MATH 205 - CALCULUS I **FINAL EXAM** **Fall 2020**

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Instructions: Though calculators can be used for the entire exam, all problems require you to show your work. If you use your calculator, you must still show the appropriate integral, derivative, or antiderivative. To receive full credit, answers must have proper justification. {Each questions is worth 3 points}

1. Determine $\lim\limits_{x\to -5}\dfrac{x^2-4x-45}{x^2+7x+10}$. $\overset{\text{L'Hop}}{\Rightarrow} \lim\limits_{x\to -5}\dfrac{2x-4}{2x+7} = \dfrac{-14}{-3} = \dfrac{14}{3}$

DS: 0/0

$\lim\limits_{x\to -5}\dfrac{(x-9)(x+5)}{(x+2)(x+5)} = \lim\limits_{x\to -5}\dfrac{x-9}{x+2} = \dfrac{-14}{-3} = \dfrac{14}{3}$

2. Use L'Hopital's Rule to determine $\lim\limits_{x\to 0}\dfrac{2x^2-\cos(4x)+1}{8x^2}$ $\overset{\text{L'Hop}}{\Rightarrow} \lim\limits_{x\to 0}\dfrac{4x+4\sin(4x)}{16x} \overset{\text{L'Hop}}{\Rightarrow} \star$

DS: 0/0 DS: 0/0

$\star \Rightarrow \lim\limits_{x\to 0}\dfrac{4+16\cos(4x)}{16} = \dfrac{20}{16} = \dfrac{5}{4}$

3. Using the definition of the derivative, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, determine the first derivative of $f(x)=\sqrt{x+3}$

$\lim\limits_{h\to 0}\dfrac{\sqrt{(x+h)+3}-\sqrt{x+3}}{h}$

4. Determine an equation of the tangent line to $y=\cos(x)+\sin(2x)$ at (π, -1).

$y' = -\sin x + 2\cos(2x)$

$-\sin \pi + 2\cos(2\pi)$

$m = 2$

$y+1 = 2(x-\pi)$

5. Determine $\dfrac{d}{dx}\left(8x^{10}-9\tan x\right)$

$80x^9 - 9\sec^2 x$

6. Determine $g'(x)$ if $g(x) = 8x^4\cos(3x)$

$g'(x) = 32x^3(\cos(3x)) + 8x^4(-3\sin(3x))$

$g'(x) = 32x^3\cos(3x) - 24x^4\sin(3x)$

7. Determine $k'(x)$ if $k(x) = \dfrac{\arctan(x)}{x^2 + \sec(x)}$

$k'(x) = \dfrac{(x^2+\sec(x))\left(\frac{1}{1+x^2}\right) - (\arctan(x))(2x + \sec x\tan x)}{(x^2+\sec(x))^2}$

8. Use Implicit Differentiation to determine $\dfrac{dy}{dx}$ for $4x^2 - 8y^3 + 6x + 10y = 0$

$8x - 24y^2\frac{dy}{dx} + 6 + 10\frac{dy}{dx} = 0$

$\dfrac{-8x-6}{10-24y^2} = \dfrac{dy}{dx}$

9. Determine $\int (6x + \cos x - e^x)dx$

$3x^2 + \sin x - e^x + C$

10. Determine $\int 6x\sec^2(x^2 - 4)dx \Rightarrow \int \sec^2(u)\,3du$

let $u = x^2 - 4$

$du = 2x\,dx$

$3du = 6x\,dx$

$= 3\int \sec^2(u)du$

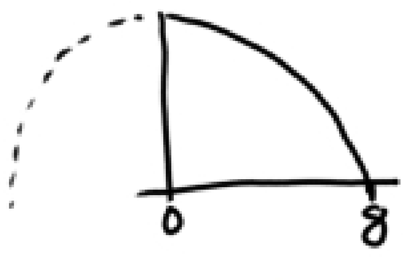
$= 3\tan u + C$

$= 3\tan(x^2-4) + C$

11. Determine $\int_0^8 \sqrt{64 - x^2}\,dx$ [Hint: Use Geometry]

$\dfrac{\pi r^2}{4}$

$= \int_0^8 (64 - x^2)^{1/2}dx$



$\dfrac{64\pi}{4}$

16π

12. Use symmetry to determine $\int_{-1}^{1} (\overset{\text{odd}}{x^5} - \overset{\text{odd}}{2x^3} + \overset{\text{even}}{8x^2})dx$

$\int_{-1}^{1} x^5dx - \int_{-1}^{1} 2x^3dx + 2\int_0^1 8x^2dx$

$= 2\left[\frac{8}{3}x^3\right]_0^1$

$= \frac{16}{3}$

For question 13 – 22, use $f(x) = x^3 - 2x^2 - 15x + 6$

AROC= $\frac{f(b)-f(a)}{b-a}$

13. What is the average rate of change of $f(x)$ over [-3, 4]?

$= \frac{-22-6}{4+3} = \frac{-28}{7} = -4$

$f(-3) = (-3)^3 - 2(-3)^2 - 15(-3) + 6$
$-27 - 18 + 45 + 6$
6

$f(4) = (4)^3 - 2(4)^2 - 15(4) + 6$
$64 - 32 - 60 + 6$
-22

14. What is the instantaneous rate of change of $f(x)$ at $x = 2$?

$f'(x) = 3x^2 - 4x - 15$
$f'(2) = 12 - 8 - 15$
IROC $= -11$

15. Determine $f'''(x)$. $f(x) = x^3 - 2x^2 - 15x + 6$

$f'(x) = 3x^2 - 4x - 15$
$f''(x) = 6x - 4$
$f'''(x) = 6$

16. For what value(s) of x on [-3, 4] will the instantaneous rate of change of $f(x)$ be the same as the average rate of change of $f(x)$ on [-3, 4].

$3x^2 - 4x - 15 = -4$
$3x^2 - 4x - 11 = 0$

$\frac{4 \pm \sqrt{16 - 4(3)(-11)}}{6} = \frac{4 \pm \sqrt{16+132}}{6} = \frac{4 \pm \sqrt{148}}{6} = \frac{4+\sqrt{148}}{6} \approx 2.69 \quad \frac{4-\sqrt{148}}{6} \approx -1.36$

$x = \frac{4 \pm \sqrt{148}}{6}$

a) What theorem guarantees the existence of at least one x value that satisfies question 16.

Intermediate Value Theorem

17. