



# Virtual and Augmented Reality

CS-GY 9223/CUSP-GX 6004

<https://nyu-icl.github.io/courses/2022fall-vr-ar>

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# Final Presentation Next Week

Submit **EO Sun**: We need to prepare

You shall have voice-over

No need for ultra-high res: 720P-1080P mp4 is fine enough

Report/code to open soon (don't worry about them now)

1-1s Today

# Spatial Sound

- what is sound? how do we synthesize it?
- the human auditory system
- stereophonic sound
- spatial audio of point sound sources
- surround sound
- ambisonics

# What is Sound?

- “sound” is a pressure wave propagating in a medium
- speed of sound is  $c = \sqrt{K/\rho}$  where  $c$  is velocity,  $\rho$  is density of medium and  $K$  is elastic bulk modulus
- in air, speed of sound is 340 m/s
- in water, speed of sound is 1,483 m/s

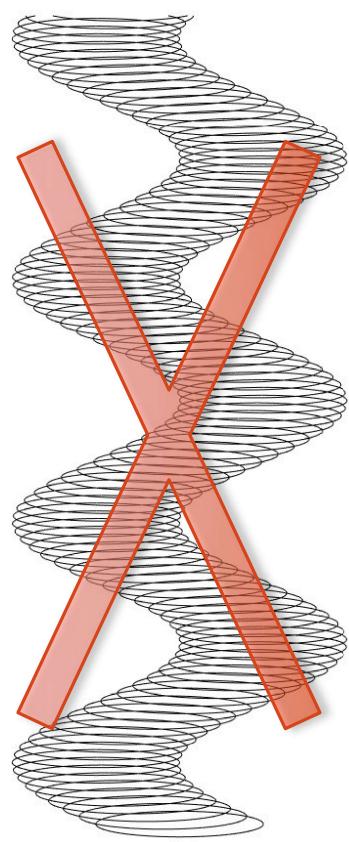
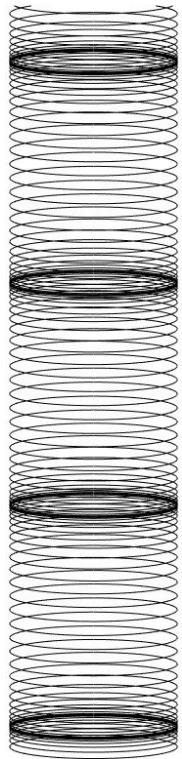
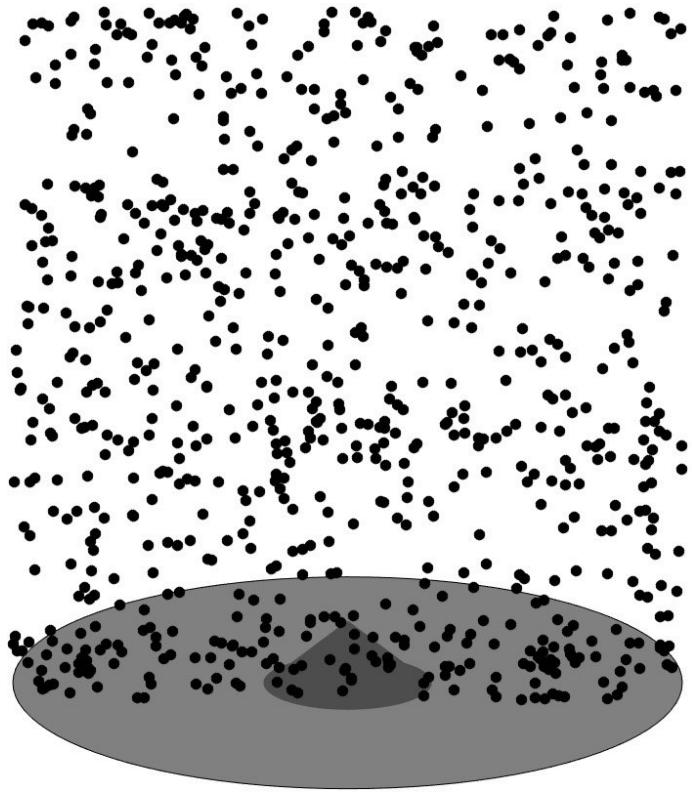
# How do we Synthesize Sound?



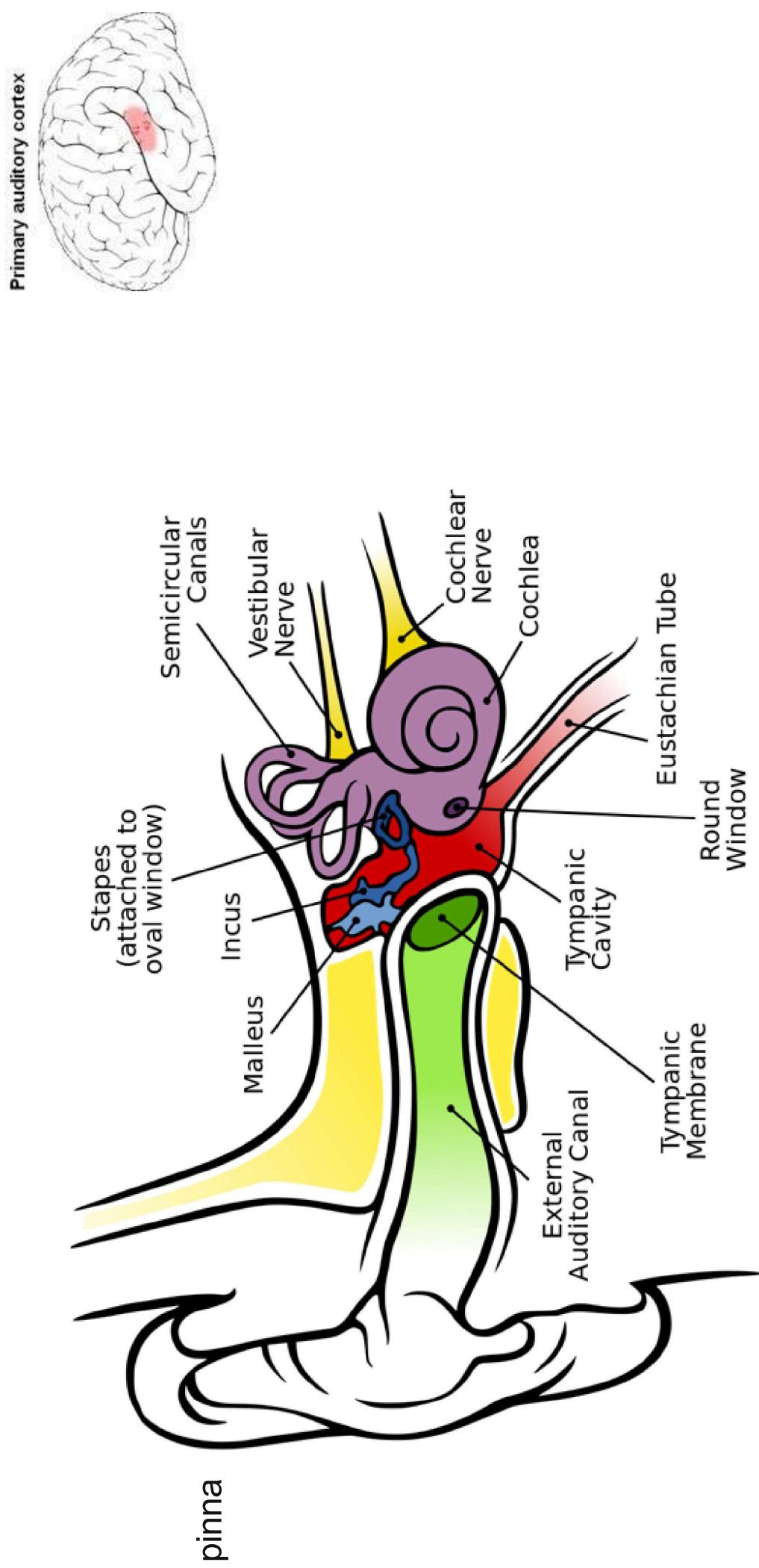
<https://www.youtube.com/watch?v=aDrs6EieFCM>

# Producing Sound

- Sound is longitudinal vibration of air particles
- Speakers create wavefronts by physically compressing the air, much like one could a slinky

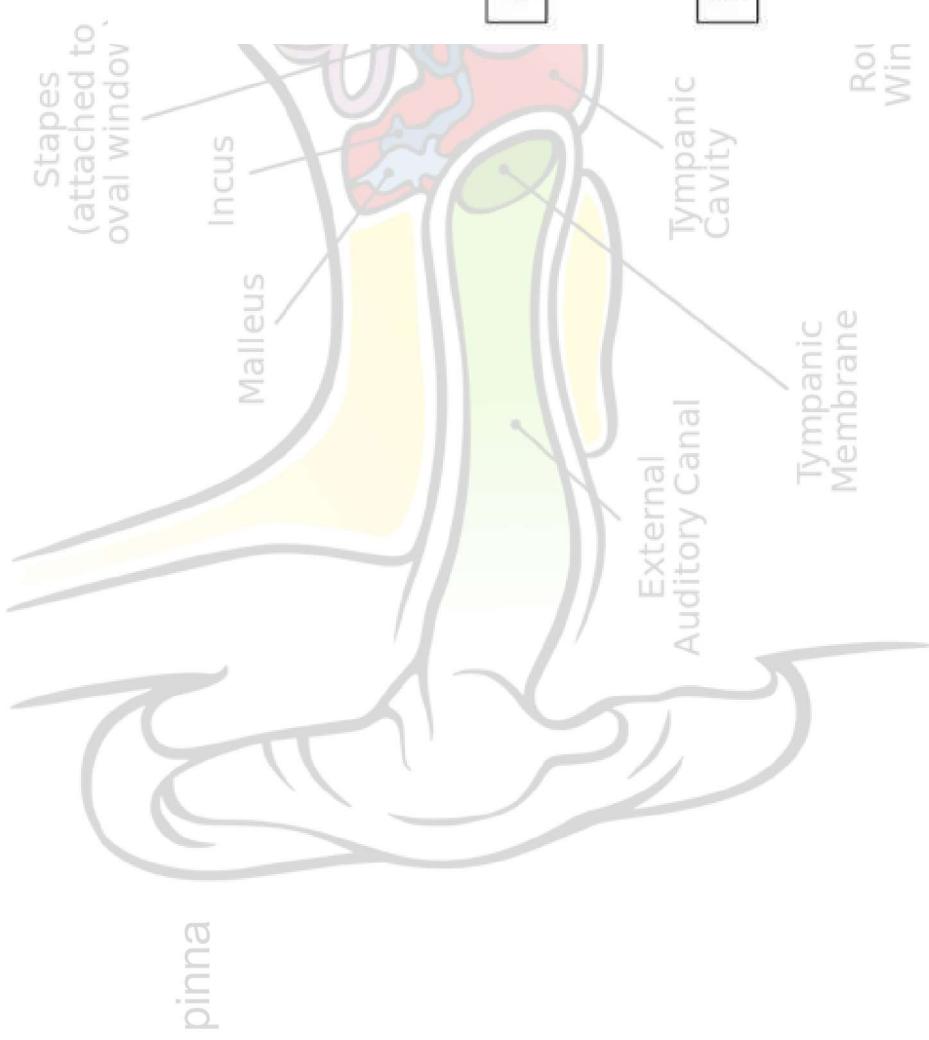


# The Human Auditory System

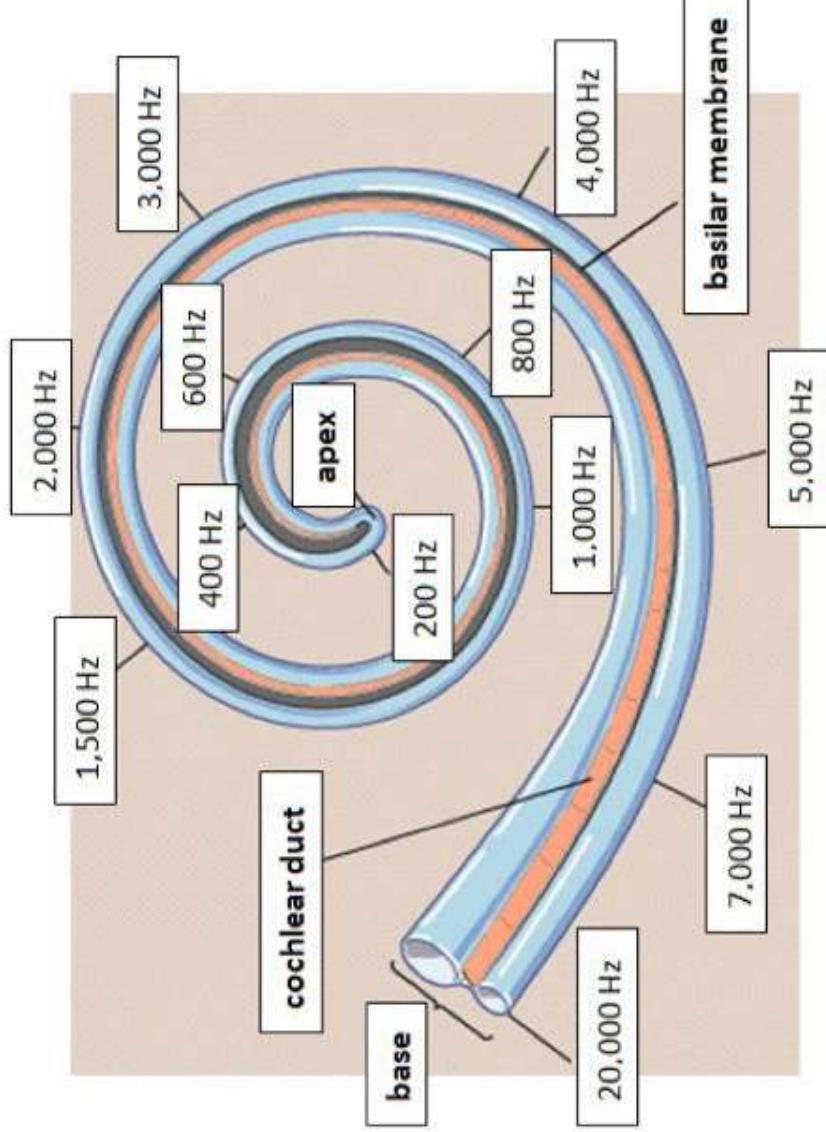


# The Human Auditory System

- hair receptor cells pick up vibrations



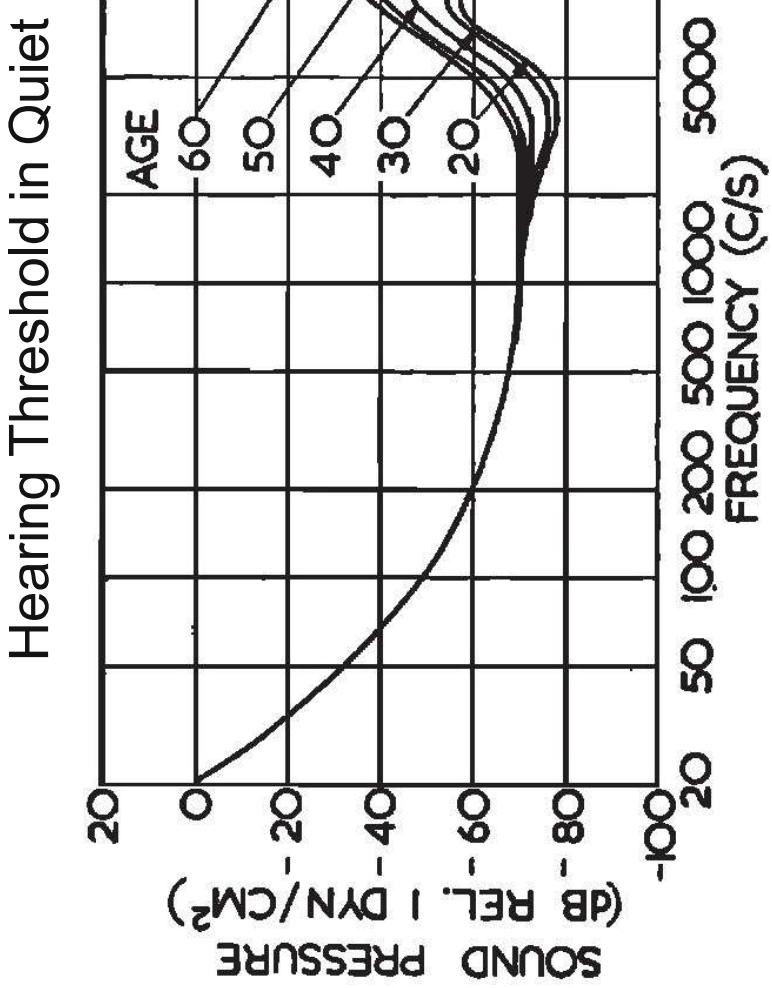
cochlea



wikipedia

# The Human Auditory System

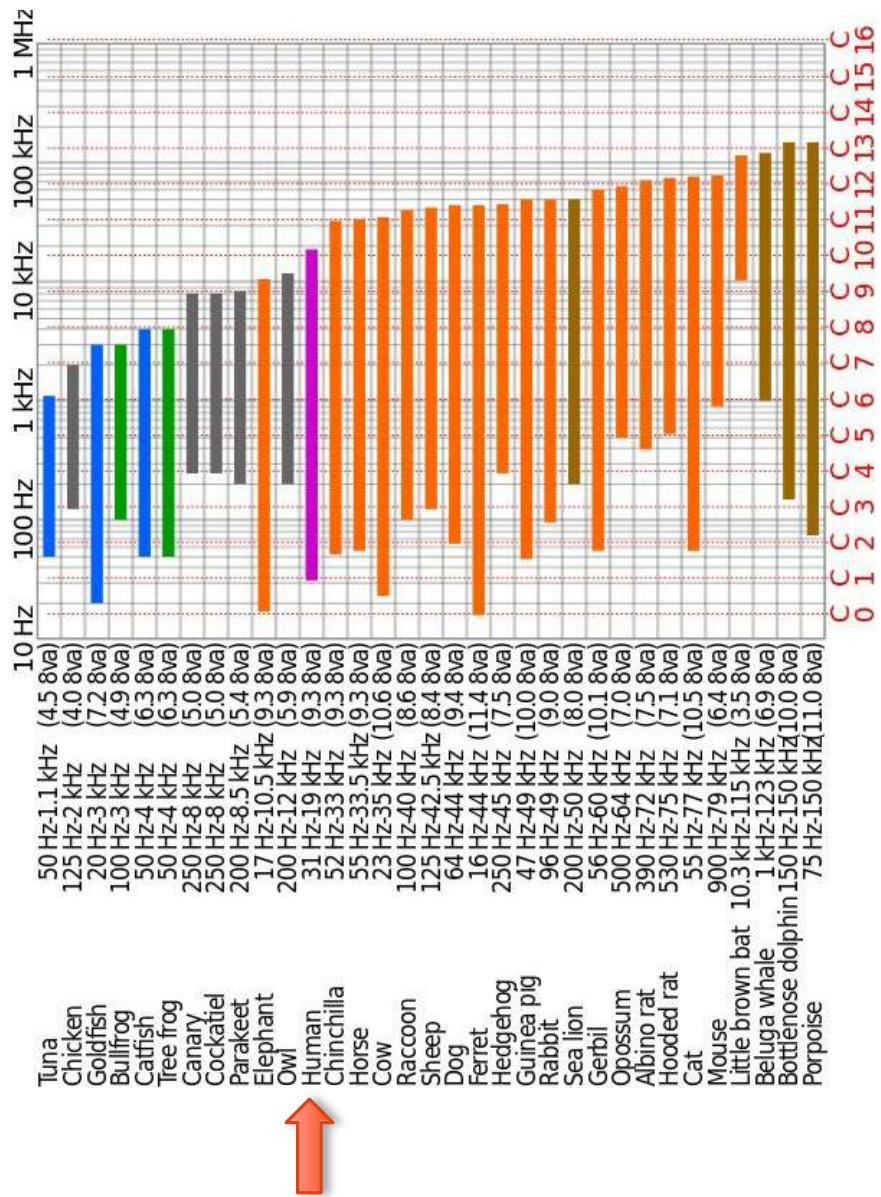
- Human hearing range:  
~20–20,000 Hz
- Variation between individuals
- Degrades with age



