LABORATORY MANUAL

CZ2003: Computer Graphics and Visualization
SW Lab or Your Own Computer

Making Images with Mathematics

Lab Experiments 1 - 5

SESSION 2022/2023
SEMESTER 1
COMPUTER SCIENCE COURSE

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY

MAKING IMAGES WITH MATHEMATICS

1. OBJECTIVE

In this coursework you will learn how to visualize curves, surfaces and solid shapes defined by simple mathematical formulas. Each of 5 experiments takes 2 hours. Upon completion of these experiments you will know:

- How to define shapes by parametric and implicit functions, and
- How to transform and animate shapes.

2. EQUIPMENT

You have to install the latest version of *ShapeExplorer* from the course site. The software is available for Windows, MacOS and Linux (Ubuntu). **You can work on the lab assignments using your either the computers in the lab and your own computers.**

3. INTRODUCTION

In this labs, you will learn how to define geometric shapes by mathematical formulas which are used to compute coordinates of all the points belonging to the shapes. You will be using ***ShapeExplorer*** (Fig. 1) which is an interactive software tool designed to display shapes defined by *parametric* and *implicit* function scripts and colors defined by *explicit* functions. It can run on Windows, MacOS and Linux (Ubuntu) computers. Shape Explorer is just one interactive window where you can type the definitions scripts and other parameters. The definitions can be saved in a proprietary format and loaded later to the software to continue working with them. Shape Explorer supports only one shape visualization at the time. Its purpose is to work as a quick all-in-one multi-platform visualization tool.

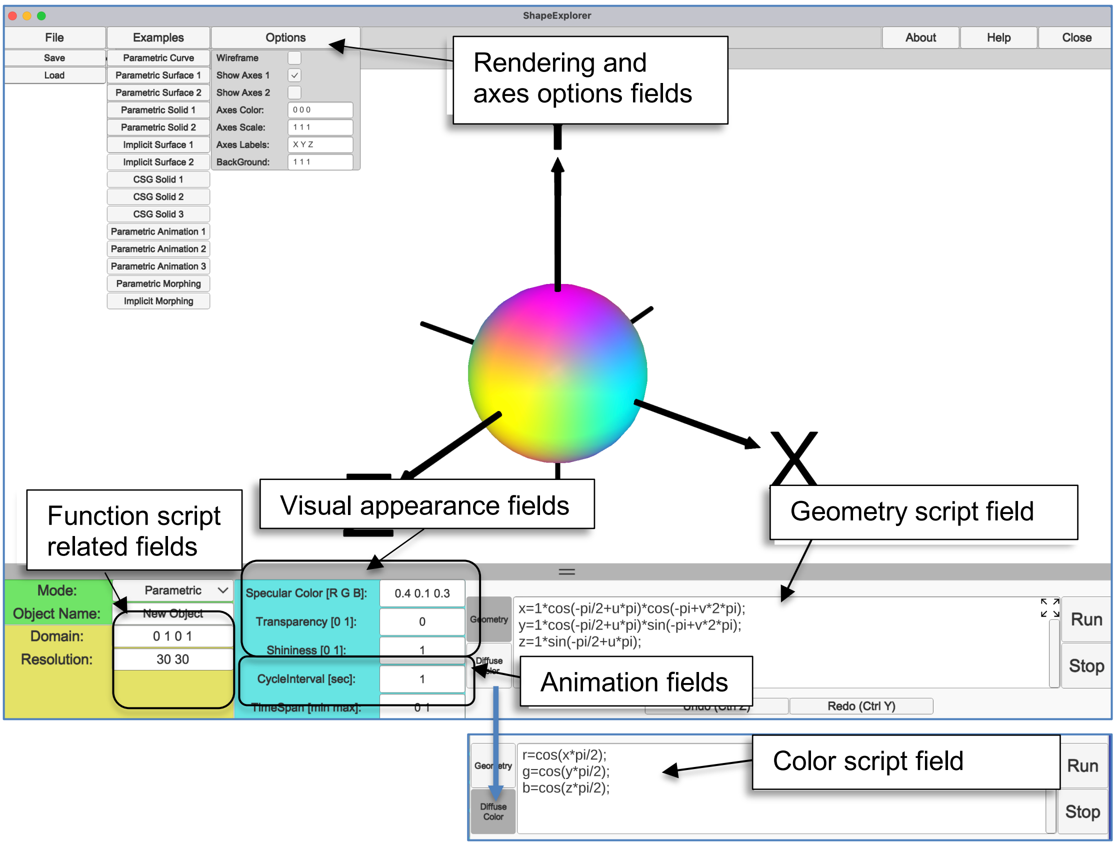


Figure 1. ShapeExplorer at a glance.

Parametric functions in ShapeExplorer are explicit functions of up to three variables u, v, w (which are called parameters) and time t. They can define Cartesian coordinates x, y, z of curves, surfaces and solid objects. To define a curve, only one parameter u, v or w has to be used, to define a surface–any two parameters u,v , v,w or u,w are required, for solid objects–all three parameters u, v, w have to be used. When t is added, these objects will become time-dependent. The examples of defining parametrically curves, surfaces and sold objects are given in Figs. 2, 3 and 4.

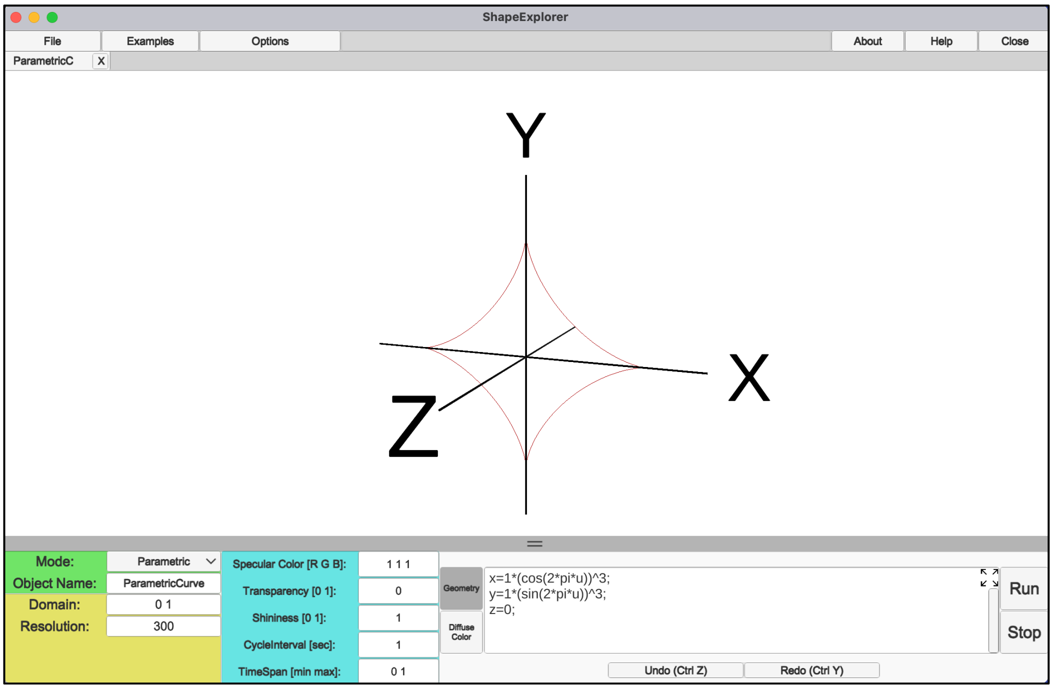


Figure 2. Displaying a parametric curve.

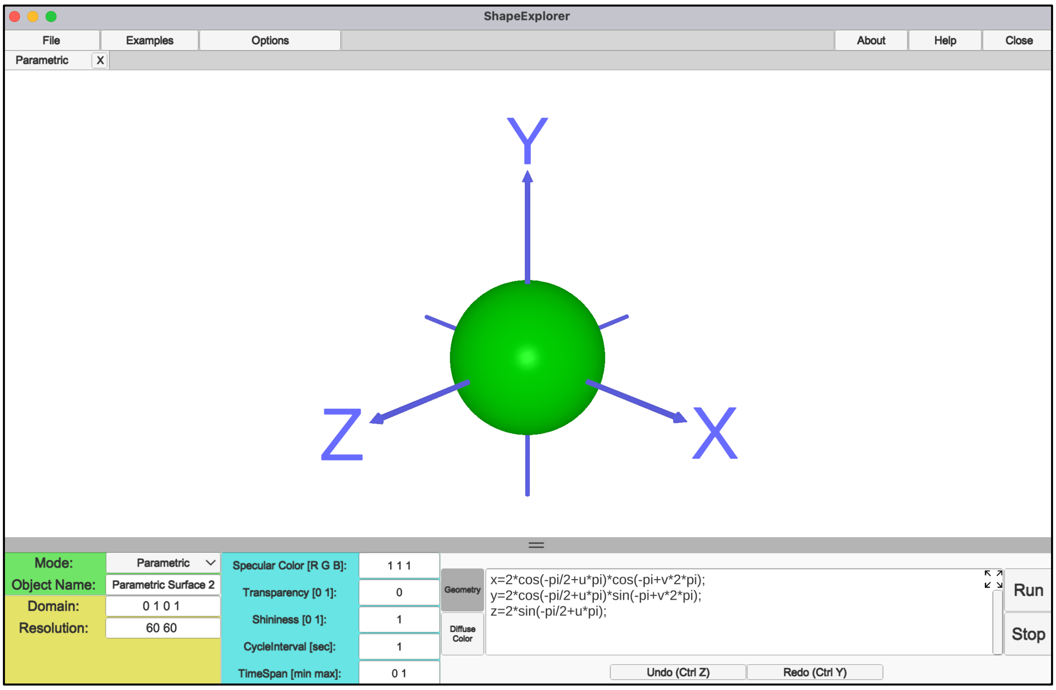


Figure 3. Displaying a parametric surface.

