MATH 205 – Calculus I **Exam 4**

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For the following question, you must show complete and accurate work to receive full credit. All derivatives, integrals and antiderivatives must be written out and solved via algebraic means. Proper notation must be used to receive full credit. You may use your calculator to verify the answer but not to generate the answer. Each question is worth 4 points unless otherwise noted.

Determine the following indefinite integrals: **{5 points each}**

1. $\int (5x^4 + 1)dx$

$F(x) = x^5 + x + C$

2. $\int \frac{1}{1+x^2} dx$

$F(x) = \tan^{-1} x$

Evaluate the following definite integrals: {**5 points each**}

3. $\int_1^8 \sqrt[3]{x}\, dx = \int_1^8 x^{1/3} dx$

$\frac{3}{4} x^{4/3} \Big|_1^8$

$\frac{3\sqrt[3]{x^4}}{4} \Big|_1^8$

$\frac{3\sqrt[3]{8^4}}{4} - \frac{3\sqrt[3]{1^4}}{4}$

$\frac{3(16)}{4} - \frac{3}{4}$

$12 - 3/4 = 45/4$

4. $\int_0^{\frac{\pi}{3}} \cos 3x dx$

$\frac{\sin 3x}{3} \Big|_0^{\frac{\pi}{3}}$

$\frac{\sin \pi}{3} - \frac{\sin 3(0)}{3}$

$0 - 0$

0

5. $\int_{e^8}^{e^{10}} \frac{7}{x} dx$

$7\ln|x| \Big|_{e^8}^{e^{10}}$

$7\ln e^{10} - 7\ln e^8$

$70 - 56$

14

6. $\int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \sqrt{7+x^4} dx$ = Zero Width Interval

$= 0$

7. Write the limit of the Riemann sum, $\lim_{\Delta x \to 0} \sum_{k=1}^{n} (\ln(x_k^*) + 8\sec^3(x_k^*))\Delta x_k$, on [-1, 7] as a definite integral.

$\int_{-1}^{7} (\ln x + 8\sec^3 x)\,dx$

8. Use the Fundamental Theorem of Calculus to find $\frac{d}{dx}\int_4^x (\sqrt{\sin(t)} + 8e^t)\,dt$

$\sqrt{\sin x} + 8e^x$

9. Use the Fundamental Theorem of Calculus to find $\frac{d}{dx}\int_{-3}^{\sin x} (t^3 - 6t)\,dt$

$(\sin x)^3 - 6\sin x$

10. Estimate the area bounded by $g(x) = x^{\cos x}$ and the x-axis on [0, 2 π] by using a **Midpoint Riemann Sum** 4 rectangles of uniform width. Show all work including a graph of the function, the rectangles you create, and the intermediate values for the height and area of the rectangles. Your final answer may be rounded at 3 decimal places.

$x_k = a + (k - \frac{1}{2})\Delta x$

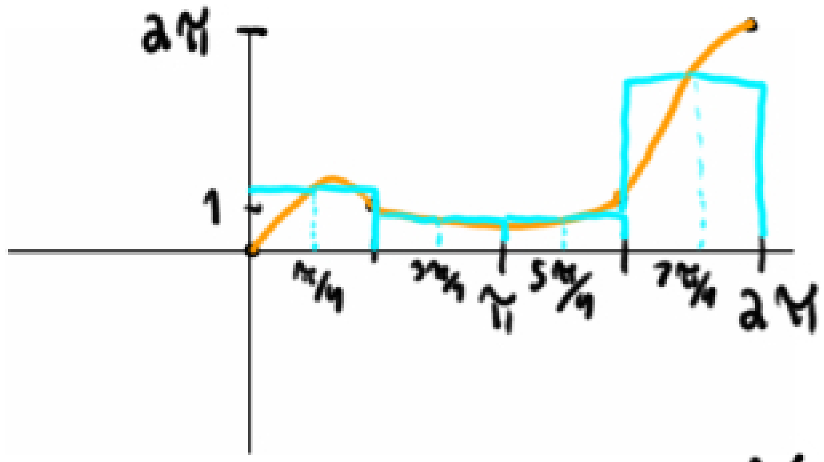
$\Delta x = \frac{2\pi - 0}{4} = \frac{\pi}{2}$