D. W. Robinson and R. S. Dadson, 1957

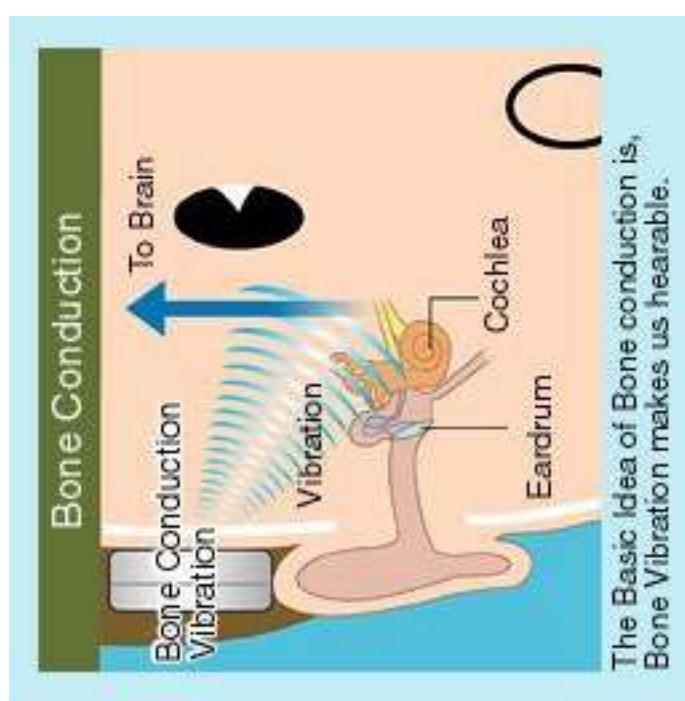
# The Human Auditory System

- human hearing range:  
~20 – 20,000 Hz
- variation between individuals and changes with age



# Bone Conduction

- can stimulate eardrum mechanically to create the illusion of audio, e.g. with bone conduction

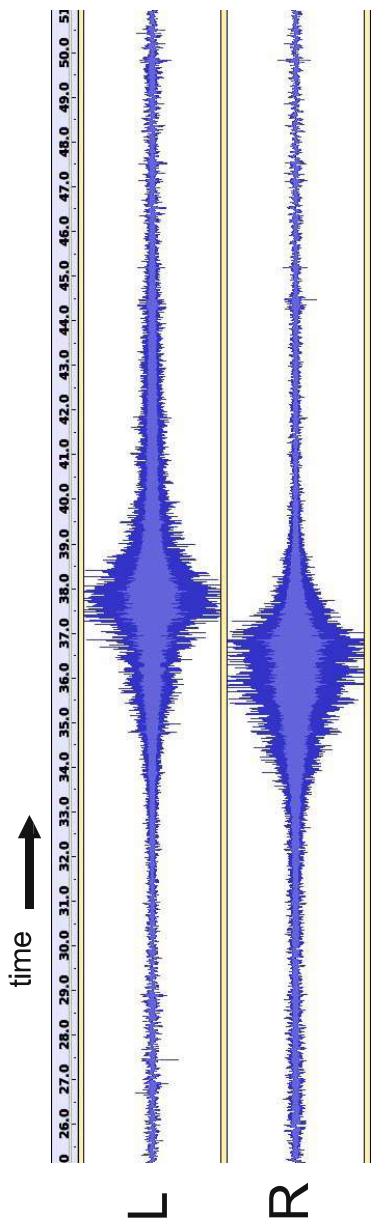
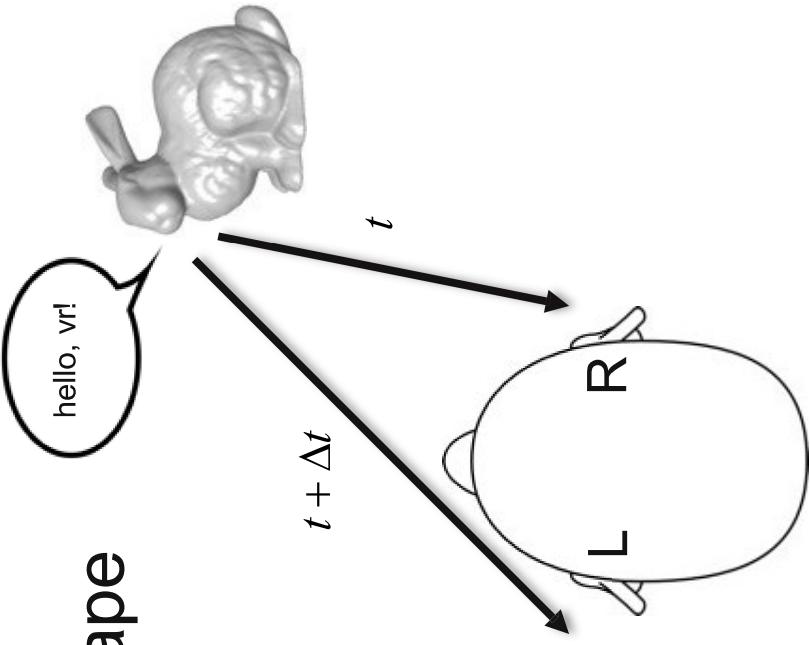


<http://www.goldendance.co.jp/English/boneconduct/01.html>

the verge

# Stereophonic Sound

- mainly captures differences between the ears:
  - interaural time difference
  - amplitude differences from body shape (nose, head, neck, shoulders, ...)



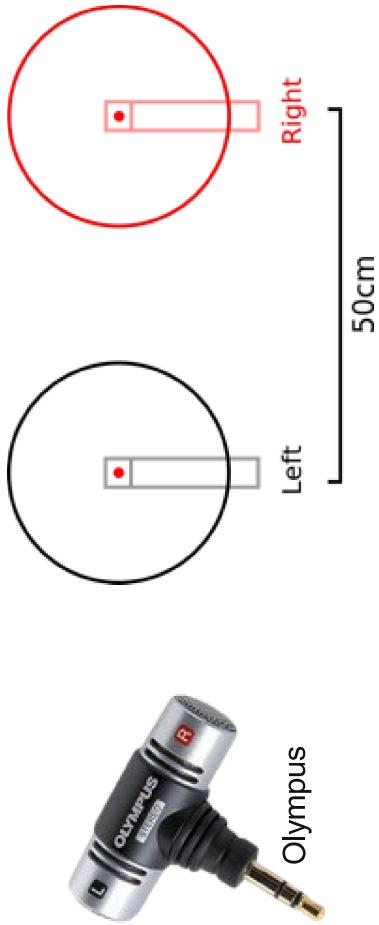
wikipedia

# Stereophonic Sound Recording

- use two microphones

- A-B techniques captures differences in time-of-arrival

wikipedia



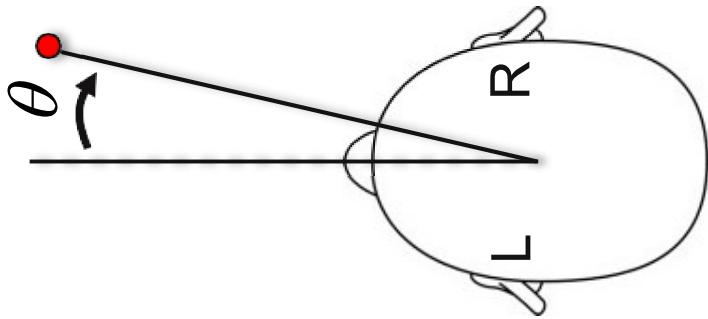
X-Y technique

- other configurations work too,  
capture differences in amplitude



# Head-related Impulse Response (HRIR)

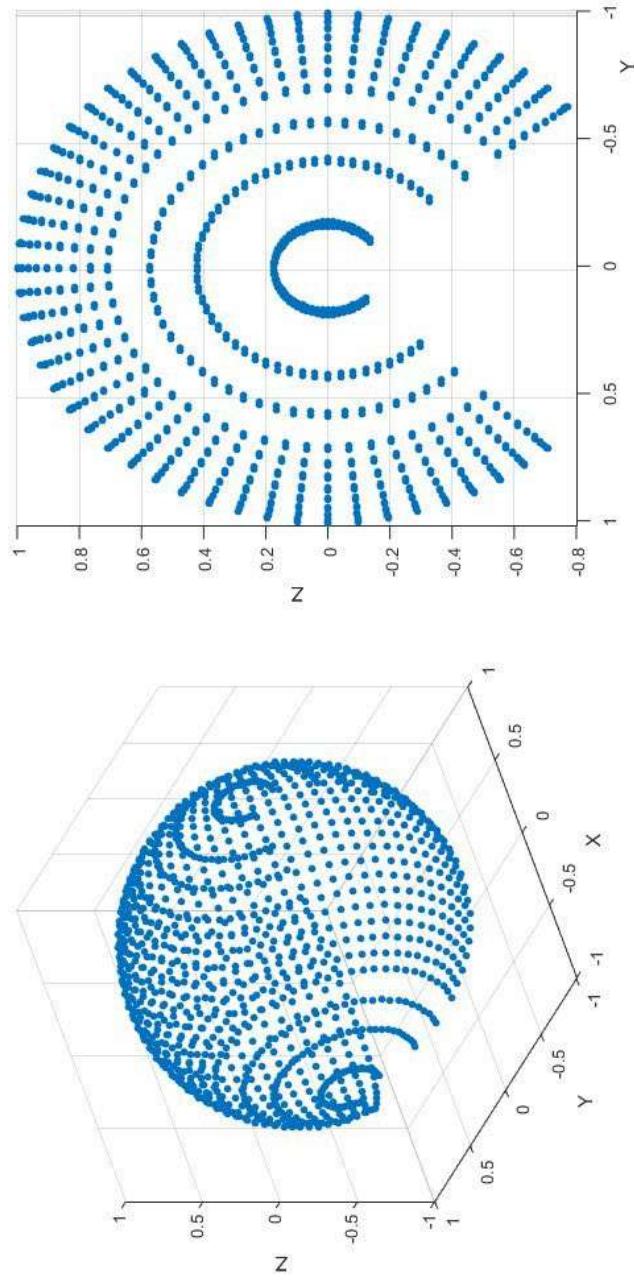
- models phase and amplitude differences for all possible sound directions parameterized by azimuth  $\theta$  and elevation  $\phi$
- can be measured with two microphones in ears of mannequin & speakers all around



Zhong and Xie, "Head-Related Transfer Functions and Virtual Auditory Display"

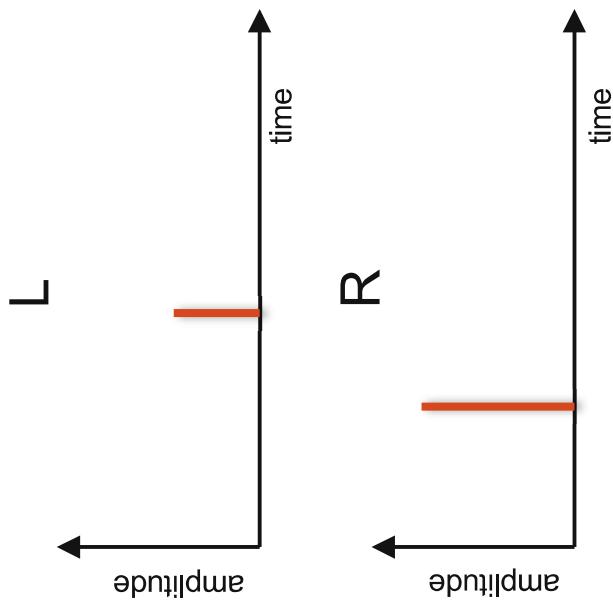
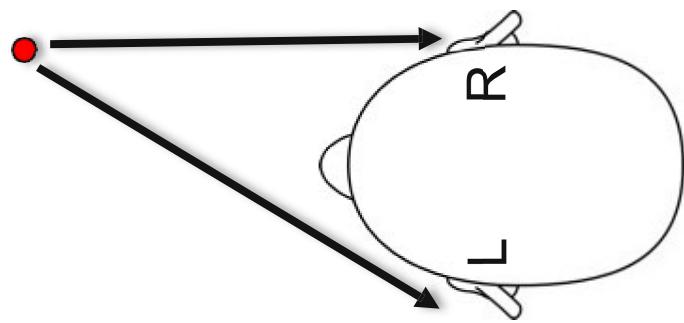
# Head-related Impulse Response (HRR)

- CIPIC HRTF database: <http://interface.cipic.ucdavis.edu/sound/hrtf.html>
- elevation:  $-45^\circ$  to  $230.625^\circ$ , azimuth:  $-80^\circ$  to  $80^\circ$
- need to interpolate between discretely sampled directions



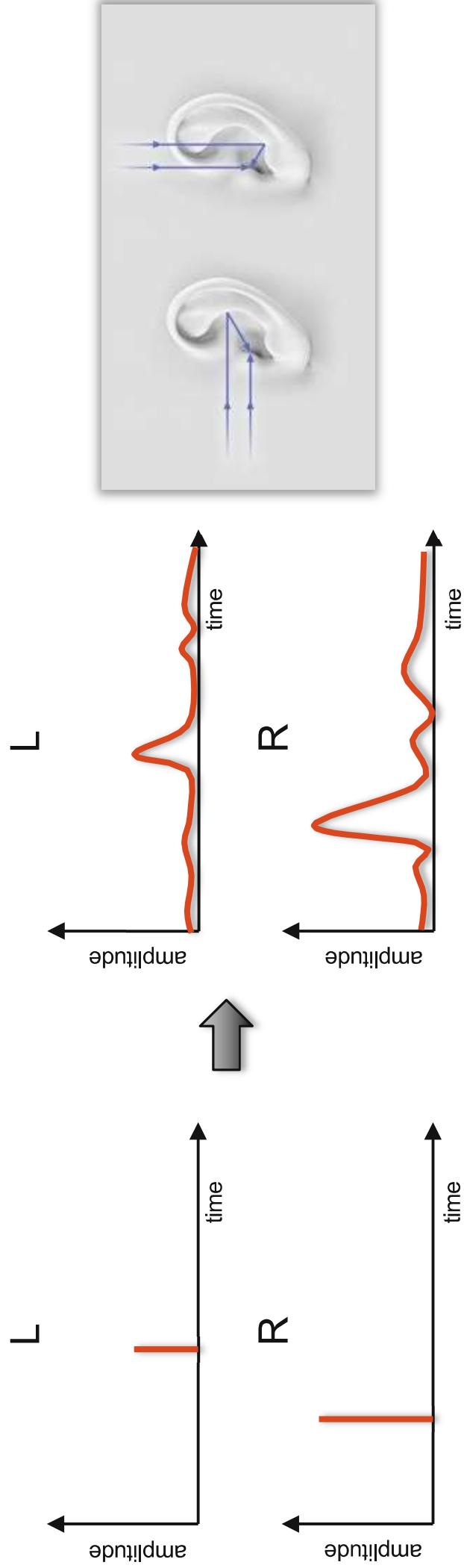
# Head-related Impulse Response (HRIR)

- measuring the HRIR
  - ideal case: scaled & shifted Dirac peaks



# Head-related Impulse Response (HRIR)

- measuring the HRIR
  - ideal case: scaled & shifted Dirac peaks
  - in practice: more complicated, includes scattering in the ear, shoulders etc.



