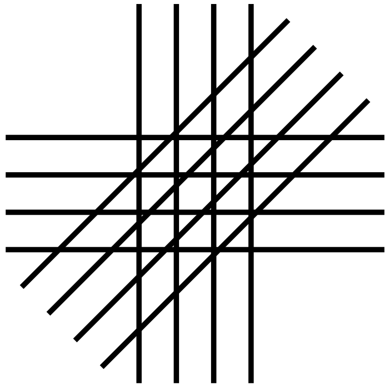
Variational Inference for Demixing Odors/ Orientations

Demixing Odors/ Orientations



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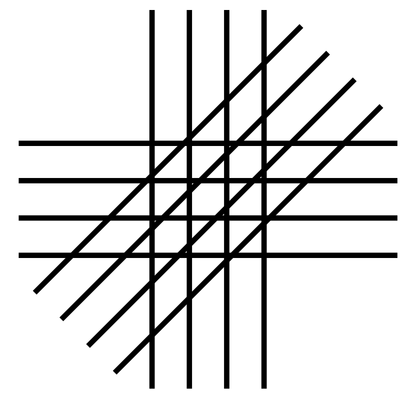

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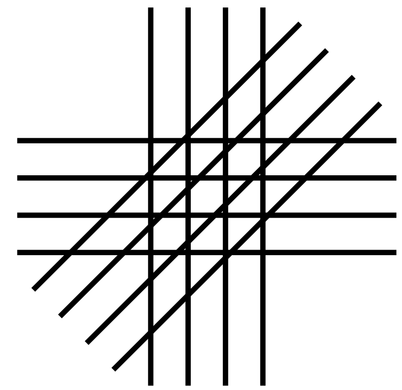

