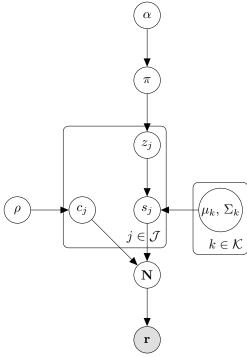
Variational Inference Notes



 $s_{j}|z_{j} \sim \mathcal{N}(\mu_{z_{j}}, \Lambda_{z_{j}})$ $N_{ij}|\{s_{j}\} \sim \text{Poisson}(c_{j}f_{i}(s_{j}))$ $r_{i}|N_{ij} \sim \delta(r_{i} - \sum_{j} N_{ij})$ $\pi \sim \text{Dir}(\alpha)$ $c \sim \text{Dir}(\rho)$

$$\log Q(z) = \langle \log P(z, s, c, \mu, \Lambda, N, r) \rangle_{Q(s)Q(N)Q(c)Q(\mu,\Sigma)}$$

$$= \langle \log(\operatorname{Mult}(z; \pi)\operatorname{Dir}(\pi; \alpha) \prod_{j=1}^{J} \mathcal{N}(s_j; \mu_{z_j}, \Lambda_{z_j})\operatorname{Dir}(c, \rho) \dots$$

$$\prod_{j=1}^{J} ((\delta(r_i - \sum_{j} N_{ij})\operatorname{Poisson}(c_j f_i(s_j))) \dots$$

$$\prod_{k=1}^{K} \mathcal{N}(\mu_k; m_k, (\beta_0 \Lambda_k)) \mathcal{W}(\Lambda_k; \mathbf{W}_0, \nu_0)) \rangle_{Q(s)Q(N)Q(c)Q(\mu, \Lambda)}$$

$$\log Q(z_j = k) = \langle \log(\operatorname{Mult}(z; \pi)) \rangle_{Q(\pi)} + \langle \log(\prod_{j=1}^{J} \mathcal{N}(s_j; \mu_{z_j}, \Lambda_{z_j})) \rangle_{Q(s)Q(\mu, \Lambda)})$$

$$= \frac{D}{2} \langle \log(|\Lambda_k|) \rangle_{Q(\Lambda)} - \frac{1}{2} \langle (s_j - \mu_k)^T \Lambda_k(s_k - \mu_k) \rangle_{Q(s)Q(\mu, \Lambda)} + \langle \log \pi_k \rangle_{Q(\pi)}$$

$$= \frac{D}{2} (\sum_{d=1}^{D} \Psi(\frac{\nu_k^* + 1 - d}{2}) + \log(|\mathbf{W}_k^*|))$$

$$- \frac{1}{2} (D\beta_k^{*-1} + \nu_k^*((\mu_k^* - m_j^*)^T \mathbf{W}_k^*(\mu_k^* - m_j^*)) + \operatorname{Tr}(\mathbf{W}_k^* \Lambda_j^{*-1}))$$

$$+ \Psi(\alpha_k) - \Psi(\sum_{k=1}^{K} \alpha_k)$$

$$q_{jk}^* = \frac{Q(z_j = k)}{\sum_{k=1}^{K} Q(z_j = k)}$$

$$(1)$$

$$\log Q(c) = \langle \sum_{j} (\rho_{j} - 1) \log c_{j} \rangle + \sum_{ij} \langle N_{ij} \log c_{j} \rangle$$

$$= \sum_{j} [\log c_{j}] [\rho_{j} - 1 + \sum_{i} \langle N_{ij} \rangle]$$

$$Q(c) \sim \operatorname{Dir}(\rho_{j} - 1 + \sum_{i} \langle N_{ij} \rangle) = \operatorname{Dir}(\gamma_{j}^{*}) \qquad (2)$$

$$\gamma_{j}^{*} = \rho_{j} - 1 + \sum_{i} r_{i} p_{ij}^{*}$$

$$\langle \log c_{j} \rangle = \Psi(\rho_{j} - 1 + \sum_{i} r_{i} p_{ij}^{*}) - \Psi(\sum_{j} (\rho_{j} - 1 + \sum_{i} r_{i} p_{ij}^{*}))$$

$$\log Q(s_{j}) = \langle \log \mathcal{N}(\mu_{z_{j}}, \Lambda_{z_{j}}) \rangle_{Q(z)Q(\mu,\Lambda)} + \sum_{i=1}^{I} \langle N_{ij} \log f_{i}(s_{j}) \rangle_{Q(N)}$$