# Head-related Impulse Response (HRIR)

- measuring the HRIR
  - need one temporally-varying function for each angle
  - total of  $2 \cdot N_\theta \cdot N_\phi \cdot N_t$  samples, where  $N_{\theta,\phi,t}$  is the number of samples for azimuth, elevation, and time, respectively

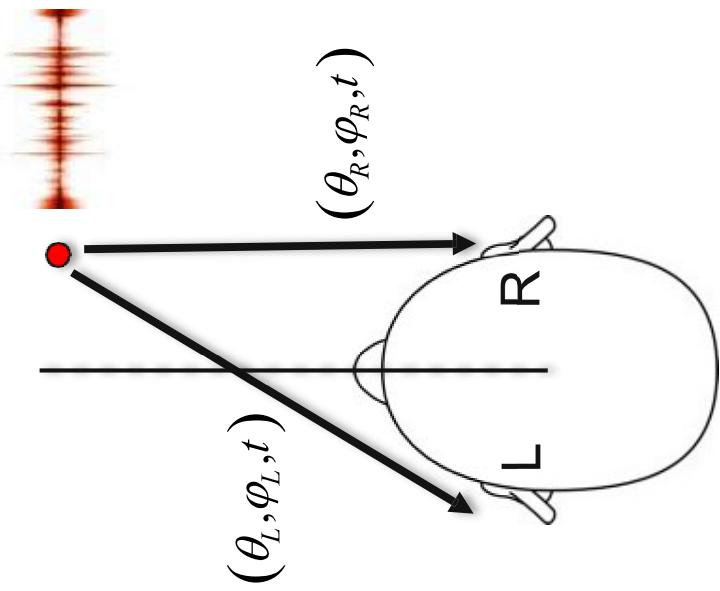
$$hrir\_l(\theta, \phi, t)$$

$$hrir\_r(\theta, \phi, t)$$

# Head-related Impulse Response (HRIR)

applying the HRIR:

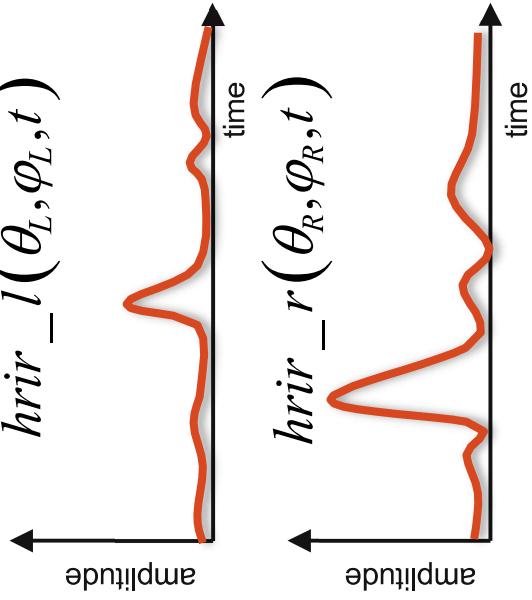
- given a mono sound source  $s(t)$  and it's 3D position
  1. compute  $(\theta_L, \varphi_L)$  and  $(\theta_R, \varphi_R)$  relative to center of listener



# Head-related Impulse Response (HRIR)

applying the HRIR:

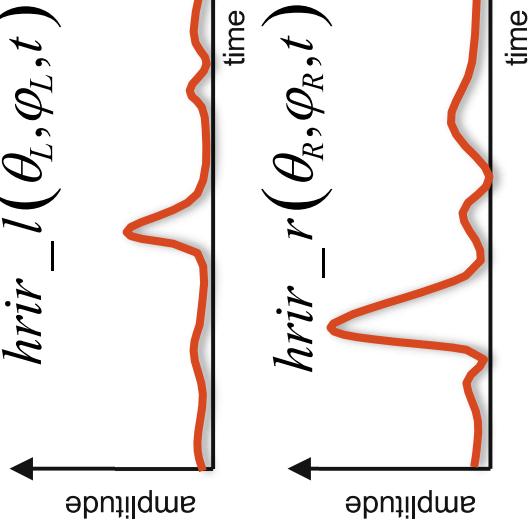
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1. compute  $(\theta_L, \phi_L)$  and  $(\theta_R, \phi_R)$  relative to center of listener
  2. look up measured HRIR for left and right ear at these angles



# Head-related Impulse Response (HRIR)

applying the HRIR:

- given a mono sound source  $s(t)$  and it's 3D position
- 1. compute  $(\theta_L, \phi_L)$  and  $(\theta_R, \phi_R)$  relative to center of listener
- 2. look up measured HRIR for left and right ear at these angles
- 3. convolve signal with HRIRs to get response for each ear as



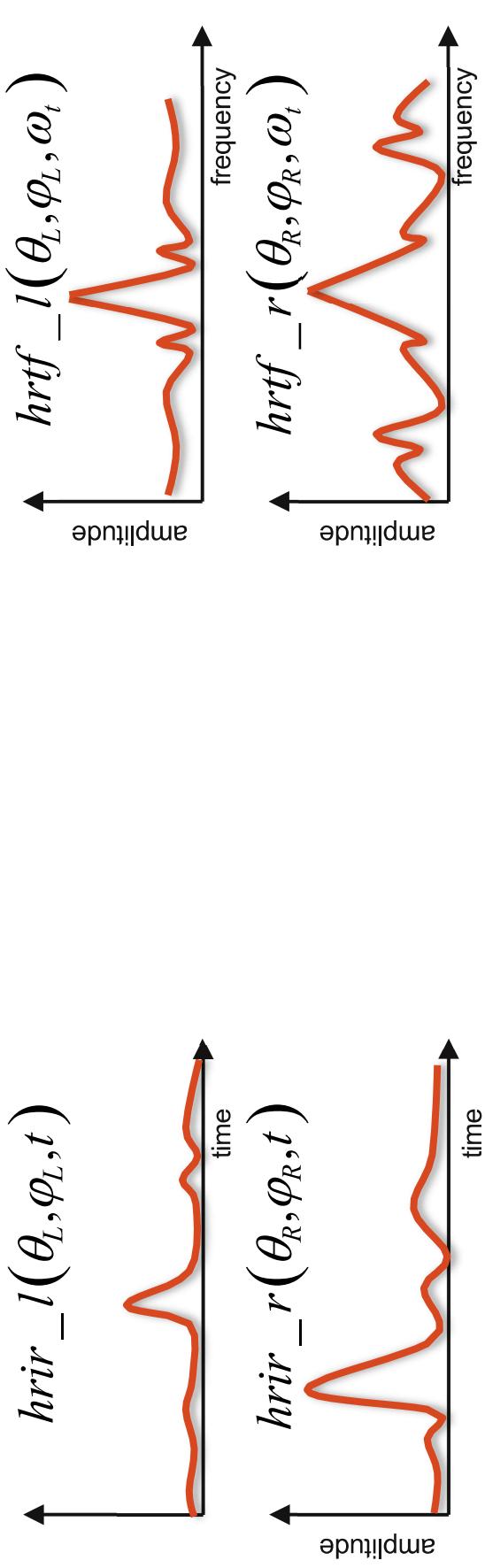
$$s_L(t) = \text{hrir\_l}(\theta_L, \phi_L, t) * s(t)$$

$$s_R(t) = \text{hrir\_r}(\theta_R, \phi_R, t) * s(t)$$

# Head-related Transfer Function (HRTF)

- HRTF is Fourier transform of HRIR! (you'll find the term HRTF more often than HRIR)

$$\begin{aligned} S_L(t) &= hrir\_l(\theta_L, \varphi_L, t) * s(t) \\ S_R(t) &= hrir\_r(\theta_R, \varphi_R, t) * s(t) \end{aligned}$$



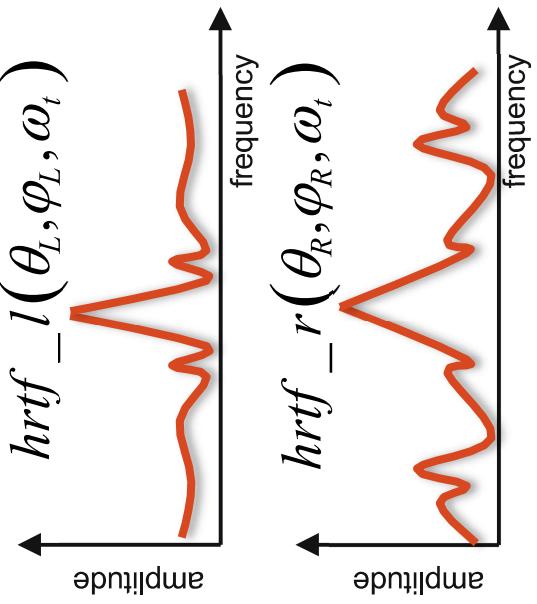
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↑

convolution theorem  
 $hrtf\_l(\theta_L, \phi_L, \omega_t)$

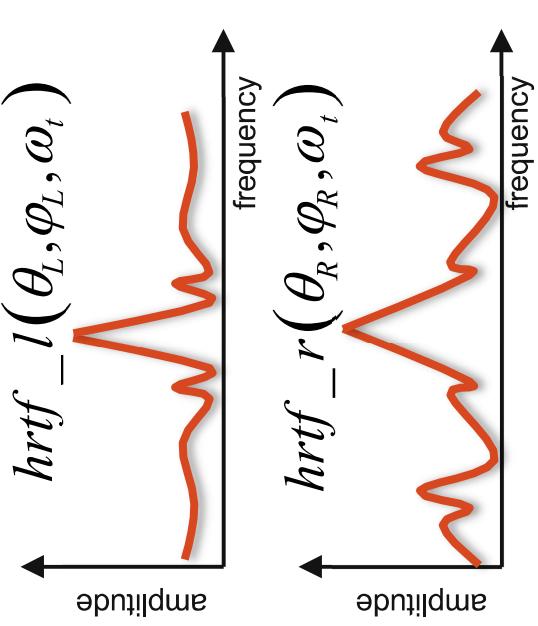


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↑



- properties of HRTF:

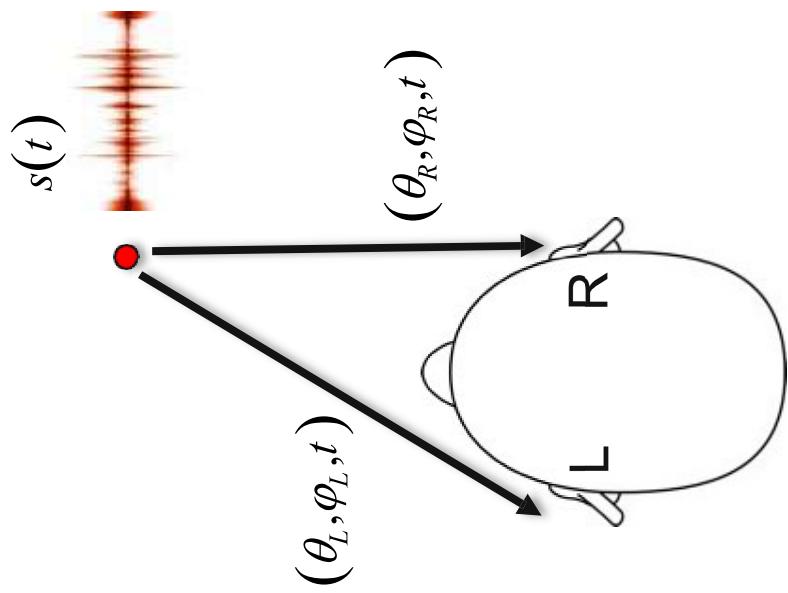
- complex-valued
- symmetric (because HRIR is real-valued)

# Head-related Transfer Function (HRTF)

$$\begin{aligned} s_L(t) &= F^{-1} \left\{ hrtf\_l(\theta_L, \phi_L, \omega_t) \cdot F \{s(t)\} \right\} \\ s_R(t) &= F^{-1} \left\{ hrtf\_r(\theta_R, \phi_R, \omega_t) \cdot F \{s(t)\} \right\} \end{aligned}$$

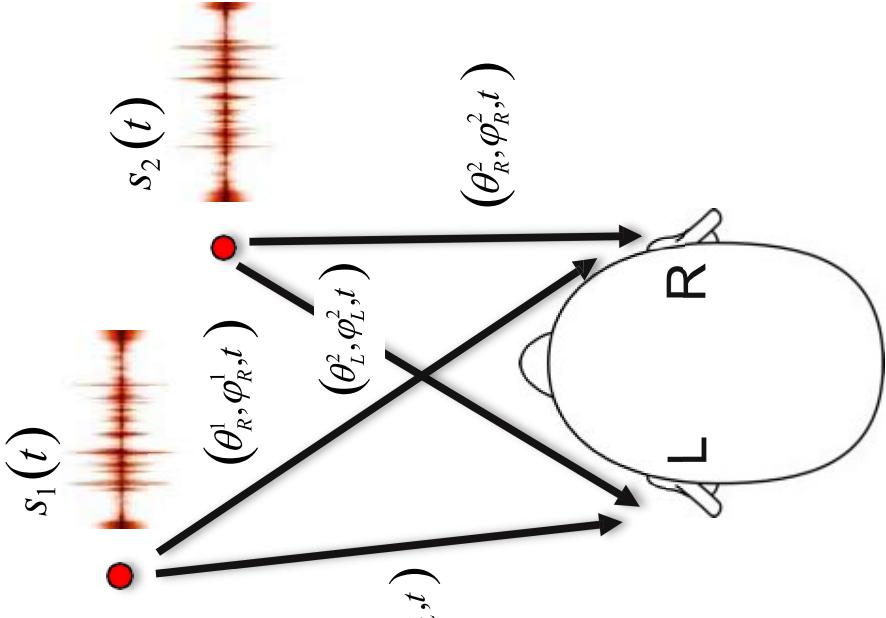
# Spatial Sound of 1 Point Sound Source

- given  $s(t)$  and 3D position, follow instructions from last slides by convolving Fourier transform of  $s$  with HRTFs for each each



# Spatial Sound of N Point Sound Sources

- superposition principle holds, so just sum the contributions of each



$$S_L(t) = \sum_{i=1}^N F^{-1} \left\{ hrtf\_l(\theta_L^i, \phi_L^i, \omega_t) \cdot F \{ s_i(t) \} \right\}$$

$$S_R(t) = \sum_{i=1}^N F^{-1} \left\{ hrtf\_r(\theta_R^i, \phi_R^i, \omega_t) \cdot F \{ s_i(t) \} \right\}$$

# Surround Sound

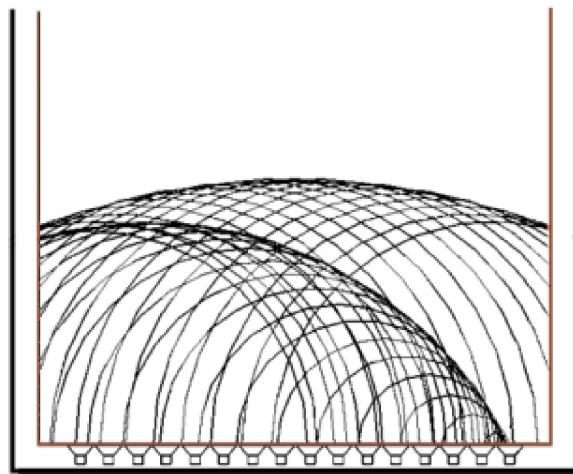
- approximate continuous wave field with discrete set of speakers



- most common:
  - 5.1 surround sound = 5 (channels) . 1 (bass)  
→ 6 channels total

# Surround Sound

- approximate continuous wave field with discrete set of speakers
- can also use more speakers for “wave field synthesis” (i.e. audio hologram)



<http://spatialaudio.net/>

# Surround Sound

- approximate continuous wave field with discrete set of speakers
- can also use more speakers for “wave field synthesis” (i.e. audio hologram)
- for wave field synthesis, phase of speakers needs to be synchronized, i.e. a phased array!

# Surround Sound & HRTF

- for all speaker-based (surround) sound, we don't need an HRTF because the ears of the listener will apply them!
- speaker setup usually needs to be calibrated

# Spatial Audio for VR

- VR/AR requires us to **re-think** audio, especially spatial audio!
- could use 5.1 surround sound and set up “virtual speakers” in the virtual environment – can use existing content, but not super easy to capture new content; also doesn’t capture directionality from above/below

# Spatial Audio for VR

Two primary approaches:

## 1. Real-time sound engine

- render 3D sound sources via HRTF in real-time, just as discussed in the previous slides
- used for games and synthetic virtual environments
- a lot of libraries available: FMOD, OpenAL, ...

# Spatial Audio for VR

Two primary approaches:

2. Spatial sound recorded from real environments
  - most widely used format now: ambisonics
  - simple microphones exist
  - relatively easy mathematical model
  - only need 4 channels for starters
  - used in YouTube VR and many other platforms

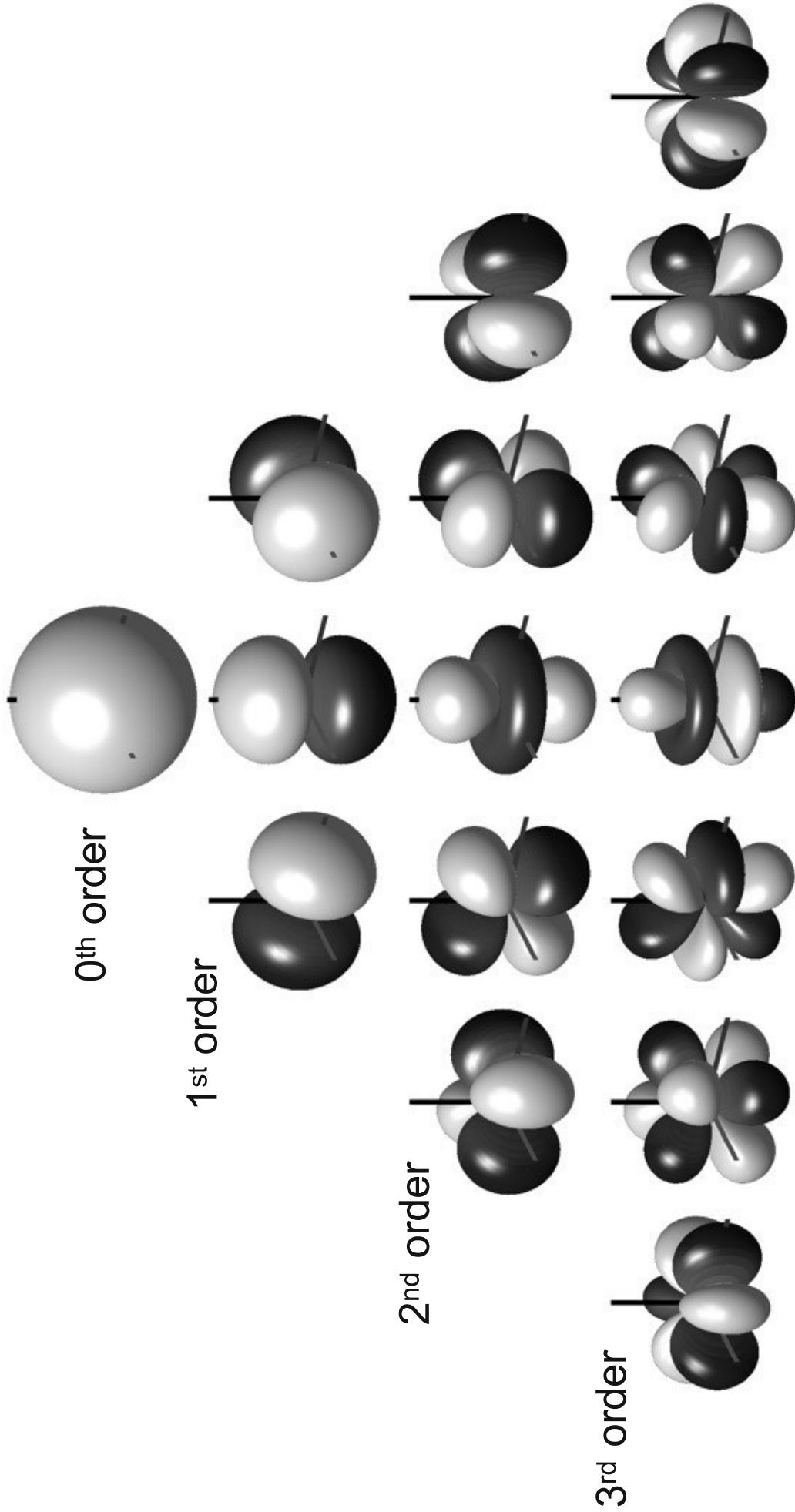
# Ambisonics

- idea: represent sound incident at a point (i.e. the listener) with some directional information
- using all angles  $\theta, \varphi$  is impractical – need too many sound channels (one for each direction)
- some lower-frequency (in direction) components may be sufficient → directional basis representation to the rescue!

