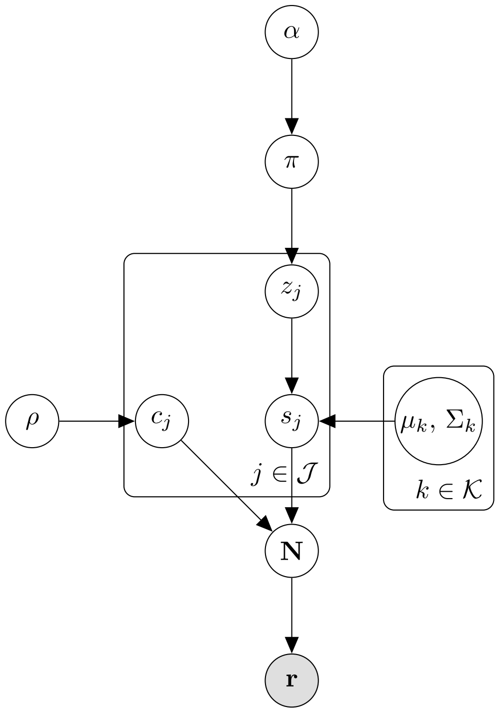
Notes from meeting



$s_j | z_j \sim \mathcal{N}(\mu_{z_j}, \Lambda_{z_j})$
$N_{ij} | \{s_j\} \sim \mathrm{Poisson}(c_j f_i(s_j))$
$r_i | N_{ij} \sim \delta(r_i - \sum_j N_{ij})$
$\pi \sim \mathrm{Dir}(\alpha)$
$c \sim \mathrm{Dir}(\rho)$

$$
\begin{aligned}
\log Q(z) &= \langle \log P(z,s,c,\mu,\Lambda,N,r)\rangle_{Q(s)Q(N)Q(c)Q(\mu,\Sigma)} \\
&= \langle \log(\mathrm{Mult}(z;\pi)\mathrm{Dir}(\pi;\alpha)\prod_{j=1}^{J}\mathcal{N}(s_j;\mu_{z_j},\Lambda_{z_j})\mathrm{Dir}(c,\rho)... \\
&\qquad \prod_{j=1}^{J}((\delta(r_i-\sum_j N_{ij})\mathrm{Poisson}(c_j f_i(s_j)))... \\
&\qquad \prod_{k=1}^{K}\mathcal{N}(\mu_k;m_k,(\beta_0\Lambda_k))\mathcal{W}(\Lambda_k;\mathbf{W}_0,\nu_0))\rangle_{Q(s)Q(N)Q(c)Q(\mu,\Lambda)} \\
\log Q(z_j=k) &= \langle \log(\mathrm{Mult}(z;\pi)\rangle_{Q(\pi)} + \langle \log(\prod_{j=1}^{J}\mathcal{N}(s_j;\mu_{z_j},\Lambda_{z_j}))\rangle_{Q(s)Q(\mu,\Lambda)}) \\
&= \frac{D}{2}\langle \log(|\Lambda_k|)\rangle_{Q(\Lambda)} - \frac{1}{2}\langle (s_j-\mu_k)^T\Lambda_k(s_k-\mu_k)\rangle_{Q(s)Q(\mu,\Lambda)} + \langle \log \pi_k\rangle_{Q(\pi)} \\
&= \frac{D}{2}(\sum_{d=1}^{D}\Psi(\frac{\nu_k^*+1-d}{2}) + \log(|\mathbf{W}_k^*|)) \\
&\qquad -\frac{1}{2}(D{\beta_k^*}^{-1} + \nu_k^*((\mu_k^*-m_j^*)^T\mathbf{W}_k^*(\mu_k^*-m_j^*)) + \mathrm{Tr}(\mathbf{W}_k^*{\Lambda_j^*}^{-1})) \\
&\qquad + \Psi(\alpha_k) - \Psi(\sum_{k=1}^{K}\alpha_k) \\
q_{jk}^* &= \frac{Q(z_j=k)}{\sum_{k=1}^{K}Q(z_j=k)}
\end{aligned}
\tag{1}
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\begin{aligned}
\log Q(c) &= \langle \sum_j (\rho_j - 1) \log c_j \rangle + \sum_{ij} \langle N_{ij} \log c_j \rangle \\
&= \sum_j [\log c_j][\rho_j - 1 + \sum_i \langle N_{ij} \rangle] \\
Q(c) &\sim \mathrm{Dir}(\rho_j - 1 + \sum_i \langle N_{ij} \rangle) = \mathrm{Dir}(\gamma_j^*) \\
\gamma_j^* &= \rho_j - 1 + \sum_i r_i p_{ij}^* \\
\langle \log c_j \rangle &= \Psi(\rho_j - 1 + \sum_i r_i p_{ij}^*) - \Psi(\sum_j (\rho_j - 1 + \sum_i r_i p_{ij}^*))
\end{aligned}
\tag{2}
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\begin{aligned}
\log Q(s_j) &= \langle \log \mathcal{N}(\mu_{z_j}, \Lambda_{z_j}) \rangle_{Q(z)Q(\mu,\Lambda)} + \sum_{i=1}^{I} \langle N_{ij} \log f_i(s_j) \rangle_{Q(N)} \\
