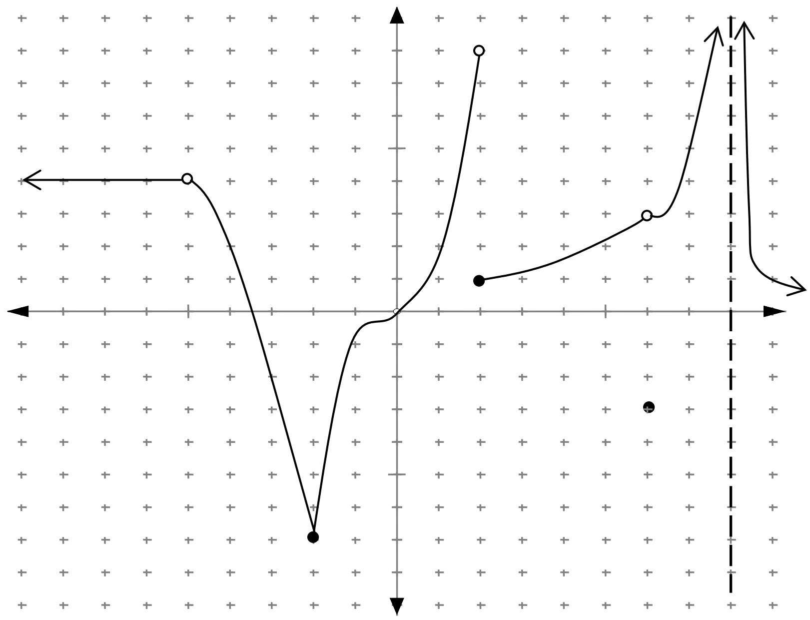
MATH 205 – Calculus I **Exam #1** **Week #4**

Name: ______________________________ Date: _______

Instructions: Though calculators can be used for the entire exam, all problems require you to show your work. Any answer without proper justification will receive ***ZERO*** credit. Only **EXACT** answers will receive full credit unless otherwise noted. Proper Interval and Limit Notation must be used to receive credit. Each question is worth 5 points.

1. Fill out the following table for the function f shown below:



c	$f(c)$	$\lim_{x\to c^-} f(x)$	$\lim_{x\to c^+} f(x)$	$\lim_{x\to c} f(x)$	Is $f(x)$ continuous at x = c?
-5					
-2					
2					
6					
8					

2. Find the average rate of change of $f(x) = \cos(x) + x$ on $\left[0, \frac{\pi}{2}\right]$.

3. Create a table of at least 6 entries to determine $\lim_{x \to 0} \frac{8\tan(x)}{\sin(x)}$. {Round each entry to six decimal places.}

x	$\frac{8\tan(x)}{\sin(x)}$

$\lim_{x \to 0} \frac{8\tan(x)}{\sin(x)} =$ ______

Determine the following limits and justify your answers using the properties of limits and algebraic means. Do not rely on tables or graphs.

4. $\lim_{x \to 3}(8x^2 + 7x - 5)$

5. $\lim_{x \to 4^-} \frac{x+3}{x^2-16}$

6. $\lim_{x \to \frac{7\pi}{6}} (sec^2 x - tan^2 x)$

7. $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}$

8. $\lim_{x \to -\infty} \frac{4\sin(x)}{e^{4x}}$

9. $\lim_{x \to 1} \begin{cases} \frac{x^2-3x+4}{x-1}, & x \neq 1 \\ 5, & x = 1 \end{cases}$

10. Assume $\lim_{x \to b} f(x) = -1$ and $\lim_{x \to b} g(x) = 12$, determine $\lim_{x \to b} \left[\frac{2g(x)}{f(x)-5}\right]^{\frac{3}{2}}$

11. Use the Intermediate Value Theorem to show $\frac{\sqrt{x^3+1}}{x^2+3} = \frac{5}{12}$ has a solution on [0, 3]

Use $k(x) = \frac{x^2+2x-15}{2x^2+x-21}$ for question 12 – 13:

12. Determine all the asymptotes (vertical and horizontal), if any, of $k(x)$. Justify the type of asymptote using the concept of limits.

13. Determine a piecewise function, $g(x)$, that "fixes" any **removable** discontinuities in $k(x)$.

14. There are three instances where **differentiability** of a function (at a point) can fail. List and give an example, either graphically or algebraically, of each of these instances.

15. Use the limit of the difference quotient to determine the instantaneous rate of change of $f(x) = \frac{4}{x-6}$ at $x = 15$.

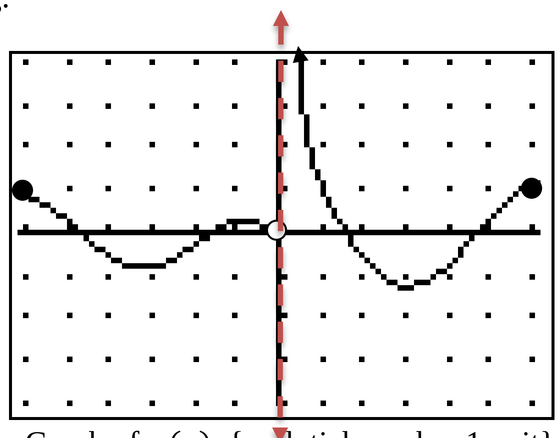
Use $f(x) = 3x^2 - 5x + 7$ for questions 16 – 18:

16. Determine the slope of tangent line to $f(x)$ at $x = -4$ by using the limit of the difference quotient.

17. Determine the equation of the tangent line to the graph of $f(x)$ at $x = -4$.

18. Where will $f'(x) = 13$

19. Use the graph of $g(x)$, whose domain is $[-6,0)\cup(0,6]$, to determine each of the following:



Graph of $g(x)$ {each tick mark = 1 unit}

a) Estimate $g'(2)$.

b) Where on (-6, 6) will $g(x)$ have horizontal tangent lines?

c) What type of discontinuity does $g(x)$ have at $x = 0$?

20. For each of the following, circle the correct answer: (T = True, F = False). If false, provide justification.

T F : Differentiability at a point on $f(x)$ **DOES NOT** guarantee continuity at the same point on $f(x)$.

T F : The derivative at a point of $f(x)$ is the same as the slope of the **tangent** line that passes through said point on the graph of $f(x)$.

T F : If $\lim_{x\to 3^-} f(x) = 7$ and $\lim_{x\to 3^+} f(x) = -2$, then $f(x)$ has a hole at $x = 3$.

T F : The limit of the difference quotient is used when determining an average rate of change.

T F : Zero is an even number.