$$= -\frac{1}{2} (\langle (s_{j} - \mu_{j})^{T} \Lambda_{z_{j}}^{-1} (s_{j} - \mu_{j}) \rangle_{Q(z)Q(\mu,\Lambda)} + \sum_{i=1}^{I} \langle N_{ij} \rangle_{Q(N)} \log f_{i}(s_{j}))$$

$$= -\frac{1}{2} (\sum_{k=1}^{K} q_{jk}^{*} \nu_{k}^{*} (s_{j} - \mu_{k})^{T} \mathbf{W}_{k}^{*} (s_{j} - \mu_{k}) - \frac{1}{2} \sum_{i=1}^{I} 2 \frac{||s_{j} - s_{i}^{\text{pref}}||^{2}}{2\sigma^{2}})$$

$$Q(s_{j}) \sim \mathcal{N}(m_{j}^{*}, \Lambda_{j}^{*})$$

$$\Lambda_{j}^{*} = \sum_{l=1}^{K+I} \Lambda_{l}$$

$$m_{j}^{*} = \Lambda_{j}^{*} \sum_{l=1}^{K+I} (\Lambda_{l} m_{l})$$

$$m_{l} \in \{m_{k}\}_{k=1}^{K}, \{s_{i}^{\text{pref}}\}_{i=1}^{I}$$

$$\Lambda_{l} \in \{q_{kj}^{*} \nu_{k}^{*} \mathbf{W}_{k}^{*}\}_{k=1}^{K}, \{\frac{2r_{i}p_{ij}}{2\sigma^{2}}I\}_{i=1}^{I}$$

$$\langle \log f_{i}(s_{j}) \rangle_{Q(s_{j})} = \frac{-(||s_{j} - s_{j}^{\text{pref}}||^{2})Q(s_{j})}{2\sigma^{2}}$$

$$= \frac{-||s_{j}^{\text{pref}} - m_{j}^{*}||^{2} + \operatorname{Tr}(\Lambda_{j}^{*-1})}{2\sigma^{2}}$$

(3)

$$\log Q(N_{i}) = \log(\delta(\sum_{j=1}^{J} N_{ij} - r_{i})) + \sum_{j=1}^{J} N_{ij} [\langle \log c_{j} \rangle + \langle \log f_{i}(s_{j}) \rangle] - \sum_{j=1}^{J} \log N_{ij}!$$

$$Q(N_{ij}) \sim \text{Mult}(r_{i}, p_{ij}^{*})$$

$$p_{ij}^{*} = \frac{e^{\langle \log c_{j} \rangle + \langle \log f_{i}(s_{j}) \rangle}}{\sum_{j=1}^{J} e^{\langle \log c_{j} \rangle + \langle \log f_{i}(s_{j}) \rangle}}$$

$$= \frac{e^{\Psi(\gamma_{j}^{*}) - \Psi(\sum_{j=1}^{J} \gamma_{j}^{*}) - \frac{||s_{i}^{\text{pref}} - m_{j}^{*}||^{2} + \text{Tr}(\Lambda_{j}^{*})^{-1}}{2\sigma^{2}}}}{\sum_{j=1}^{J} e^{\Psi(\gamma_{j}^{*}) - \Psi(\sum_{j=1}^{J} \gamma_{j}^{*}) - \frac{||s_{i}^{\text{pref}} - m_{j}^{*}||^{2} + \text{Tr}(\Lambda_{j}^{*})^{-1}}{2\sigma^{2}}}$$

$$(4)$$

$$\log Q(\mu_{k}, \Lambda_{k}) = \langle \log(\prod_{j=1}^{J} \mathcal{N}(s_{j}; \mu_{z_{j}}, \Lambda_{z_{j}}) \rangle_{Q(s)Q(z)})$$

$$= \langle \sum_{j=1}^{J} [\frac{D}{2} \log |\Lambda_{z_{j}}| - \frac{1}{2} [(s_{j} - \mu_{z_{j}})^{T} \Lambda_{z_{j}}(s_{j} - \mu_{z_{j}})) \rangle_{Q(s)Q(z)}$$

$$= \langle \sum_{j=1}^{J} [\frac{D}{2} \log |\Lambda_{z_{j}}| - \frac{1}{2} [(m_{j}^{*} - m_{z_{j}})^{T} \Lambda_{z_{j}}(m_{j}^{*} - m_{z_{j}}) + \text{Tr}(\Lambda_{z_{j}} \Lambda_{j}^{*-1})] \rangle_{Q(z)}$$

