*89*

Instructions:  Though calculators can be used for the entire exam, all problems require you to show your work.   To receive full credit, answers must have proper calculus justification. Only **EXACT** answers will receive full credit unless otherwise noted.
      {Questions 1 – 12: **6** points each, Questions 13 – 22 & 24: **2** points each,
       Questions 23 and 25: **3** points each}

1. Determine $f'(x)$ if $f(x) = 7x^5 - 9x^4 + 15$

$$f'(x) = 35x^4 - 36x^3 \checkmark$$

2. Determine $g'(x)$ if $g(x) = \dfrac{\cos x}{7x^8 - 4x^3}$

$$g'(x) = \frac{(7x^8 - 4x^3)(-\sin x) - (\cos x)(56x^7 - 12x^2)}{[7x^8 - 4x^3]^2} \checkmark$$

3. Determine $k'(x)$ if $k(x) = \sin\left(\sqrt{10x^3 - 8x^{-2}}\right)$

$f'(g(x))g'$

$$k'(x) = \cos\left(\sqrt{10x^3 - 8x^{-2}}\right)\left(\frac{1}{2}(10x^3 - 8x^{-2})^{-1/2}\right)(30x^2 + 16x^{-3})$$

$$k'(x) = \cos\left(\sqrt{10x^3 - 8x^{-2}}\right)\left[\frac{1}{2\sqrt{10x^3 - 8x^{-2}}}\right](30x^2 + 16x^{-3}) \checkmark$$

4. Determine $\frac{d}{dr}(6r^4 - 21r^2 + 3)\cdot\tan r$

$$(6r^4 - 21r^2 + 3)(\sec^2 r) + (\tan r)(24r^3 - 42r) \checkmark$$

o

5. Determine $f'(x)$ if $f(x) = 8^x + \text{arcsec}(x)$

$$e^{x\ln 8}(\ln 8) + \frac{d}{dx}\text{arcsec}(x)$$

$$S'(x) = 8^x[\ln 8] + \frac{1}{|x|\sqrt{x^2-1}}$$ ✓

6. Determine $p'(x)$ if $p(x) = \ln(\sin x)$

$$p'(x) = \frac{1}{\sin(x)}(\cos x)$$

$$p'(x) = \frac{\cos x}{\sin x} \rightarrow p'(x) = \cot x$$ ✓

7. Determine $f'(x)$ if $f(x) = \tan^{-1}(\log_9(x^2 + 6x - 8))$

$$S'(x) = \frac{1}{1 + (\log_9(x^2+6x-8))^2}\left(\frac{1}{\ln(9)(x^2+6x-8)}\right)(2x+6)$$ ✓

$$S'(x) = \left[\frac{1}{1 + [\log_9(x^2+6x-8)]^2}\right]\left(\frac{2x+6}{\ln(9)(x^2+6x-8)}\right)$$

8. $\dfrac{d^{25}y}{dx^{25}} 2e^x$

$$2e^x + e^x(0)$$

$$\frac{dy}{dx} = 2e^x$$

$$\frac{d^{1...25}y}{dx^{1...25}} = 2e^x$$ ✓

9. Determine $f'''(5)$ for $f(x) = 2x^5 + 5x^3 - 3x^2 + 2$

$$S'(x) = 10x^4 + 15x^2 - 6x$$
$$S''(x) = 40x^3 + 36x - 6$$
$$S'''(x) = 120x^2 + 30$$
$$S'''(5) = 120(5)^2 + 30$$
$$S'''(5) = 120(25) + 30$$
$$S'''(5) = 3000 + 30$$
$$S'''(5) = 3030$$ ✓

0

10. Given $\dfrac{d}{dx}\sin x = \cos x$ and $\dfrac{d}{dx}\cos x = -\sin x$, prove the derivative of $f(x) = \sec x$ is

$f'(x) = \sec x \tan x$

$$S(x) = \sec x$$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{d}{dx}\frac{1}{\cos x} = \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x}\left(\frac{1}{\cos x}\right)$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$= \tan x \sec x \quad \checkmark$$

11. Using implicit differentiation and the derivatives of the trigonometric functions, prove the derivative of $g(x) = \cot^{-1} x$ is $g'(x) = \dfrac{-1}{1+x^2}$

$$\text{let } y = \cot^{-1} x$$

$$\cot y = x$$

$$-\csc^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{-1}{\csc^2 y}$$

$$\frac{dy}{dx} = \frac{-1}{1+\cot^2 y} \qquad 1+\cot^2 y = \csc^2 y$$

$$\frac{dy}{dx} = \frac{-1}{1+x^2} \quad \checkmark$$

12. Use logarithmic differentiation to determine $\dfrac{dy}{dx}$ for $y = (\cos x)^{\sin x}$

$(-3)$  WRONG Method

$$\frac{dy}{dx} = e^{\sin x \ln \cos x}\left(\frac{\sin x}{\cos x}(\sin x) + \cos x \ln \cos x\right)$$

$$\frac{dy}{dx} = (\cos x)^{\sin x}\left(\tan(x)(-\sin x) + \cos(x)\ln\cos(x)\right)$$

-3

Suppose a particle moves along a straight line and its position is given by

$$f(x) = \frac{1}{3}x^3 - 7x^2 + 33x + 13 \text{ on } [0, 14]$$

where $f(x)$ is in feet and $x$ is in seconds.

