MATH 205 – Calculus I **Exam #2** Name: ____________________

Instructions: Though calculators can be used for the entire exam, all problems require you to show your work. To receive full credit, answers must have proper calculus justification. Only **EXACT** answers will receive full credit unless otherwise noted.

{Questions 1 – 12: **6** points each, Questions 13 – 22 & 24: **2** points each, Questions 23 and 25: **3** points each}

1. Determine $f'(x)$ if $f(x) = 7x^5 - 9x^4 + 15$

$f'(x) = 35x^4 - 36x^3$

2. Determine $g'(x)$ if $g(x) = \frac{\cos(x)}{7x^8 - 4x^3}$

$$g'(x) = \frac{(7x^8 - 4x^3)(-\sin x) - \cos x\,(56x^7 - 12x^2)}{(7x^8 - 4x^3)^2}$$

3. Determine $k'(x)$ if $k(x) = \sin(\sqrt{10x^3 - 8x^{-2}})$

$k'(x) = \cos\sqrt{10x^3 - 8x^{-2}}\left(\frac{1}{2\sqrt{10x^3 - 8x^{-2}}}\right)\left(30x^2 + 16x^{-3}\right)$

4. Determine $\frac{d}{dr}(6r^4 - 21r^2 + 3)\tan r$

$= (24r^3 - 42r)\tan r + (6r^4 - 21r^2 + 3)\sec^2 r$

5. Determine $f'(x)$ if $f(x) = 8^x + \text{arcsec}(x)$

$$f'(x) = (\ln 8)\, 8^x + \frac{1}{|x|\sqrt{x^2-1}}$$

6. Determine $p'(x)$ if $p(x) = \ln(\sin x)$

$$p'(x) = \frac{1}{\sin x}(\cos x) = \cot x$$

7. Determine $f'(x)$ if $f(x) = \tan^{-1}(\log_9(x^2+6x-8))$

$$f'(x) = \frac{1}{1+(\log_9(x^2+6x-8))^2}\left(\frac{1}{(\ln 9)(x^2+6x-8)}\right)(2x+6)$$

8. $\frac{d^{25}y}{dx^{25}} 2e^x = 2e^x$

9. Determine $f'''(5)$ for $f(x) = 2x^5 + 5x^3 - 3x^2 + 2$

$$f'(x) = 10x^4 + 15x^2 - 6x$$

$$f''(x) = 40x^3 + 30x - 6$$

$$f'''(x) = 120x^2 + 30$$

$$f'''(5) = 120(5)^2 + 30 = 3030$$

10. Given $\frac{d}{dx}\sin x = \cos x$ and $\frac{d}{dx}\cos x = -\sin x$, prove the derivative of $f(x) = \sec x$ is $f'(x) = \sec x \tan x$

$$d/dx \sec x = d/dx \frac{1}{\cos x} = \frac{\cos x\,(0) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

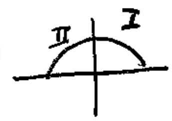
$$= \sec x \tan x$$

11. Using implicit differentiation and the derivatives of the trigonometric functions, prove the derivative of $g(x) = \cot^{-1} x$ is $g'(x) = \frac{-1}{1+x^2}$

$y = \cot^{-1}(x)$

D: $\mathbb{R}$

R: $(0, \pi)$



$1 + \cot^2 y = \csc^2 y$

let $y = \cot^{-1}(x)$

$\cot y = x$

$-\csc^2 y \; dy/dx = 1$

$$dy/dx = \frac{-1}{\csc^2 y} = \frac{-1}{1+\cot^2 y} = \frac{-1}{1+x^2}$$

12. Use logarithmic differentiation to determine $\frac{dy}{dx}$ for $y = (\cos x)^{\sin x}$

let $y = (\cos x)^{\sin x}$

$\ln y = \sin x \ln \cos x$

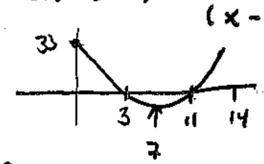
$$\frac{1}{y}\, dy/dx = \sin x \left(\frac{1}{\cos x}\right)(-\sin x) + \cos x \ln x$$

$$dy/dx = \left[-\frac{\sin^2 x}{\cos x} + \cos x \ln x\right](\cos x)^{\sin x}$$

Suppose a particle moves along a straight line and its position is given by

$v(t) = f'(t) = x^2 - 14x + 33$
$(x - 11)(x - 3)$

$$f(x) = \frac{1}{3}x^3 - 7x^2 + 33x + 13 \text{ on } [0, 14]$$



where $f(x)$ is in feet and x is in seconds.