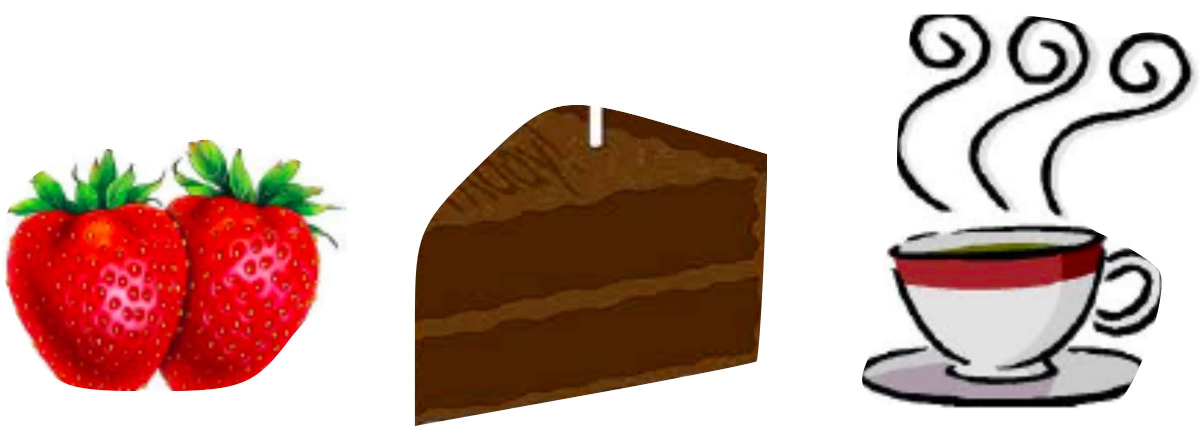

Strawberries Cake Coffee

Demixing Odors/ Orientations



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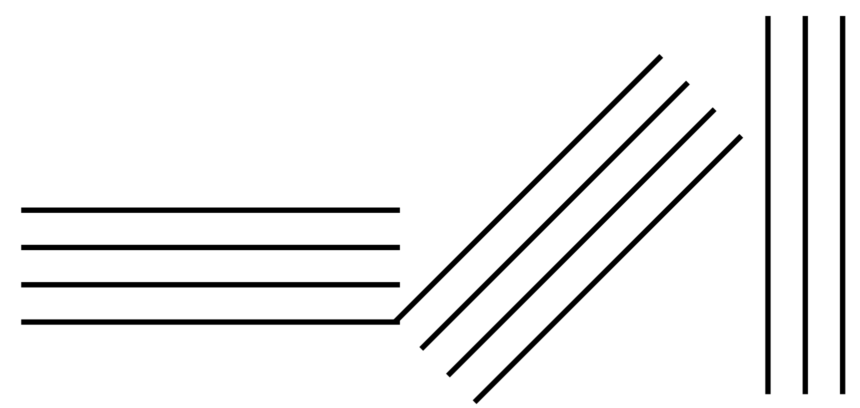

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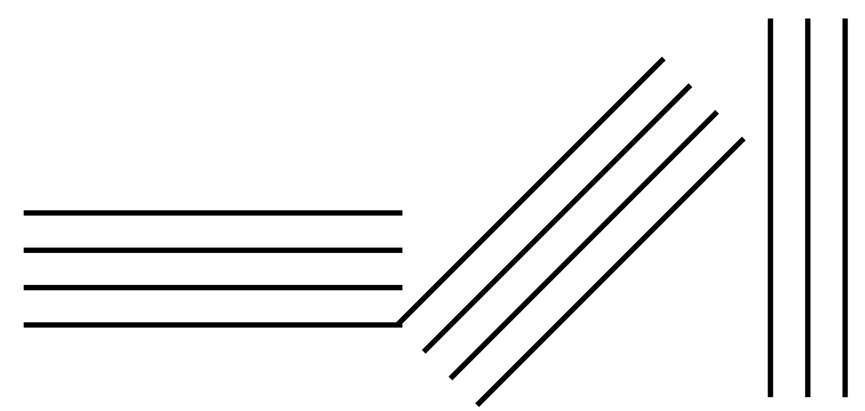
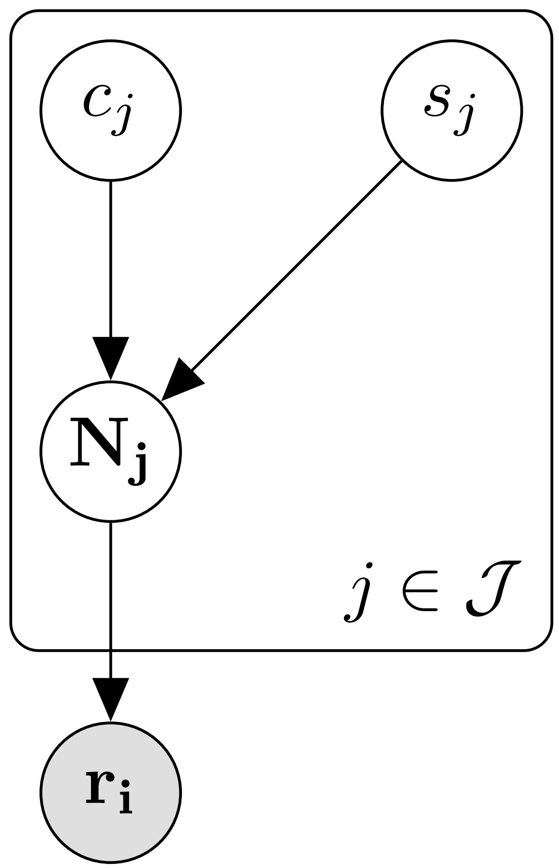