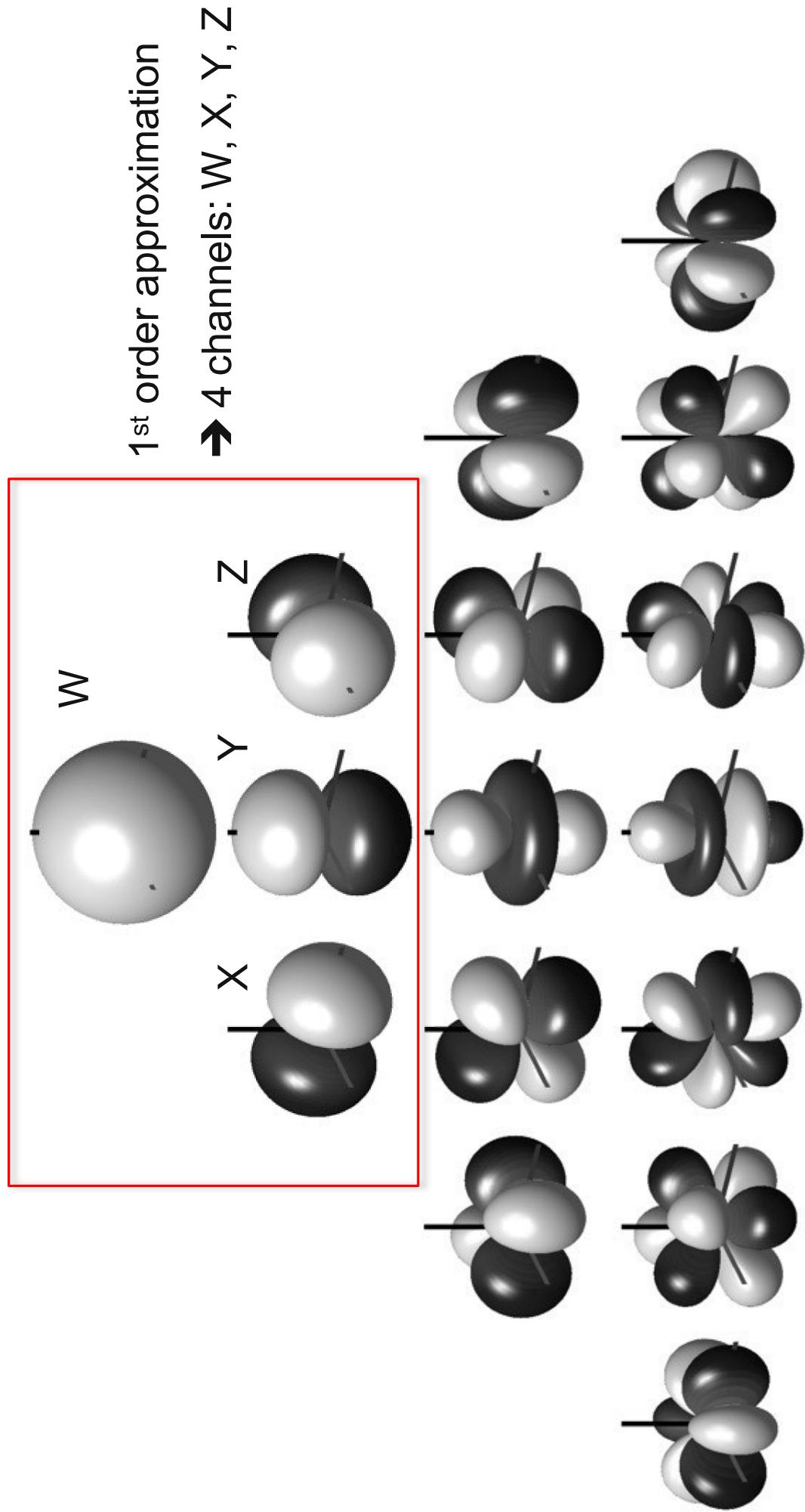
# Ambisonics – Spherical Harmonics

- use spherical harmonics!
- Orthogonal basis functions on a sphere, i.e. full-sphere surround sound
- think Fourier transform acting on the directions of a sphere

# Ambisonics – Spherical Harmonics



# Ambisonics – Spherical Harmonics



# Ambisonics – Spherical Harmonics

- can easily convert a point sound source to the 4-channel ambisonics representation
- given azimuth and elevation  $\theta, \varphi$ , compute W,X,Y,Z as

$$\begin{aligned}W &= S \cdot \frac{1}{\sqrt{2}} && \longrightarrow \text{ omnidirectional component (angle-independent)} \\X &= S \cdot \cos \theta \cos \varphi && \longrightarrow \text{ "stereo in x"} \\Y &= S \cdot \sin \theta \cos \varphi && \longrightarrow \text{ "stereo in y"} \\Z &= S \cdot \sin \varphi && \longrightarrow \text{ "stereo in z"}\end{aligned}$$

# Ambisonics – Spherical Harmonics

- can also record 4-channel ambisonics via special microphone
- same format supported by YouTube VR and other platforms

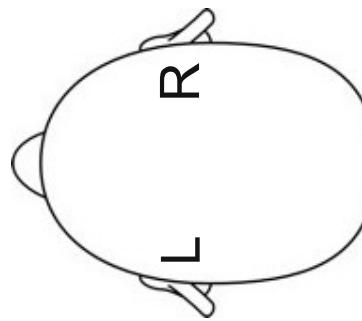


<http://www.oktava-shop.com/>

# Ambisonics – Spherical Harmonics

- easiest way to render ambisonics: convert W,X,Y,Z channels into 4 virtual speaker positions
- for a regularly-spaced square setup, this results in

LF   
RF



$$LF = (2W + X + Y)\sqrt{8}$$

$$LB = (2W - X + Y)\sqrt{8}$$

$$RF = (2W + X - Y)\sqrt{8}$$

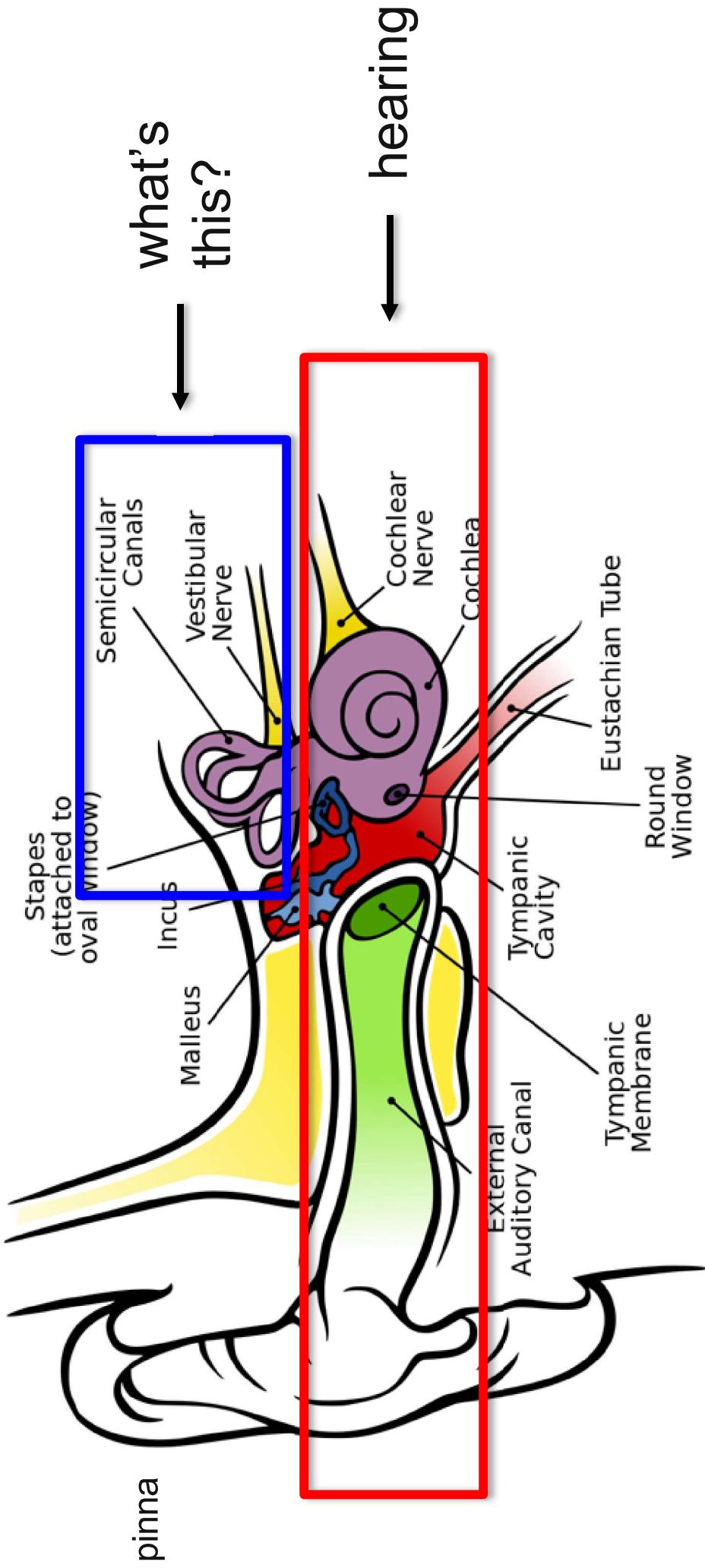
$$RB = (2W - X - Y)\sqrt{8}$$

LB   
RB

or “What else is happening in the inner ear?”

## The Vestibular System

# The Inner Ear

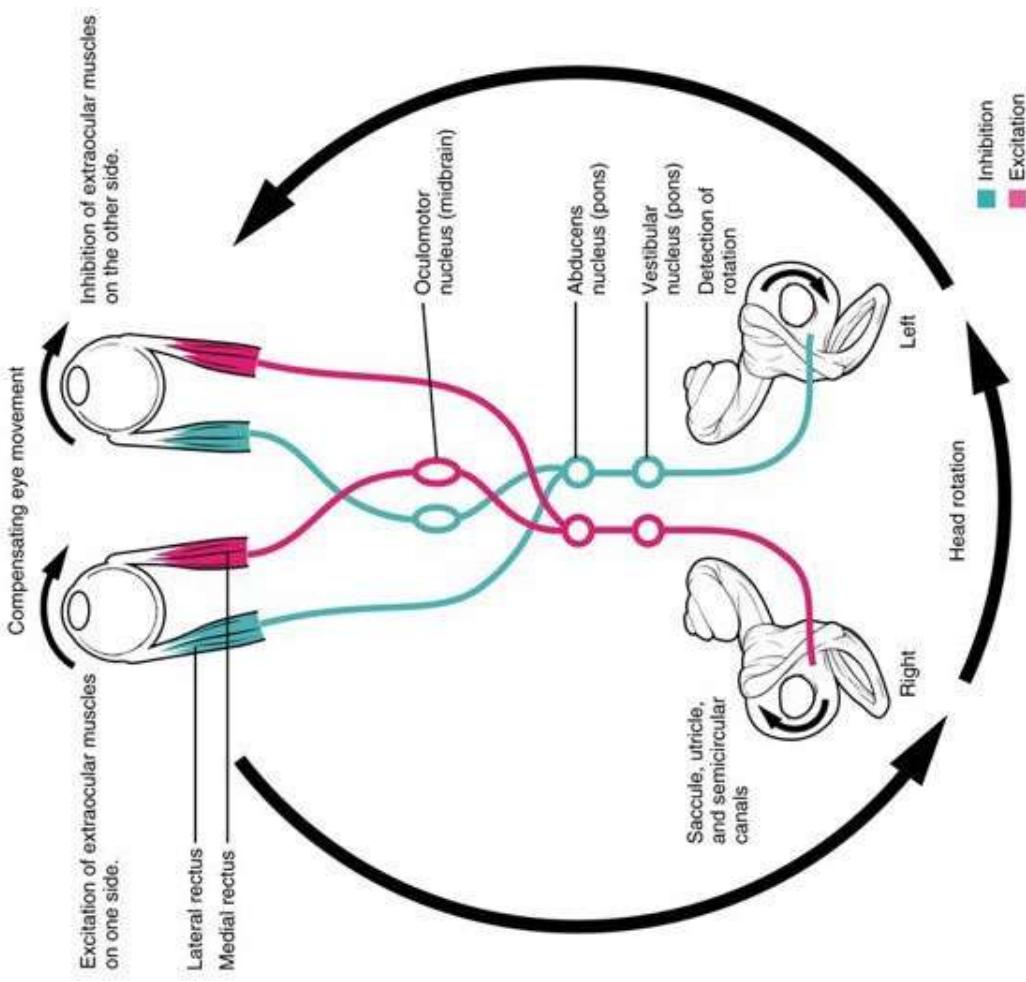


# Brief Overview of the Vestibular System

- provides sense of balance & gravity
- like IMUs – one in each ear!
- in each ear, sense linear (3 dof from otolithic organs) and angular (3 dof from 3 semicircular canals) acceleration via hair cells

# Vestibulo-Ocular Reflex (VOR)

- vestibular system and ocular system are directly coupled in a feedback system
- enables low-latency “optical image stabilization” of the visual system with head motion



# Motion Sickness

3 types of motion sickness (all related to visual-vestibular conflict theory):

1. Motion sickness caused by motion that is felt but not seen
2. Motion sickness caused by motion that is seen but not felt
3. Motion sickness caused when both systems detect motion but they do not correspond.

# Motion Sickness

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Example: car and sea sickness

# Motion Sickness

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Example: VR sickness or visually-induced motion sickness (VIMS)

# Motion Sickness

3 types of motion sickness (all related to visual-vestibular conflict theory):

1. Motion sickness caused by motion that is felt but not seen
2. Motion sickness caused by motion that is seen but not felt
3. **Motion sickness caused when both systems detect motion but they do not correspond.**

Example: motion in low gravity

# References and Further Reading

## Panoramic Imaging and VR

- M. Brown, D. Lowe “Automatic Panoramic Image Stitching using Invariant Features”, IJCV 2007
- autostitch: <http://matthewalunbrown.com/autostitch/autostitch.html>
- S. Peleg, M. Ben-Ezra, Y. Pritch “Omnistereo: Panoramic Stereo Imaging” IEEE PAMI 2001
- Konrad et al. “SpinVR: Towards Live Streaming VR Video”, ACM SIGGRAPH Asia 2017

# References and Further Reading - Spatial Sound

- Google's take on spatial audio: <https://developers.google.com/vr/concepts/spatial-audio>

## HRTF:

- Algazi, Duda, Thompson, Avendado “The CIPIC HRTF Database”, Proc. 2001 IEEE Workshop on Applications of Signal Processing to Audio and Electroacoustics
- download CIPIC HRTF database here: <http://interface.cipic.ucdavis.edu/sound/hrtf.html>

## Resources by Google:

- <https://github.com/GoogleChrome/omnitone>
- <https://developers.google.com/vr/concepts/spatial-audio>
- <https://opensource.googleblog.com/2016/07/omnitone-spatial-audio-on-web.html>
- <http://googlechrome.github.io/omnitone/#home>
- <https://github.com/google/spatial-media/>

# existing applications of haptics

entertainment



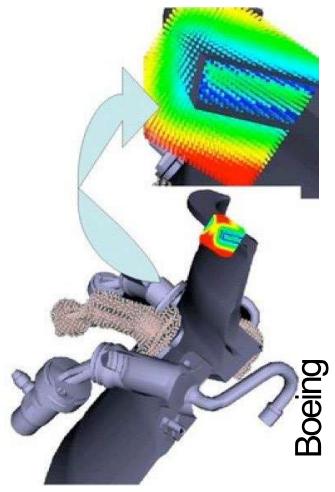
education



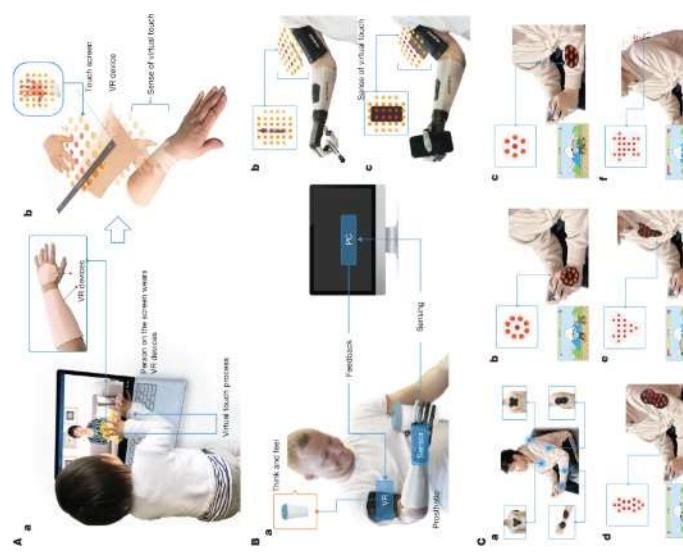
health



human-computer interfaces



© Allison Kamura, 2028



# kinesthetic vs. tactile haptic devices

Kinesthetic haptic devices display forces or motions through a tool

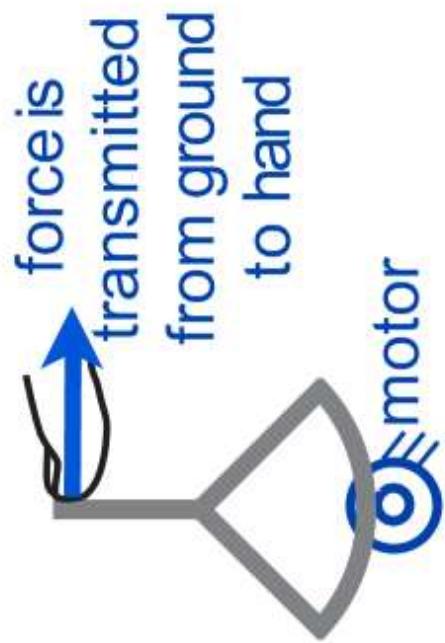
Tactile haptic devices stimulate the skin