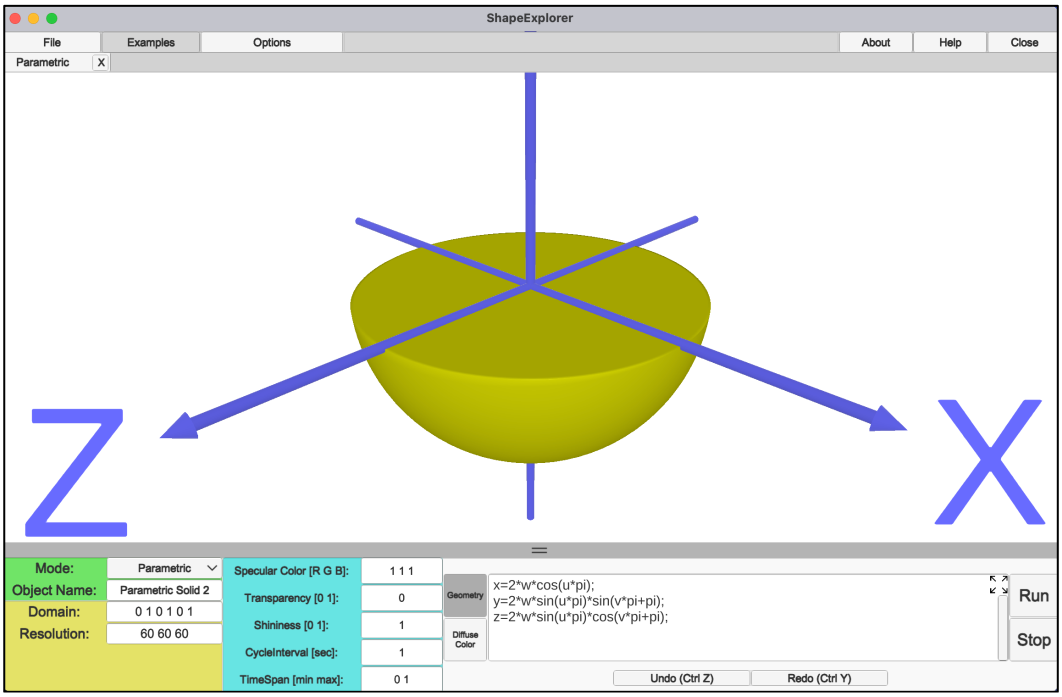


Figure 4 Displaying a parametric solid.

Implicit functions are the functions defined as $f(x, y, z, t) = 0$, where x, y, z are Cartesian coordinates and *t* is the time. You will only use them for defining surfaces. The implicit functions are equal to zero for the points located on the surface. By changing this equality into an inequality $g = f(x, y, z, t) \geq 0$, known in computer graphics as *FRep*, we define not only a surface but the space bounded by this surface, or a half-space. In this case, the function equals to zero for the points located on the surface, positive values of the function indicate points inside the solid object, and negative values are for the points which are outside the object. In the ShapeExplorer script field, only the left part of the implicit or *FRep* function has to be written – it is always assumed to be ≥ 0. The example of defining a surface is given in Fig. 5 and a definition of the solid object using set-theoretic (Boolean) operations is illustrated in Fig. 6.

Defining shapes and animations may require multiple formulas and temporary variables. ShapeExplorer supports a subset of C# language for writing definition scripts. The following mathematical functions are supported:

$abs, sqrt, exp, log, sin, cos, tan, acos, asin, atan, ceil, floor, atan2, mod, round, max, min, cosh, ssinh, tanh, log10$.

There is also $if\{\}\ else\ \{\}$ operator. Variables, x, y, z are reserved for Cartesian coordinates, while variables u, v, w are parametric coordinates. Variable t is reserved for defining the time. Al other variable can be used without declaration—they will be declared as *float*.

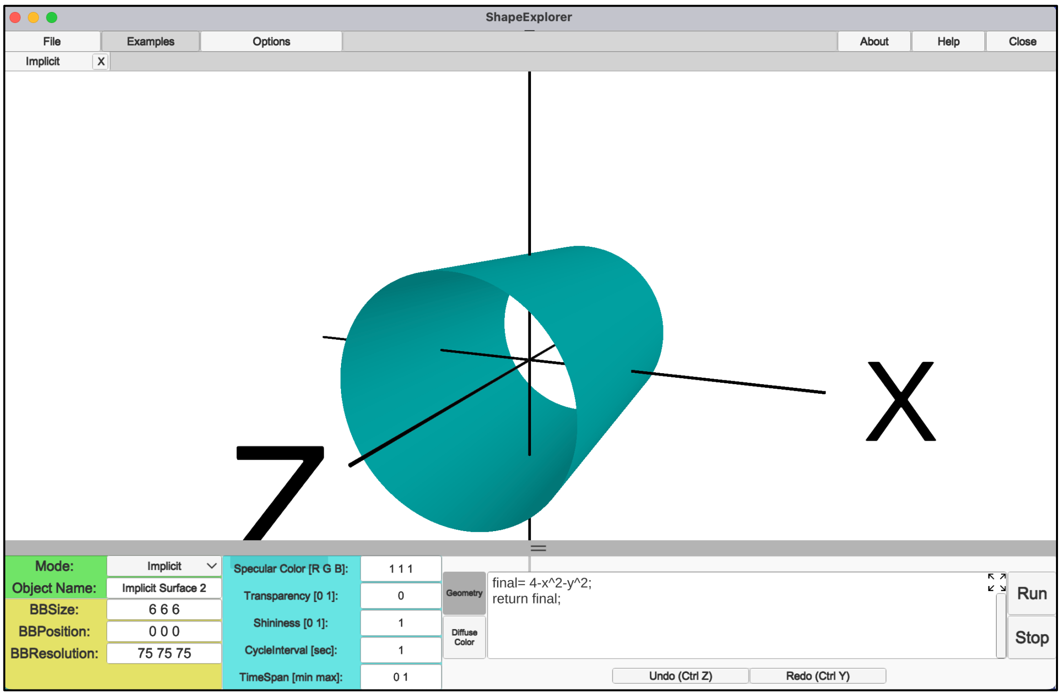


Figure 5 Displaying an implicit surface. Both sides of the surface are visible.

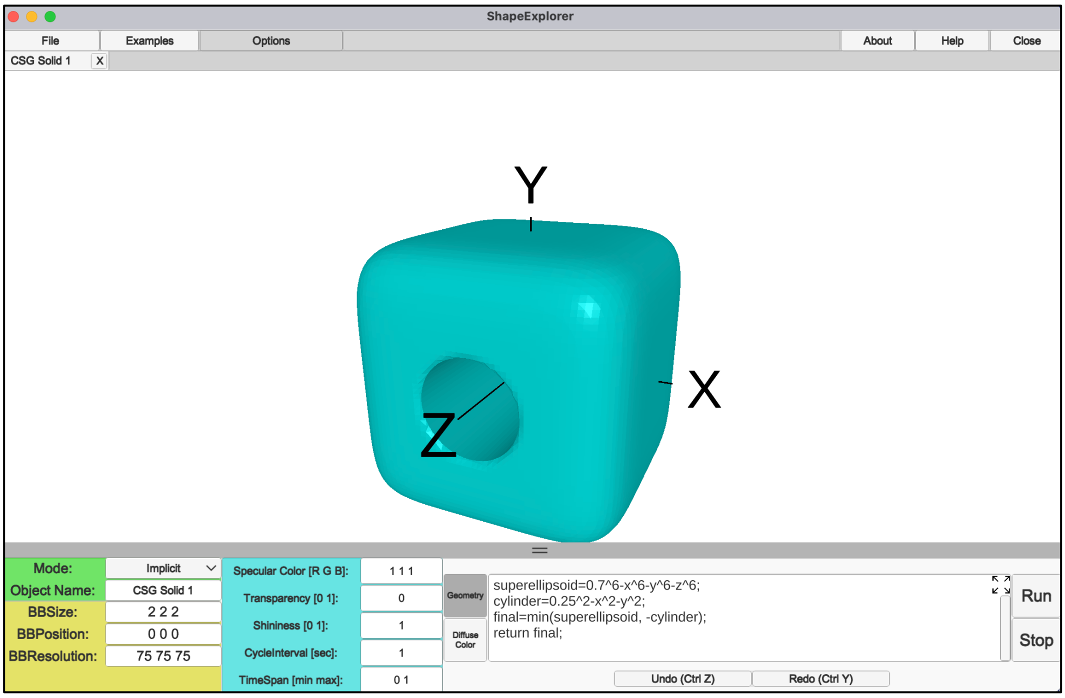


Figure 6 Displaying an implicit solid. The inner surface of the object is not visible in solids.