$x_k = \frac{\pi}{2}k - \frac{\pi}{4}$

a) Determine the summation notation for the Midpoint Riemann Sum. {**2 points**}

$\frac{\pi}{4}\sum_{k=1}^{4}\left[\left(\frac{\pi}{2}k - \frac{\pi}{4}\right)^{\cos(\frac{\pi}{2}k - \frac{\pi}{4})}\right]$

b) On the axes below, sketch the function and the rectangles you create. {**1 point**}



c) Determine the sum of the four areas: {**4 points**}

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\frac{\pi}{4}\left(g(\frac{\pi}{4}) + g(\frac{3\pi}{4}) + g(\frac{5\pi}{4}) + g(\frac{7\pi}{4})\right)$

$\frac{\pi}{4}(0.84 + 0.55 + 0.38 + 3.34)$

$= 4.010$

$\Delta x = \frac{4}{n}$

$x_k = a + k\Delta x$

$x_k = \frac{4k}{n} - 1$

11. Determine the area bounded by $f(x) = 4x^2 - 3x + 2$ and the x-axis on [-1, 3] by using **the limit of a Right Riemann Sum** with n rectangles.

$\left(\frac{4k}{n} - 1\right)$

$\frac{16k^2}{n^2} - \frac{8k}{n} + 1$

a) Determine the summation notation for the Right Riemann Sum. {**2 points**}

$$\frac{4}{n}\sum_{k=1}^{n} 4\left(\frac{4k}{n}-1\right)^2 - 3\left(\frac{4k}{n}-1\right) + 2$$

b) Simplify the summation notation as the number of rectangles increase without bound to determine the exact value of the area. {**5 points**}

$$\lim_{n\to\infty}\left[4\left(\frac{4k}{n}-1\right)^2 - 3\left(\frac{4n}{n}-3\right) + 2\right]\frac{4}{n}$$

$$\lim_{n\to\infty}\left(4\left(\frac{16k^2}{n^2} - \frac{8k}{n} + 1\right) - \left(\frac{12k}{n} - 9\right) + 2\right)\frac{4}{n}$$

$$\lim_{n\to\infty}\left(\frac{64k^2}{n^2} - \frac{32k}{n} + 4 - \frac{12k}{n} + 9 + 2\right)\frac{4}{n}$$

$$\lim_{n\to\infty}\left(\frac{64k^2}{n^2} - \frac{44k}{n} + 15\right)\frac{4}{n}$$

$$\lim_{n\to\infty}\left(\frac{64}{n^2}\sum_{k=1}^{n} k^2 - \frac{44}{n}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 15\right)\frac{4}{n}$$

$$\lim_{n\to\infty}\left(\frac{64}{n^2}\left(\frac{2n^3+3n^2+n}{6}\right) - \frac{44}{n}\left(\frac{n^2+n}{2}\right) + 15n\right)\frac{4}{n}$$

$$\lim_{n\to\infty}\left(\frac{256}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right) - \frac{176}{n^2}\left(\frac{n^2+n}{2}\right) + 60\right)$$

$$\lim_{n\to\infty}\left(\frac{256}{6}\left(\frac{2n^3}{n^3} + \frac{3n^2}{n^3} + \frac{n}{n^3}\right) - \frac{176}{2}\left(\frac{n^2}{n^2} + \frac{n}{n^2}\right) + 60\right)$$

$$\lim_{n\to\infty}\left(\frac{128}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - 88\left(1 + \frac{1}{n}\right) + 60\right)$$

$$\lim_{n\to\infty}\left(\frac{256}{3} - 88 + 60\right)$$

$$\lim_{n\to\infty}\left(\frac{256}{3} - \frac{264}{3} + \frac{180}{3}\right)$$

$$= \frac{172}{3}$$

$$\lim_{n\to\infty}\left(4\left(\frac{4k}{n}-1\right)^2 - 3\left(\frac{4k}{n}-3\right) + 2\right)\frac{4}{n} = \frac{172}{3}$$

12. Determine the function that passes through the point (-3, 14) and whose derivative is

$g(x) = 3x^2 - 4x - 7$.

