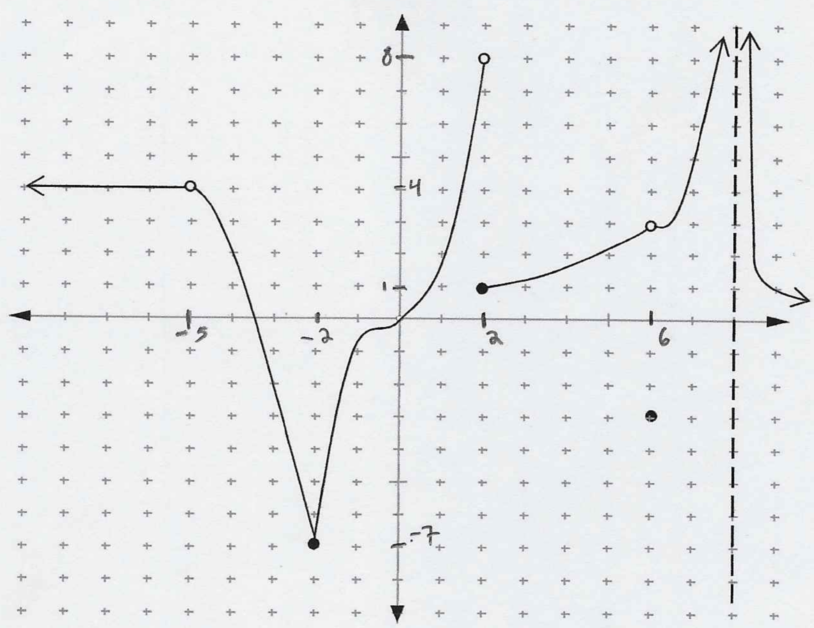
MATH 205 – Calculus I **Exam #1** **Week #4**

Name: ________________________________ Date: _______

Instructions: Though calculators can be used for the entire exam, all problems require you to show your work. Any answer without proper justification will receive ***ZERO*** credit. Only **EXACT** answers will receive full credit unless otherwise noted. Proper Interval and Limit Notation must be used to receive credit. Each question is worth 5 points.

1. Fill out the following table for the function f shown below:



c	$f(c)$	$\lim_{x\to c^-} f(x)$	$\lim_{x\to c^+} f(x)$	$\lim_{x\to c} f(x)$	Is $f(x)$ continuous at $x = c$?
-5	UNDEFINED	4	4	4	NO
-2	-7	-7	-7	-7	Yes
2	1	8	1	DNE	NO
6	-3	3	3	3	NO
8	UNDEFINED	$-\infty$	∞	∞	NO

2. Find the average rate of change of $f(x) = \cos(x) + x$ on $\left[0, \frac{\pi}{2}\right]$.

$$AROC = \frac{[\cos(\pi/2) + \pi/2] - [\cos(0) + 0]}{\pi/2 - 0} = \frac{\pi/2 - 1}{\pi/2} = 1 - \frac{2}{\pi}$$

3. Create a table of at least 6 entries to determine $\lim_{x\to 0} \frac{8\tan(x)}{\sin(x)}$. {Round each entry to six decimal places.}

x	$\frac{8\tan(x)}{\sin(x)}$
-.1	8.040167
-.01	8.000400
-.001	8.000004
0	—
.001	8.000004
.01	8.0004
.1	8.040167

$\lim_{x\to 0} \frac{8\tan(x)}{\sin(x)} =$ 8

Determine the following limits and justify your answers using the properties of limits and algebraic means. Do not rely on tables or graphs.

4. $\lim_{x\to 3}(8x^2 + 7x - 5) = 88$

Direct Substitution

5. $\lim_{x\to 4^-} \frac{x+3}{x^2-16} = -\infty$

$\frac{+}{\text{Smaller } -}$

6. $\lim_{x\to \frac{7\pi}{6}} (sec^2 x - tan^2 x)$

Trig Identity $\Rightarrow$ Constant

$\lim_{x\to 7\pi/6} (1) = 1$

7. $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$ $\quad \frac{0}{0}$ use Radical Conjugates

$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{(\sqrt{x+3}+2)}{(\sqrt{x+3}+2)}$

$\lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4} = \lim_{x\to 1} (\sqrt{x+3}+2)$

$= 4$

8. $\lim_{x\to -\infty} \frac{4\sin(x)}{e^{4x}} = DNE$

For $x<0$, $\frac{1}{e^{4x}} \Rightarrow e^{4x}$

$\therefore$ $[-4, 4] \cdot$ larger & larger pos.