For defining r, g, b diffuse colors, scripts with explicit functions of coordinates x, y, z have to be used:
$r|g|b = f(x, y, z, t);\ r, g, b \in [0,1]$. Thus the color is defined for any point of the 3D modelling space and it is then sampled by the geometry (see Fig. 7). Constant colors can also be defined as $r|g|b = value \in [0,1];$

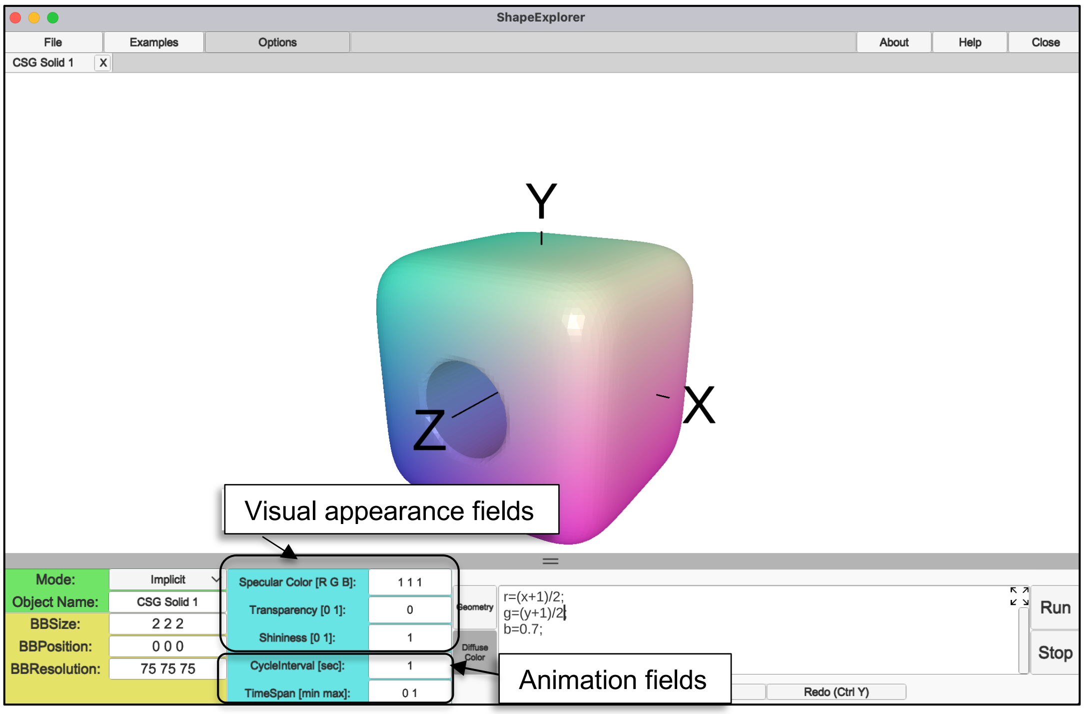


Figure 7 Defining diffuse colors as functions of coordinates.

In addition, in the Visual Appearance fields the *Specular Color* [0,1], *Shininess* [0,1] and *Transparency* [0,1] values can be defined. *Specular Color* affects overall illumination of the object. It is responsible for displaying reflections or specs on the surface of objects. Parameter *Shininess* controls the size of such specs. Colors of curves can be only defined if *Shininess* is set to 1.

The remaining Animation fields (Fig. 7) contain *TimeSpan* and *CycleInterval*. The *TimeSpan* contains minimum and maximum values of the time values between which it will be incremented in a cyclic manner. These values can be positive and negative. The *TimeSpan* interval then is mapped to the real time interval *CycleInterval* in seconds – this is how the animation will be then displayed.

Options fields (Fig. 7) contain parameters of the coordinate axes, an option to toggle to wireframe visualization mode, and background color. Coordinate axes can be scaled—shorten with the scaling parameters less than 1 or elongated with the scaling parameters greater than 1. Scaling coefficient 0 will toggle off the respective axis.

4. <u>EXPERIMENTS</u>

There are 5 lab sessions comprising **5 experiments**:

1. Parametric Curves	12 marks
2. Parametric Surfaces	12 marks
3. Parametric Solids	12 marks
4. Implicit Surfaces and Solids. Colors and 3D Textures	12 marks
5. Animation	12 marks
Total:	**60 marks**

All the experiments are personalized, i.e. each student will have different data to work with. The personalization is based on using two last digits of your matriculation number:

U1234567G

NM

which can be integer numbers from 0 to 9 where 0 will stand for 10. Therefore, the two numbers from 1 to 10 will define your personal variant of the assignments. These numbers will be further referred to as ***N*** for penultimate digit and ***M*** for the last digit.

This is an individual assignment.

In the remaining part of the manual, you will find the assignment instructions. Each of the five lab assignments will be evaluated and awarded up to 12 marks. Partial marks are indicated in the assignment instructions.

After completion of each of five lab assignment, you have to write a report following the template shown in Fig. 8.

WITHIN ONE WEEK (7*24 HOURS) AFTER THE END OF EACH OF FIVE SCHEDULED LAB SESSION you have to do the following:

1. Create a folder and name it **exactly as your name is written on your matriculation card and add as a suffix the two last matric number NM**, e.g., JAMES BOND 67.
2. Copy to this folder the scan/photo of your matriculation card with clearly readable name, photo, and at least three last characters of the matriculation number. Do not make any additional subfolders
3. Copy to this folder the PDF fie of the report and all the relevant *.*func* files.
4. Zip your assignment folder. The zipped file **must have the same name as your folder**, e.g., JAMES BOND 67.zip. Please check that your file can be unzipped to the folder with your name.
5. Submit the zipped file through the respective digital drop box in the **lab** website (note that website has been created for each lab group).
6. Check your email box regularly. The lab instructors or subject coordinator will email you if something is wrong.

CZ2003: Lab # 1 {, 2, 3, 4 or 5}

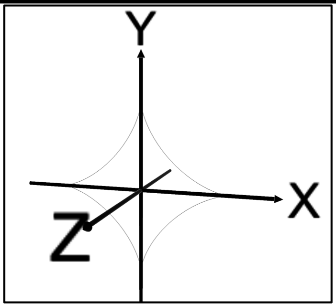
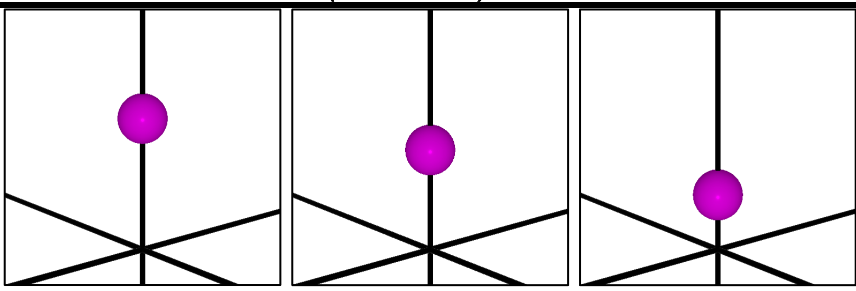
Name: < as in your matric card>	**Last two digits of the matric card:**
Q1a	 *A screenshot of the shape displayed with coordinate axes (lab 1-4) or 3 screenshots from animation (lab 5)*
	Name of the file: ***Q1a.func (for labs 1-3) or Q1.func (for lab 4 and 5)***
Q1b	*A screenshot of the shape displayed with coordinate axes (lab 1-4)*
	Name of the file: ***Q1b.func (for labs 1-3)***
Q1c	*A screenshot of the shape displayed with coordinate axes (lab 1-4)*
	Name of the file: ***Q1c.func (for labs 1-3)***
Q1c	*A screenshot of the shape displayed with coordinate axes (lab 1-4)*
	Name of the file: ***Q1d.func (for labs 1-3)***
Q2	 *A screenshot of the shape displayed with coordinate axes (lab 1-4) or 3 screenshots from animation (lab 5)*
	Name of the file: ***Q2.func***
Q3	*A screenshot of the shape displayed with coordinate axes (lab 1-4) or 3 screenshots from animation (lab 5)*
	Name of the file: ***Q3.func***

Figure 8. Template of the report.

