MATH 205 – Calculus I **SQ 5.2**

Name: Nathan Hallan Date: 11/25/2020

To receive any credit for the following problems, you must show complete and accurate work. Use proper limit notation and give exact answers unless otherwise noted.

1. Write the limit of the Riemann sum, $\lim_{\Delta x \to 0} \sum_{k=1}^{n} (\cos^{-1}(x_k^*) + 8(x_k^*)^5)\Delta x_k$, on [-8, 10] as a definite integral.

[a, b]

$$\boxed{\int_{-8}^{10} (\cos^{-1} x + 8x^5)\,dx}$$

2. Determine $\int_0^4 (x^2 + 3x + 5)dx$ using the limit of the Riemann sum, $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x_k$.

$\sum_{k=1}^{n} c = cn$

$\sum_{k=1}^{n} i = \frac{n^2+n}{2}$

$\sum_{k=1}^{n} i^2 = \frac{2n^3+3n^2+n}{6}$

$x_k = a + k\Delta x$

$\frac{4-0}{n} = \frac{4}{n}$

$x_k = \frac{4k}{n}$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(\frac{4k}{n}\right)^2 + 3\left(\frac{4k}{n}\right) + 5\right]\frac{4}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{16k^2}{n^2} + \frac{12k}{n} + 5\right]\frac{4}{n}$$

$$\lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}\sum_{k=1}^{n} k^2 + \frac{12}{n}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 5\right]$$

$$\lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}\left(\frac{2n^3+3n^2+n}{6}\right) + \frac{12}{n}\left(\frac{n^2+n}{2}\right) + 5n\right]$$

$$\lim_{n\to\infty} \left[\frac{64}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right) + \frac{48}{n^2}\left(\frac{n^2+n}{2}\right) + 20\right]$$

$$\lim_{n\to\infty} \left[\frac{64}{6}\left(\frac{2n^3}{n^3} + \frac{3n^2}{n^3} + \frac{n}{n^3}\right) + \frac{48}{2}\left(\frac{n^2}{n^2} + \frac{n}{n^2}\right) + 20\right]$$

$$\lim_{n\to\infty} \frac{32}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + 24\left(1 + \frac{1}{n}\right) + 20$$

$$\lim_{n\to\infty} \frac{64}{3} + 24 + 20$$

$$\frac{64}{3} + \frac{72}{3} + \frac{60}{3} = \boxed{\frac{196}{3}}$$