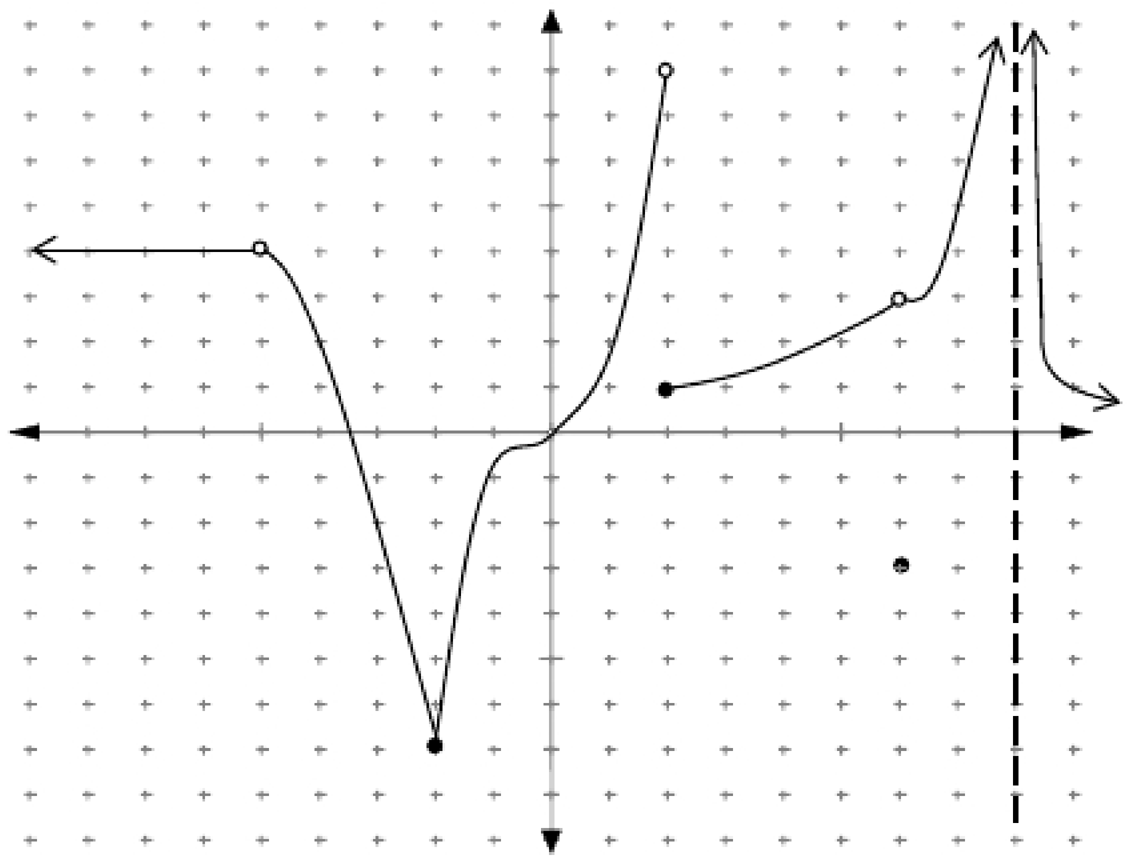
MATH 205 – Calculus I **Exam #1** **Week #4**

Name: Nathan Hallam Date: 9/21/2020

Instructions: Though calculators can be used for the entire exam, all problems require you to show your work. Any answer without proper justification will receive ***ZERO*** credit. Only **EXACT** answers will receive full credit unless otherwise noted. Proper Interval and Limit Notation must be used to receive credit. Each question is worth 5 points.

1. Fill out the following table for the function f shown below:



c	$f(c)$	$\lim_{x\to c^-} f(x)$	$\lim_{x\to c^+} f(x)$	$\lim_{x\to c} f(x)$	Is $f(x)$ continuous at $x = c$?
-5	undefined	4	4	4	no
-2	-7	-7	-7	-7	yes
2	1	8	1	DNE	no
6	-3	3	3	3	no
8	undefined	∞	∞	∞	no

2. Find the average rate of change of $f(x) = \cos(x) + x$ on $\left[0, \frac{\pi}{2}\right]$.

$$\frac{f(b)-f(a)}{b-a} \qquad \frac{\left(\cos\frac{\pi}{2} + \frac{\pi}{2}\right) - \cos(0) + 0}{\left(\pi/2\right)} = \frac{\pi/2 - 1}{\pi/2} = 0.3634$$

3. Create a table of at least 6 entries to determine $\lim_{x\to0}\frac{8\tan(x)}{\sin(x)}$. {Round each entry to six decimal places.}

x	$\frac{8\tan(x)}{\sin(x)}$
-.1	8.040167
-.01	8.000400
-.001	8.000004
0	undef
.0001	8.000000
.001	8.000004
.01	8.000400
.1	8.040167

$\lim_{x\to0}\frac{8\tan(x)}{\sin(x)} =$ 8

Determine the following limits and justify your answers using the properties of limits and algebraic means. Do not rely on tables or graphs.