4.1 Experiment 1: Parametric Curves

This assignment illustrates Module 3, and it serves a purpose to teach you how to visualize curves defined by parametric functions. To work on this assignment, you have to watch the following TEL lectures:

Module 1: Lecture 2 (Part 3) - Introduction to Computer Graphics and Foundation Mathematics {Rene Descartes and coordinate systems}
Module 3: Lecture 1 (Part 2/3) - Geometric Shapes: 2D Curves {straight-lines}
Module 3: Lecture 1 (Part 3/3) - Geometric Shapes: 2D Curves {straight-lines}
Module 3: Lecture 2 (Part 1/3) - Geometric Shapes: 2D Curves {circle}
Module 3: Lecture 2 (Part 2/3) - Geometric Shapes: 2D Curves {circle and beyond}
Module 3: Lecture 2 (Part 3/3) - Geometric Shapes: 2D Curves {ellipse and summary}
Module 3: Lecture 3 - Geometric Shapes: 3D Curves

Assignment instructions:
Create folder **Lab1**. Download into it from the course-site the file **ParametricCurve.func** (Fig. 9). Use it as a reference for the following exercises with curves. For each of the displayed curves, you have to select a sampling resolution providing for smooth curve visualization. Each of the curves has to be displayed with 1 sec and using ***red*** color with ***black*** coordinate axes on the ***white*** background.

1. Using functions $x(u), y(u), u \in [0,1]$, define parametrically in 4 separate files and display:

   a. A straight line segment spanning from the point with coordinates (-***N***,-***M***) to the point with coordinates (***M***,***N***).

   b. A circular arc with radius ***N***, centered at point with coordinates (***N***,***M***) with the angles $[\frac{\pi}{\boldsymbol{N}}, 2\pi]$.

   c. An origin-centered 2D spiral curve which starts at the origin, makes ***N***+***M*** revolutions clockwise (as counted from the positive axis X) and reaches eventually the radius 2****M***.

   d. A 3D cylindrical helix with radius ***N*** which is aligned with axis *Z*, makes ***M*** counterclockwise revolutions about axis *Z* while spanning from $z_1 = -\boldsymbol{N}$ to $z_2 = \boldsymbol{M}$.

(4 marks)

2. Based on the explicit definition of the curve with number ***M*** (Table 1), derive its parametric representation $x(u), y(u), u \in [0,1]$, and then modify it to scale and translate the curve so that it will make ***N*** full periodic oscillations* within the given *x*-domain (Fig. 10). Display the curve.

(4 marks)

3. A so-called "butterfly curve" is defined in polar coordinates by:

$$r = e^{\sin(\alpha)} - 2\cos(4\alpha) + \sin^5\left(\frac{2\alpha - \pi}{24}\right) \quad \alpha \in [0, 2\pi\boldsymbol{N}]$$

   Derive its parametric representation in Cartesian coordinates as $x(u), y(u), u \in [0,1]$ and display the curve with the centre at coordinates (***N***, ***M***).

(4 marks)

* *Oscillation of a curve is its repetitive variation about a central value or between two states.*

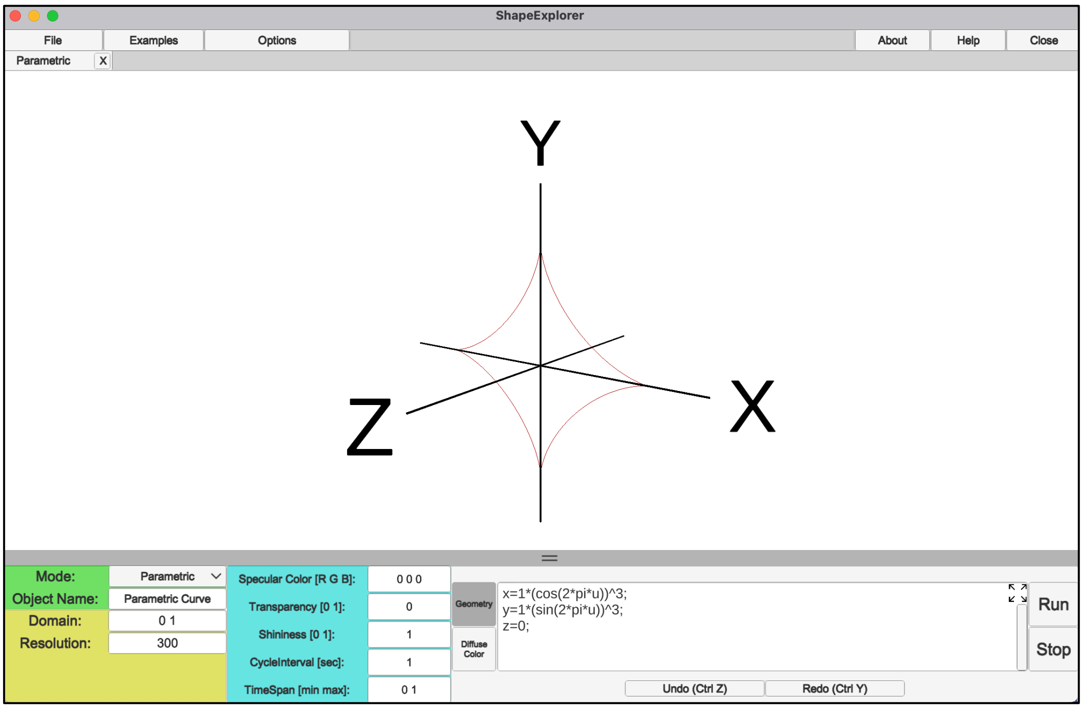


Figure 9. Displaying the shape defined in ParametricCurve.func.

Table 1. Trigonometric curves defined explicitly.

M	Formula of the curve and the target domain.
1	$y = \sin x + \cos^3 x$ $x \in [-N, 2N]$
2	$y = \cos(2x) + \sin(3x)$ $x \in [-N, N]$
3	$y = \sin^2 x - \cos^3(2x)$ $x \in [-2N, N]$
4	$y = 5\sin^3 x - \cos x$ $x \in [-N, 3N]$
5	$y = \lvert\sin x\rvert - 2\sin^3(2x)$ $x \in [-1.5N, 2N]$
6	$y = \sin(x)\cos(3x)$ $x \in [-N, 1.5N]$
7	$y = 2\sin(3x) + \cos x$ $x \in [-1.5N, 2.5N]$
8	$y = 2\sin(x) + \sin(2x)$ $x \in [-0.7N, 2N]$
9	$y = \sin(3x) + \sin x$ $x \in [-N, 1.8N]$
10	$y = \sin(2x) + \cos x$ $x \in [-1.3N, 2N]$

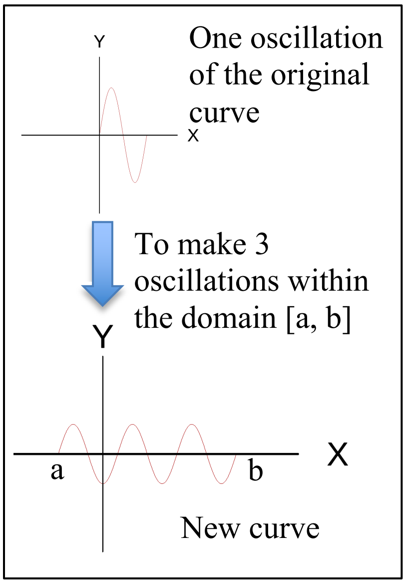


Figure 10. A curve makes 3 oscillations in domain [a,b]

4.2 Experiment 2: Parametric Surfaces

This assignment illustrates Module 3, and it serves a purpose to teach you how to visualize surfaces defined by parametric functions. To work on this assignment, you have to watch the following TEL lectures:

Module 3: Lecture 4 (Part 1/4) - Geometric Shapes: Surfaces {classification, polygon mesh}
Module 3: Lecture 4 (Part 2/4) - Geometric Shapes: Surfaces {plane}
Module 3: Lecture 4 (Part 3/4) - Geometric Shapes: Surfaces {plane parametrically and bilinear surface}
Module 3: Lecture 4 (Part 4/4) - Geometric Shapes: Surfaces {bilinear surfaces and summary}
Module 3: Lecture 5 (Part 1/2) - Geometric Shapes: Quadric Surfaces and Sweeping {sphere}
Module 3: Lecture 5 (Part 2/2) - Geometric Shapes: Quadric Surfaces and Sweeping {other quadrics and sweeping}

Assignment instructions:
Create folder **Lab2**. Download into it the file **ParametricSurface.func** from the course-site (Fig. 11). Use it as a reference for the following exercises with parametric surfaces. For each of the displayed surfaces, you have to select a sampling resolution providing for smooth surface visualization. Each of the surfaces has to be displayed with 1 sec using ***shiny green*** color with ***black*** coordinate axes on the ***white*** background.

1. In 4 separate files, define parametrically using functions $x(u,v), y(u,v), z(u,v)$, $u,v \in [0,1]$ and display:

   a. A part of plane passing through the points with coordinates (***N***, ***M***, 0), (0, ***M***, ***N***), (***N***, 0, ***M***).

   b. A triangular polygon with the vertices at the points with coordinates (***N***, ***M***, 0), (0, ***M***, ***N***), (***N***, 0, ***M***).

   c. An origin-centered ellipsoid with the semi-axes ***N***, ***M***, (***N***+***M***)/2

   d. A cylindrical surface with radius *N* which is aligned with axis Z, and spans from $z_1 = -\boldsymbol{N}$ to $z_1 = \boldsymbol{M}$.

