\hat{x}_{cp}$). Therefore, the method uses the full C_n from Eq. (1.36) and f'' (calculated from the mentioned static lookup table), to resolve the contribution to C_m from the unsteady TE separation.

$$C_m = C_n f''_m + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \tag{1.43}$$

In this case, the two treatments [**??**] seem somewhat equivalent, with the exception that **?** proposes 21 (7 for each f'' related to C_n, C_c, and C_m)different multipliers for T_f depending on the state of the airfoil aerodynamics (e.g.,increasing AOA and above a critical C_{n1}, increasing AOA and below a critical C_{n1}).

1.3 Dynamic Stall

During DS there is shear layer roll-up at the LE, vortex formation, and vortex travel over the upper surface of the airfoil to be subsequently shed in the wake.

The main condition to be met for the shear layer roll up is:

$$\begin{array}{lll} C_n' > C_{n1} & \text{for} & \alpha \geq \alpha_0 \\ C_n' < C_{n2} & \text{for} & \alpha < \alpha_0 \end{array} \tag{1.44}$$

The normal force coefficient contribution from the additional lift associated with the low pressure LE vortex can be written as **?**:

$$C^v_{n\,t} = C^v_{n\,t-1} \exp\left(-\frac{\Delta s}{T_V}\right) + (C_{V\,t} - C_{V\,t-1}) \exp\left(-\frac{\Delta s}{2T_V}\right) \tag{1.45}$$

T_V is the time constant associated with the vortex lift decay process; it is used in the expression of C^v_n. It depends on Re,M, and airfoil class.. T_V gets modified via a multiplier σ_3 to account for various stages of the process as discussed later, here suffice to say that:

$$T_V = T_{V0}/\sigma_3 \tag{1.46}$$

C_V represents the contribution to the normal force coefficient due to accumulated vorticity in the LE vortex. C_V is modeled proportionally to the difference between the attached and separated circulatory contributions to C_n:

$$C_V = C^c_{n_{\alpha,q}}(s, M) - C^c_{n_{\alpha,q}}(s, M) \left(\frac{1+\sqrt{f''}}{2}\right)^2 = C^c_{n\alpha}(s, M)\alpha_e \left(1 - \frac{1+\sqrt{f''}}{2}\right)^2 \tag{1.47}$$

If **?** is used, then C_V writes as:

$$C_V = C^c_{n\alpha}(s, M)\alpha_e \left(1 - \frac{1+2\sqrt{f''}}{3}\right)^2 \tag{1.48}$$

If $\tau_V > T_{VL}$ and if α_f is not moving away from stall (i.e.,$[(\alpha_f - \alpha_0) * (\alpha_{f\,t} - \alpha_{f\,t-1})] > 0)$, then the vorticity is no longer allowed to accumulate, in which cases Eq.(1.45) rewrites as:

$$\begin{array}{lll} & C^v_{n\,t} & = C^v_{n\,t-1} \exp\left(-\frac{\Delta s}{T_{V0}/\sigma_3}\right) \\ \text{with} & \sigma_3 & = 2 \end{array} \tag{1.49}$$

where the decay of the normal force (due to vorticity at the LE) is accelerated at twice the original rate and no further accretion of vorticity is allowed. Eq. (1.49) should also be used when conditions in Eq. (1.44) are not met. Note that T_{VL} represents the time constant associated with the vortex advection process; it represents the non-dimensional time in semi-chords, needed for a vortex to travel from LE to TE; it is used in the expression of C^v_n. It depends on Re, M (weakly), and airfoil. Value's range = [6; 13].

Finally the total normal force can be written as:

$$C_n = C^{fs}_n + C^v_n = C^c_{n\alpha}(s, M)\alpha_e \left(\frac{1+\sqrt{f''}}{2}\right)^2 + C^{nc}_{n_{\alpha,q}}(s, M) + C^v_n \tag{1.50}$$

Again, if **?** is used, then the correction factor for the separated flow treatment is slightly modified as in Eq.(1.48).

Note that multiple vortices can be shed at a given shedding frequency corresponding to:

$$T_{sh} = 2\frac{1-f''}{St_{sh}} \tag{1.51}$$

Therefore τ_V is reset to 0 if $\tau_V = 1 + \frac{T_{sh}}{T_{VL}}$.

1.3.1 Tangential Force

The along-chord force coefficient gets modified by the presence of the LE vortex as [**?**]:

$$C_c = \eta_e C_c^{pot}\sqrt{f''} + C_n^v \tan\left(\alpha_e\right)\left(1-\tau_V\right) \tag{1.52}$$

Note: **?** does not contain the vortex contribution to C_c based on experimental validation.

The original [**?**] model had C_c written as:

$$C_c = \begin{cases} \eta_e C_c^{pot}\sqrt{f''}\sin\left(\alpha_e+\alpha_0\right) & , \quad C_n' \leq C_{n1} \\ \hat{k}_1 + C_c^{pot}\sqrt{f''}f''^{\hat{k}_2}\sin\left(\alpha_e+\alpha_0\right) & , \quad C_n' > C_{n1} \end{cases} \tag{1.53}$$

1.3.2 Pitching Moment

? offers a form for the $\hat{x}_{cp}^v$, which is the center-of-pressure distance from the 1/4-chord, in percent chord, during the LE vortex advection process:

$$\begin{aligned} C_m^v &= -\hat{x}_{cp}^v C_n^v \\ \hat{x}_{cp}^v(\tau_V) &= \bar{\bar{x}}_{cp}\left(1-\cos\left(\frac{\pi\tau_V}{T_{VL}}\right)\right) \end{aligned} \tag{1.54}$$

