4.3 What Derivatives Tell Us

MATH 205

Increasing and Decreasing Functions

- Let f be a function defined on interval I with x_1 and x_2 two points in I

1. If $f(x_1) < f(x_2)$ whenever $x_1 < x_2$, then f is said to be increasing on I

   - If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f'(x) > 0$ at each x in (a, b), then f is increasing on $[a, b]$

2. If $f(x_1) > f(x_2)$ whenever $x_1 < x_2$, then f is said to be decreasing on I

   - If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f'(x) < 0$ at each x in (a, b), the f is decreasing on $[a, b]$

First Derivative Test for Local Extrema

- Suppose f is a continuous function and c is a critical point of f and f is differentiable in some neighborhood containing c (but not necessarily at c). Moving from left to right across c:

1. If $f'(x)$ changes from negative to positive at c, then f has a local minimum at c.
2. If $f'(x)$ changes from positive to negative at c, then f has a local maximum at c.
3. If $f'(x)$ does not change sign at c, then f has no local extremum at c.

Using the First Derivative

1. Sketch a function *f* continuous on $(-\infty, \infty)$ that satisifies:

   i. $f' < 0$ on $(-5,-2), (-2,4)$ and $(8,\infty)$

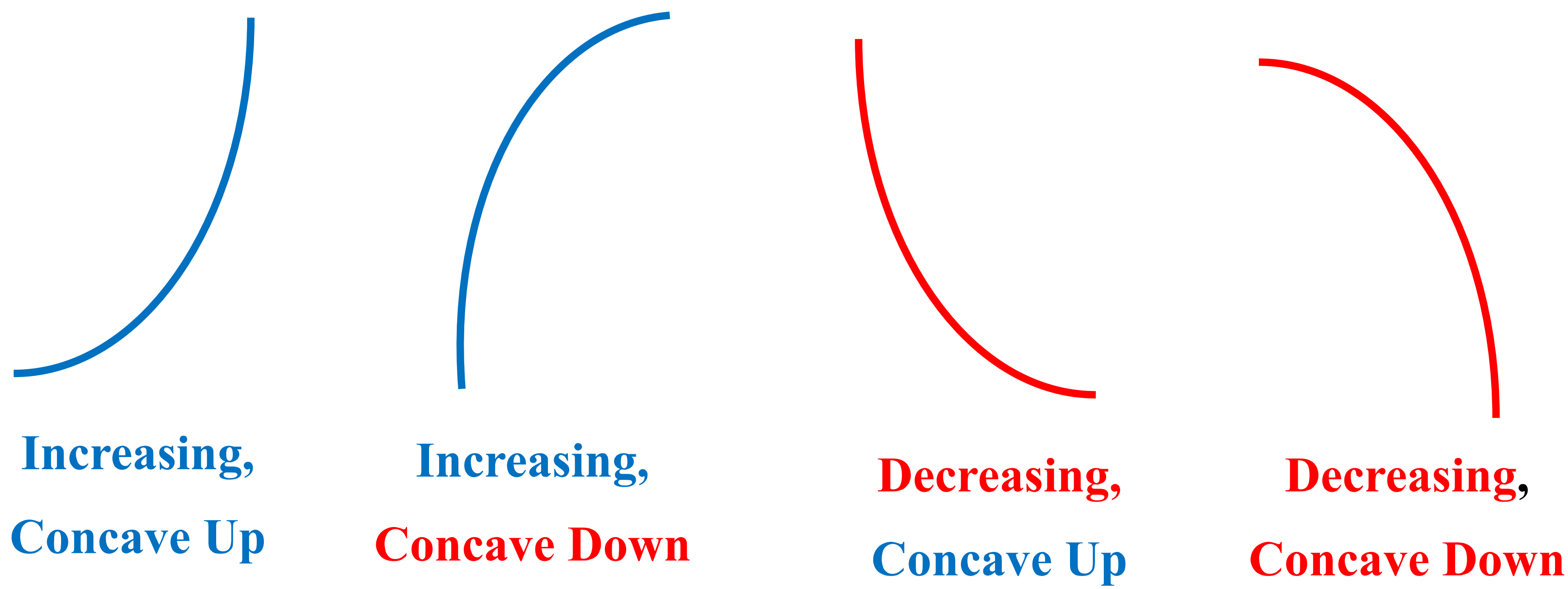
   ii. $f' > 0$ on $(-\infty,-5)$ and $(4,8)$

   iii. $f'(8)$ is undefined and $f'(-5) = f'(-2) = f'(4) = 0$

Using the First Derivative

- Find the intervals on which $g(x) = xe^{-x}$ is increasing and decreasing.

Types of concavity



1. Describe f' for each of the above cases.
2. Describe f'' for each of the above cases.

So, lets formalize

Let f be a differentiable function, then f is:

1. Concave up on (a, b) if f' is increasing.
2. Concave down on (a, b) if f' is decreasing.

If f is a twice-differentiable function then f is:

1. Concave up on (a, b) if f'' is positive.
2. Concave down on (a, b) if f'' is negative.

Inflection Points

- An Inflection point is where the graph of a function changes concavity {provided a tangent line exists at that point.}
- Look for inflection points where the second derivative does not exist or is equal to zero.
- The second derivative must change sign, as you move from left to right across the suspected point, for an inflection point to exist.

Second Derivative Test for Local Extrema

- Suppose f'' is continuous on (a, b) that contains $x = c$

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.
3. If $f'(c) = 0$ and $f''(c) = 0$, then we know nothing for sure. A maximum, minimum or nothing may exist.

Putting it all together

3. Given $h(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$, determine the inflection points, concavity, extrema, and increasing/decreasing intervals.

Putting it all together

4. Given the following table, sketch a graph of $f(x)$ where $f(x)$ is continuous for all x.

x	$x<-3$	-3	$-3<x<1$	1	$1<x<4$	4	$4<x<6$	6	$6<x<9$	9	$x>9$
f		4		7		3		1		0	
f'	-	0	+	DNE	-	0	-	DNE	-	0	+
f''	+	0	+	DNE	+	0	-	DNE	+	0	+