(4 marks)

2. Define parametrically by functions $x(u,v), y(u,v), z(u,v), u,v \in [0,1]$ a surface obtained by translational sweeping of the curve defined in polar coordinates by:

$$r = \boldsymbol{N} - (\boldsymbol{M} + 5)\cos\alpha \quad \alpha \in [0,2\pi]$$

The curve has to be first placed in coordinate plane X*Y* and then swept along axis *Z* so that the surface will span from $z_1 = -\boldsymbol{N}$ to $z_1 = \boldsymbol{M}$.

(4 marks)

3. Define parametrically by functions $(u,v), y(u,v), z(u,v)$, $u,v \in [0,1]$ a surface created by rotational sweeping of the same polar curve $r = \boldsymbol{N} - (\boldsymbol{M} + 5)\cos\alpha$ $\alpha \in [0,2\pi]$. The curve has to be first placed in coordinate plane *ZY*, then translated by (0, 0, -***N***) and finally subjected to rotational sweeping about axis Y clockwise by angle $\frac{\pi}{N}$ with the offset angle $\frac{3\pi}{2M}$ (the rotation angle and offset are counted from axis Z).

(4 marks)

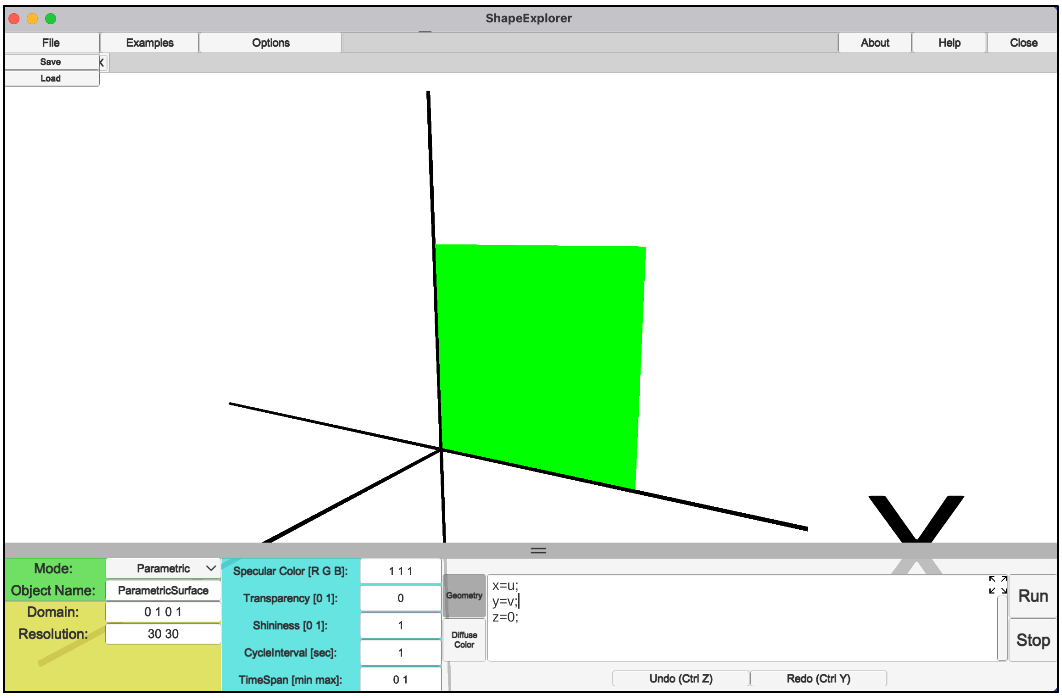


Figure 11. Displaying the shape defined in ParametricSurface.func.

4.3 Experiment 3: Parametric Solids

This assignment illustrates Module 3, and it serves a purpose to teach you how to visualize surfaces defined by parametric functions. To work on this assignment, you have to watch the following TEL lectures:

Module 3: Lecture 6 (Part 1/2) - Geometric Shapes: Solid Objects {parametric solids}
Module 3: Lecture 6 (Part 2/2) - Geometric Shapes: Solid Objects {solids by sweeping}

Assignment instructions:
Create folder **Lab3**. Download into it the file **ParametricSolid.wrl** from the course-site (Fig. 12). Use it as a template for the following exercises with parametric solids. For each of the solids, you have to select a sampling resolution providing for smooth solid surface visualization. Each of the solids has to be displayed with 1 sec using ***shiny*** *yellow* color with ***black*** coordinate axes on the ***white*** background.

1. Define parametrically using functions $x(u,v,w),\ y(u,v,w),\ z(u,v,w)$, $u,v,w \in [0,1]$ in 4 separate files and display:

   a. A solid box with the sides parallel to the coordinate planes and the coordinates of two opposite vertices (***N***, 0, ***M***), (***N+M***, ***M,*** 2(***N+M***)).

   b. A solid three-sided pyramid with the vertices of the base with coordinates (0,0,0), (***N***,0,0), (0,0,***M***), and the apex at (0,***N+M***,0).

   c. A lower half of the origin-centered solid sphere with radius ***N***.

   d. An upper half of the solid torus which axis is the vertical axis Y. The radius of the torus tube is $\frac{\boldsymbol{N}}{5}$. The distance from axis Y to the center of the torus tube is ***N***.

   (4 marks)

2. Define parametrically using functions $x(u,v,w),\ y(u,v,w),\ z(u,v,w)$, $u,v,w \in [0,1]$ a solid cylinder with a cylindrical hole in it. The radius of the cylinder is ***N+M***, the radius of the hole is ***N***. The cylinder is parallel to axis Z and spans from $z_1 = -\boldsymbol{N}$ to $z_1 = \boldsymbol{M}$. Its axis is passing through the point with coordinates (***N***, ***M***, 0).

   (4 marks)

3. Define parametrically using functions $x(u,v,w),\ y(u,v,w),\ z(u,v,w),\ u,v,w \in [0,1]$ a solid object created by translational sweeping of the surface defined by

$$y = \boldsymbol{M}cos(2\pi\sqrt{(-1+2u)^2 + (-1+2v)^2})$$
$$x, y \in [-10\boldsymbol{N}, 10\boldsymbol{M}], u, v \in [0,1]$$

   The sweeping has to be done parallel to the vertical axis Y so that the lowest point of the surface moves from $y = -\boldsymbol{N}$ to $y = \boldsymbol{M}$.

   (4 marks)

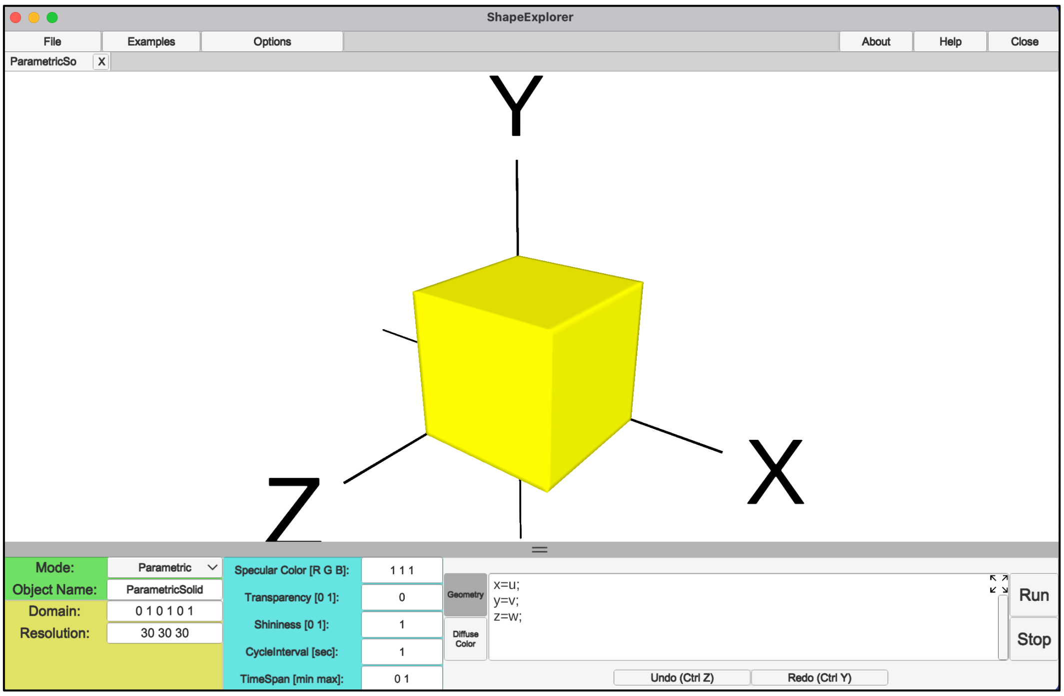


Figure 12. Displaying the shape defined in ParametricSolid.func.