Finally, the final expression for the total pitching moment can ve written as:

$$C_m = C_{m0} - C_{n_{\alpha,q}}^c(s,M)(\hat{x}_{cp}-0.25) + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) + C_m^v \tag{1.55}$$

If **?**'s approach is used then Eq.(1.55) rewrites as:

$$C_m = C_m\left(\alpha_f\right) + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) + C_m^v \tag{1.56}$$

and if **?**'s treatment is used then the total moment becomes:

$$C_m = C_n f_m'' + C_{m_q}^c(s,M) + C_{m_\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) + C_m^v \tag{1.57}$$

Chapter 2

Inputs, Outputs, Parameters, and States

2.1 Init_Inputs

The Init_Inputs to the LBM are:

- Airfoil static tables of C_l C_d C_m
- f values as a function of α extracted from the airfoil tables using Kirchoff's Eq.(1.29); optionally, if CENER's treatment is used, a third value for f extracted from CENER's approximation $C_m = f_m C_n$ and the C_n C_m values.
- bbb
- ccc

2.2 Inputs u

The Inputs to the LBM are:

- α
- Re
- q
- M

2.3 Outputs y

The Outputs from the LBM are:

- C_n
- C_c
- C_m

2.4 States x

The States from the LBM are:

- α_e
- α_f
- s
- $\dot{\alpha}$ or K_α
- $\dot{q}$ or K_q
- K'_α
- K'_q
- K''_q
- K'''_q
- D_p
- D_f
- $k_{m,q}(M)$, $k_\alpha(M)$, $k_q(M)$
- T_I
- f'
- f''
- $\hat{x}_{cp}$
- C_n^v
- C_V
- α'_f $D_{\alpha f}$ - (only if **?** treatment is used)

2.5 Parameters p

The Parameters from the LBM are airfoil specific quantities:

- $A_1 b_1 A_2 b_2 A_5 b_5$
- T_p, fairly independent of airfoil type
- T_f
- T_V
- T_{VL}
- α_0
- $C_{n\alpha}$
- k_0, k_1, k_2, k_3
- C_{n1}, C_{n2}
- $\bar{\bar{x}}_{cp}$
- St_{sh}

2.6 Dynamic stall Implementation

2.6.1 DS_Init

2.6.2 DS_UpdateStates

The model is of the parsimonius, open loop, Kelvin-chain kind. Outputs of one subsystems go into inputs of the next subsystems. There are no differential equations to solve. There is no solver per se, for this reason states are discrete states only.

Main Logical Flags

- IF $C'_n > C_{n1}$ ($C'_n < C_{n2}$ for $\alpha < \alpha_0$) THEN
  leading edge separation flag (LESF)=TRUE: this means LE separation can occur.
  ELSE LESF=False: this means reattachment can occur.

- IF ${f''}_t < {f''}_{t-1}$ THEN
  trailing edge separation flag (TESF)=True: this means TE separation is in progress.
  ELSE TESF=False: this means TE reattachment is in progress.

- IF $0 < \tau_V \leq 2T_{VL}$ THEN
  vortex advection flag (VRTX)=True: this means vortex advection is in progress.
  ELSE VRTX=False: this means vortex is in wake.
- IF $\tau_V \geq 1 + \dfrac{T_{sh}}{T_{VL}}$ THEN
  τ_V is reset to 0.

T_f modifications

The following conditional statements operate on a multiplier σ_1 that affects T_f, i.e.,the actual T_f is given by Eq.(1.34).

$$T_f = T_{f0}/\sigma_1 \qquad \text{(1.34 revisited)}$$

where T_{f0} is the initial value of T_f.

$\sigma_1 = 1$ (initialization default value)

$\Delta_{\alpha 0} = \alpha\text{-}\alpha_0$

IF TESF=True THEN: (separation)

IF $K_\alpha \Delta_{\alpha 0} < 0$ THEN $\sigma_1 = 2$ (accelerate separation point movement)

ELSEIF LESF=False THEN $\sigma_1 = 1$ (default value, LE separation can occur)

ELSEIF $f''_{n-1} \leq 0.7$ THEN $\sigma_1 = 2$ (accelerate separation point movement if separation is occurring)

ELSE σ_1 =1.75 (accelerate separation point movement)

ELSE: (reattachment, this means TESF=False)

IF LESF = False THEN $\sigma_1 = 0.5$ (default: slow down reattachment)

IF VRTX=True AND $0 \leq \tau_V \leq T_{VL}$ THEN
$\sigma_1 = 0.25$ - No flow reattachment if vortex shedding in progress

IF $K_\alpha \Delta_{\alpha 0} > 0$ THEN $\sigma_1 = 0.75$

Note the last three conditional statements are separate IFs.

T_V modifications

For T_V, an analogous set of conditions is used to set the proper value of the time constant depending on subsystem stages:

σ_3=1 (initialization default value)

IF $T_{VL} \leq \tau_V \leq 2T_{VL}$ THEN
  σ_3=3 (post-shedding)
  IF TESF=False THEN
    σ_3=4 (accelerate vortex lift decay)
    IF VRTX=True AND $0 \leq \tau_V \leq T_{VL}$ THEN:
      IF $K_\alpha \Delta_{\alpha 0} < 0$ THEN σ_3=2 (accelerate vortex lift decay)
      ELSE σ_3=1 (default)

ELSEIF $K_\alpha \Delta_{\alpha 0} < 0$ THEN σ_3=4 (vortex lift must decay fast)

IF TESF=False AND $K_q \Delta_{\alpha 0} < 0$ THEN σ_3=1 (default)

2.6.3 DS_CalcOutput

Bibliography