$V(t) = x^2 - 14x + 33$

$a(t) = 2x - 14$

13. Determine when the particle is moving to the left.

left neg
Right pos

The Particle is moving left on the interval $(3, 11)$ ✓
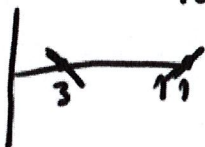
$V(4) = 4^2 - 14(4) + 33$    $V(2) = 4 - 28 + 33$
$16 - 56 + 33$                        $V(2) = 9$
$-40 + 33$
$-7$



14. Determine when the particle changes direction.

$0 = x^2 - 14x + 33$   $0 = \dfrac{+14 \pm \sqrt{14^2 - 4(33)}}{2} = \dfrac{14 \pm \sqrt{196 - 132}}{2} = \dfrac{14 \pm \sqrt{64}}{2}$

$x = 3, x = 11$ ✓      $\dfrac{6}{2}$ & $\dfrac{22}{2}$ ← $\dfrac{14 \pm 8}{2}$

15. Determine the interval(s) on which the particle is slowing down.

$[0, 3) \cup (7, 11)$ ✓

$\dfrac{d}{b}$

16. Determine the particle's average *speed* on [2, 8].
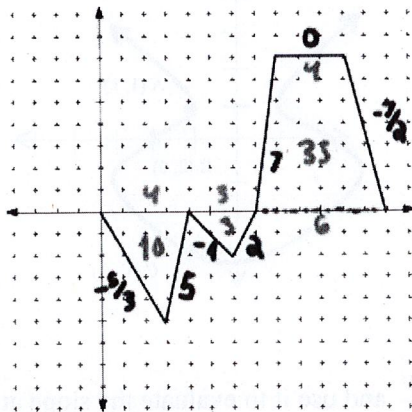
$\dfrac{f(8) - f(2)}{8 - 2} = -9 \text{ ft/s}$

17. Determine the particle's displacement from time $t = 0$ to $t = 14$.

$\dfrac{f(14) - f(6)}{14} = \frac{1}{3} \text{ ft}$

18. Determine the total distance traveled by the particle from time $t = 0$ to $t = 14$.

$\dfrac{|f(14)| + }{|f(6)|} = 4\frac{2}{3} \text{ ft}$

-6

For questions 19 - 22, use the graph below that gives a particle's **VELOCITY** (m/sec) at time *t* (secs). {Scale is 1 tick mark = 1 unit}

This is the VELOCITY GRAPH!

It is not, I repeat, **IS NOT**, the position graph!

$-13 + 35 = 22$ right

19. When is the particle moving to the right?

$(7, 13)$ ✓

20. When is the particle speeding up?

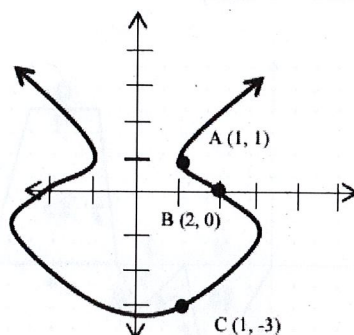$[0, 3) \cup (4, 6) \cup (7, 8)$ ✓

21. When does the particle change directions?

$x = 7$ ✓

22. If the particle starts 10 meters to the left of zero, estimate the particle's ending position?

The particle will be around 12 units right of the origin. ★

−2

For questions 23 and 24, refer to the equation $y^3 + y^2 - 5y - x^2 = -4$ whose graph is shown to the below: {If you cannot determine (23), I will supply it for use in (24)}



A (1, 1)

B (2, 0)

C (1, -3)

23. Find a formula for $\dfrac{dy}{dx}$ and use it to evaluate the slope at point C.

$$3y^2 \frac{dy}{dx} + 2y \frac{dy}{dx} - 5 \frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5} \checkmark$$

24. a) Find an equation of the **tangent** line to the curve at point C.

$$\frac{2}{2 \cdot 2 \cdot 6 - 5} = \frac{2}{16} = \frac{1}{8} \qquad y + 3 = \frac{1}{8}(x - 1) \checkmark$$

b) Find an equation of the **normal** line to the curve at point C.

$$y + 3 = -8(x - 1) \checkmark$$

25. Suppose $\left(f^{-1}\right)'(5) = \dfrac{\sqrt{3}}{\pi}$ at (5, -2), determine $f'(-2)$.

$$\left(f^{-1}\right)'(5) = \frac{1}{f'(-2)} = \frac{\sqrt{3}}{\pi}$$

$$f'(-2) = \frac{\pi}{\sqrt{3}} \checkmark$$

0