3.3 Differentiation Rules

MATH 205

But I love the Difference Quotient!

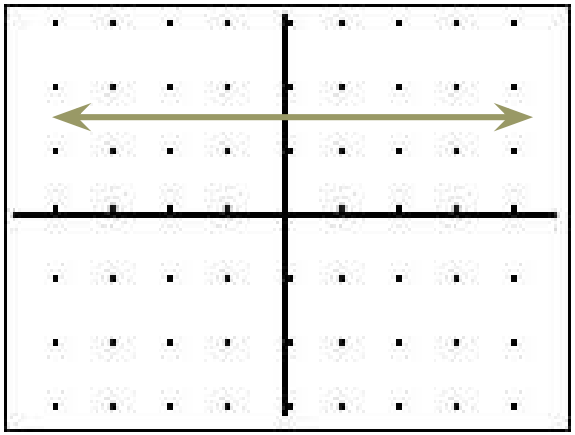
- Ok, as you have probably figured out by now, because some of your classmates have been alluding to it, there are quicker ways to find a derivative than using the limit of the difference quotient.
- These are called the rules of derivation.
- Yes, you have to know them cold! No notecards.

The derivative of the constant function

- Since the derivative function is a rule to determine the slope at any point of a given function, we can see graphically that the derivative of a constant function must be zero.

- Thus, if c is any real number

$$\frac{d}{dx}c = 0$$



Derivative Power Rule

- For any natural number, n, $\frac{d}{dx}x^n = nx^{n-1}$
  - Later, it will be shown that the power rule works for any real number n.

- Determine each of the following:

1. $\frac{d}{dx}x^5$
2. $f'(x)$ if $f(x) = x^{204}$
3. $\frac{d}{dx}x^8$
4. $\frac{d}{dx}\pi^5$

Constant Multiplier Rule and the Derivative of e^x

- For any real number c,

$$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x) = cf'(x)$$

- Determine the following derivatives:

5. $\frac{d}{dx}7x^8$

6. $\frac{d}{dt}(-10t^{13})$

7. Determine $\frac{d}{dx}e^x$

Derivative Sum (Difference) Rule

- If $f(x)$ and $g(x)$ are differentiable, then $(f+g)(x)$ is differentiable at all points where both $f(x)$ and $g(x)$ are differentiable.

$$\frac{d}{dx}(f \pm g)(x) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x) = f'(x) \pm g'(x)$$

Determine the following derivatives:

8. $\frac{d}{dx}(7x^4 - 8x^3 + 10x - 11)$

Derivative Sum (Difference) Rule

9. y' for $y = (4x^2 + 7x)(-2x^3 - 5x^2 + 9)$

10. Determine where $f(x) = x^3 - 9x^2 + 11x + 21$ has horizontal tangents.

Second, and Higher, Derivatives

- Since it is often desirable to find the rate of change of a rate of change, we may have to find multiple derivatives of a given function.

$$f'' \Leftrightarrow \frac{d^2y}{dx^2} \Leftrightarrow \text{The second derivative of } f(x)$$

$$f''' \Leftrightarrow \frac{d^3y}{dx^3} \Leftrightarrow \text{The third derivative of } f(x)$$

$$f^{(n)} \Leftrightarrow \frac{d^ny}{dx^n} \Leftrightarrow n^{th} \text{ derivative of } f(x)$$

Determine each of the following

11. $\frac{d^2y}{dx^2}(8x^3 + 6x^2 - 10x + 17)$

12. Determine $f'''(x)$ if $f(x) = 8x^3 - 15x^2 + 7x + 15$

Determine each of the following

13. $\frac{d^2y}{dx^2}(9x^{10} + 7e^x)$

14. $f^{(100)}(8x + 14)$