$\therefore$ oscillates btwn large Neg & Pos Numbers

9. $\lim_{x\to 1} \begin{cases} \frac{x^2-3x+4}{x-1}, & x \neq 1 \\ 5, & x = 1 \end{cases}$ $\frac{\ }{0}$ $\leftarrow$ Irrelevant $= DNE$

$\lim_{x\to 1^-} \frac{x^2-3x+4}{x-1} = \frac{+}{\text{Small} -} = -\infty$

$\lim_{x\to 1^+} \frac{x^2-3x+4}{x-1} = \frac{+}{\text{Small}} = \infty$

10. Assume $\lim_{x\to b} f(x) = -1$ and $\lim_{x\to b} g(x) = 12$, determine $\lim_{x\to b}\left[\frac{2g(x)}{f(x)-5}\right]^{\frac{3}{2}}$

$$\left[\lim_{x\to b}\frac{2g(x)}{f(x)-5}\right]^{3/2} = \left[\frac{2\lim_{x\to b} g(x)}{\lim_{x\to b} f(x) - \lim_{x\to b} 5}\right]^{3/2} = \left[\frac{2(12)}{-1-5}\right]^{3/2} = [-4]^{3/2}$$

$[-4]^{3/2}$ is an imaginary #, $\therefore$ $\lim_{x\to b}\left[\frac{2g(x)}{f(x)-5}\right]^{3/2}$ DNE

11. Use the Intermediate Value Theorem to show $\frac{\sqrt{x^3+1}}{x^2+3} = \frac{5}{12}$ has a solution on [0, 3]

let $f(x) = \sqrt{x^3+1}$ &
$g(x) = x^2+3$

both $f(x)$ & $g(x)$ are cts on $[0,3]$ and $g(x) \neq 0$ on $[0,3]$

$\therefore \frac{f(x)}{g(x)} = \frac{\sqrt{x^3+1}}{x^2+3}$ is cts on $[0,3]$

$x=0$: $\frac{\sqrt{0^3+1}}{0^2+3} = 1/3 = 4/12$

$x=3$: $\frac{\sqrt{27+1}}{9+3} = \frac{\sqrt{28}}{12}$

$1/3 < 5/12 < \frac{\sqrt{28}}{12}$

$\therefore$ By the IVT, $\frac{\sqrt{x^3+1}}{x^2+3} = 5/12$ has at least one solution on $[0, 3]$

Use $k(x) = \frac{x^2+2x-15}{2x^2+x-21}$ for question 12 – 13:

12. Determine all the asymptotes (vertical and horizontal), if any, of $k(x)$. Justify the type of asymptote using the concept of limits.

$$k(x) = \frac{(x-3)(x+5)}{(2x+7)(x-3)} = \frac{x+5}{2x+7}$$

Horizontal Asy: $\lim_{x\to\infty} k(x) = 1/2 = \lim_{x\to-\infty} k(x)$ $\therefore$ $y = 1/2$ is the horizontal Asymptote

Vertical Asy: $\lim_{x\to -7/2^+} k(x) = \infty$, $\lim_{x\to 3^+} k(x) = \frac{8}{13} = \lim_{x\to 3^-} k(x)$ $\therefore$ $x = -7/2$ is a vertical Asymptote

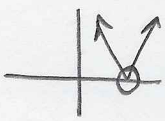
$\frac{+}{\text{small } +}$

13. Determine a piecewise function, $g(x)$, that "fixes" any **removable** discontinuities in $k(x)$.

$$g(x) = \begin{cases} k(x), & x \neq 3 \\ 8/13, & x = 3 \end{cases}$$

14. There are three instances where **differentiability** of a function (at a point) can fail. List and give an example, either graphically or algebraically, of each of these instances.