&= -\frac{1}{2}(\langle (s_j - \mu_j)^T \Lambda_{z_j}^{-1} (s_j - \mu_j) \rangle_{Q(z)Q(\mu,\Lambda)} + \sum_{i=1}^{I} \langle N_{ij} \rangle_{Q(N)} \log f_i(s_j)) \\
&= -\frac{1}{2}(\sum_{k=1}^{K} q_{jk}^* \nu_k (s_j - \mu_k)^T \mathbf{W}_k (s_j - \mu_k) - \frac{1}{2} \sum_{i=1}^{I} 2 \frac{||s_j - s_i^{\mathrm{pref}}||^2}{L}) \\
Q(s_j) &\sim \mathcal{N}(m_j^*, \Lambda_j^*) \\
\Lambda_j^* &= \sum_{l=1}^{K+I} \Lambda_l \\
m*_j &= \Lambda_j^* \sum_{l=1}^{K+I} (\Lambda_l m_l) \\
m_l &\in \{m_k\}_{k=1}^{K}, \{s_i^{\mathrm{pref}}\}_{i=1}^{I} \\
\Lambda_l &\in \{q_{kj}^* \nu_k \mathbf{W}_k\}_{k=1}^{K}, \{\frac{2 r_i p_{ij}}{K+I} I\}_{i=1}^{I} \\
\langle \log f_i(s_j) \rangle_{Q(s_j)} &= \frac{-\langle ||s_j - s_i^{\mathrm{pref}}||^2 \rangle_{Q(s-j)}}{K+I} \\
&= \frac{-||s_i^{\mathrm{pref}} - m_j^*||^2 + \mathrm{Tr}({\Lambda_j^*}^{-1})}{K+I}
\end{aligned}
\tag{3}
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\begin{aligned}
\log Q(N_i) &= \log(\delta(\sum_{j=1}^{J} N_{ij} - r_i)) + \sum_{j=1}^{J} N_{ij}[\langle \log c_j \rangle + \langle \log f_i(s_j) \rangle] - \sum_{j=1}^{J} \log N_{ij}! \\
Q(N_{ij}) &\sim \mathrm{Mult}(r_i, p_{ij}^*) \\
p_{ij}^* &= \frac{e^{\langle \log c_j \rangle + \langle \log f_i(s_j) \rangle}}{\sum_{j=1}^{J} e^{\langle \log c_j \rangle + \langle \log f_i(s_j) \rangle}} \\
&= \frac{e^{\Psi(\gamma_j^*) - \Psi(\sum_{j=1}^{J} \gamma_j^*) - \frac{-||s_i^{\mathrm{pref}} - m_j^*||^2 + \mathrm{Tr}(\Lambda_j^{*\,-1})}{K+I}}}{\sum_{j=1}^{J} e^{\Psi(\gamma_j^*) - \Psi(\sum_{j=1}^{J} \gamma_j^*) - \frac{-||s_i^{\mathrm{pref}} - m_j^*||^2 + \mathrm{Tr}(\Lambda_j^{*\,-1})}{K+I}}}
\end{aligned}
\tag{4}
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\begin{aligned}
\log Q(\mu_k, \Lambda_k) &= \langle \log(\prod_{j=1}^{J} \mathcal{N}(s_j; \mu_{z_j}, \Lambda_{z_j})\rangle_{Q(s)Q(z)}) \\
&= \langle \sum_{j=1}^{J} [fracD2 \log |\Lambda_{z_j}| - \frac{1}{2}[(s_j - \mu_{z_j})^T \Lambda_{z_j} (s_j - \mu_{z_j})\rangle_{Q(s)Q(z)} \\
&= \langle \sum_{j=1}^{J} [fracD2 \log |\Lambda_{z_j}| - \frac{1}{2}[(m_j^* - m_{z_j})^T \Lambda_{z_j} (m_j^* - m_{z_j}) + \mathrm{Tr}(\Lambda_{z_j} {\Lambda_j^*}^{-1})]]\rangle_{Q(z)} \\