# kinesthetic vs. tactile haptic devices

Kinesthetic haptic devices  
are usually **grounded**

Tactile haptic  
devices can more  
easily be **wearable**





## Tactile Devices

Stimulate skin to create contact sensations



## Hybrid Devices

Attempt to combine tactile and kinesthetic feedback



## Kinesthetic Devices

Apply forces to guide or inhibit body movement

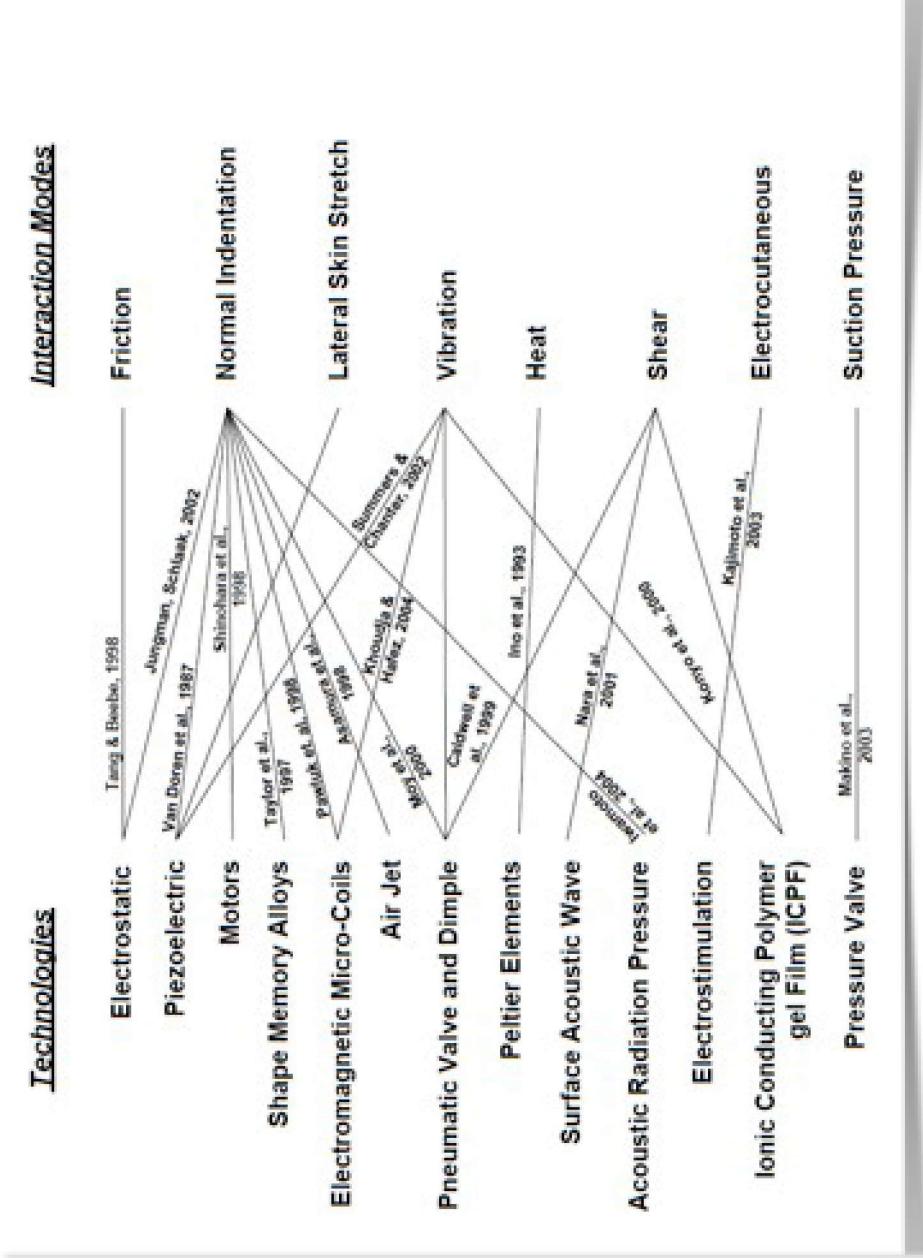
K. Kuchenbecker

tactile  
(cutaneous)  
device basics

# tactile feedback

- goal is to stimulate the **skin** in a programmable manner to create a desired set of sensations
- *sometimes distributed* tactile feedback is provided
- tactile feedback is generated by a **tactile device**, sometimes called a **tactile display**
- can aim to recreate real sensations, create novel ones, or communicate information

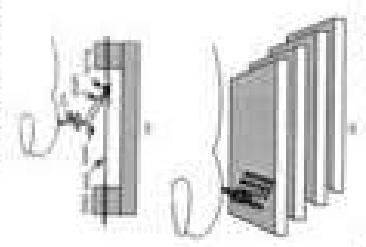
# technologies and interaction modes



Jerome Pasquero, Survey on Communication through Touch, Technical Report:TR-CIM 06.04, 2006



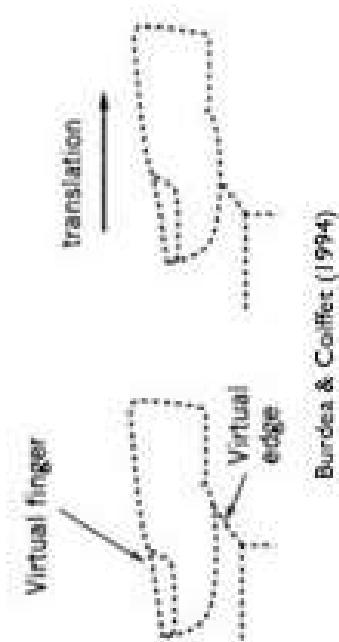
Wagner & Howe (2002)



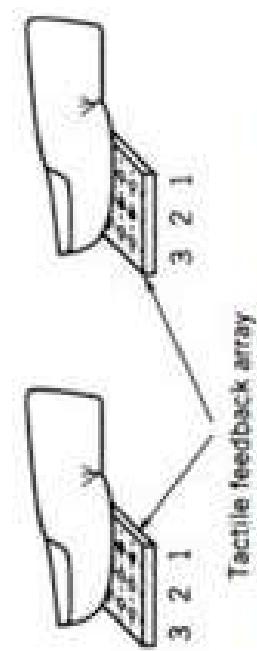
Kontarinis et al. (1995)



Russell



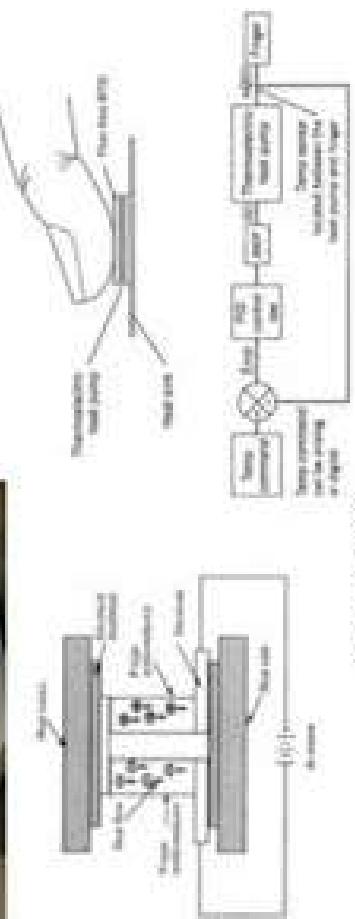
Burdea & Coiffet (1994)



Katsumata et al. (1995)



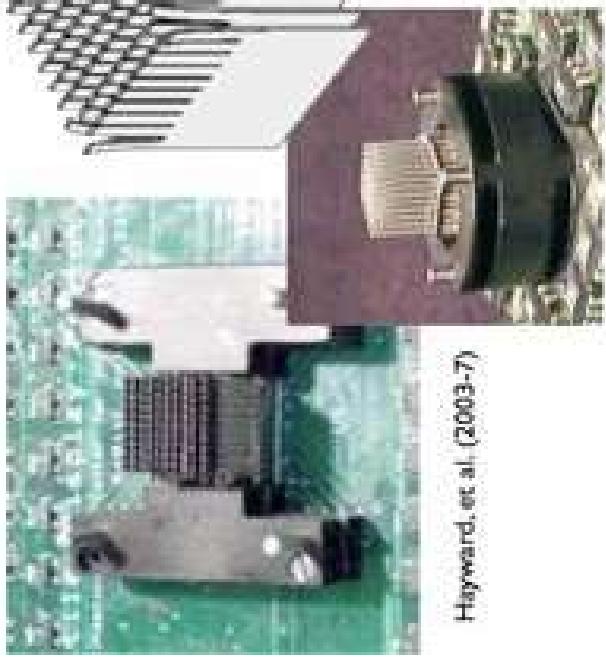
Zehnma (1993)



Sadash et al. (2002-5)



Hannan et al. (2003-7)



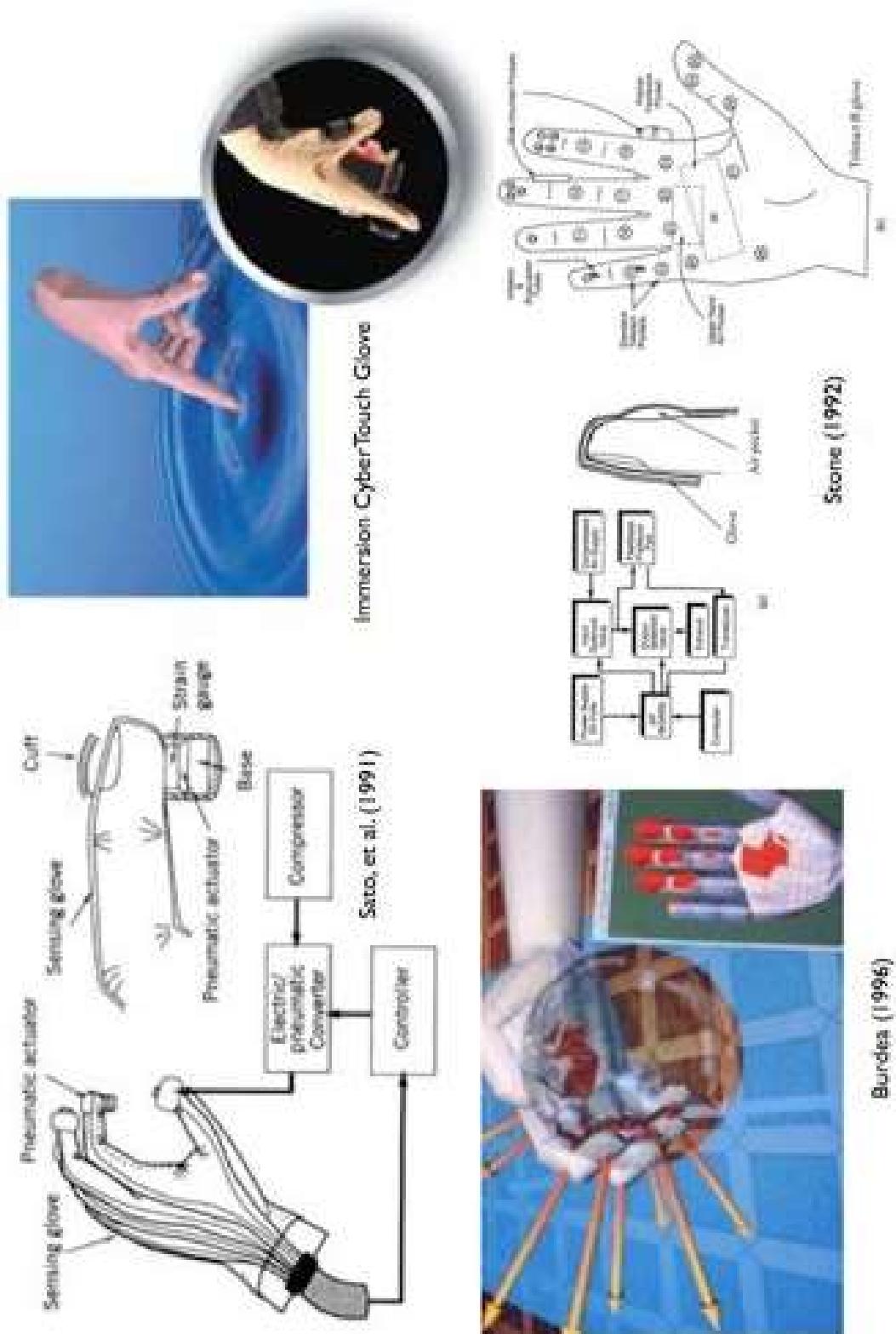
Wright & Callahan (2007)



Gessamine et al. (2006)



Samsung et al.



**kinesthetic  
(force-feedback)  
device basics**

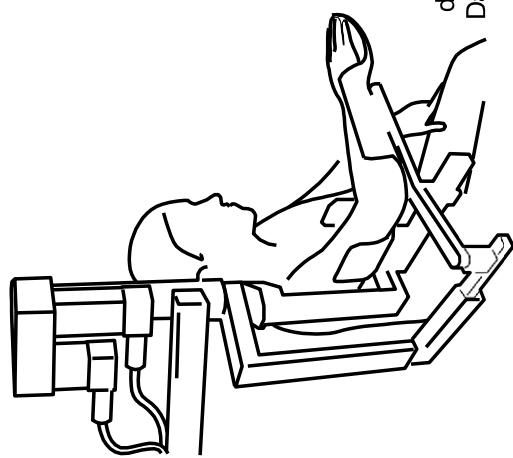
# typical kinesthetic device configurations

manipulandum grasp



drawing by Jorge Cham

exoskeleton



drawing by  
David Grow

# manipulators (expensive)



Omega  
from Force Dimension  
delta configuration  
3 degrees of freedom



Phantom Premium 1.5  
from SensAble/Geomagic  
5-bar + rotation  
3 degrees of freedom



Virtuose  
from Haption  
additional “wrist”  
6 degrees of freedom

all images from Wikimedia Commons

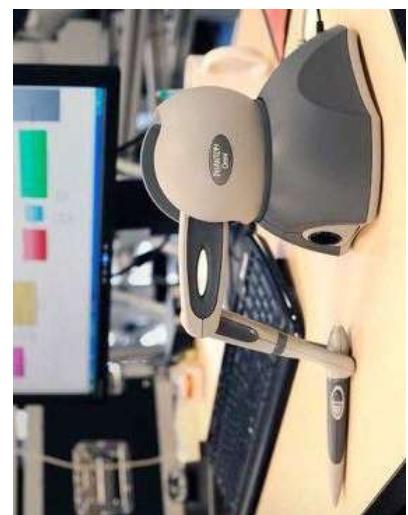
# manipulandums (cheaper)



Sidewinder  
from Microsoft

spherical mechanism  
2 degrees of freedom

image from Wikimedia Commons



Phantom Omni/T Touch  
from SensAble/Geomagic

5-bar + rotation  
3 degrees of freedom

photographed by Akiko Nabeshima



Falcon  
from Novint

delta configuration  
3 degrees of freedom

image from Wikimedia Commons

# Grip/grasp



Custom haptic gripper for  
**Phantom Premium**

© 2007 IEEE. Reprinted, with permission,  
from L.N.Yerner and A.M.Okamura..  
Effects of Translational and Gripping Force  
Feedback are Decoupled in a 4-Degree-  
of-Freedom Telemanipulator, World  
Haptics Conference, pp. 286-291, 2007



Single-finger Cybergasp  
from **Cyberglove Systems**

photograph courtesy  
Stanford Center for Design Research



daVinci Surgical System  
from **Intuitive Surgical, Inc.**  
(no programmable force  
feedback on gripper)

photographed by Akiko Nabeshima

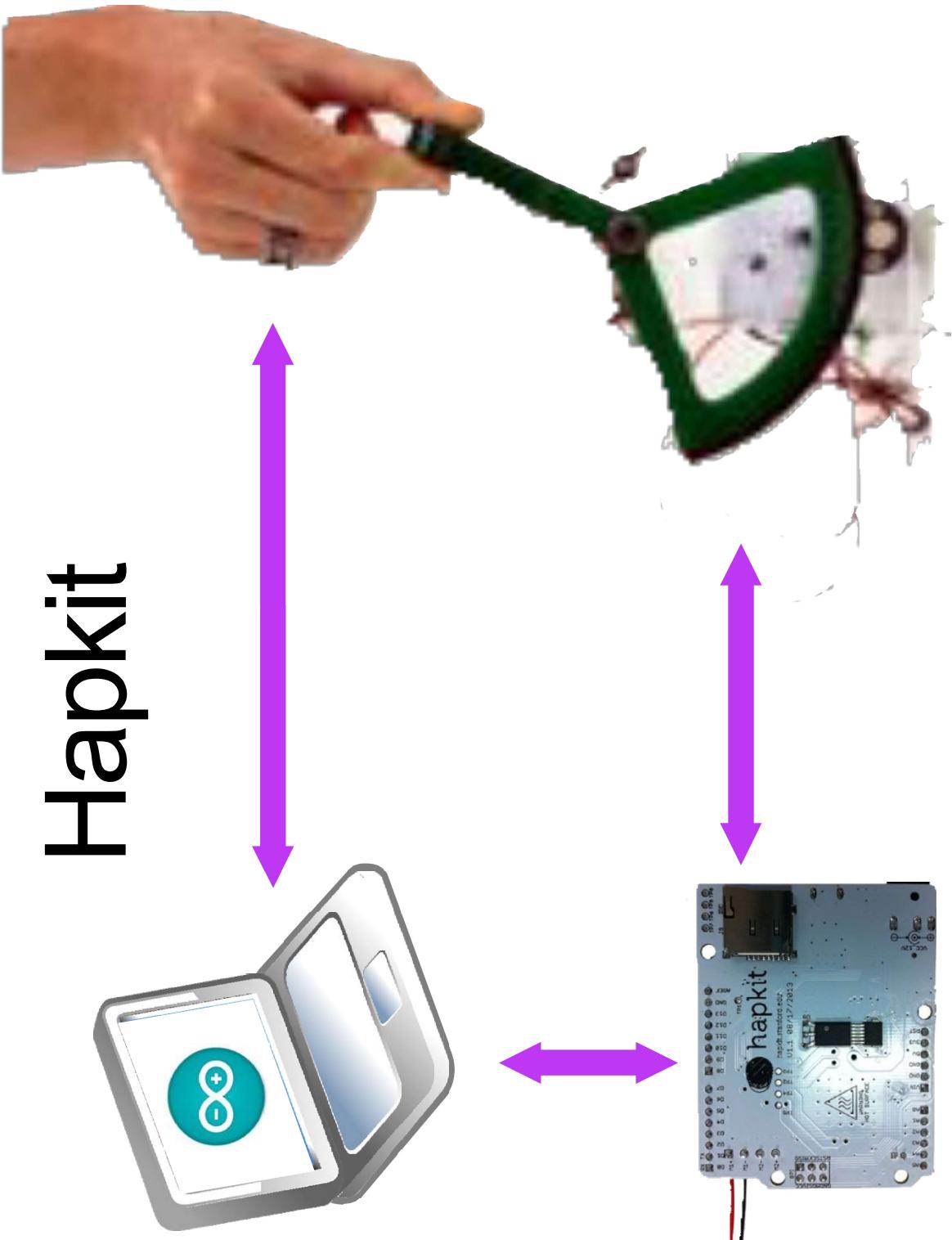
# Exoskeletons



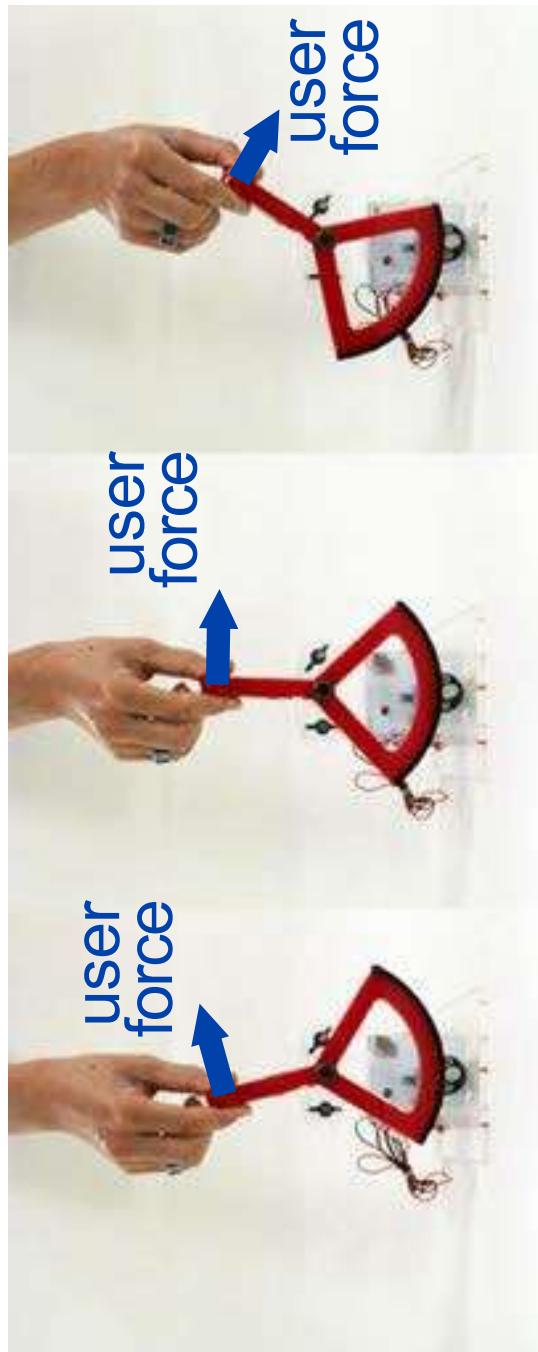
KINARM Exoskeleton  
from BKIN Technologies



