MATH 205 – Calculus I **Exam #2** Name: Nathan Hallam

Instructions: Though calculators can be used for the entire exam, all problems require you to show your work. To receive full credit, answers must have proper calculus justification. Only **EXACT** answers will receive full credit unless otherwise noted.

{Questions 1 – 12: **6** points each, Questions 13 – 22 & 24: **2** points each, Questions 23 and 25: **3** points each}

1. Determine $f'(x)$ if $f(x) = 7x^5 - 9x^4 + 15$

$$f'(x) = 35x^4 - 36x^3$$

2. Determine $g'(x)$ if $g(x) = \frac{\cos x}{7x^8 - 4x^3}$

$$g'(x) = \frac{(7x^8-4x^3)(-\sin x) - (\cos x)(56x^7 - 12x^2)}{[7x^8-4x^3]^2}$$

3. Determine $k'(x)$ if $k(x) = \sin\left(\sqrt{10x^3 - 8x^{-2}}\right)$

$f'(g(x))g'$

$$k'(x) = \cos\left(\sqrt{10x^3-8x^{-2}}\right)\left(\frac{1}{2}\left(10x^3-8x^{-2}\right)^{-1/2}\right)(30x^2+16x^{-3})$$

$$k'(x) = \cos\left(\sqrt{10x^3-8x^{-2}}\right)\left[\frac{1}{2\sqrt{10x^3-8x^{-2}}}\right](30x^2+16x^{-3})$$

4. Determine $\frac{d}{dr}(6r^4 - 21r^2 + 3)\cdot\tan r$

$$(6r^4-21r^2+3)(\sec^2 r) + (\tan r)(24r^3 - 42r)$$

5. Determine $f'(x)$ if $f(x) = 8^x + \text{arcsec}(x)$

$$e^{x\ln 8}(\ln 8) + \frac{d}{dx}\text{arcsec}(x)$$

$$f'(x) = 8^x[\ln 8) + \frac{1}{|x|\sqrt{x^2-1}}$$

6. Determine $p'(x)$ if $p(x) = \ln(\sin x)$

$$p'(x) = \frac{1}{\sin(x)}(\cos x)$$

$$p'(x) = \frac{\cos x}{\sin x} \rightarrow p'(x) = \cot x$$

7. Determine $f'(x)$ if $f(x) = \tan^{-1}(\log_9(x^2+6x-8))$

$$f'(x) = \frac{1}{1+(\log_9(x^2+6x-8))^2}\left(\frac{1}{\ln(9)(x^2+6x-8)}\right)(2x+6)$$

$$f'(x) = \left[\frac{1}{1+[\log_9(x^2+6x-8)]^2}\right]\left(\frac{2x+6}{\ln(9)(x^2+6x-8)}\right)$$

8. $\frac{d^{25}y}{dx^{25}} 2e^x$

$$2e^x + e^x(0) \qquad \frac{d^{1\ldots25}y}{dx^{1\ldots25}} = 2e^x$$

$$\frac{dy}{dx} = 2e^x$$

9. Determine $f'''(5)$ for $f(x) = 2x^5 + 5x^3 - 3x^2 + 2$

$$f'(x) = 10x^4 + 15x^2 - 6x$$

$$f''(x) = 40x^3 + 30x - 6$$

$$f'''(x) = 120x^2 + 30$$

$$f'''(5) = 120(5)^2 + 30$$

$$f'''(5) = 120(25) + 30$$

$$f'''(5) = 3000 + 30$$

$$f'''(5) = 3030$$

10. Given $\frac{d}{dx}\sin x = \cos x$ and $\frac{d}{dx}\cos x = -\sin x$, prove the derivative of $f(x) = \sec x$ is $f'(x) = \sec x \tan x$

$$f(x) = \sec x$$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{d}{dx}\frac{1}{\cos x} = \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x}\left(\frac{1}{\cos x}\right)$$

$$= \tan x \sec x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

11. Using implicit differentiation and the derivatives of the trigonometric functions, prove the derivative of $g(x) = \cot^{-1} x$ is $g'(x) = \frac{-1}{1+x^2}$

let $y = \cot^{-1} x$

$$\cot y = x$$

$$-\csc^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{-1}{\csc^2 y}$$

$$\frac{dy}{dx} = \frac{-1}{1+\cot^2 y}$$

$$\frac{dy}{dx} = \frac{-1}{1+x^2}$$

$1+\cot^2 y = \csc^2 y$

12. Use logarithmic differentiation to determine $\frac{dy}{dx}$ for $y = (\cos x)^{\sin x}$

$$\frac{dy}{dx} = e^{\sin x \ln \cos x}\left(\frac{\sin x}{\cos x}(\sin x) + \cos x \ln \cos x\right.$$

$$\frac{dy}{dx} = (\cos x)^{\sin x}\left(\tan(x)(-\sin x) + \cos(x)\ln\cos(x)\right)$$

Suppose a particle moves along a straight line and its position is given by

$$f(x) = \frac{1}{3}x^3 - 7x^2 + 33x + 13 \text{ on } [0, 14]$$

where $f(x)$ is in feet and x is in seconds.

