Capital asset pricing model

CAPM –

two regressions ; multivariate normal distribution

$\Downarrow$

conditional distribution

theory of population regression

probability

$\updownarrow$ LLN, CLT, ...

sample regressions

Two types of population regressions

CAPM

i.i.d. all gross returns are i.i.d.

across time but not across assets

$$X_t = \begin{pmatrix} R_{f,t} \\ R_{m,t} \\ R_{1,t} \\ R_{2,t} \\ \vdots \\ R_{n,t} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} ER_f \\ ER_m \\ ER_1 \\ \vdots \\ ER_n \end{pmatrix}, \Sigma \right)$$

CAPM – $X_0, X_1, \ldots, X_{T-1}$

time series regression:

$$R^{e,i}_t = R^i_t - R_{f,t}$$

$$R^{e,m}_t = R^m_t - R_{f,t}$$

e ~ "excess return"

$$R^{e,i}_t = \alpha_i + \beta_i R^{e,m}_t + \underbrace{\varepsilon^i_t}_{\text{l.s.}} \qquad \varepsilon^i_t \perp R^{e,m}_t$$

$t = 0, \dots, T-1$

$i = 1, \dots, n$

β_i ~ l.s. regression coefficient

$$\Longrightarrow \boxed{\beta_1, \beta_2, \dots, \beta_n}$$

why run this?

a "theory" tells you to run this regression

& gives you an interpretation of the β_i, α_i.

Cross-section regression: , across dif

$i = 1, \dots, n$

Process the data:

(1) Use time series regression to compute

$$\beta_1, \beta_2, \dots, \beta_n$$

(2) calculate with a time series average —

$$E_T R^{e,i} = \frac{1}{T} \sum_{t=0}^{T-1} R^{e,i}_t$$

time series average excess return

$i = 1, \dots, n$

run this cross-section regression

$$E_T R^{e,i} = \tilde{\alpha}_i + \beta_i \lambda \qquad \tilde{\alpha}_i \perp \beta_i$$

regressors

regression coefficient

Cross section regressions

tests the CAPM.

What asset pricing theory underlies the CAPM?

Sharpe, Mossin /// Hansen - Scott Richard

the CAPM is implied by

$$\boxed{E_t(m_{t+1} R^i_{t+1}) = 1}$$

regression theory

CAPM ⟵ link to Lucas model

Cauchy-Schwartz inequality

$\Updownarrow$

$R^2 < 1$