4.4 Experiment 4: Implicit Surfaces and Solids. Colors and 3D Textures.

This assignment illustrates Module 3, and it serves a purpose to teach you how to visualize surfaces and solid objects defined by implicit functions and using CSG operations. To work on this assignment, you have to watch the following TEL lectures:

Module 3: Lecture 7 (Part 1/3) - Geometric Shapes: Constructive Solid Geometry {from implicit to inequality}
Module 3: Lecture 7 (Part 2/3) - Geometric Shapes: Constructive Solid Geometry {CSG definitions}
Module 3: Lecture 7 (Part 3/3) - Geometric Shapes: Constructive Solid Geometry {examples}
Module 3: Lecture 8 - Geometric Shapes: Blobby Shapes
Module 5: Illumination and Texture Mapping

Assignment instructions:
Create folder **Lab4**. Download into it the files **ImplicitSurface.func** and **CSGsolid.func** from the course-site (Figs. 13 and 14). Use them as a reference for the following exercises with implicit surfaces and CSG solids. For each of the shapes, you must use an **origin-centred** bounding box with the **size** $\mathbf{30} \times \mathbf{30} \times \mathbf{30}$, and select a sampling resolution providing for smooth surface visualization. Each of the shapes has to be displayed within 2 seconds using ***cyan*** color with ***black*** coordinate axes on the ***white*** background.

1. Define by implicit function $f(x, y, z) = 0$ a plane $z = \boldsymbol{M}$ and paint it with a color pattern as it is displayed in Table 2 according to your value of ***N***.

   (4 marks)

2. Define by functions $f(x, y, z) \geq 0$ a solid box with coordinates:

   $$-15 \leq x \leq 15, \quad -15 \leq y \leq 15, \quad -10 \leq z \leq 0.$$

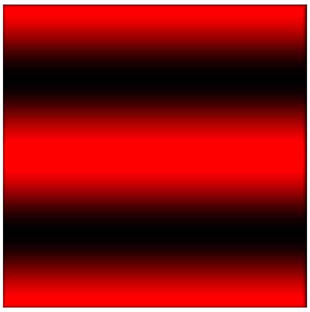
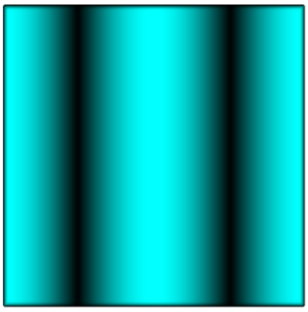
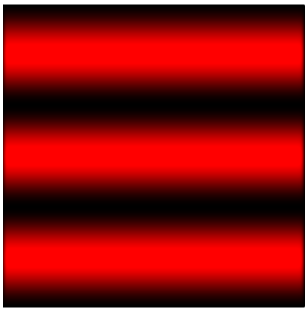
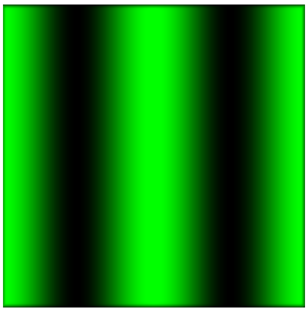
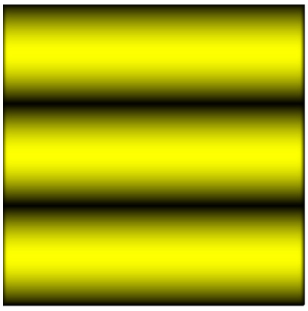
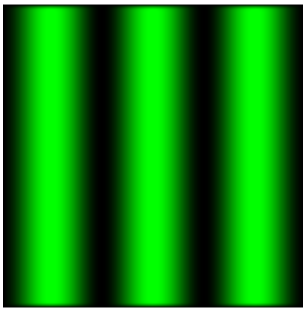
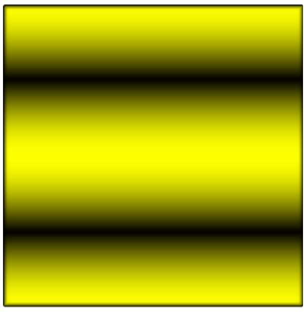
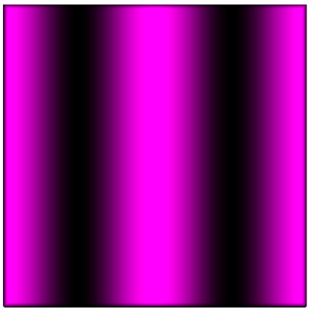
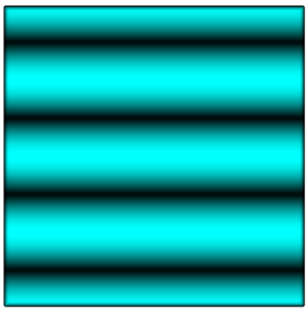
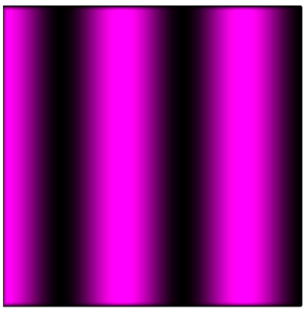
   Add a 3D displacement texture to its front face (i.e. to the face with *z* = 0) making it look as displayed in Table 3 according to your value of ***M***. Maximum displacement from the front face is 10. Display the 3D textured box.

   (4 marks)

3. With reference to Table 4, construct one complex shape using set-theoretic operations following the sketch number ***M***. Design and apply variable colors to it so that the *r,g,b* values remain within [0,1] interval on the surface if the shape.

   (4 marks)

Table 2. Color pattern

N	Color pattern	N	Color pattern
1		6	
2		7	
3		8	
4		9	
5		10	

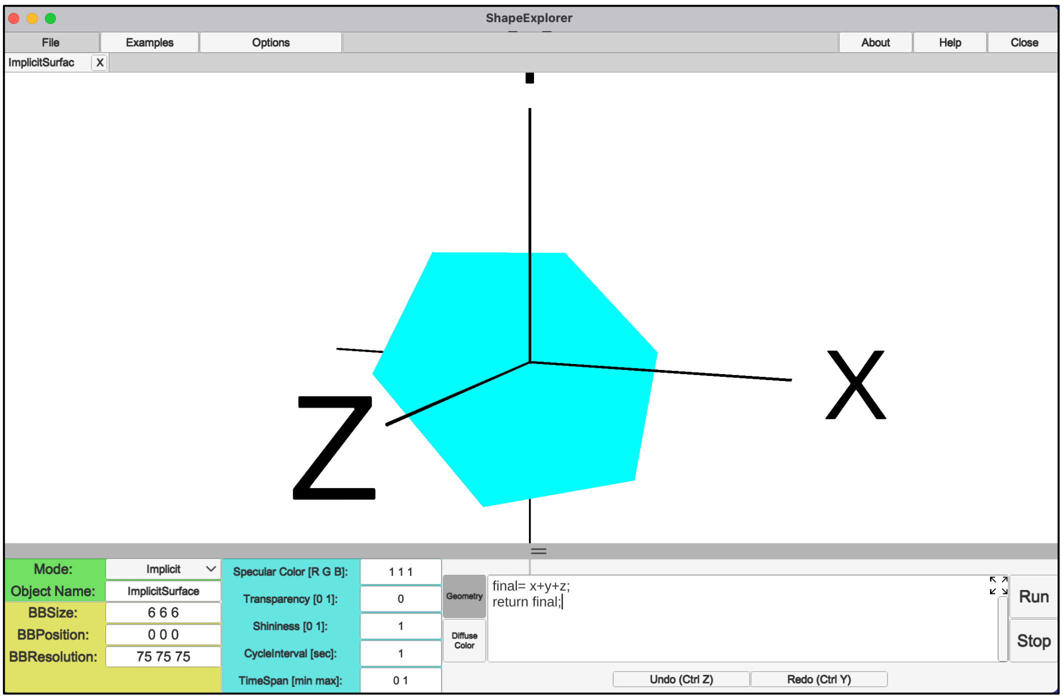


Figure 13. Displaying the shape defined in ImplicitSurface.func.

