Homework 4

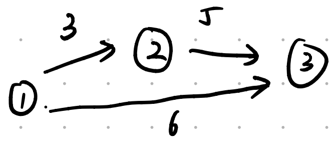
Chapter 4 Exercise 2

(a) True. consider if $\{e_1, e_2, e_3, \ldots e_m\}$ is ordered from small to big, $\{e_1^2, e_2^2, e_3^2, \ldots, e_m^2\}$ is also ordered from small to big.

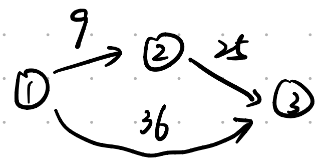
If we use kruskal algorithm to generate the minimum spanning Tree of a graph, it only cares about the relative order of costs (sorting in ascending cost order) and does not care about actual values.

We Input new cost. and the edges will be sorted in the same order, resulting in same minimum spanning Tree T.

(b) False, there is a counterexample.



before



after

Let Path 1 = (1,2), (2,3)
cost 1 = 3+5 = 8
Path 2 = (1,3)
cost 2 = 6

Let Path 1 = (1,2)(2,3)
cost 1 = 9+25 = 34
Path 2 = (1,3)
Cost 2 = 36.

cost 1 > cost 2 ⇒ Shortest Path is (1,3)

cost 1 < cost 2.
⇒ Shortest path is (1,2)(2,3).

So, shortest path didn't remain the same after squaring the cost of a directed graph.

Chapter 4 Exercise 8

Prove: First of all, as the graph G is connected, by using the greedy algorithms (like kruskal, prim.etc), we can find at least one Minimum Spanning Tree.

Let's prove such a MST is unique.

Suppose there are two minimum Spanning Trees, A and B. Let e be the edge in just one of A, B with the smallest cost, connecting node P and node Q.

Suppose e (P-Q) is in A but not B. Then B must contain a path from P to Q. which is not simply the edge e (P-Q) So if we add e (P-Q) to B, then we get a cycle. If all the other edges in the cycle were in A, then A would have a cycle. which is contradict to A is a Tree. So it can not. Therefore, the cycle must have an edge f not in A. Hence, by the definition of e and the fact that all edge-costs are different, the cost of f must be bigger than the cost of e. So if we replace f by e we get a spanning tree with smaller total cost of B.

It is contradict to B is a minimum Spanning Tree.

Similarly, supose e (P-Q) is in B but not A, we can find it is contradict to A is a minimum Spanning Tree.

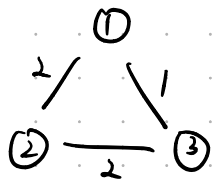
In Conclusion. G has a unique minimum spanning Tree.

Chapter 4 Exercise 22

This statement is False, there is a counterexample.

We use (p, q, w) represent an undirect edge between p and q whose cost is w.

Consider $G = (V, E)$ $V = \{1, 2, 3\}$ $E = \{(1,2,2), (1,3,1), (2,3,2)\}$



The edge set $E_1 = \{(1,2,2), (2,3,2)\}$ is not a MST. since there is a spanning tree edge set $E_2 = \{(1,3,1), (2,3,2)\}$ whose total weight is 3.

Note that spanning tree set $E_3 = \{(1,3,1), (1,2,2)\}$ is also a MST edge set.

$(1,2,2) \in E_3$. $(2,3,2) \in E_2$. but E_1 is not a MST edge set.

In conclusion, the statement is false.