3.10 Derivatives of Inverse Trigonometric Functions

MATH 205

Derivatives of the Inverse Trigonometric Functions

- Implicit Differentiation is the method I'll use to determine the derivatives of the inverse trig functions

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}arc\sec x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$$

A few Inverse Trig Derivatives

1. $\dfrac{d}{dx} x^4 \sin^{-1}(x)$

Practice

2. $\dfrac{d}{dx}\ \dfrac{\tan x - \cos x}{arc\csc x}$

Practice

3. $\frac{d}{dx}\arccos(4x^3+7x-9)$

Practice

4. $\dfrac{d}{dx} x^{\arctan x}$

Derivative of the Inverse Functions

- Given f is differentiable and has an inverse on I. If x_0 is a point of I at which $f'(x_0) \neq 0$, then f^{-1} is differentiable at $y_0 = f(x_0)$ and

$$\left(f^{-1}\right)'(y_0) = \frac{1}{f'(x_0)}, \text{ where } y_0 = f(x_0)$$

General Inverse Functions

5. Determine $\left(f^{-1}\right)'(4)$ if $f(x) = 7x^{10} + 6x^7 - 9$

6. Suppose the slope of the curve $y = f^{-1}(x)$ at (9, 13) is $\frac{-11}{3}$. Find $f'(13$