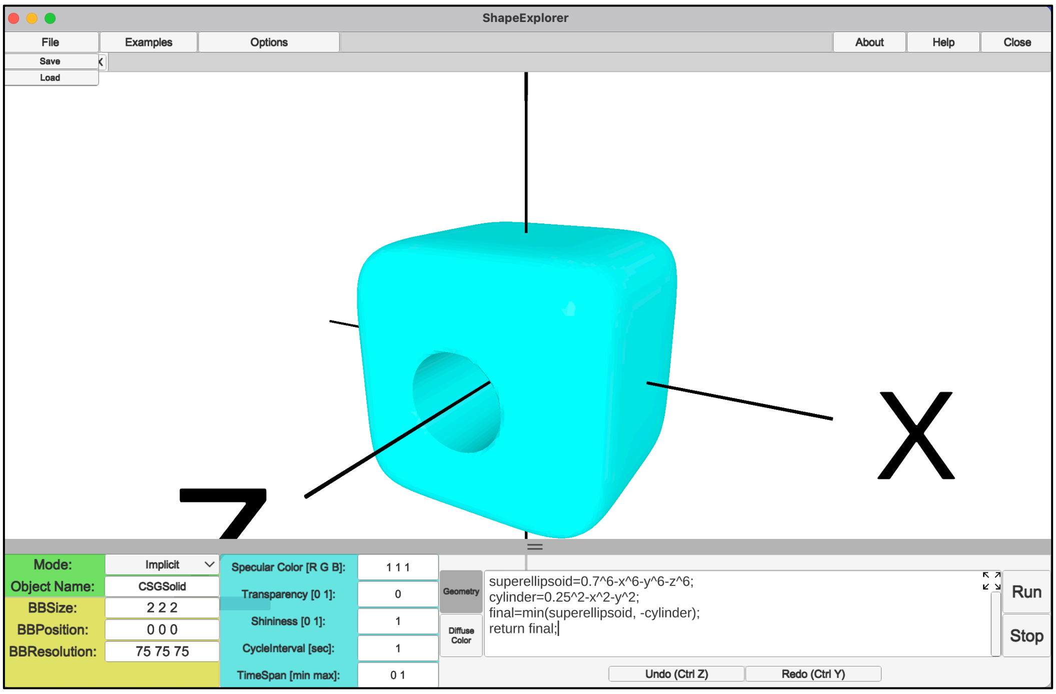


Figure 14. Displaying the shape defined in CSGSolid.func.

Table 2. 3D displacement textures applied to the front face of the box

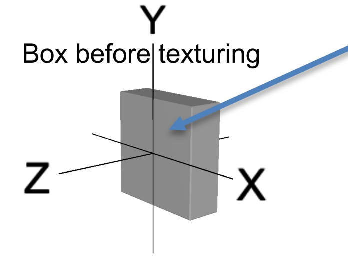
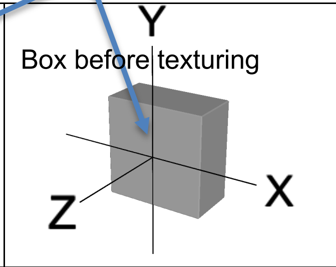
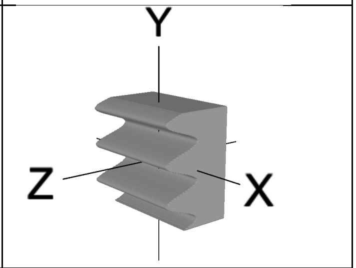
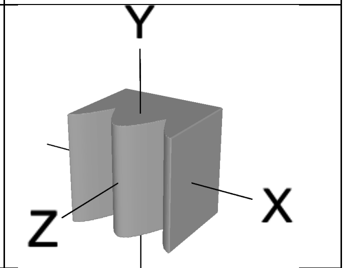
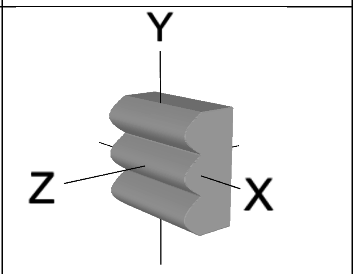
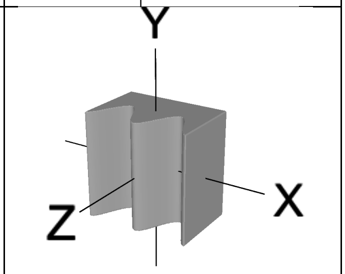
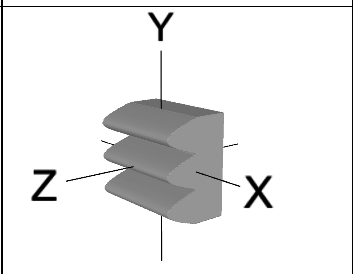
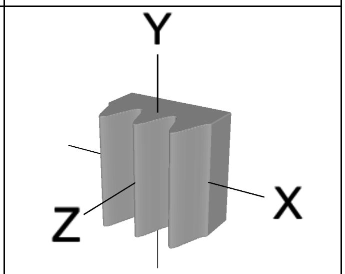
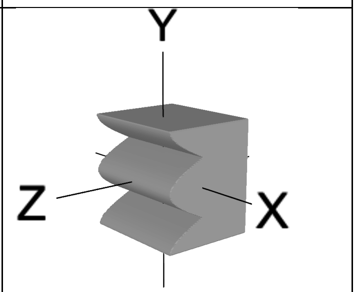
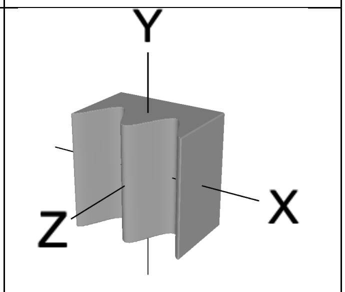
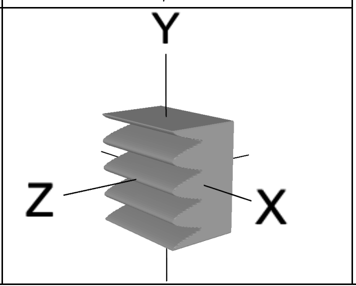
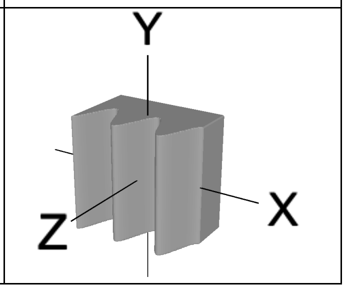
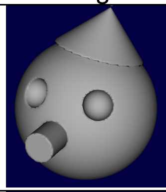
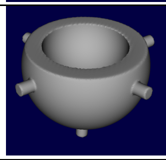
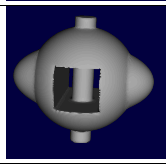
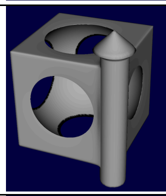
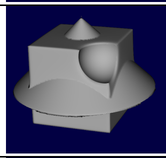
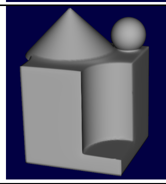
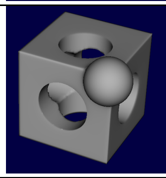
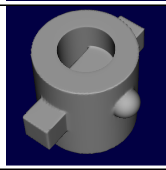
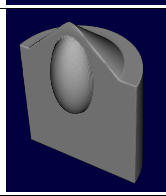
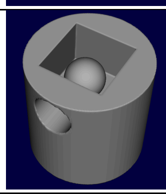
M		M	
1		6	
2		7	
3		8	
4		9	
5		10	

Table 3. CSG designs to follow.

M	Boolean function description	Sketch of the design
1	$(((Sphere_1 \cup Cylinder) \setminus Sphere_2) \setminus Sphere_3) \cup Cone$	
2	$(((Sphere_1 \cup Cylinder_1 \cup Cylinder_2 \cup Cylinder_3) \setminus PlaneHalfSpace) \setminus Sphere_2$	
3	$((Sphere \cup Ellipsoid) \setminus Box)) \cup Cylinder$	
4	$(Box \setminus Sphere) \cup Cylinder \cup Cone$	
5	$(Box \cup Cone) \setminus Sphere$	
6	$(Box \setminus Cylinder) \cup Cone \cup Sphere$	
7	$(Box \setminus Cylinder_1 \setminus Cylinder_2 \setminus Cylinder_3) \cup Sphere$	
8	$(Cylinder_1 \setminus Cylinder_2) \cup Sphere \cup Box$	
9	$((Cylinder \cup Cone) \setminus PlaneHalfSpace) \cup Ellipsoid$	
10	$((Cylinder_1 \setminus Box_1) \setminus Cylinder_2) \cup Sphere$	

4.5 Experiment 5: Animation

This assignment illustrates Module 4 and 5, and it serves a purpose to teach you how to create moving objects, and morphing between two surfaces defined parametrically. To work on this assignment, you have to watch the following TEL lectures:

Week 8 - *2D Transformations*
Week 9 - *3D Transformations*
Week 10 - *Motions and Morphing*

Assignment instructions:

Create folder **Lab5**. Download into it the file **ParametricAnimation.func** (Fig. 15), and use it as a reference for the following exercises. For each of the shapes, you have to select a sampling resolution providing for smooth surface visualization and animation. The shapes have to be displayed with ***black*** coordinate axes on the ***white*** background.

1. Using parametric functions $x(u,v,t), y(u,v,t), z(u,v,t), u,v \in [0,1]$, write a function script defining a 15 sec animation in which a ***purple*** sphere with radius ***N*** initially resting at the height of ***N***+***M***, is dropped down and begins to bounce up (***N***+***M***) times on the horizontal plane $y = 0$. The bouncing has to be done within 10 sec with reduction of an altitude so that eventually, at the end of the 10 sec period, the sphere has to come to complete stop and rest on the plane till the end of the 15 sec animation cycle. Display the bouncing sphere together with the coordinate axes.

