# 3.7 The Chain Rule

MATH 205

# Review

- Find $\frac{d}{dx}(4x+5)^2$ by
  1. Foiling and using sum/difference rule
  2. Using the product rule

- There is a third method for this type of problem and it is called the chain rule!

- Find $\frac{d}{dx}\left(8x^5-6x^3+18x-8\right)^6$

# The Chain Rule

- If $f$ is differentiable at $y = g(x)$ and $g$ is differentiable at $x$, the $(f \circ g)(x) = f(g(x))$ is differentiable at $x$ and

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

or if $y = f(u)$ and $u = g(x)$ then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \text{ where } \frac{dy}{du} \text{ is evaluated at } u = g(x)$$

- The derivative of the outer function in terms of the inner function times the derivative of the inner function.

# Time To Break It Down

- For each of the following:
  - Determine the inner function $u(x)$
  - Determine the outer function $f(u)$
  - Determine $f'(u)$ and $u'(x)$
  - Determine $f'(x)$

1. $f(x) = (4\sin x - 3x^2)^5$

2. $f(x) = \tan(x^2 - 6x + 1)$

# Chain Rule

3. $\frac{d}{d\theta}\tan(\cos\theta)$

# Chain Rule

4. $\dfrac{d}{dx}\dfrac{\sin\left(3x^4-10x\right)}{4x^9-6e^{5x}}$

# Chain Rule

5. $\frac{d}{dx}\sqrt{7x^5 - 6x^3 + 8}$

# Chain Rule

6. $\dfrac{d}{dx}\cot x\left(4x^{7}\sec x\right)^{-10}$

# Chain Rule

7. $\dfrac{d}{d\theta}\left(\sin^2\theta + \cos^2\theta\right)$

# Chain Rule

8. $\frac{d}{dx}\cos\left(\sqrt{\sin\left(5x^3+6x^2-11\right)}\right)$

# Application

9. Determine the equation of the tangent line to $y$ at $x = \frac{\pi}{3}$ if $y = 7\cos^2 x$

# Application

The displacement of a mass on a spring suspended from the ceiling is given by $y = 10e^{\frac{-t}{2}} \cos\left(\frac{\pi t}{8}\right)$

10. Graph the displacement function

11. Determine and graph $v(t)$.