4.2 The Mean Value Theorem

MATH 205

Rolle's Theorem

- Named for the French mathematician Michel Rolle, 1652 – 1719, though he was not the first to use it nor to prove it in regards to differential calculus.
  - Earliest documented: Indian mathematician, Bhaskara II, 1114 - 1185
  - Calculus Proof: Augustin-Louis Cauchy, 1789 -1857

is continuous on $[a, b]$ and differentiable on
then there exists at least one number, c, in
$) = 0$

$7x - 12$ has exactly 1 real solution.

The Speeding Ticket.

- A person traveling from New Hampshire to Michigan gets on the New York Throughway at Albany and gets off at Buffalo, a distance of 250 miles. After the toll is paid, the driver is pulled over and given a ticket for speeding on the throughway. How did the toll booth attendant know to have the police ticket the driver?

The Mean Value Theorem

- The MVT:

  If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one point c in (a, b), such that

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

- The average rate of change = the instantaneous rate of change

  - Proof

Find the value(s) of c that satisfy the MVT for:

1. $f(x) = 3x^2 - 6x + 5, \quad [-1, 4]$

Find the value(s) of c that satisfy the MVT for:

2. $g(x) = \sin x, \quad \left[0, 2\pi\right]$

Find the value(s) of c that satisfy the MVT for:

3. $k(x) = 3x + \dfrac{2}{x^2},\ [3, 6]$

Find the value(s) of c that satisfy the MVT for:

4. $p(x) = |x|, \; [-2, 2]$

Consequences of the MVT

- If f is differentiable and $f'(x) = 0$ at all points of an interval I, then f is a constant function on I.

- If $f'(x) = g'(x)$ for all x on an interval I, then $f(x) - g(x) = C$ on I, where C is a constant.
  - Functions with equal derivatives differ by a constant

- If f is continuous on I and differentiable at all interior points of I then
  - If $f'(x) > 0$ at all interior points of I, the $f(x)$ is increasing on I
  - If $f'(x) < 0$ at all interior points of I, the $f(x)$ is decreasing on I