0 45 90

Graphical Model



s_j is stimulus j

c_j is contrast of grating j

r_i is response of neuron i

N_{ij} is the response of neuron i to grating j

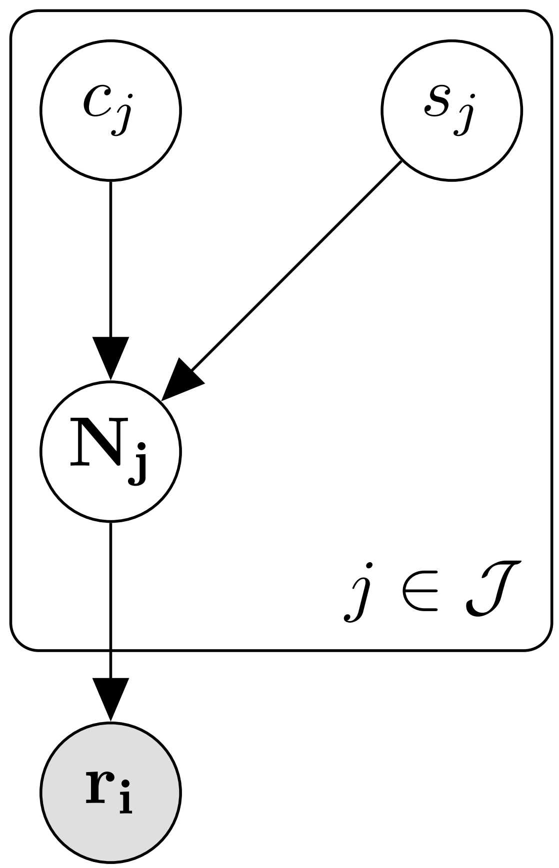
Tuning Curves and Assumptions

$f_i(s)$ is tuning curve of neuron i

Tuning curves cover the space so $\sum_i f_I(s_j)$ is independent of s_j

The tuning curves are von Mises: $f_i(s_j) = e^{\kappa(\cos(s_j - s_i^{pref}) - 1}$

Probabilistic Model



$$P(N|c,s) = \prod_i \mathrm{Poisson}(N_j; f_i(s_j)c_j)$$
$$P(c) = \mathrm{Gamma}(c_j; \alpha, \beta)$$
$$P(s) = \mathrm{Unif}(0, 2\pi)$$

Inference

$$P(\mathbf{c},\mathbf{s}|\mathbf{r}) = \sum_N P(\mathbf{N},\mathbf{c},\mathbf{s}|\mathbf{r})$$
$$= \prod_i (\delta(r_i - \sum_j N_{ij}) \prod_j \frac{(f_i(s_j)c_j)^{N_{ij}} e^{-f_i(s_j)c_j}}{N_{ij}!})(\prod_j \frac{\beta^\alpha}{\Gamma(\alpha)} c_j^{\alpha-1} e^{\beta c_j})$$

This looks pretty ugly so…variational inference?
Let's approximate the above distribution with:

$$Q(\mathbf{N},\mathbf{c},\mathbf{s}|\mathbf{r}) = Q(\mathbf{N}|\mathbf{r})Q(\mathbf{c}|\mathbf{r})Q(\mathbf{s}|\mathbf{r})$$

In order to minimize KL-divergence,
we're going to need:

$$\log P(\mathbf{N},\mathbf{c},\mathbf{s}|\mathbf{r}) = \sum_i [\log(\delta(r_i - \sum_j N_{ij})) - \sum_j [-\log N_{ij}! + N_{ij}\log(f_i(s_j)c_j) - f_i(s_j)c_j]]$$
$$+ \sum_j [(\alpha-1)\log c_j - \beta c_j - \log(\beta^{-\alpha}\Gamma(\alpha))]$$

Factorized distributions

$$
\begin{aligned}
\log Q(\mathbf{N}|\mathbf{r}) &= \langle \log P(\mathbf{N}, \mathbf{c}, \mathbf{s}|\mathbf{r}) \rangle_{Q(\mathbf{c}|\mathbf{r})Q(\mathbf{s}|\mathbf{r})} \\
&= \sum_i [\log(\delta(r_i - \sum_j N_{ij})) - \sum_j [N_{ij} \log(f_i(\langle s_j \rangle_{Q(\mathbf{s}|\mathbf{r})}) \langle c_j \rangle_{Q(\mathbf{c}|\mathbf{r})}) - \log N_{ij}!]]
\end{aligned}
$$

