

4.9 Antiderivatives

Query:

- Suppose that the velocity of an object is modeled by the equation $v(t) = 12t^2 + 10t - 7$, $v(t)$ is measured in feet per second, and t is in seconds.
- If the initial position of the object, at time $t = 0$, is 13 feet, find the equation that models the object's position.
- Find the general equation that models the object's position if the initial position is not known.

Antiderivatives or the act of going backwards.

- When we try to retrieve a function's equation from its derivative, it is called antidifferentiation.
- Definition: A function F is **an** antiderivative of f on an interval I , if $F'(x) = f(x)$ for all x in I .
- Find an antiderivative for:

1. $f(x) = 8x^3$

2. $g(x) = \cos x$

3. $k(x) = e^x + \sin x$

4. $p(x) = \frac{1}{x}$

Family Affairs

- Because the derivative of a constant is zero, adding any arbitrary constant to any antiderivative will result in another antiderivative.
- Every function that has an antiderivative will actually have an infinite number of antiderivatives called a *family*.
- If F is an antiderivative of f on an interval I , then the most general antiderivative of f is $F(x) + C$, where C is an arbitrary constant.

Basic Antiderivative Rules

- Polynomials
- Trigonometric
- Exponential/Logarithmic
- Inverse Trigonometric

Notation

- The Indefinite Integral is the set that contains all the antiderivatives of a function
- The symbol for the indefinite integral of f with respect to x is: $\int f(x)dx$
 - \int is called the integral sign
 - $f(x)$ is the integrand
 - The dx means with respect to the variable x
 - Later, differential dx will mean the change in x

A few more rules

	Function	General
	Antiderivative	
i.	Constant Multiplier:	$kf(x) \quad kF(x) + C$
ii.	Negative Rule:	$-f(x) \quad -F(x) + C$
iii.	Sum/Difference Rule:	$f(x) \pm g(x) \quad F(x) \pm G(x) + C$

Determine the general antiderivative for each of the following

5. $f(x) = x^4 + \sin 5x + 2$

6. $g(x) = \frac{7}{\sqrt[3]{x}} - \csc x \cot x$

7. $k(x) = \frac{1}{x} + 8x^7$

Determine each of the following:

8. $\int (x+10)dx$

9. $\int x^{\frac{-4}{5}} dx$

Determine each of the following:

10. $\int \frac{w^7 + \sqrt{w}}{5} dw$

11. $\int 8 \cos(3\theta) d\theta$

Determine each of the following:

12. $\int \cos y (\tan y + \sec y) dy$

13. $\int (1 + \cot^2 \theta) d\theta$

Determine each of the following:

14. $\int \frac{1}{\sqrt{25-x^2}} dx$

15. $\int (8^x - 5x^3 + \tan(4x) \sec(4x)) dx$

Initial Value Problems and Differential Equations

- An equation that involves the differentiation of an unknown variable is called a differential equation, i.e.
 $dy/dx = \cos x$
- Some differential equations can be solved by the method of separation of variables.
- As the name implies, separate the derivatives and antidifferentiate both sides of the equation. $dy = \cos x \, dx$
$$y = \sin x + C$$
- To find the value of C , we must be given some initial value!

Problems

16. Find an antiderivative of $f(x) = \sec^2 x$ that satisfies $(-\pi/4, 7)$.
17. Find the curve whose slope at (x, y) is $4x^3 - 6x^2$ and is required to pass through the point $(2, 5)$.

Problems

18. During the Pittsfield Balloon rally, a hot air balloon ascending at 12 ft/sec is at a height of 80 ft above the ground when a sand bag is cut loose and falls. How long does it take for the sandbag to reach the ground?