&= \sum_{j=1}^{J} q_{jk}^* [fracD2 \log |\Lambda_{z_j}| - \frac{1}{2}[(m_j^* - m_{z_j})^T \Lambda_{z_j} (m_j^* - m_{z_j}) + \mathrm{Tr}(\Lambda_{z_j} {\Lambda_j^*}^{-1})] \\
Q(\mu_k, \Lambda_k) &\sim \mathcal{N}(\mu_k; \mu_k^*, (\beta_k^* \Lambda_k))\mathcal{W}(\Lambda_k; \mathbf{W}_k^*, \nu_k^*) \\
\sum_{j=1}^{J} q_{jk}^* \mathrm{Tr}(\Lambda_k {\Lambda_j^*}^{-1}) &= \mathrm{Tr}(\Lambda_k \sum_{j=1}^{J} q_{jk}^* {\Lambda_j^*}^{-1}) \\
\mathbf{W}_k^* &= (\sum_{j=1}^{J} (q_{jk}^* {\Lambda_j^*}^{-1}))^{-1} \\
|\Lambda_{z_j}|^{\frac{\nu_k^* - 1}{2}} &= |\Lambda_{z_j}|^{\frac{D}{2} + \sum_{j=1}^{J} q_{jk}^*} \\
\nu_k^* &= D + 2 \sum_{j=1}^{J} q_{jk}^* \\
C &= \Lambda_k \sum_{j=1}^{J} q_{jk}^* \\
X &= \Lambda_k \sum_{j=1}^{J} q_{jk}^* m_j^* \\
\mu_k^* &= \frac{X}{C} = \frac{\sum_{j=1}^{J} q_{jk}^* m_j^*}{\sum_{j=1}^{J} q_{jk}^*} \\
(\beta_k^* \Lambda_k)^{-1} &= C \\
\beta_k^* &= \frac{1}{\sum_{j=1}^{J} q_{jk}^*}
\end{aligned}
\tag{5}
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\begin{aligned}
\frac{dq_{jk}^*}{dt} &= -q_{jk}^* + \frac{Q(z_j = k)}{\sum_{k=1}^{K} Q(z_j = k)} \\
\frac{dm_j^*}{dt} &= -m_j^* \Lambda_j^* (\sum_{k=1}^{K} (q_{kj}^* \nu_k \mathbf{W}_k m_k) + \sum_{i=1}^{I} (\frac{2 r_i p_{ij}}{K+I} I s_i^{\mathrm{pref}} \\
\frac{d\Lambda_j^*}{dt} &= -\Lambda_j^* + \sum_{k=1}^{K} (q_{kj}^* \nu_k \mathbf{W}_k) + \sum_{i=1}^{I} (\frac{2 r_i p_{ij}}{K+I} I) \\
\frac{dp_{ij}^*}{dt} &= -p_{ij}^* + \frac{e^{\Psi(\gamma_j^*) - \Psi(\sum_{j=1}^{J} \gamma_j^*) - \frac{-||s_i^{\mathrm{pref}} - m_j^*||^2 + \mathrm{Tr}({\Lambda_j^*}^{-1})}{K+I}}}{\sum_{j=1}^{J} e^{\Psi(\gamma_j^*) - \Psi(\sum_{j=1}^{J} \gamma_j^*) - \frac{-||s_i^{\mathrm{pref}} - m_j^*||^2 + \mathrm{Tr}({\Lambda_j^*}^{-1})}{K+I}}} \\
\frac{d\gamma_j^*}{dt} &= -\gamma_j^* + \rho_j - 1 + \sum_{i} r_i p_{ij}^* \\
\frac{d\mu_k^*}{dt} &= -\mu_k^* + \frac{\sum_{j=1}^{J} q_{jk}^* m_j^*}{\sum_{j=1}^{J} q_{jk}^*} \\
\frac{d\beta_k^*}{dt} &= -\beta_k^* + \frac{1}{\sum_{j=1}^{J} q_{jk}^*} \\
\frac{d\mathbf{W}_k^*}{dt} &= -\mathbf{W}_k^* + \frac{1}{\sum_{j=1}^{J} (q_{jk}^* {\Lambda_j^*}^{-1})} \\
\frac{d\nu_k^*}{dt} &= -\nu_k^* + D + 2 \sum_{j=1}^{J} q_{jk}^*
\end{aligned}
\tag{6}
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