

# 3.11 Related Rates

MATH 205

# Problem 1 (of Many)

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- A bicycle 4 miles east of an intersection is traveling at 9 mph towards the intersection. At the same time, a second bicycle is 3 miles south of the intersection traveling at 10 mph away from the intersection. Is the distance between the bicycles increasing or decreasing at that moment and at what rate?

# Strategy for Related Rates Problems

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- I. Draw a picture and define all variables and constants
- II. Identify what it is you are trying to determine.
- III. Identify the values you currently have
  - I. The numerical values of the items in step 1.
- IV. Create an equation that relates the different rates.
- V. Differentiate step 4 with respect to the independent variable.
  - I. This will quite often be time.
- VI. Solve for the item you are trying to determine.
- VII. Interpret your results.

## Problem 2

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- Sand is falling in a conical pile at the rate of 12 cubic feet/minute. The half of the vertical cross-section of the pile is always an isosceles right triangle.
  1. How fast is the height of the pile increasing when the height is 4 feet?
  2. How fast is the lateral surface area of the pile increasing when the height is 4 feet?

# Problem 3

- Two sticks, each 3 feet long, are hinged together and are leaning against a wall so that the sticks and the floor form an isosceles triangle. The sticks slide apart and at the moment the triangle is equilateral, the angle at the hinge is increasing at the rate of  $\frac{1}{3}$  radian/second.
  1. At what rate is the area of the triangle changing at that moment?
  2. How fast is the length of the third side changing at that moment?

## Problem 4

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- At a certain moment during the compression stroke in a diesel engine, one of the cylinders contains 50 cubic inches of gas vapor under a pressure of 400 pounds/square inch and the volume is decreasing at the rate of 100 cubic inches/second. The volume  $v$  and the pressure  $p$  are related by  $pv^{1.4} = \text{constant}$ . At what rate is the pressure increasing at that moment?

## Problem 5

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- A baseball diamond is a square whose sides are 90 feet. Suppose that a player is running from second base to third base at 30 ft/sec when he is 20 feet from the third. At what rate is the player's distance from home plate changing at that instant?

## Problem 6

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- Suppose a camera, 3000 feet from the base of a rocket launch pad, is covering the takeoff of a new rocket. If the rocket is rising vertically at the 880 ft/sec when it is 4000 feet above the launch pad, how fast must the camera's angle of elevation change at that instant to keep the rocket in view?



# Problem 7

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- Suppose a MLB base runner has a 6-foot lead off first base. The pitcher winds ups and delivers a 98mph fastball for a perfect strike right down the middle of the plate. How fast is the distance between the runner and the ball changing when the ball is half-way to the plate?

## Problem 8

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- Suppose that a 40 foot high conical water tank, with a radius of 18 feet at the top, is being drained at the rate of 300 gallons per minute. How fast is the height of the water in the tank changing when the radius is 11 feet?
- {1 gallon  $\approx$  0.1339 cubic feet}

## Problem 9

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- A spherical balloon is to be deflated so that its radius decreases at a constant rate of  $0.15$  cm/min. At what rate must the air be removed when the radius is  $9$  cm?

## Problem 10

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- As the sun sets behind a 120-ft building, the building's shadow grows. How fast is the shadow growing (in feet per second) when the sun's rays make an angle of  $45^\circ$  with the ground?