Harvard      DARPA  
images from Wikimedia Commons



# Hapkit



# Hapkit



# Trends driving haptics

- Networking - constant connectivity
- Ubiquity of computing devices - beyond sparse visual real estate
- Multitasking- doing more things at once may benefit from multiple channels of communication
- Virtualization – fostering presence

# Rendering friction

(in one degree of freedom)

## surface properties

- typical haptic display general shape very well, but don't feel "realistic"
  - add surface properties to increase realism
- how you do it depends on
  - the surface model
  - complexity of the surface property
  - frequency response of your haptic device

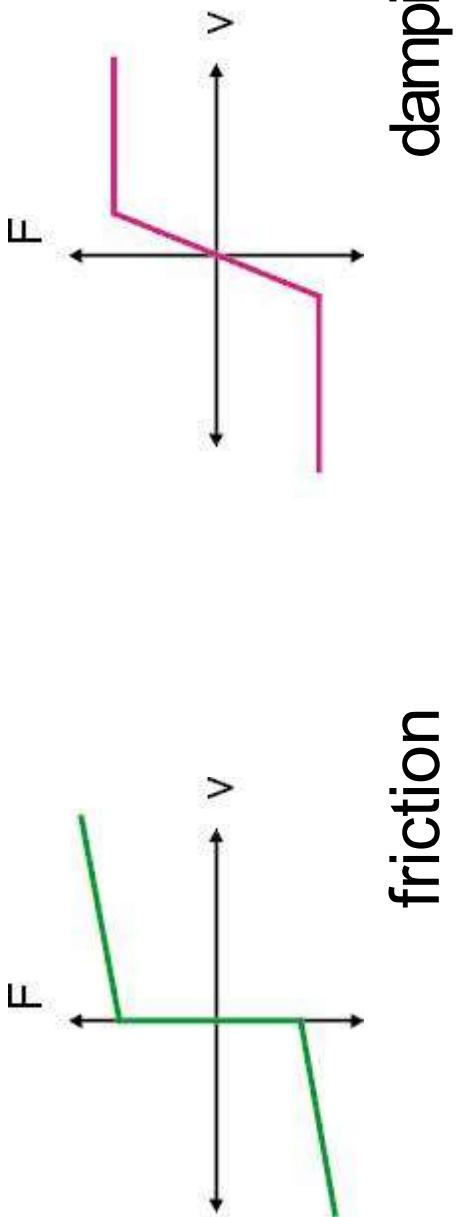
# damping for virtual walls

- a pure springforce for a wall may seem to “active”
- add a dissipative term, where b is the damping coefficient
  - only damps when going into the wall
- this can also create vibrations upon wall impact

$$F = \begin{cases} k\Delta x + b\dot{x} & \text{for } \dot{x} > 0 \\ k\Delta x & \text{for } \dot{x} < 0 \end{cases}$$

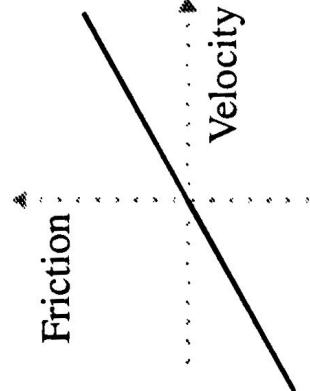
## “frictional” damping

- surfaces can feel unnaturally slippery
- friction would help, but it is difficult to implement
- you can add damping to motion parallel to surface

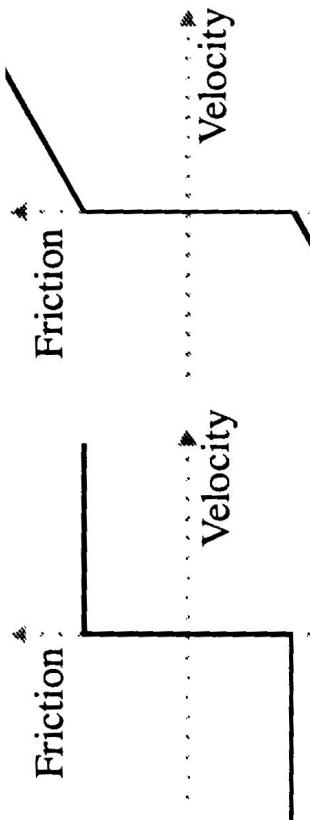


# friction display

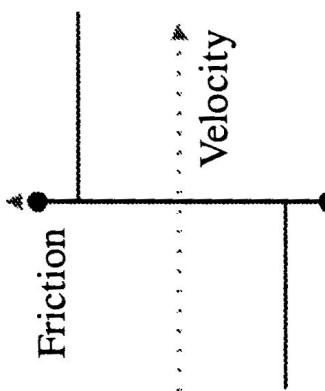
difficult to render because it is non-linear



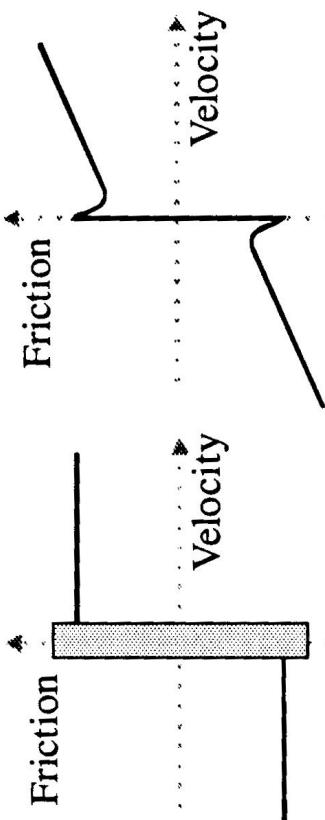
(a) damping



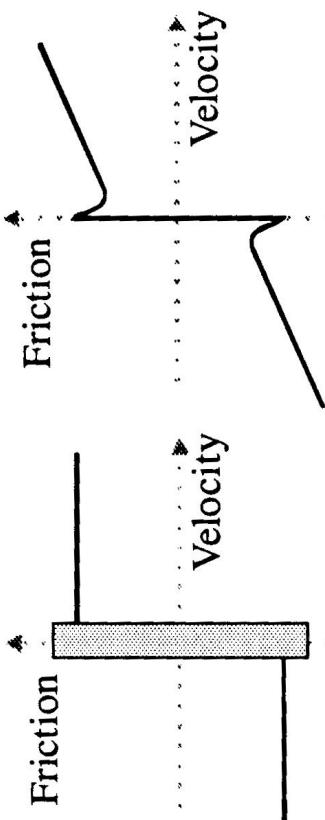
(b) Coulomb



(c) a & b



(d) stiction



(e) static

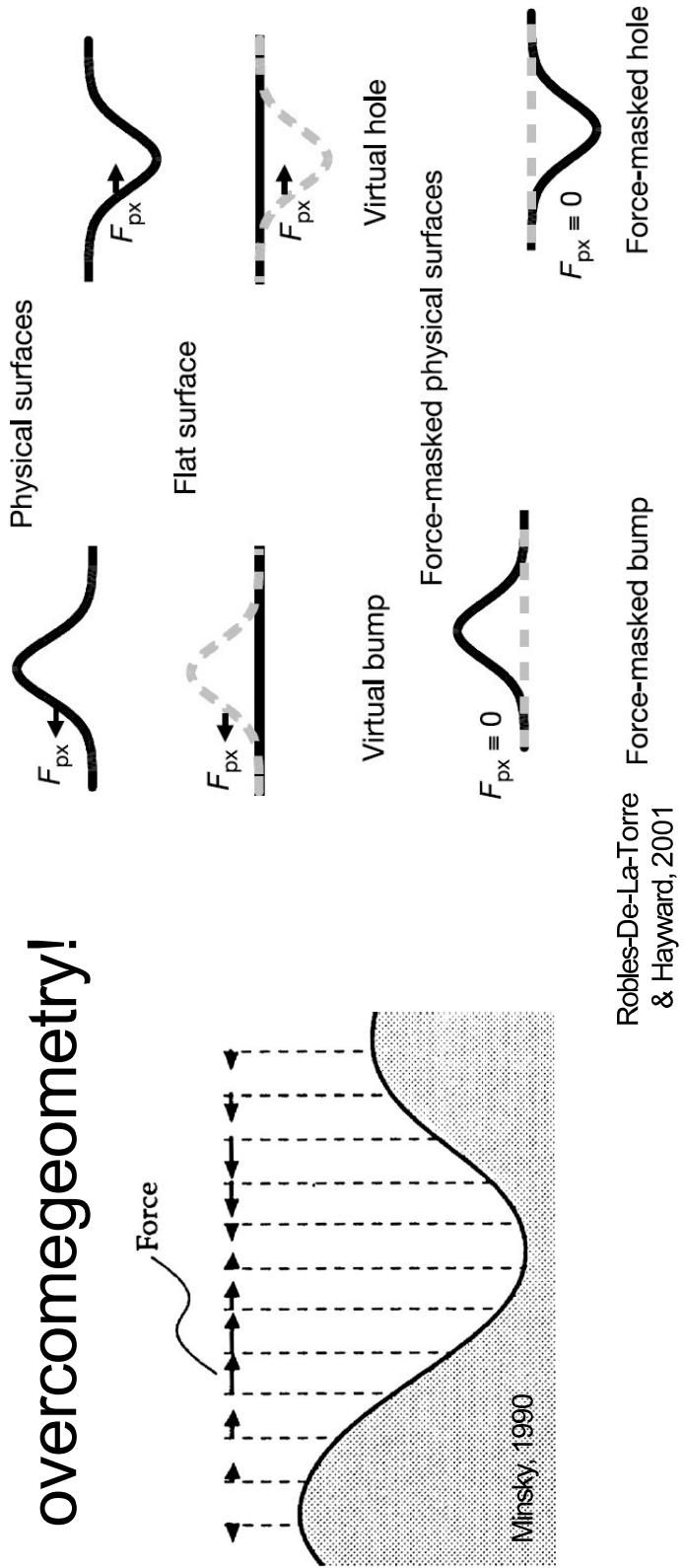
(f) Stribeck effect  
Richard & Cukosky

# Rendering bumps and textures

(in one degree of freedom)

# bumps and valleys

- as a user moves “up” a bump, motion is opposed.
- done in 2D, springforce proportional to height of bump
- force information can overcome geometry!

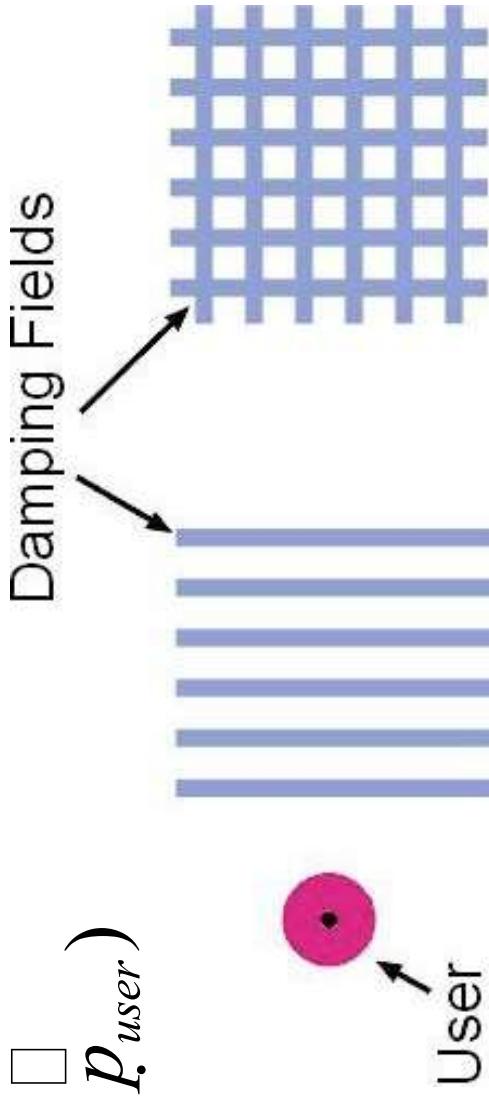


# damping textures

if  $p_{user}$  is inside a damping area

$$F = b v_{user}$$

( $v_{user} = p_{user}$ )



note that vibrations occur due to discontinuity in force

# **simulating and rendering dynamic objects (in one degree of freedom)**

# dynamic simulation of “rigid” bodies

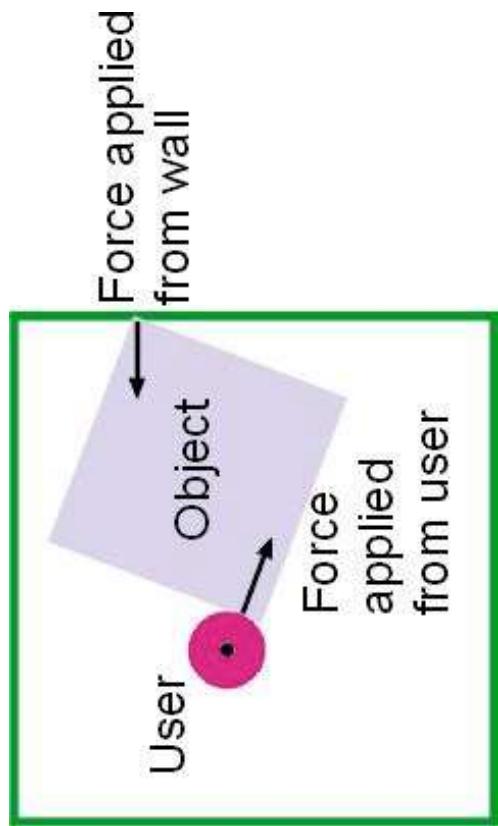
- assumptions
  - you have an impedance-type force-feedback display
  - you are using a linear stiffnessmodel of the surface
- basic approach:
  1. save the state of the moving object
  2. sum the forces on the object
  3. calculate the new state

# object state

- the “state” of an object is used to describe its current condition
  - made up of variables that will change with time, such as
    - position
    - velocity
    - acceleration
  - rarely: other parameters such as mass, shape, etc. that might be changed by dynamic interaction
- © Allison M. Kamura, 2020

# calculating forces on an object

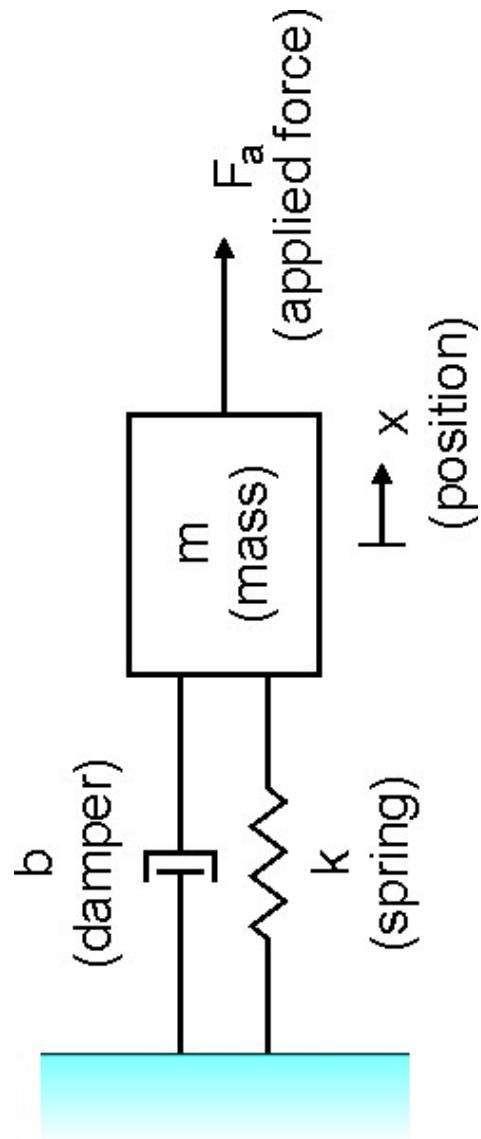
- for forces from the user's hand pushing, this is equal and opposite to the force fed back to the user
- for forces from other objects in the VE, use the same idea: force is proportional to penetration