4. $\lim_{x\to3}(8x^2+7x-5)$

$8[\lim_{x\to3} x]^2 + 7\lim_{x\to3} x - \lim_{x\to3} 5$

$8(3)^2+7(3)-5$

$8(9)+21-5$

$72+16$

88

$\boxed{\lim_{x\to3} f(x)=88}$

5. $\lim_{x\to4^-}\frac{x+3}{x^2-16}$

$\frac{\lim_{x\to4} x+\lim_{x\to4} 3}{-[\lim_{x\to4} x]^2-\lim_{x\to4} 16} = \frac{4+3}{-(4)^2-16} = \frac{7}{-32}$

$\boxed{\lim_{x\to4} f(x) = -\frac{7}{32}}$

6. $\lim_{x\to\frac{7\pi}{6}}(sec^2 x - tan^2 x)$

$\sec^2(\lim_{x\to\frac{7\pi}{6}} x) - \tan^2(\lim_{x\to\frac{7\pi}{6}} x)$

$\lim_{x\to\frac{7\pi}{6}} 1 + \cancel{\tan^2(\lim_{x\to\frac{7\pi}{6}} x)} - \cancel{\tan^2(\lim_{x\to\frac{7\pi}{6}} x)}$

$\boxed{\lim_{x\to\frac{7\pi}{6}} f(x)=1}$

7. $\lim_{x\to1}\frac{x-1}{\sqrt{x+3}-2}$

$\frac{\lim_{x\to1} x-\lim_{x\to1} 1}{[\lim_{x\to1} x+\lim_{x\to1} 3]^{1/2}-\lim_{x\to1} 2}$

$\frac{(\cancel{\lim_{x\to1} x-\lim_{x\to1} 1})([\lim_{x\to1} x+\lim_{x\to1} 3]^{1/2}+\lim_{x\to1} 2)}{(\cancel{\lim_{x\to1} x-\lim_{x\to1} 1})}$

$[\lim_{x\to1} x+\lim_{x\to1} 3]^{1/2}+\lim_{x\to1} 2$

$\sqrt{(1)+3}+2$

$2+2$

4

$\boxed{\lim_{x\to1} f(x)=4}$

8. $\lim_{x\to-\infty}\frac{4\sin(x)}{e^{4x}}$

$\frac{4\sin(\lim_{x\to-\infty} x)}{[\lim_{x\to-\infty} e]^{4\lim_{x\to-\infty} x}}$

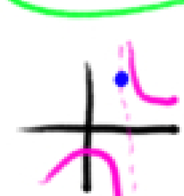
$\frac{4\cdot}{\text{really small \#}} =$

$\boxed{\lim_{x\to-\infty} f(x)=DNE}$

oscilates

9. $\lim_{x\to1}\begin{cases}\frac{x^2-3x+4}{x-1}, & x\neq1\\ 5, & x=1\end{cases}$

$\boxed{\lim_{x\to1} f(x) = DNE}$



$\frac{[\lim_{x\to1^-} x]^2-\lim_{x\to1^-}3x+\lim_{x\to1^-}4}{\lim_{x\to1^-} x-\lim_{x\to1^-}1}$

$\frac{(0.9)^2-3(0.9)+4}{0.9-1}$

$\lim_{x\to1^-} f(x) = -\infty$

$\frac{[\lim_{x\to1^+} x]^2-3\lim_{x\to1^+} x+\lim_{x\to1^+}4}{\lim_{x\to1^+} x-\lim_{x\to1^+}1}$

$\frac{1.1^2-3.3+4}{1.1-1}$

$\lim_{x\to1^+} f(x)=\infty$

10. Assume $\lim_{x\to b} f(x) = -1$ and $\lim_{x\to b} g(x) = 12$, determine $\lim_{x\to b}\left[\frac{2g(x)}{f(x)-5}\right]^{\frac{3}{2}} = h(x)$

