Project 1: computing all kinematic informations of a real manipulator

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Question 1: basics

Solutions:

To write down all these requested functions, we need formulas below:

Definition 3.7. Given a vector $x = [x_1\ x_2\ x_3]^{\mathrm{T}} \in \mathbb{R}^3$, define

$$[x] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \qquad (3.30)$$

$$\mathcal{V}_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix} \in \mathbb{R}^6, \qquad [\mathcal{V}_s] = \begin{bmatrix} [\omega_s] & v_s \\ 0 & 0 \end{bmatrix} = \dot{T}T^{-1} \in se(3), \qquad (3.74)$$

Proposition 3.15. *The inverse of a transformation matrix $T \in SE(3)$ is also a transformation matrix, and it has the following form:*

$$T^{-1} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R^{\mathrm{T}} & -R^{\mathrm{T}}p \\ 0 & 1 \end{bmatrix}. \qquad (3.64)$$

The above derivation essentially provides a constructive proof of the Chasles–Mozzi theorem. That is, given an arbitrary $(R, p) \in SE(3)$, one can always find a screw axis $\mathcal{S} = (\omega, v)$ and a scalar θ such that

$$e^{[\mathcal{S}]\theta} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}, \qquad (3.90)$$

i.e., the matrix

$$[\mathcal{S}]\theta = \begin{bmatrix} [\omega]\theta & v\theta \\ 0 & 0 \end{bmatrix} \in se(3)$$

is the matrix logarithm of $T = (R, p)$.

Definition 3.20. Given $T = (R, p) \in SE(3)$, its **adjoint representation** $[\mathrm{Ad}_T]$ is

$$[\mathrm{Ad}_T] = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times 6}.$$

All the required functions have been posted in codes.

Question 2: forward kinematics

Here is the formula I use to write down the function:

Continuing with this reasoning and now allowing all the joints $(\theta_1, \ldots, \theta_n)$ to vary, it follows that

$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_{n-1}]\theta_{n-1}} e^{[\mathcal{S}_n]\theta_n} M. \qquad (4.14)$$

the required function has been posted in codes.

Question 3: Jacobians

Here is the formula I use to write down the function:

$$\mathcal{V}_s = \underbrace{\mathcal{S}_1}_{J_{s1}} \dot{\theta}_1 + \underbrace{\mathrm{Ad}_{e^{[\mathcal{S}_1]\theta_1}}(\mathcal{S}_2)}_{J_{s2}} \dot{\theta}_2 + \underbrace{\mathrm{Ad}_{e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2}}(\mathcal{S}_3)}_{J_{s3}} \dot{\theta}_3 + \cdots$$

The **space Jacobian** $J_s(\theta) \in \mathbb{R}^{6\times n}$ relates the joint rate vector $\dot{\theta} \in \mathbb{R}^n$ to the spatial twist $\mathcal{V}_s$ via

$$\mathcal{V}_s = J_s(\theta)\dot{\theta}. \tag{5.10}$$

The ith column of $J_s(\theta)$ is

$$J_{si}(\theta) = \mathrm{Ad}_{e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_{i-1}]\theta_{i-1}}}(\mathcal{S}_i), \tag{5.11}$$

for $i = 2, \ldots, n$, with the first column $J_{s1} = \mathcal{S}_1$.

the required function has been posted in codes.

A note about Hint Q2&Q3:

I have posted some functions written when doing homework in the coding file. Here I will explain some examples How the function is written and how to use them:

(Normally, every extra function has a note in the coding file.)

1. From w, p → Twist

Usually we face a problem that we have to write down Twists(S) of joints from the given structure. Normally, it is easy for people to read w and p from a rotation joint. Then you have to use v = - w × p to get linear velocity. So I wrote a function that uses w and p as parameters to get the Twist corresponding to the joint

Function name: getTwistfromwp(w,p)

2. Body Jacobian

There is the following transformation relationship between Spatial Jacobian and Body Jacobian:

Since we also have $\mathcal{V}_b = J_b(\theta)\dot{\theta}$ for all $\dot{\theta}$, it follows that $J_s(\theta)$ and $J_b(\theta)$ are related by

$$J_b(\theta) = \mathrm{Ad}_{T_{bs}}(J_s(\theta)) = [\mathrm{Ad}_{T_{bs}}]J_s(\theta). \tag{5.22}$$

The space Jacobian can in turn be obtained from the body Jacobian via

$$J_s(\theta) = \mathrm{Ad}_{T_{sb}}(J_b(\theta)) = [\mathrm{Ad}_{T_{sb}}]J_b(\theta). \tag{5.23}$$

