3.2 The Derivative Function

MATH 205

Derivative at a point versus the Derivative Function

- But suppose, instead of looking at a single point, we wish to find a rule to determine the instantaneous rate of change anywhere on a given function.

- If such a rule exists, we would have a function whose outputs would be the original functions IROCs for any input.

- The derivative of a function $f(x)$ with respect to the variable x is the function $f'(x)$ whose value at x is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists.

A few more definitions

- Differentiable at a point:

  If f' exists at $x = a$, then f is differentiable at $x = a$.

- Differentiable (function):

  If f' exists for all x in the domain of f, then f is called differentiable.

- The act of taking a derivative is differentiation.
  - The reverse is called Antidifferentiation.

Notation

There are many ways to write the derivative of a function:

I. $f'(x)$ "f prime of x"

II. y' "y prime"

III. $\frac{dy}{dx}$ "The derivative of y with respect to x"

IV. $\frac{df}{dx}$ "the derivative of f with respect to x"

V. $\frac{d}{dx}f(x)$ "the derivative of $f(x)$"

VI. $D(f)(x)$ "the derivative of $f(x)$"

VII. $D_x f(x)$ "the derivative, with respect to x, of $f(x)$"

Practice

1. $\frac{d}{dx}(x^2 - 3x + 1)$

Practice

2. $y'(x)$ for $y = \frac{2}{x+5}$

Practice

3. $f'(x) = \sqrt{3x+5}$

Practice

4. $f'(0)$ for $f(x) = |x|$

Where does differentiability fail?

- Though a function may be continuous at a point, it might not be differentiable at the point.

$$\lim_{x\to c} f(x) = f(c) \text{ but } \lim_{h\to 0}\frac{f(c+h)-f(c)}{h} \text{ may not exist}$$

- Continuity DOES NOT guarantee differentiability.

Three behaviors that ruin differentiability

I. Corner: $f(x) = |x|$

- Cusp: $g(x) = x^{2/3}$

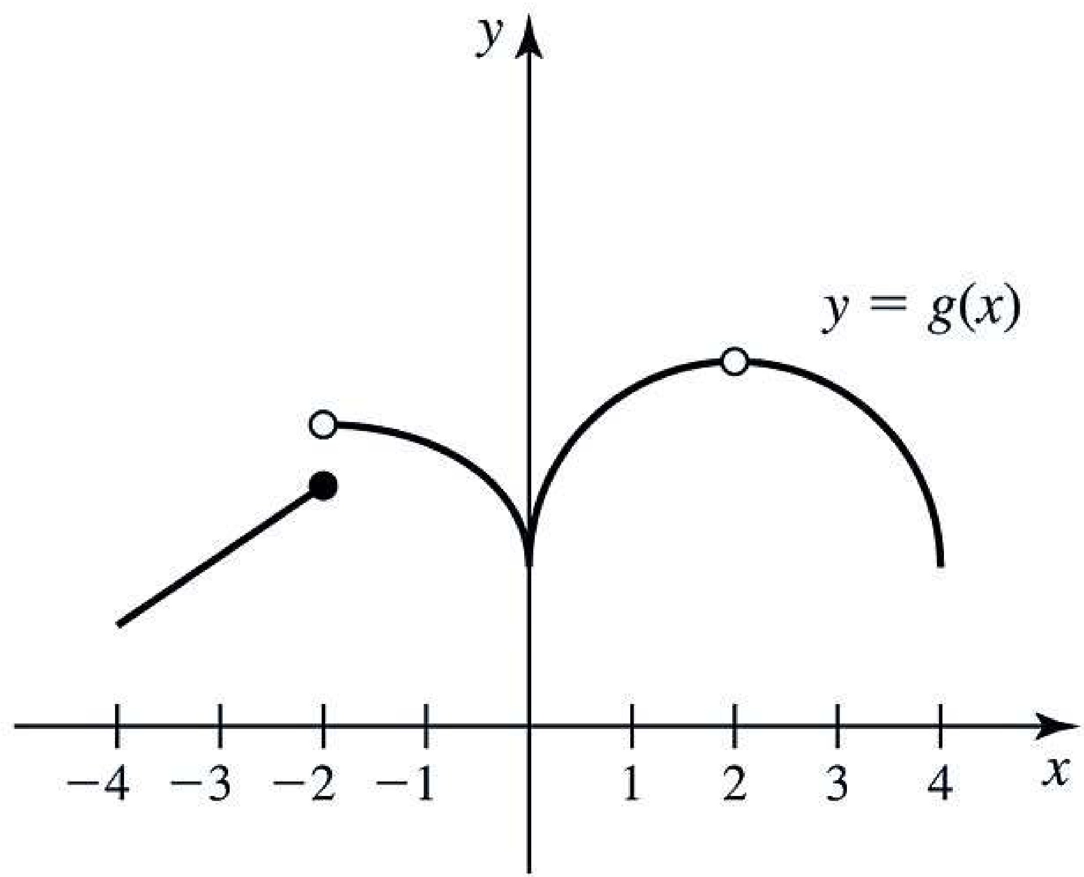
II. Vertical Tangent: $k(x) = \sqrt[3]{x}$

III. Discontinuity: $h(x) = \dfrac{x^2}{x}$

So, what can we conclude?

- Continuity at a point does not guarantee differentiability at that point!
- If a function is differentiable at a point, then it is continuous at that point.
  - A differentiable function is a continuous function
- In terms of functions defined on closed intervals, we can discuss right/left-handed differentiability just as we described right/left-handed continuity.

Graphically



Derivative Plotter