5.5 Substitution Rule

MATH 205

Ok, what do I do with this?

$$\int \left(\sqrt[3]{x^3 + 2x - 7}\right)\left(3x^2 + 2\right)dx$$

- Well, if I could simplify or replace the integrand, it may be easier to find an antiderivative.

Substitution Rule

- If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

- We are substituting u for $g(x)$ and du for $g'(x)dx$.
- Closest thing we have to an Integral chain rule
- Must re-substitute back to a function in terms of x at the end.

Determine the Indefinite Integral

1. $\int x^2\sqrt{x^3-4}\,dx$

Determine the Indefinite Integral

2. $\int \frac{1}{(5x-4)^3}\,dx$

Determine the Indefinite Integral

3. $\int \left(\sin^4\theta \cos\theta\right) d\theta$

Determine the Indefinite Integral

4. $\int \sec^2(6x)\,dx$

Determine the Indefinite Integral

5. $\int x^{2/3} \cos\left(8x^{5/3} + 13\right) dx$

Determine the Indefinite Integral

6. $\int 2\sin x \cos x dx \;\; \{3\text{ ways}\}$

Determine the Indefinite Integral (may be more difficult, maybe not)

7. $\int x\sqrt{6x-12}\,dx$

Determine the Indefinite Integral (may be more difficult, maybe not)

8. $\int \frac{4x^5 - 6x^3 + 7x + 9}{x^2} dx$

Determine the Indefinite Integral (may be more difficult, maybe not)

9. $\int \sin^2 x dx$

Determine the Indefinite Integral
(may be more difficult, maybe not)

10. $\int \tan x dx$

What about Definite Integrals

- If g' is continuous on $[a, b]$ and f is continuous on the range of $g(x) = u$, then

$$\int_a^b f(g(x)) \cdot g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

- As with indefinites, we try to find a substitution to make finding the antiderivative easier.
- We can also change the limits of integration in terms of u so we don't have to substitute back at the end.

Evaluate the Definite Integral

11. $\int_{0}^{\ln 4} e^{x}\sqrt{e^{x}}\,dx$

Evaluate the Definite Integral

12. $\displaystyle\int_{-1}^{1} \frac{4x}{10x^2 - 11}\,dx$

Evaluate the Definite Integral

13. $\displaystyle\int_{0}^{2} \frac{\sin x}{(1+\cos x)^{5}}\,dx$

Evaluate the Definite Integral

14. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 4\csc^2\theta\cot\theta\,d\theta$

Evaluate the Definite Integral

15. $\int_{0}^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta \, d\theta$

Evaluate the Definite Integral

16. $\displaystyle\int_{-2}^{12} \frac{x^2 - 6x + 9}{(x-3)^2}\,dx$

Evaluate the Definite Integral

17. $\displaystyle\int_0^1 \frac{5x+6}{x^2+1}\,dx$

Solve the following Initial Value Problem (Differential Equation)

- 18. $\frac{ds}{dt} = 8t(3t^2 - 4)^4 \quad s(1) = 10$

Solve the following Initial Value Problem (Differential Equation)

19. $\dfrac{d\theta}{dt} = 2\cos t \sin^2(t) \quad \theta\left(\dfrac{\pi}{3}\right) = 0$