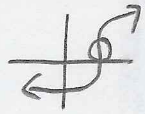
① Corner/Cusp



$f(x) = |x|$

at $x = 0$

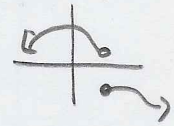
② Vertical tangent



$f(x) = x^{2/3}$

at $x = 0$

③ any pt of discontinuity



$f(x) = \frac{x^2}{x}$

at $x = 0$

15. Use the limit of the difference quotient to determine the instantaneous rate of change of $f(x) = \frac{4}{x-6}$ at $x = 15$.

$$\lim_{h \to 0} \frac{\frac{4}{(15+h)-6} - \frac{4}{(15-6)}}{h} = \lim_{h \to 0} \frac{\frac{4}{h+9} - \frac{4}{9}}{h} = \lim_{h \to 0} \frac{36 - 4(h+9)}{h(h+9)(9)}$$

$$= \lim_{h \to 0} \frac{-4h}{h(h+9)(9)} = \lim_{h \to 0} \frac{-4}{(h+9)(9)} = \frac{-4}{81}$$

Use $f(x) = 3x^2 - 5x + 7$ for questions 16 – 18:

16. Determine the slope of tangent line to $f(x)$ at $x = -4$ by using the limit of the difference quotient.

$$m_{tan} = \lim_{h \to 0} \frac{[3(x+h)^2 - 5(x+h) + 7] - [3x^2 - 5x + 7]}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5x - 5h + 7 - 3x^2 + 5x - 7}{h} = \lim_{h \to 0} \frac{6xh + 3h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{h(6x - 5 + 3h)}{h} = \lim_{h \to 0} (6x-5) = 6x - 5$$

at $x = -4$:

$m_{tan} = 6(-4) - 5 = -29$

17. Determine the equation of the tangent line to the graph of $f(x)$ at $x = -4$.

Slope: $m_{tan} = -29$

pt: $(-4, f(-4)) = (-4, 75)$

$y - 75 = -29(x+4)$

sigh

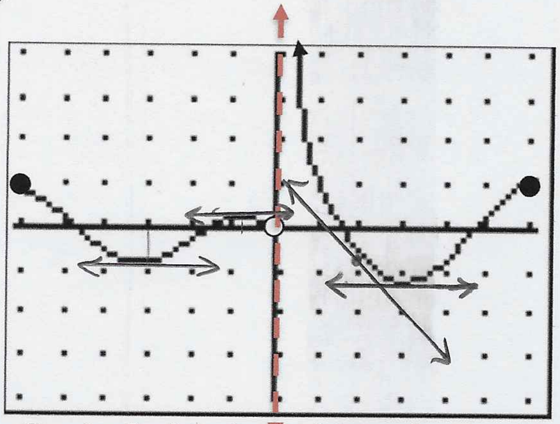
$y = -29x - 41$

18. Where will $f'(x) = 13$

$13 = 6x - 5$

$x = 3$

19. Use the graph of $g(x)$, whose domain is $[-6,0) \cup (0,6]$, to determine each of the following:



Graph of $g(x)$ {each tick mark = 1 unit}

a) Estimate $g'(2)$. $m_{tan} \approx -1$

b) Where on (-6, 6) will $g(x)$ have horizontal tangent lines?

at $x = -3, -1/2, 3$

c) What type of discontinuity does $g(x)$ have at $x = 0$? Asymptotic

20. For each of the following, circle the correct answer: (T = True, F = False). If false, provide justification.

T (F): Differentiability at a point on $f(x)$ **DOES NOT** guarantee continuity at the same point on $f(x)$.

Differentiability Does guarantee Continuity. Continuity does not guarantee Differentiability

(T) F : The derivative at a point of $f(x)$ is the same as the slope of the **tangent** line that passes through said point on the graph of $f(x)$.

T (F): If $\lim_{x \to 3^-} f(x) = 7$ and $\lim_{x \to 3^+} f(x) = -2$, then $f(x)$ has a hole at $x = 3$.

Jump Discontinuity

T (F): The limit of the difference quotient is used when determining an average rate of change. USED For IROC

(T) F : Zero is an even number.