13. Determine when the particle is moving to the left. $v(t) < 0$

$(3, 11)$ sec

14. Determine when the particle changes direction.

at $x = 3, 11$ sec

15. Determine the interval(s) on which the particle is slowing down. $v(t)$ Moving towards x-axis

$[0, 3) \cup (7, 11)$ sec

16. Determine the particle's average *speed* on [2, 8].

Speed = |velocity|

Avg Speed = $\frac{\text{Dist}}{\text{time}}$

distance $(2, 3)$ moving Right: $f(3) - f(2) = 58 - 53\,^2/_3 = 4\,^1/_3$

$(3, 8)$ moving left: $|f(8) - f(3)| = |-^1/_3 - 58| = 58\,^1/_3$

distance $= 62\,^2/_3$ Avg Speed $= \dfrac{62\,^2/_3}{6} = 10\,^4/_9$ ft/sec

17. Determine the particle's displacement from time $t = 0$ to $t = 14$.

$f(14) - f(0) = 17\,^2/_3 - 13 = 4\,^2/_3$ ft

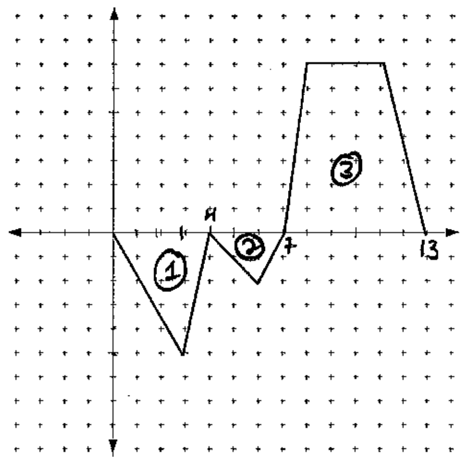
18. Determine the total distance traveled by the particle from time $t = 0$ to $t = 14$.

$[f(3) - f(0)] + |f(11) - f(3)| + [f(14) - f(11)]$

$[58 - 13] + |-27\,^1/_3 - 58| + [17\,^2/_3 - (-27\,^1/_3)]$

$45 + 85\,^1/_3 + 45 = 175\,^1/_3$ ft

For questions 19 - 22, use the graph below that gives a particle's ***VELOCITY*** (m/sec) at time t (secs). {Scale is 1 tick mark = 1 unit}



This is the VELOCITY GRAPH!

It is not, I repeat, **IS NOT**, the position graph!

19. When is the particle moving to the right? $v(t) > 0$

$(7, 13]$

20. When is the particle speeding up? $v(t)$ moving away from x-axis

$[0,3) \cup (4,6) \cup (7,8)$

21. When does the particle change directions? at $t = 7$

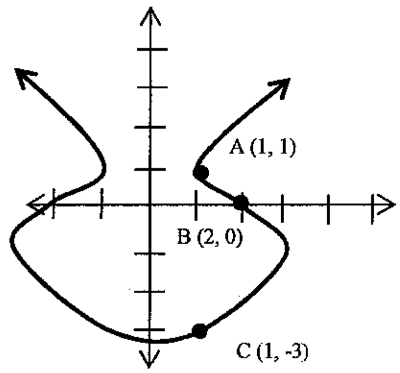
22. If the particle starts 10 meters to the left of zero, estimate the particle's ending position? $A_{\Delta_1} + A_{\Delta_2} + A_{\text{Trapezoid}} + (-10)_{\text{starting point}}$

$\frac{1}{2}(4)(-5) + \frac{1}{2}(3)(-2) + \frac{1}{2}(6+3)(7) + (-10)$

$(-10) + (-3) + (31.5) + (-10) = 8.5$

The particle will end up 8.5 units to the right of zero.

For questions 23 and 24, refer to the equation $y^3 + y^2 - 5y - x^2 = -4$ whose graph is shown to the below: {If you cannot determine (23), I will supply it for use in (24)}



23. Find a formula for $\frac{dy}{dx}$ and use it to evaluate the slope at point C.

$$3y^2\, dy/dx + 2y\, dy/dx - 5\, dy/dx - 2x = 0$$

$$dy/dx = \frac{2x}{3y^2+2y-5}$$

Slope at $(1,-3)$: $\quad m = \frac{2(1)}{3(-3)^2+2(-3)-5} = \frac{2}{16} = \frac{1}{8}$

24. a) Find an equation of the **tangent** line to the curve at point C.

$y+3 = 1/8(x-1)$ $\qquad$ $y = 1/8x - 25/8$

b) Find an equation of the **normal** line to the curve at point C.

$y+3 = -8(x-1)$ $\qquad$ $y = -8x+5$

25. Suppose $(f^{-1})'(5) = \frac{\sqrt{3}}{\pi}$ at (5, -2), determine $f'(-2)$.

$f'(-2) = \pi/\sqrt{3}$