© Allison M. Kamura, 2028

# second order dynamic systems

## mass-spring-damper



$x = 0$  at equilibrium

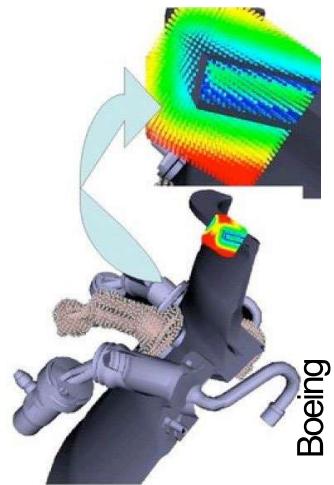
© Allison M. Kamura, 2028

# Haptics

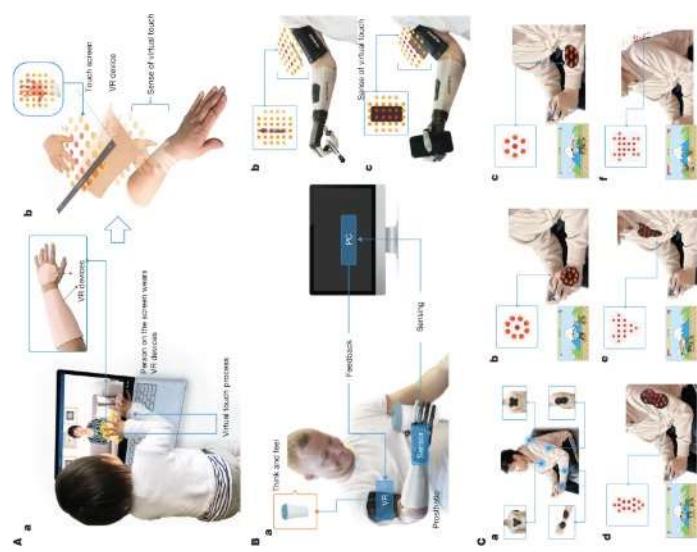
entertainment education health



human-computer interfaces



© Allison M. Okamura, 2020



# kinesthetic vs. tactile haptic devices

Kinesthetic haptic devices display forces or motions through a tool

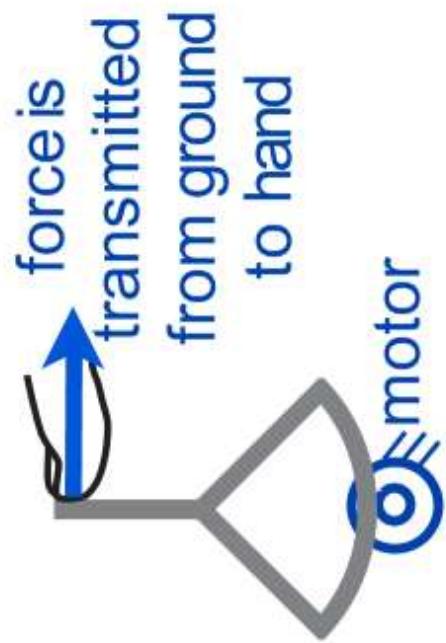
Tactile haptic devices stimulate the skin



# kinesthetic vs. tactile haptic devices

Kinesthetic haptic devices  
are usually **grounded**

Tactile haptic  
devices can more  
easily be **wearable**



## Tactile Devices



Stimulate skin to create contact sensations



## Hybrid Devices

Attempt to combine tactile and kinesthetic feedback



## Kinesthetic Devices

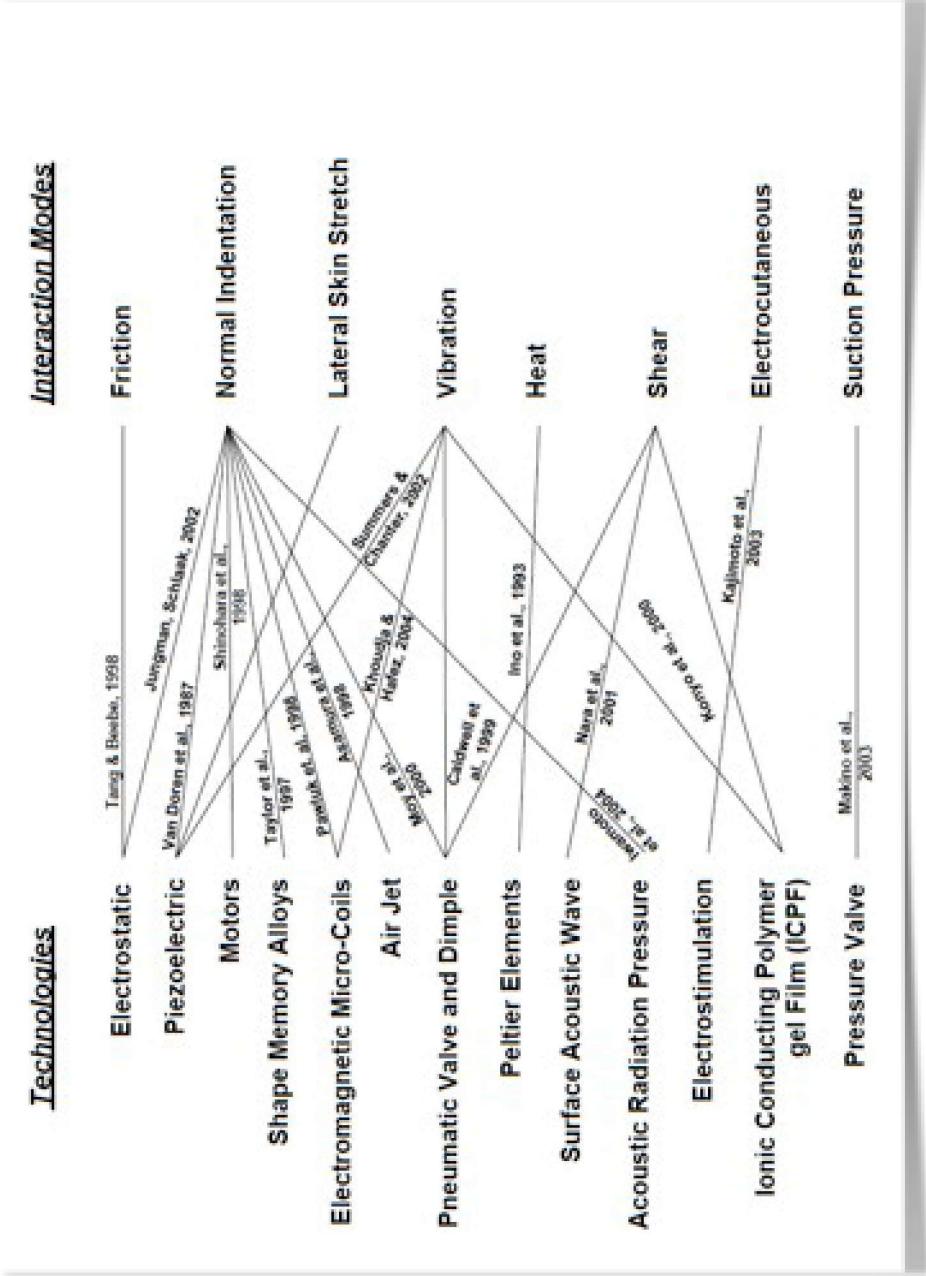
Apply forces to guide or inhibit body movement

# tactile (cutaneous) device basics

# tactile feedback

- goal is to stimulate the **skin** in a programmable manner to create a desired set of sensations
- *sometimes distributed* tactile feedback is provided
  - tactile feedback is generated by a **tactile device**, sometimes called a **tactile display**
- can aim to recreate real sensations, create novel ones, or communicate information

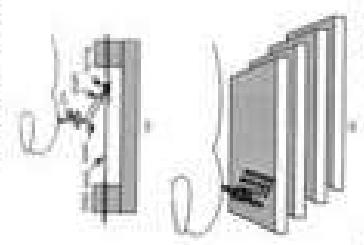
# technologies and interaction modes



Jerome Pasquero, Survey on Communication through Touch, Technical Report:TR-CIM 06.04, 2006

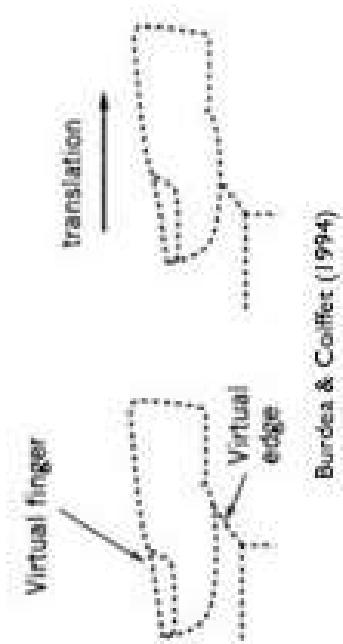


Wagner & Howe (2002)

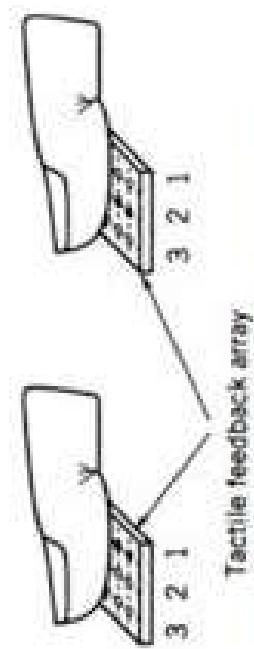


Kontarinis, et al. (1995)

Russell



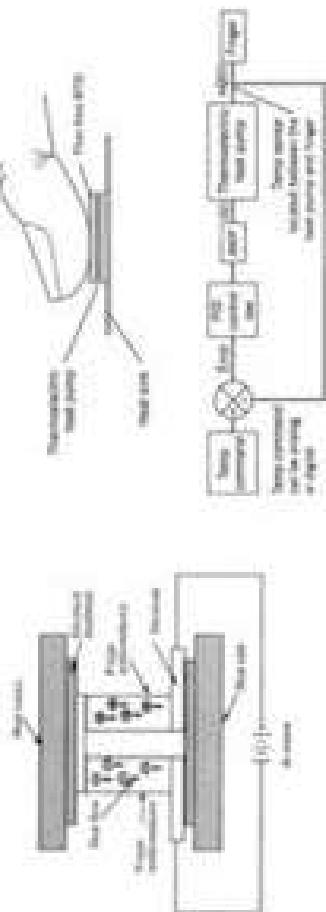
Burdea & Coiffet (1994)



Kacmarcik, et al. (1995)



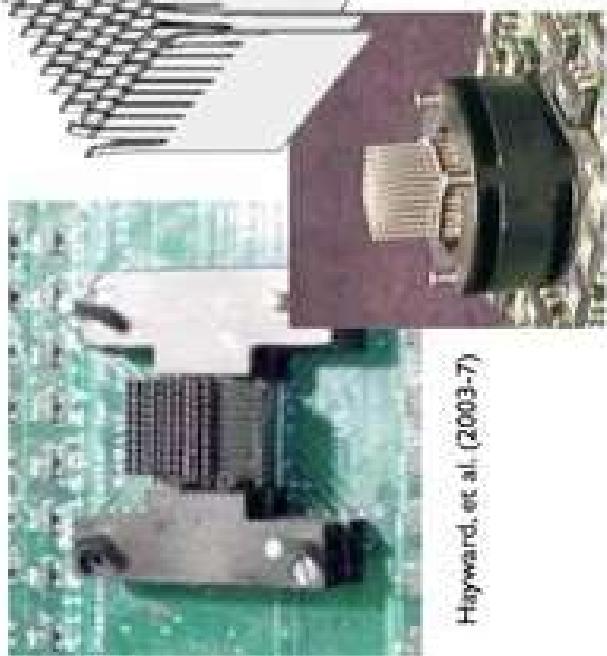
Zelchins (1993)



Sadash et al. (2002-5)



Hwang et al. (2003-7)



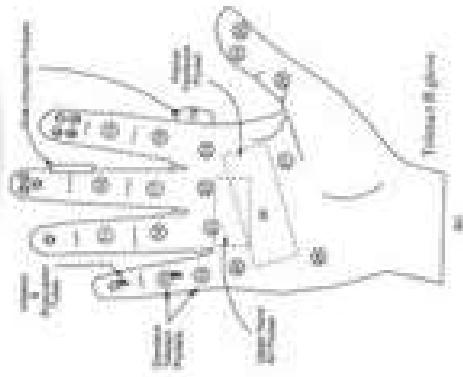
Wright & Coggrave (2007)



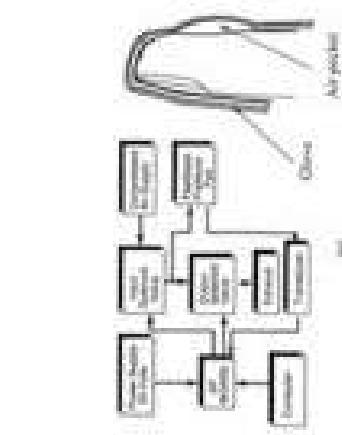
Glassmire thesis (2006)



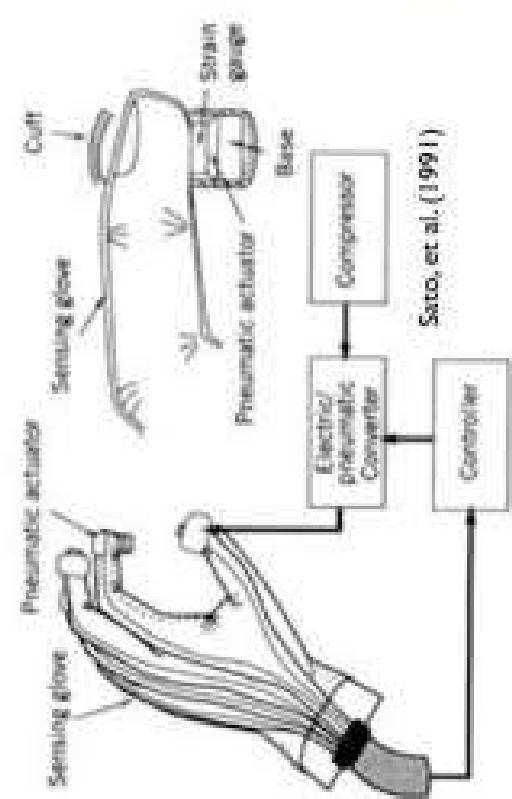
Santos et al.



Stone (1992)



Burdia (1995)



Sato, et al. (1991)



Immersion CyberTouch Glove

# **kinesthetic (force-feedback) device basics**

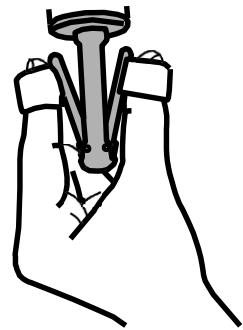
# typical kinesthetic device configurations

manipulandum



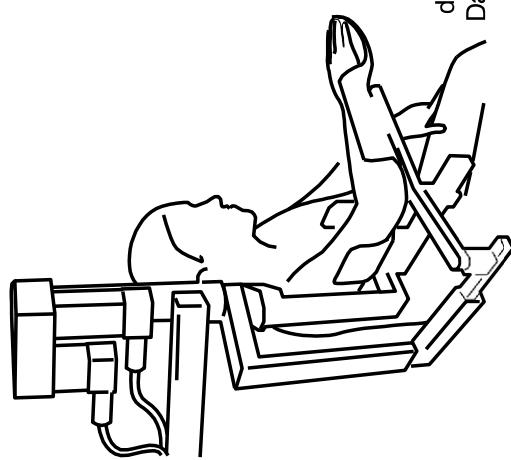
drawing by Jorge Cham

grasp



drawing by Tricia Gibo

exoskeleton



drawing by  
David Grow

# manipulandum (expensive)



Omega  
from Force Dimension  
delta configuration  
3 degrees of freedom



Phantom Premium 1.5  
from SensAble/Geomagic  
5-bar + rotation  
3 degrees of freedom



Virtuose  
from Haption  
additional “wrist”  
6 degrees of freedom

all images from Wikimedia Commons  
© Allison M. Okamura, 2020

# manipulandums (cheaper)



Falcon  
from Novint

**delta configuration**  
**3 degrees of freedom**

image from Wikimedia Commons



Phantom Omni/Touch  
from SensAble/Geomagic

**5-bar + rotation**  
**3 degrees of freedom**

photographed by Akiko Nabeshima



Sidewinder  
from Microsoft

**spherical mechanism**  
**2 degrees of freedom**

image from Wikimedia Commons

# Grip/grasp



Custom haptic gripper for  
**Phantom Premium**

© 2007 IEEE. Reprinted, with permission,  
from L.N.Yerner and A.M. Okamura...  
Effects of Translational and Gripping Force  
Feedback are Decoupled in a 4-Degree-  
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Single-finger Cybergloves  
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photograph courtesy  
Stanford Center for Design Research



da Vinci Surgical System  
from Intuitive Surgical, Inc.  
(no programmable force  
feedback on gripper)