This is just another multinomial:

$$
Q(\mathbf{N}|\mathbf{r}) = \prod_i \delta(r_i - \sum_j N_{ij}) r_i! \prod_j \frac{(f_i(\langle s_j \rangle_{Q(\mathbf{s}|\mathbf{r})}) \langle c_j \rangle_{Q(\mathbf{c}|\mathbf{r})}))^{N_{ij}}}{N_{ij}!}
$$

Factorized distributions

$$\begin{aligned}
\log Q(\mathbf{c}|\mathbf{r}) &= \langle \log P(\mathbf{N}, \mathbf{c}, \mathbf{s}|\mathbf{r}) \rangle_{Q(\mathbf{N}|\mathbf{r})Q(\mathbf{s}|\mathbf{r})} \\
&= \sum_{ij} [\langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} \log c_j - f_i(\langle s_j \rangle_{Q(\mathbf{s}|\mathbf{r})}) c_j] \\
&\quad + \sum_j [(\alpha - 1) \log c_j - \beta c_j - \log(\beta^{-\alpha} \Gamma(\alpha))] \\
&= \sum_{ij} [(\alpha + \langle N_{ij} \rangle_{Q(\mathbf{c}|\mathbf{r})} - 1) \log c_j - (\beta + f_i(\langle s_j \rangle_{Q(\mathbf{s}|\mathbf{r})})) c_j] \\
&\quad - \sum_j \log(\beta^{-\alpha} \Gamma(\alpha))
\end{aligned}$$

This is just another Gamma distribution:

$$Q(\mathbf{c}|\mathbf{r}) = \prod_j \mathrm{Gamma}(c_j | \alpha + \langle N_{ij} \rangle_{Q(\mathbf{c}|\mathbf{r})}, \beta + f_i(\langle s_j \rangle_{Q(\mathbf{s}|\mathbf{r})}))$$

Factorized distributions

$$\log Q(\mathbf{s}|\mathbf{r}) = \langle \log P(\mathbf{N}, \mathbf{c}, \mathbf{s}|\mathbf{r}) \rangle_{Q(\mathbf{N}|\mathbf{r})Q(\mathbf{c}|\mathbf{r})}$$
$$= \sum_{ij} [\langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} \log(f_i(s_j)) - f_i(s_j) \langle c_j \rangle_{Q(\mathbf{c}|\mathbf{r})}] = \sum_{ij} [\langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} \log(f_i(s_j))]$$

With the last equality because of the space covering assumption. Then this is just another von Mises

$$Q(\mathbf{s}|\mathbf{r}) \propto \prod_j e^{\kappa(\sum_{ij} [\langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} \cos(s_j - s_i^{\text{pref}})}$$
$$= \prod_j e^{\kappa(\sum_j \langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} (\cos s_j \cos s_i^{\text{pref}} + \sin s_j \sin s_i^{\text{pref}}))}$$
$$= \prod_j e^{\kappa(\sum_j (\cos s_j \sum_i \langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} \cos s_i^{\text{pref}} + \sin s_j \sum_i \langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} \sin s_i^{\text{pref}})})$$
$$= \prod_j e^{\kappa(\cos s_j \cos(\sum_i \langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} s_i^{\text{pref}}) + \sin s_j \sin(\sum_i \langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} s_i^{\text{pref}})}$$
$$= \prod_j e^{\tilde{\kappa} \cos(s_j - \hat{s})}$$

where $\hat{s} = \sum_i \langle N_{ij} \rangle_{Q(\mathbf{N}|\mathbf{r})} s_i^{\text{pref}}$