$G(x) = x^3 - 2x^2 - 7x + C$

$G(-3) = (-3)^3 - 2(-3)^2 - 7(-3) + C$

$14 = -27 - 18 + 21 + C$

$38 = C$

$G(x) = x^3 - 2x^2 - 7x + 38$

13. Suppose James Bond is driving an Aston Martin DB5 equipped with a passenger ejector seat (as in "Goldfinger"). As he crosses a bridge that spans a 450-ft deep mountain gorge, he "accidently" pushes the eject button and launches a double agent into the air. If the initial velocity of the ejection seat is 45 ft/sec, how long will it take for the double agent to hit the bottom of the gorge?

$x_n = 45$

{Acceleration due to gravity ≈ -32 ft/sec/sec }

SVA

{Angle of launch is negligible and remember: Moles can't fly}

-32 ft/sec^2

$-32t$ ft/sec

$-16t^2$ ft

$x_0 = 45$

$22.8122 = x_1 = 45 - \dfrac{450 - 16t^2}{-32t}$

$12.62269 = x_2$

$7.281067 = x_3$

$5.54872 = x_4$

X

14. Use Newton's Method to determine the root of $f(x) = \ln(x) - 2x^3 + 4$

a) Determine the formula to find the $n + 1$ root {**2 points**}

$x_{n+1} = x_n - \ln(x_n) - 2(x_n)^3 + 4$

b) Use an initial value of $x = 2$ and your calculator to approximate the sought after root. Show each successive value of x and terminate the process when successive iterations agree to five (5) decimal places. {**5 points**}

$\frac{1}{x} - 6x^2$

$x = 2$

$x_1 = 1.51886 = 2 - \dfrac{\ln 2 - 16 + 4}{1/2 - 24}$

$x_2 = 1.32241$

$x_3 = 1.28690$

$x_4 = 1.28578$

$x_5 = 1.28578$

For questions 15 – 17, use L'Hopital's Rule to determine each of the limits:

15. $\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$ $\overset{\text{LHop}}{\Rightarrow} \left(\frac{x}{3}\right)^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

DS=0/0 DS $=\frac{1}{12}$

$\lim_{x\to 8}\frac{\sqrt[3]{x^1}-2}{x-8}=\frac{1}{12}$

16. $\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\sec x = \frac{\frac{\pi}{2}-x}{\cos x} = 0$

DS $=0(1)$ DS $=\frac{0}{1}$

$=0$ $=0$

17. $\lim_{x\to 0}(1+2x)^{\frac{5}{x}} =$ Let $y=(1+2x)^{5/x}$

DS $=1^0$

$\ln y=\frac{5}{x}\ln(1+2x)$

$\lim_{x\to 0}\frac{5}{x}\ln(1+2x) \overset{\text{LHop}}{\Rightarrow}$

$\frac{10}{x+2x^2}+\ln(1+2x)\left(\frac{-5}{x^2}\right)$

DS=0/0

$\Rightarrow \lim_{x\to 0}\frac{\frac{10}{x+2x^2}+\ln(1+2x)\left(\frac{-5}{x^2}\right)}{\frac{-10(1+4x)}{(x+2x^2)^2}+\frac{2}{1+2x}\left(\frac{-5}{x^2}\right)+\frac{10x}{x^4}(\ln(1+2x))}$