$\lim_{x\to b^-}\left[\frac{2(12)}{-1-5}\right]^{3/2}$ $\lim_{x\to b}\left[\frac{24}{-6}\right]^{3/2}$ $\lim_{x\to b}\left[-4\right]^{3/2}$ $\lim_{x\to b} h(x) = \text{undef}$

11. Use the Intermediate Value Theorem to show $\frac{\sqrt{x^3+1}}{x^2+3} = \frac{5}{12}$ has a solution on [0, 3]

$\frac{\sqrt{0^3+1}}{0^2+3} = \frac{1}{3}$ $\frac{\sqrt{3^3+1}}{3^2+3} = \frac{\sqrt{28}}{12}$

$f(a) < f(c) < f(b)$

$1/3 < 5/12 < \frac{\sqrt{28}}{12}$

$0, 1/3$

$3, \sqrt{28}/12$

Using the IVT - There exists at least one point C at $y = 5/12$ between the points $(0, 1/3)$ and $(3, \sqrt{28}/12)$ on the interval $[0,3]$

Use $k(x) = \frac{x^2+2x-15}{2x^2+x-21}$ for question 12 – 13:

12. Determine all the asymptotes (vertical and horizontal), if any, of $k(x)$. Justify the type of asymptote using the concept of limits.

$\frac{x^2+2x-15}{2x^2+x-21} = \frac{1}{2}$

$HA = y = 1/2$

$\frac{(x+5)(x-3)}{(2x+7)(x-3)}$

$\frac{x+5}{2x+7}$

$2x+7=0$

$2x=-7$

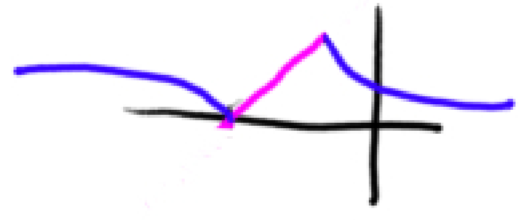
$VA = x = -7/2$

$\lim_{x\to -7/2^-} \frac{x+5}{2x+7}$ $\frac{-3.6+5}{-7.1+7} = \frac{+}{-}$ $\lim_{x\to -7/2} k(x) = -\infty$

$\lim_{x\to -7/2^+} \frac{x+5}{2x+7}$ $\frac{-3.4+5}{-6.9+7} = \frac{+}{+}$ $\lim_{x\to -7/2} k(x) = \infty$

13. Determine a piecewise function, $g(x)$, that "fixes" any **removable** discontinuities in $k(x)$.

$$k(x) = \begin{cases} \frac{x^2+2x-15}{2x^2+x-21} & (-\infty,-5), (-3,\infty) \\ x+5 & [-5,-3] \end{cases}$$



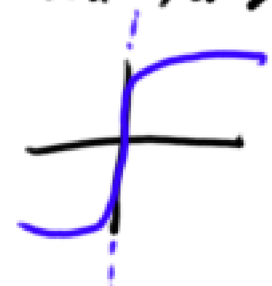
14. There are three instances where **differentiability** of a function (at a point) can fail. List and give an example, either graphically or algebraically, of each of these instances.

cusp

$f(x) = |x|$

discontinuity
- asymptote
- hole
- jump
- oscillating

vertical tangent



15. Use the limit of the difference quotient to determine the instantaneous rate of change of $f(x) = \frac{4}{x-6}$ at $x = 15$.

$$\frac{f(x+h)-f(x)}{h}$$

$$(x+h-6)(x-6)$$
$$x^2-6x+xh-6h-6x+36$$

$$\lim_{h\to 0} \frac{\frac{4}{x+h-6} - \left(\frac{4}{x-6}\right)}{h} = \lim_{h\to 0} \frac{\frac{\cancel{4x}-\cancel{24}-\cancel{4x}-4h+\cancel{24}}{x^2+xh-6h-12x+36}}{h} = \lim_{h\to 0} \frac{\frac{-4\cancel{h}}{x^2+\cancel{xh}-\cancel{6h}-12x+36}}{\cancel{h}} = \frac{-4}{x^2-12x+36}$$

$$\boxed{\frac{-4}{81}} = \frac{-4}{15^2-12(15)+36}$$

Use $f(x) = 3x^2 - 5x + 7$ for questions 16 – 18:

16. Determine the slope of tangent line to $f(x)$ at $x = -4$ by using the limit of the difference quotient.

$$\lim_{h\to 0} = \frac{3(x+h)^2-5(x+h)+7-(3x^2-5x+7)}{h}$$

$$\lim_{h\to 0} = \frac{\cancel{3x^2}+6xh+3h^2-\cancel{5x}-5h+\cancel{7}-\cancel{3x^2}+\cancel{5x}-\cancel{7}}{h}$$

$$\lim_{h\to 0} \frac{6x+3\cancel{h}^2-5\cancel{h}}{\cancel{h}} = f'(x) = 6x-5$$

$f'(-4) = 6(-4)-5$

$f'(-4) = -24-5$

$\boxed{f'(-4) = -29}$

17. Determine the equation of the tangent line to the graph of $f(x)$ at $x = -4$.

$3(-4)^2-5(-4)+7$
$48+20+7$
$68+7$
75

$(-4, 75)$

$\boxed{y-75 = -29(x+4)}$

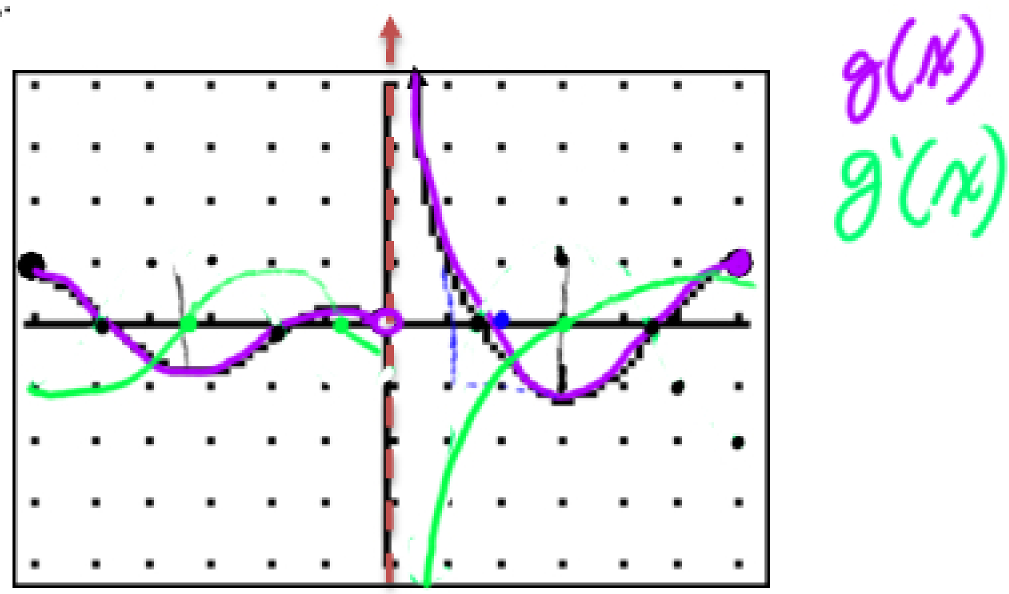
18. Where will $f'(x) = 13$

$f'(13) = 6x-5$
$13 = 6x-5$
$18 = 6x$
$3 = x$

$f'(13) = 6(3)-5$

$\boxed{\text{f'(x) will equal 13 when x equals 3.}}$

19. Use the graph of $g(x)$, whose domain is $[-6,0) \cup (0,6]$, to determine each of the following:



Graph of $g(x)$ {each tick mark = 1 unit}

a) Estimate $g'(2)$.

-1

b) Where on (-6, 6) will $g(x)$ have horizontal tangent lines?

y=0

c) What type of discontinuity does $g(x)$ have at $x = 0$?

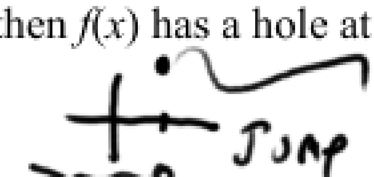
removable from left.
vert. Asymptote from right.

20. For each of the following, circle the correct answer: (T = True, F = False). If false, provide justification.

T F : Differentiability at a point on $f(x)$ **DOES NOT** guarantee continuity at the same point on $f(x)$.

T F : The derivative at a point of $f(x)$ is the same as the slope of the **tangent** line that passes through said point on the graph of $f(x)$.

T F : If $\lim_{x\to 3^-} f(x) = 7$ and $\lim_{x\to 3^+} f(x) = -2$, then $f(x)$ has a hole at $x = 3$.



T F : The limit of the difference quotient is used when determining an average rate of change.

T F : Zero is an even number.