$V(t) = x^2 - 14x + 33$

$a(t) = 2x - 14$

13. Determine when the particle is moving to the left.

left neg
Right pos

The Particle is moving left on the interval $(3, 11)$

$V(4) = 4^2 - 14(4) + 33$
$16 - 56 + 33$
$-40 + 33$
-7

$V(2) = 4 - 28 + 33$
$V(2) = 9$

14. Determine when the particle changes direction.

$0 = x^2 - 14x + 33 \quad 0 = \frac{+14 \pm \sqrt{14^2 - 4(33)}}{2} = \frac{14 \pm \sqrt{196 - 132}}{2} = \frac{14 \pm \sqrt{64}}{2}$

$x = 3, x = 11$

$\frac{6}{2}$ & $\frac{22}{2} \leftarrow \frac{14 \pm 8}{2}$

15. Determine the interval(s) on which the particle is slowing down.

$[0, 3) \cup (7, 11)$

16. Determine the particle's average ***speed*** on [2, 8].

$\frac{d}{t}$

$\frac{f(8) - f(2)}{8 - 2} = -9 \text{ ft/s}$

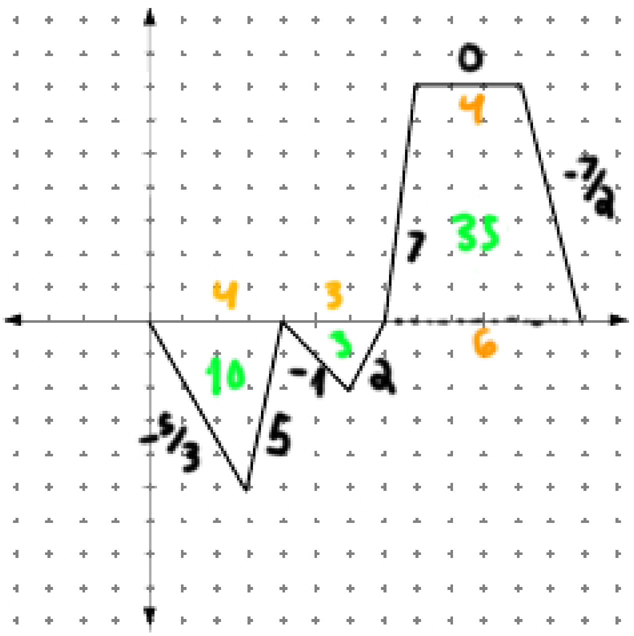
17. Determine the particle's displacement from time $t = 0$ to $t = 14$.

$\frac{f(14) - f(0)}{14} = 1/3 \text{ ft}$

18. Determine the total distance traveled by the particle from time $t = 0$ to $t = 14$.

$|f(14)| + |f(0)| = 4\,3/3 \text{ ft}$

For questions 19 - 22, use the graph below that gives a particle's ***VELOCITY*** (m/sec) at time *t* (secs). {Scale is 1 tick mark = 1 unit}



This is the VELOCITY GRAPH!

It is not, I repeat, **IS NOT**, the position graph!

$-13+35=22$ right

19. When is the particle moving to the right?

$(7, 13]$

20. When is the particle speeding up?

$[0,3) \cup (4,6) \cup (7,8)$

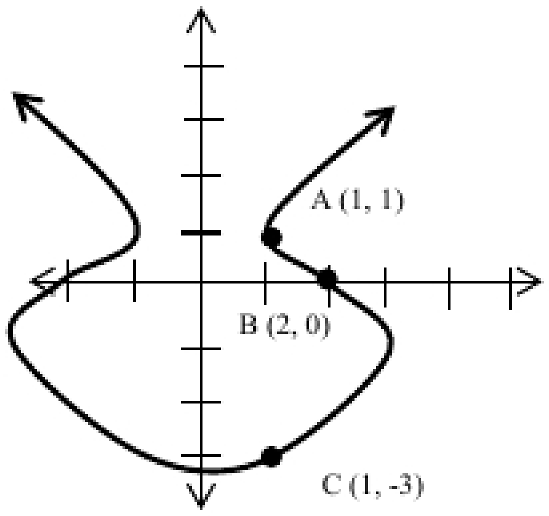
21. When does the particle change directions?

$x=7$

22. If the particle starts 10 meters to the left of zero, estimate the particle's ending position?

The particle will be around 12 units right of the origin.

For questions 23 and 24, refer to the equation $y^3 + y^2 - 5y - x^2 = -4$ whose graph is shown to the below: {If you cannot determine (23), I will supply it for use in (24)}



23. Find a formula for $\frac{dy}{dx}$ and use it to evaluate the slope at point C.

$$3y^2\frac{dy}{dx}+2y\frac{dy}{dx}-5\frac{dy}{dx}-2x=0$$

$$\frac{dy}{dx}=\frac{2x}{3y^2+2y-5}$$

24. a) Find an equation of the **tangent** line to the curve at point C.

$$\frac{2}{27-6-5}=\frac{2}{16}=\frac{1}{8} \qquad y+3=\frac{1}{8}(x-1)$$

b) Find an equation of the **normal** line to the curve at point C.

$$y+3=-8(x-1)$$

25. Suppose $\left(f^{-1}\right)'(5)=\frac{\sqrt{3}}{\pi}$ at (5, -2), determine $f'(-2)$.

$$(f^{-1})'(5)=\frac{1}{f'(-2)}=\frac{\sqrt{3}}{\pi}$$

$$f'(-2)=\frac{\pi}{\sqrt{3}}$$