$$= \sum_{j=1}^{J} q_{jk}^{*} |\frac{D}{2} \log |\Lambda_{z_{j}}| - \frac{1}{2} [(m_{j}^{*} - m_{z_{j}})^{T} \Lambda_{z_{j}}(m_{j}^{*} - m_{z_{j}}) + \text{Tr}(\Lambda_{z_{j}} \Lambda_{j}^{*-1})]$$

$$Q(\mu_{k}, \Lambda_{k}) \sim \mathcal{N}(\mu_{k}; \mu_{k}^{*}, (\beta_{k}^{*} \Lambda_{k})) \mathcal{W}(\Lambda_{k}; \mathbf{W}_{k}^{*}, \nu_{k}^{*})$$

$$\sum_{j=1}^{J} q_{jk}^{*} \text{Tr}(\Lambda_{k} \Lambda_{j}^{*-1}) = \text{Tr}(\Lambda_{k} \sum_{j=1}^{J} q_{jk}^{*} \Lambda_{j}^{*-1})$$

$$\mathbf{W}_{k}^{*} = (\sum_{j=1}^{J} (q_{jk}^{*} \Lambda_{j}^{*-1}))^{-1}$$

$$|\Lambda_{z_{j}}|^{\frac{\nu_{k}^{*}-1}{2}} = |\Lambda_{z_{j}}|^{\frac{D}{2} + \sum_{j=1}^{J} q_{jk}^{*}}$$

$$\nu_{k}^{*} = D + 2 \sum_{j=1}^{J} q_{jk}^{*} + 1$$

$$C = \Lambda_{k} \sum_{j=1}^{J} q_{jk}^{*}$$

$$X = \Lambda_{k} \sum_{j=1}^{J} q_{jk}^{*} m_{j}^{*}$$

$$\mu_{k}^{*} = \frac{X}{C} = \frac{\sum_{j=1}^{J} q_{jk}^{*} m_{j}^{*}}{\sum_{j=1}^{J} q_{jk}^{*}}$$

$$(\beta_{k}^{*} \Lambda_{k})^{-1} = C$$

$$\beta_{k}^{*} = \frac{1}{\sum_{j=1}^{J} q_{jk}^{*}}$$
(5)

$$\frac{dq_{jk}^*}{dt} = -q_{jk}^* + \frac{Q(z_j = k)}{\sum_{k=1}^K Q(z_j = k)}$$

$$\frac{dm_j^*}{dt} = -m_j^* + \Lambda_j^* \left(\sum_{k=1}^K (q_{kj}^* \nu_k^* \mathbf{W}_k^* \mu_k^*) + \sum_{i=1}^I \left(\frac{2r_i p_{ij}}{2\sigma^2} I s_i^{\text{pref}} \right) \right)$$

$$\frac{d\Lambda_j^*}{dt} = -\Lambda_j^* + \sum_{k=1}^K (q_{kj}^* \nu_k^* \mathbf{W}_k^*) + \sum_{i=1}^I \left(\frac{2r_i p_{ij}}{2\sigma^2} I \right)$$

$$\frac{dp_{ij}^*}{dt} = -p_{ij}^* + \frac{e^{\Psi(\gamma_j^*) - \Psi(\sum_{j=1}^J \gamma_j^*) - \frac{||s_i^{\text{pref}} - m_j^*||^2 + \text{Tr}(\Lambda_j^{*^{-1}})}{2\sigma^2}}}{\sum_{j=1}^J e^{\Psi(\gamma_j^*) - \Psi(\sum_{j=1}^J \gamma_j^*) - \frac{||s_i^{\text{pref}} - m_j^*||^2 + \text{Tr}(\Lambda_j^{*^{-1}})}{2\sigma^2}}$$

$$\frac{d\gamma_j^*}{dt} = -\gamma_j^* + \rho_j - 1 + \sum_i r_i p_{ij}^*$$

$$\frac{d\mu_k^*}{dt} = -\mu_k^* + \frac{\sum_{j=1}^J q_{jk}^* m_j^*}{\sum_{j=1}^J q_{jk}^*}$$

$$\frac{d\beta_k^*}{dt} = -\beta_k^* + \frac{1}{\sum_{j=1}^J q_{jk}^*}$$

$$\frac{dW_k^*}{dt} = -W_k^* + \left(\sum_{j=1}^J (q_{jk}^* \Lambda_j^{*^{-1}})\right)^{-1}$$

$$\frac{d\nu_k^*}{dt} = -\nu_k^* + D + 2\sum_{j=1}^J q_{jk}^* + 1$$