DS=0/0

Limit DNE

18. Given the graph of $A(x)=\int_2^x f(t)dt$ shown below, determine each of the following:

a) $A(5)$

$\frac{1}{2}(3)(7)+\frac{1}{2}(1)(2)+\frac{1}{2}(1)(2)=$

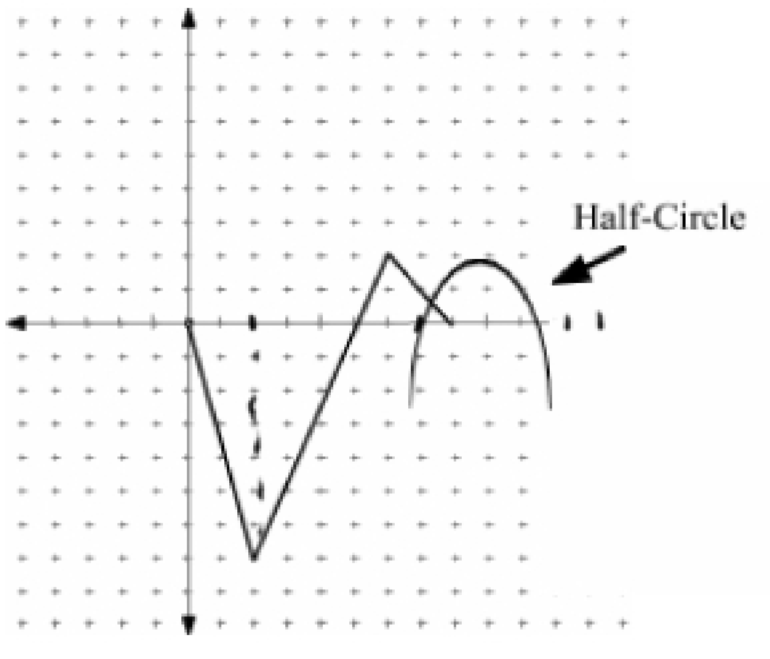
$21/2+1+1=25/2$

b) $A(12)$

$\frac{27}{2}+\frac{1}{2}+\pi(2.5)^5=\frac{25\pi}{4}+13$

c) $A(2)$

$A=0$



d) $A(0) = -\frac{1}{2}(7)(2) = -7$

19. Use symmetry to determine $\int_{-8}^{8} (\underbrace{\sin^7 x}_{odd} + \underbrace{3x^4}_{even} - \underbrace{10x^3}_{odd})dx$

$\int_{-8}^{8} \sin^7 x\,dx + 2\int_0^8 3x^4\,dx - \int_{-8}^{8} 10x^3\,dx$

$\frac{6}{5}(x^5)\Big|_0^8$

$\frac{6(8^5)}{5} - 0 = \frac{196608}{5}$

20. Determine the average value of $g(x) = x^2 + 3x + 1$ on [-2, 6].

$\bar{f}\ \frac{1}{6-(-2)}\int_{-2}^{6}(x^2+3x+1)\,dx$

$\frac{1}{8}\left[\frac{x^3}{3} + \frac{3x^2}{2} + x\right]\Big|_{-2}^{6}$

$\frac{1}{8}\left[\frac{6^3}{3} + \frac{3(6)^2}{2} + 6\right] - \left[\frac{-2^3}{3} + \frac{3(-2)^2}{2} - 2\right] = \frac{997}{3}$

21. Determine the value(s) of x, if any exists, at which $g(x) = e^x$ on [0, 3] equals its average value. {Mean Value Theorem for Integrals}

$\bar{f}\ \frac{1}{3}\int_0^3 e^x$

$\frac{1}{3}e^x\Big|_0^3$

$\frac{1}{3}e^3 - \frac{1}{3}e^0$

$\frac{e^3}{3} - \frac{1}{3}$

$\frac{e^3-1}{3}$

$e^x = \frac{e^3-1}{3}$

$x = \ln\left(\frac{e^3-1}{3}\right)$