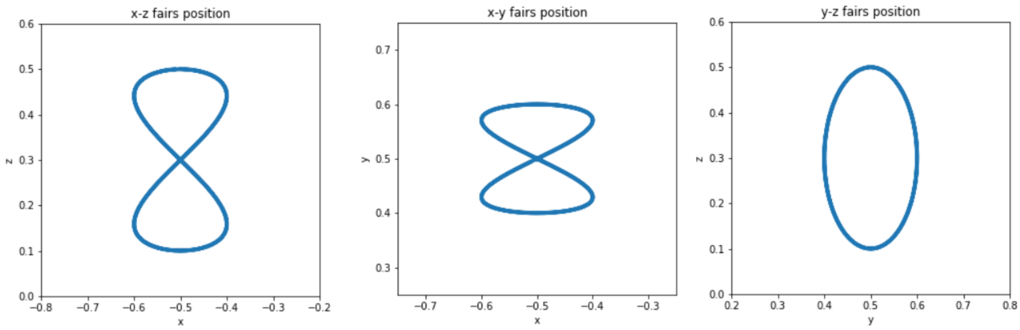
So the task is to write down a function that uses Jspace and FK(theta=0) as parameters to get Body Jacobian.

Function name: getJbody(theta,M)

Question 4: displaying hand trajectories

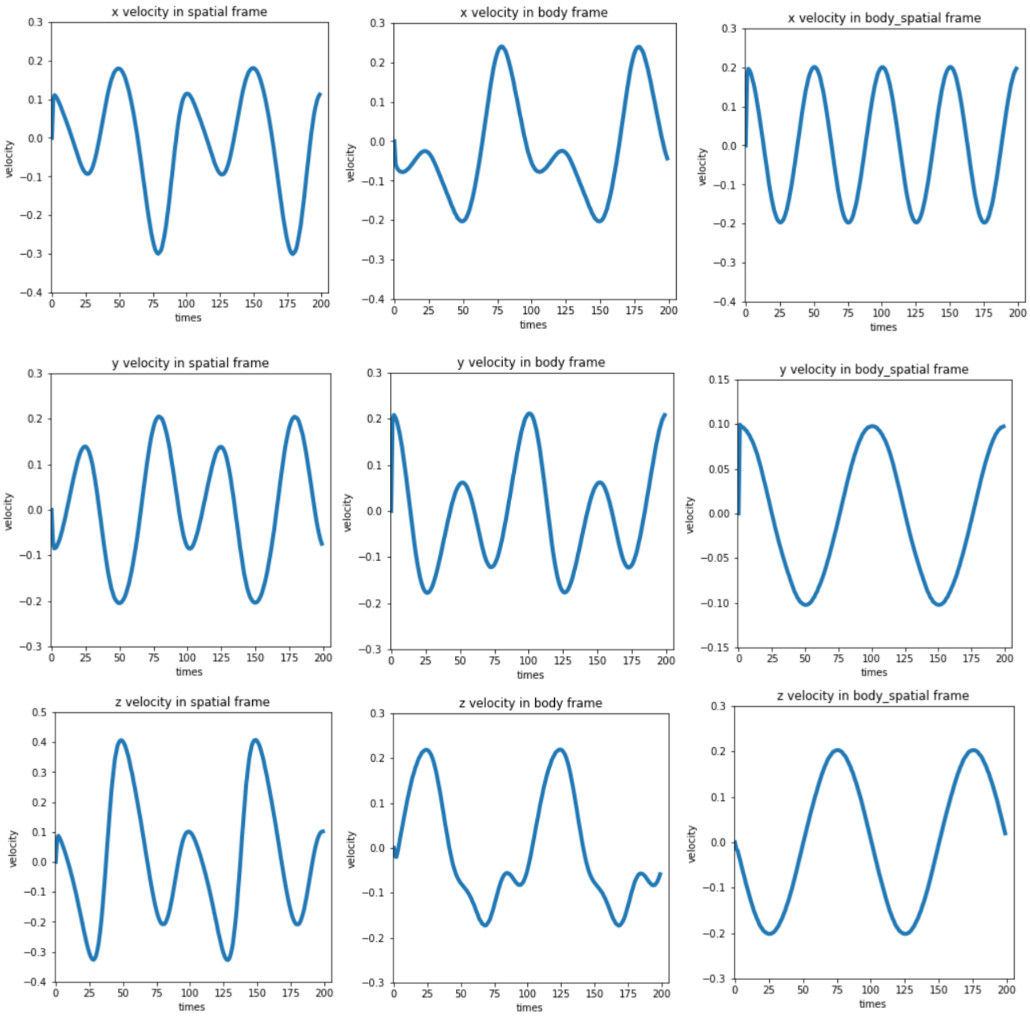
Hand trajectory: a trajectory similar to simple harmonic motion

Here are all the plots of x-y, x-z, y-z:



Question 5: computing velocities

Here are all the plots:



Notes:

Obviously, the plots in the third kind frame is the most intuitive. This is a standard simple harmonic motion, which fits more closely with the corresponding position image.