

# 3.4 Product and Quotient Rules

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MATH 205



# Derivative Product Rule

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- If  $f(x)$  and  $g(x)$  are differentiable at  $x$ , then  $(f \bullet g)(x)$  is differentiable.

$$\frac{d}{dx}(f \bullet g)(x) = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x) = f(x)g'(x) + g(x)f'(x)$$

- Product Rule: first \* derivative of the second + the second \* the derivative of the first.

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1.  $\frac{d}{dx}(7x^3-6x+10)(8x^5+6x^4-17x^2)$

2.  $\frac{d}{dx}x^4e^x$



# Derivative Quotient Rule

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- If  $f(x)$  and  $g(x)$  are differentiable at  $x$ , and  $g(x) \neq 0$ , then  $(f/g)(x)$  is differentiable at  $x$ ,

$$\frac{d}{dx} \left( \frac{f}{g} \right) (x) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{(g(x))^2} = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$$

- Low D-High minus High D-Low over Low Squared



## Determine the following derivatives

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3.  $\frac{d}{dx} \frac{7x^5 - 10x^3 + 18}{-3x^4 + 7x}$

4.  $\frac{d}{dx} \frac{5x - 8}{9x + e^x}$

# Power Rule for Negative Integers

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- Time to show  $\frac{d}{dx} x^{-n} = -n x^{-n-1}$ , where  $n$  is a natural number.

5.  $\frac{d}{dx} x^{-7}$

6.  $\frac{d}{dx} \frac{8}{x^{12}}$

7.  $\frac{d}{dx} \left( 10x^4 - 12x^3 + 6x - \frac{2}{x^5} \right)$



# Combinations

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8. Determine  $g'(x)$  if  $g(x) = (7x^3 - 6x^2 + \frac{3}{x^6})(10e^{4x})$

9.  $\frac{d}{dx}e^{3x}$



# Practice

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☐ 10.  $\frac{d^2y}{dx^2} [(8x^3 + 6x^2 - 10x + 17)e^{4x}]$





# More Practice

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11.  $\frac{d^2y}{dx^2} \frac{7x^2 + 3x - 1}{4x - 5}$

# Applications

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12. Determine the equation of the tangent line to  $f(x) = \frac{2x^3 - 3x}{x^2 + 4}$  at  $x = 2$ .

- Starting with a full tank of gas, the distance traveled by a particular car is  $D(g) = 0.05g^2 + 35g$  where  $D$  is measured in miles and  $g$  is the amount of gas consumed in gallons.
13. Determine  $\frac{dD}{dg}$  and explain its meaning.
14. Determine  $D'(0.5)$  and  $D'(10)$ . What do these results tell us?
15. What is the range of the car if it has a 12-gal tank?

# A Bit Abstract

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Given the following table:

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|-----|--------|--------|---------|---------|
| 0   | -1     | 1      | 2       |         |
| -1  | -3     | -1     | -2      |         |
| 2   | 5      | 0      |         | 11      |
|     |        |        |         | -4      |

Determine the derivatives of each of the following

16.  $8f(x) + 4g(x)$  at  $x = 2$

17.  $f(x)g(x)$  at  $x = -1$

18.  $\frac{f(x) + 7}{g(x)}$  at  $x = 0$

19.  $\frac{f(x)e^x + x^2g(x)}{f(x)}$  at  $x = 0$