3.1 Introducing the Derivative

MATH 205

What is a tangent line?

- In regards to a circle, a tangent line is a line that intersects the circle at only one point.

- In regards to a general curve, a tangent line at a point of a graph is the line whose slope is the limit of the slope of the secant line {as the points move closer together} and passes through the given point, provided the limit exists.

The Difference Quotient

- Recall from section 2.1, the average rate of change (slope of the secant line) of a function was given by the formula

$$\frac{f(x_1+h)-f(x_1)}{h}$$

- This is also known as the Difference Quotient

- If we take the limit as h approaches zero, then we can determine the instantaneous rate of change (slope of the tangent line), provided this limit exists.

Rates of Change and the Tangent Line

- The average rate of change in f on the interval $[a, a + h]$ is the slope of the corresponding secant line: $$m_{\sec} = \frac{f(a+h) - f(a)}{h}$$

- The instantaneous rate of change in f at $x = a$ is the slope of the tangent line at $(a, f(a))$ {provided the limit exists}

$$m_{\tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- Alternate Forms: AROC on $[a, x]$: $$m_{\sec} = \frac{f(x) - f(a)}{x - a}$$
  IROC at $x = a$: $$m_{\tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The Tangent Line

- The tangent line, at $x = a$, is the unique line through $(a, f(a))$, with the equation

$$y - f(a) = f'(a)(x - a)$$

- $m_{tan} = f'(a)$

Lets investigate $f(x) = x^2 - 2x + 3$

1. Determine the slope at $x = 4$.
2. Determine the equation of the tangent line at $x = 4$.
3. Determine the slope at $x = a$.
4. Where is the slope equal to 7?
5. Where is the slope equal to 0?

Lets investigate $g(x) = \sqrt{3x+1}$

6. Determine the rate of change of $g(x)$ at $x = 5$.

7. Determine the equation of the tangent line to $g(x)$ at $x = 5$.

8. Determine the rate of change of $g(x)$ at $x = a$.

9. Describe the behavior of the slopes of the tangent lines as a increases.

10. Where does the rate of change equal -1?

What the heck is a Derivative?

a. The instantaneous rate of change of a function at a point.
b. The slope of the function at a point.
c. The slope of the tangent line to a point on the graph of a function.
d. The limit of the difference quotient of f at $x = a$:

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h} \text{ or } \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

- All these mean the same thing: The DERIVATIVE of a function at a point

The Derivative of a function f at $x = a$

- The derivative of a function $f(x)$ at $x = a$, called $f'(a)$ is $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

  provided this limit exists.

- The limit of the difference quotient is the derivative.

  { f' is read as "f prime". This is a variation of Newton's notation.}

Practice

- Determine $f'(3)$ if $f(x) = \frac{4}{x-7}$
- Determine the equation of the tangent line to $f(x) = x^2 - 3x$ at $x = 5$.
- Suppose the chart shows the growth of an insect where $M(t)$ is the mass t weeks after birth. Estimate $M'(1)$ and