   (4 marks)

2. Using parametric functions $x(u,v,t), y(u,v,t), z(u,v,t), u,v \in [0,1]$, write a function script defining a 15 sec animation cycle in which a ***blue*** origin-centred disk with radius ***N*** located in coordinate plane XY, within ***N*** sec rotates by $\frac{\pi}{2}$ about an axis parallel to axis *Y* and passing through the point with coordinates (***M***, 0, 0). Then the disk instantly changes its color to ***red*** and within 15-***N*** sec rotates back to its original position. Display the rotating disk together with the coordinate axes.

   (4 marks)

3. With reference to Table 4, define parametrically using (u,v,t), $y(u,v,t)$, $z(u,v,t)$, $u,v \in [0,1]$, $t \in [0,15]$ and display, together with the coordinate axes, continuous back-and-forth morphing transformation between surfaces ***M*** and (***N+M***). One cycle of the animation from surface ***M*** to surface ***N+M*** and back to surface ***M*** has to take 15 sec. During the animation cycle the morphing surface has to also smoothly change its color from ***red*** to ***green*** and then back to ***red***.

   (4 marks)

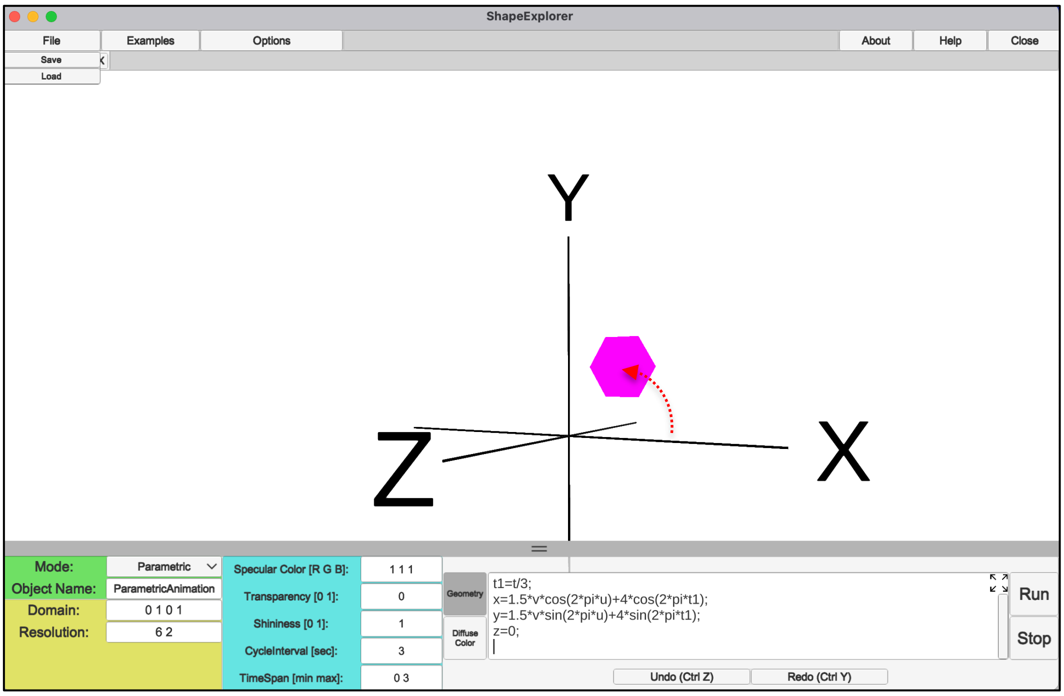


Figure 15. Displaying the animation defined in ParametricAnimation.func.

Table 4 Parametric surfaces

#	formulas	parameters
1	$x=1.6(\cos(\varphi))^3$ $y=1.6(\cos(\theta)\sin(\varphi))^3$ $z=1.6\sin(\theta)\sin(\varphi)$	$0\le\theta\le2\pi,\ 0\le\varphi\le\pi$
2	$x=1.5a\cos(\theta)$ $y=1.5a\sin(\theta)\cos(\theta)$ $z=1.5a(\sin(2a\pi))^5$	$0\le\theta\le2\pi,\ 0\le a\le1$
3	$x=b\cos(\alpha)$ $y=b\sin(\alpha)$ $z=b\sin(2b\pi)\sin\alpha$	$0\le\alpha\le2\pi,\ 0\le b\le1$
4	$x=\cos(\varphi)\sin(\varphi)$ $y=\cos(a\pi)\sin(\varphi)$ $z=\sin(a\pi)\sin(\varphi)$	$0\le a\le2,\ \ 0\le\varphi\le\pi$
5	$x=\cos(\varphi)$ $y=\cos(2\theta)\sin(\varphi)$ $z=\sin(2\theta)\cos(\varphi)$	$0\le\theta\le\pi,\ \ 0\le\varphi\le\pi$
6	$x=(1+0.25\cos(\theta))\cos(b\pi)$ $y=(1+0.25\cos(\theta))\sin(b\pi)$ $z=0.25\sin(\theta)$	$0\le\theta\le2\pi,\ \ 0\le b\le2$
7	$x=\cos(0.5\theta)(\sin(\varphi))^3$ $y=\sin(0.5\theta)(\sin(\varphi))^3$ $z=\cos(\varphi)$	$0\le\theta\le4\pi,\ \ 0\le\varphi\le\pi$

8	$x=2(\cos\theta)^3(\sin 4\varphi)^3$ $y=2(\sin\theta)^3(\sin 4\varphi)^3$ $z=2(\cos 4\varphi)^3$	0≤θ≤2π, 0≤φ≤π/4
9	$x=0.5\cos(4\theta)(-3+\cos(\varphi)(1+\cos(\varphi)))$ $y=0.5\sin(\varphi)(1+\cos(\varphi))$ $z=0.5\sin(4\theta)(-3+\cos(\varphi)(1+\cos(\varphi)))$	0≤θ≤π/2, 0≤φ≤2π
10	$x=0.5((\varphi-0.5\sin(\varphi))-3)$ $y=0.5\cos(4a\pi)(1-0.5\cos(\varphi))$ $z=0.5\sin(4a\pi)(1-0.5\cos(\varphi))$	0≤α≤0.5, 0≤φ≤2π
11	$x=2.5\varphi/(1+\varphi^3)$ $y=2.5\cos(\theta)\varphi^2/(1+\varphi^3)$ $z=2.5\sin(\theta)\varphi^2/(1+\varphi^3)$	0≤θ≤2π, 0≤φ≤2π
12	$x=0.25(\varphi-\sin(2\varphi)-2\pi)$ $y=0.5\cos(0.5\theta)(2-\cos(2\varphi))$ $z=0.5\sin(0.5\theta)(2-\cos(2\varphi))$	0≤θ≤4π, 0≤φ≤2π
13	$x=0.5(\cos(\varphi)+0.5\cos(2\varphi))$ $y=0.5\cos(6\theta)(2+\sin(\varphi)-0.5\sin(2\varphi))$ $z=0.5\sin(6\theta)(2+\sin(\varphi)-0.5\sin(2\varphi))$	0≤θ≤π/3, 0≤φ≤2π
14	$x=0.1(3\cos(\varphi/3)+0.8\cos(\varphi))$ $y=0.2\cos(\theta)(5+3\sin(\varphi/3)-0.8\sin(\varphi))$ $z=0.2\sin(\theta)(5+3\sin(\varphi/3)-0.8\sin(\varphi))$	0≤θ≤2π, 0≤φ≤6π
15	$x=0.15(4\cos(\varphi/4)-\cos(\varphi))$ $y=0.15\cos(\theta)(6+4\sin(\varphi/4)-\sin(\varphi))$ $z=0.15\sin(\theta)(6+4\sin(\varphi/4)-\sin(\varphi))$	0≤θ≤2π, 0≤φ≤8π
16	$x=\cos(2\pi\sin(\varphi))$ $y=\cos(4\pi\theta)\sin(\varphi)$ $z=\sin(4\pi\theta)\sin(\varphi)$	0≤θ≤0.5, 0≤φ≤π
17	$x=\sin(\theta/6)$ $y=\cos(\varphi)$ $z=\sin(\theta/6)\sin(\varphi)$	0≤θ≤12π, 0≤φ≤2π
18	$x=(\cos(0.25\theta)+1)\cos(\varphi)$ $y=\sin(0.25\theta)\cos(\varphi)$ $z=\sin(\varphi)$	0≤θ≤8π, 0≤φ≤2π
19	$x=0.5(\cos(\varphi)+2)\cos(8\theta)$ $y=0.5(\sin(\varphi)+16\theta/\pi-2)$ $z=-0.5(\cos(\varphi)+2)\sin(8\theta)$	0≤θ≤π/4, 0≤φ≤2π
20	$x=2\theta\sin(\varphi)$ $y=0.2\varphi\sin(10\theta)$ $z=0.2\varphi\cos(10\theta)$	0≤θ≤π/5, 0≤φ≤2π