photographed by Akiko Nabeshima

# Exoskeletons



KINARM Exoskeleton  
from BKIN Technologies



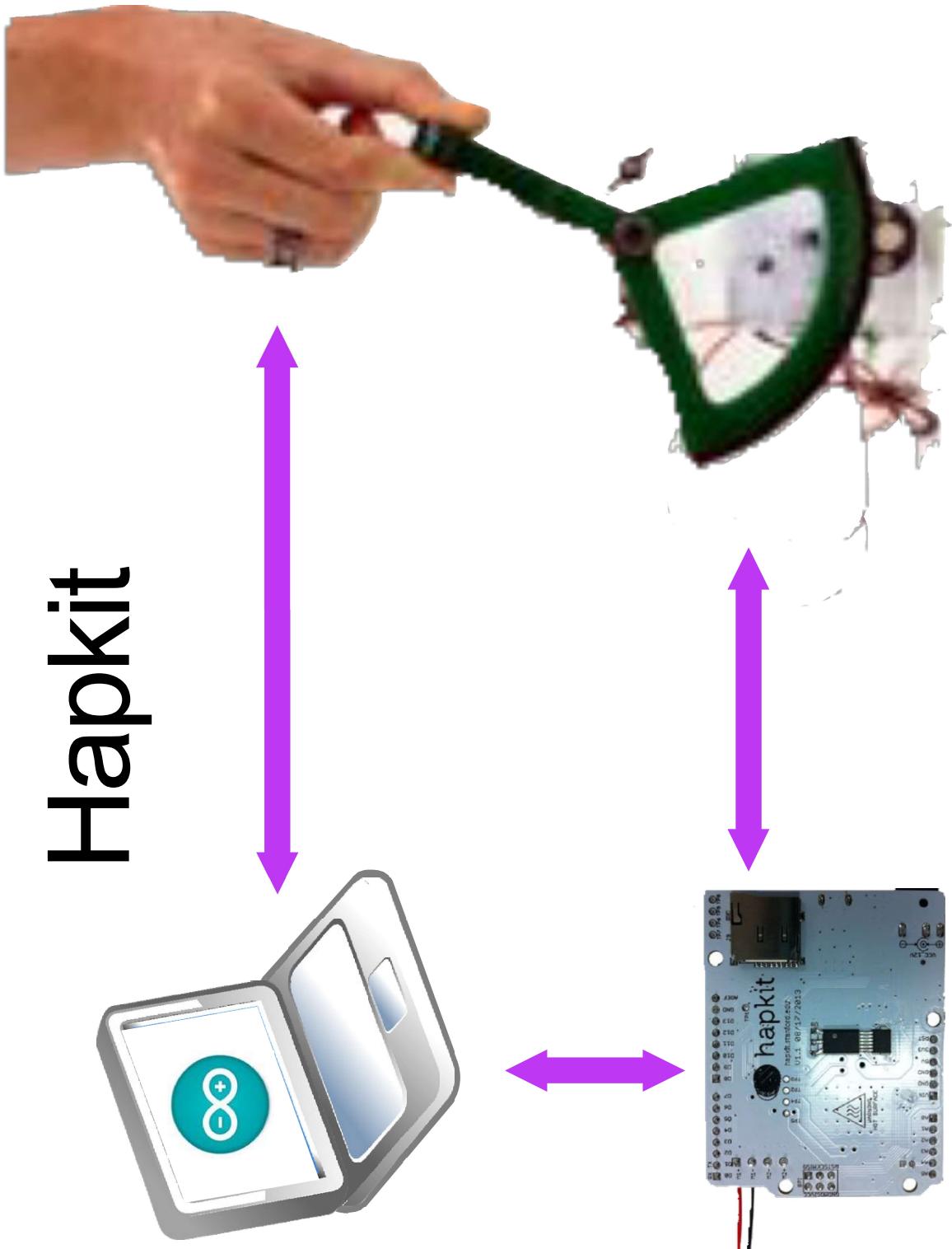
Harvard

images from Wikimedia Commons

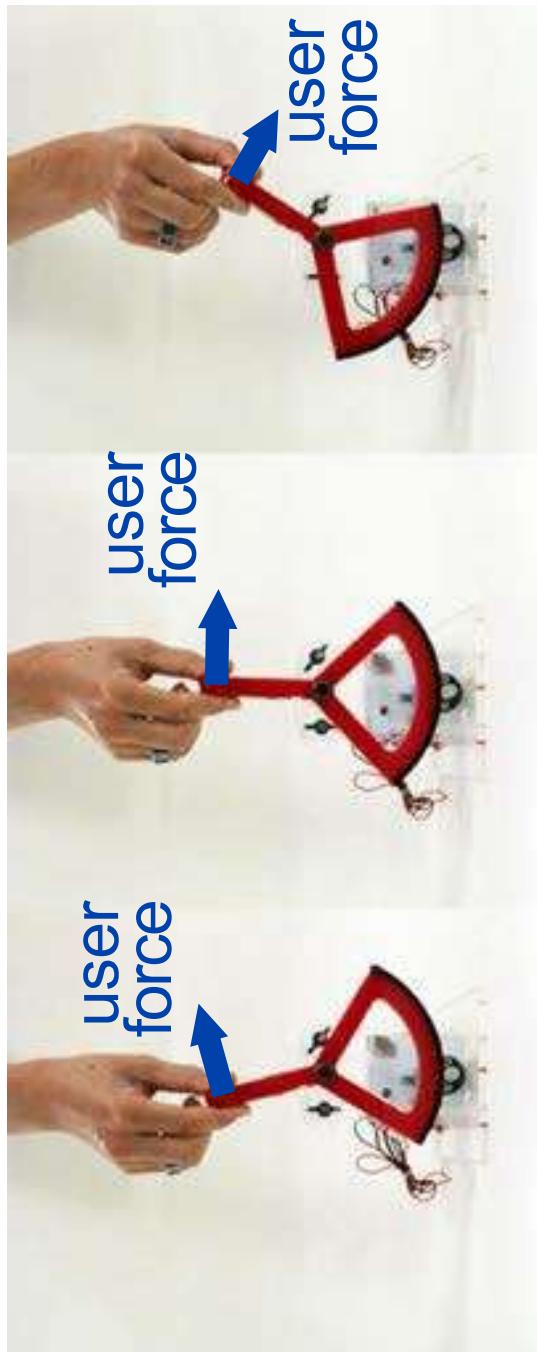


DARPA

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# Hapkit



# Hapkit



# Trends driving haptics

- Networking - constant connectivity
- Ubiquity of computing devices - beyond sparse visual real estate
- Multitasking- doing more things at once may benefit from multiple channels of communication
- Virtualization – fostering presence

# **rendering friction**

(in one degree of freedom)

# surface properties

- typical haptic display general shape very well, but don't feel “realistic”
  - add surface properties to increase realism
- how you do it depends on
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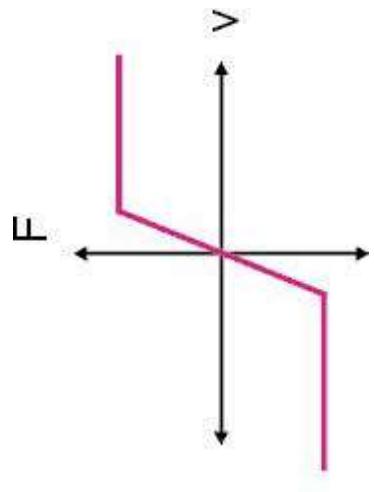
# damping for virtual walls

- a pure springforce for a wall may seem to “active”
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  - only damps when going into the wall
- this can also create vibrations upon wall impact

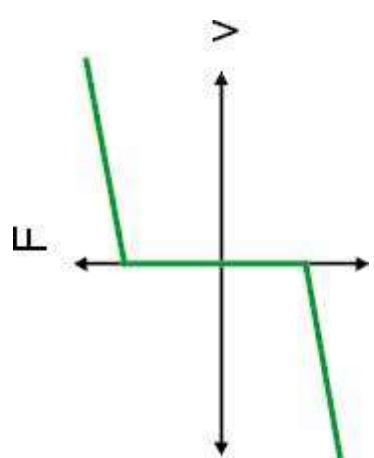
$$F = \begin{cases} k\Delta x + b\dot{x} & \text{for } \dot{x} > 0 \\ k\Delta x & \text{for } \dot{x} < 0 \end{cases}$$

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- surfaces can feel unnaturally slippery
- friction would help, but it is difficult to implement
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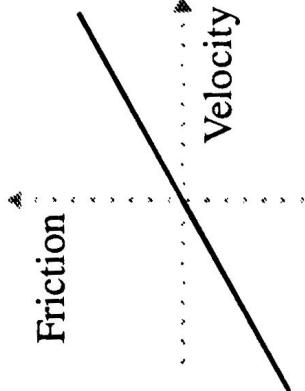
friction



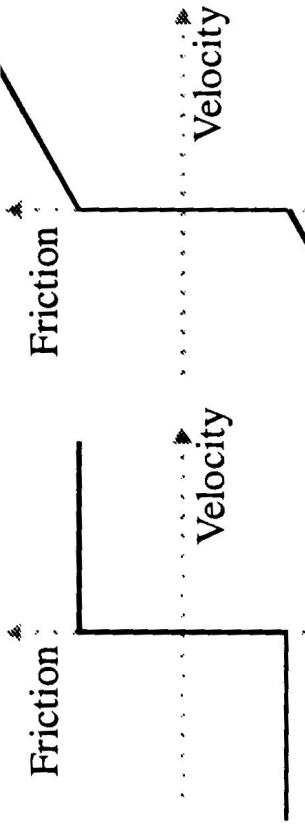
damping

# friction display

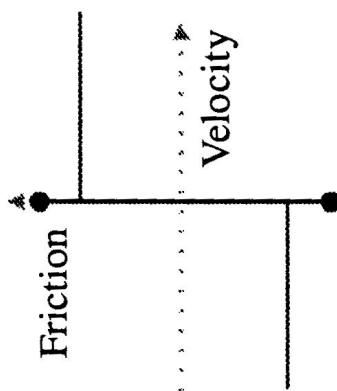
difficult to render because it is non-linear



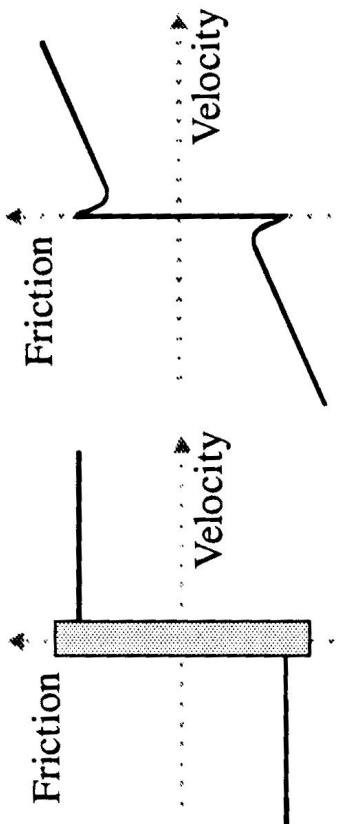
(a) damping



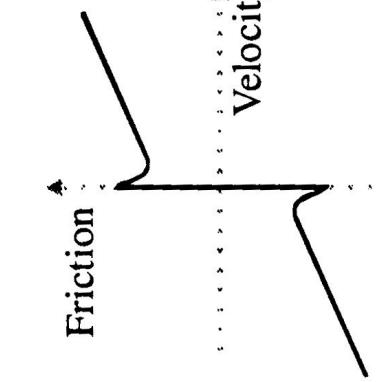
(b) Coulomb



(d) stiction



(e) static



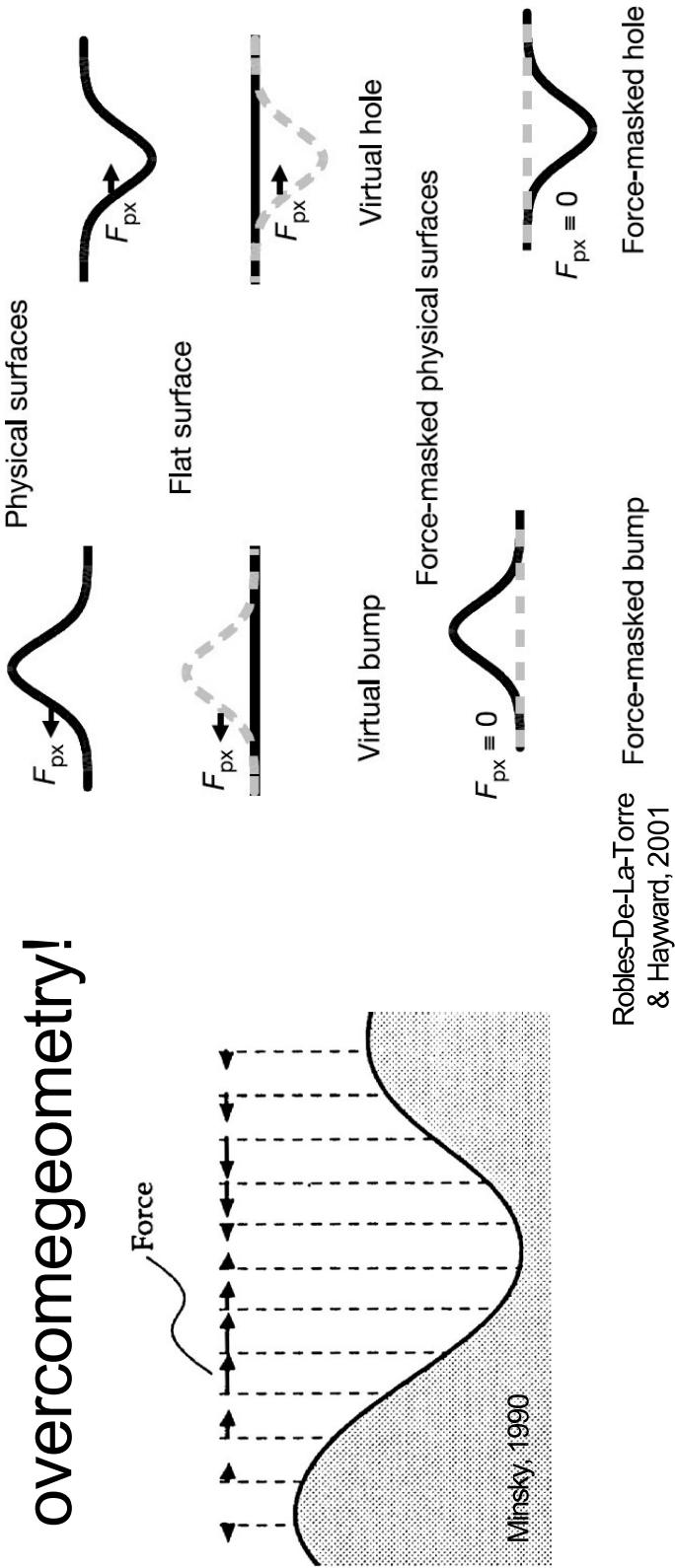
(f) Stribeck effect

# **rendering bumps and textures**

(in one degree of freedom)

# bumps and valleys

- as a user moves “up” a bump, motion is opposed.
- done in 2D, springforce proportional to height of bump
- force information can overcome geometry!



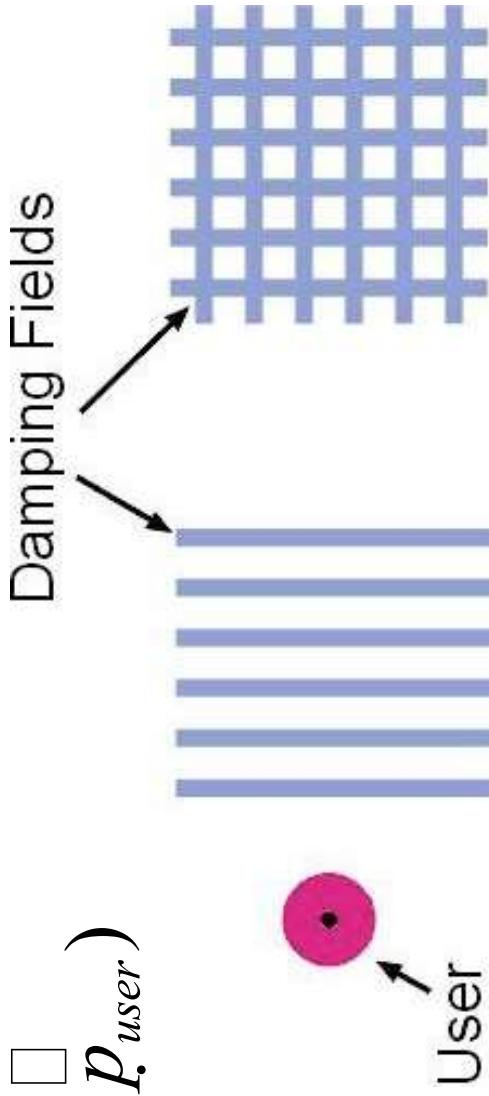
Robles-De-La-Torre & Hayward, 2001

© Allison M. Okamura, 2020

# damping textures

if  $p_{user}$  is inside a damping area

$$F = b v_{user} \quad (v_{user} = p_{user})$$



note that vibrations occur due to discontinuity in force

# simulating and rendering dynamic objects (in one degree of freedom)

# dynamic Simulation of “rigid” bodies

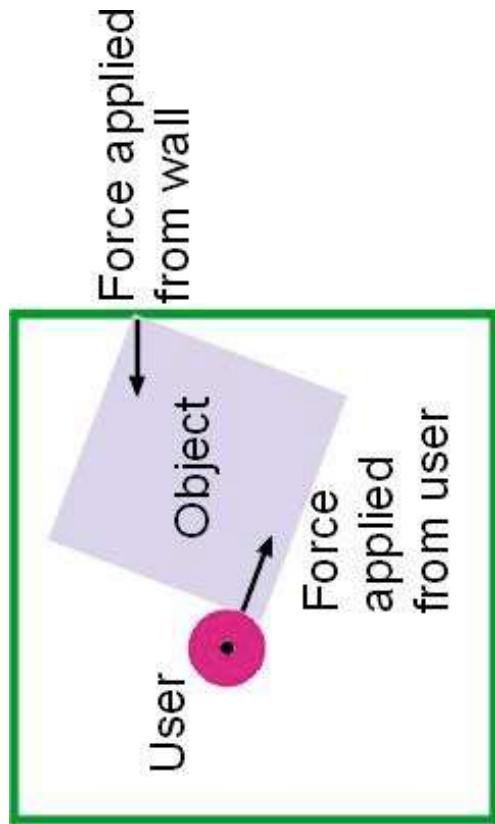
- assumptions
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  1. save the state of the moving object
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# Object state

- the “state” of an object is used to describe its current condition
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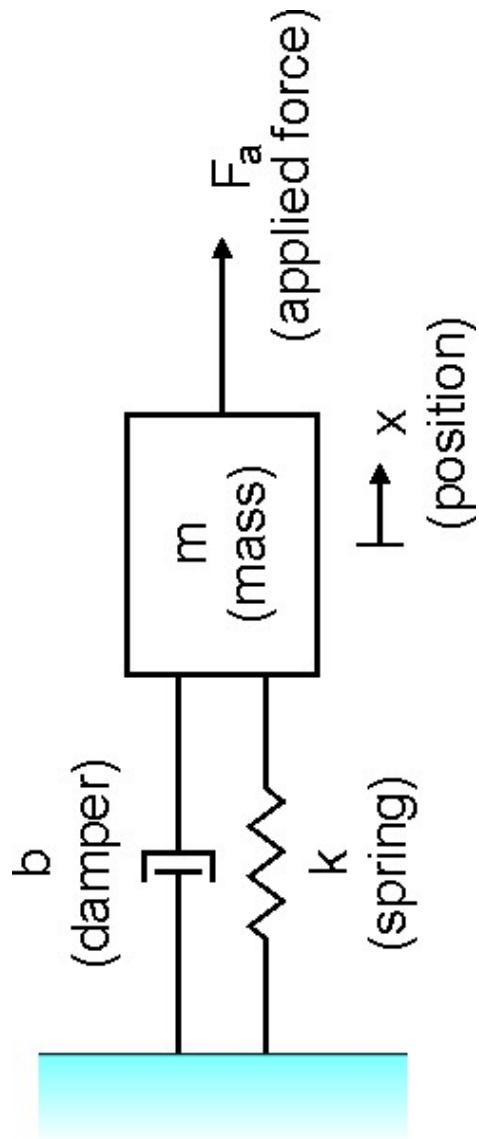
# calculating forces on an object

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# second order dynamic systems

## mass-spring-damper



$x = 0$  at equilibrium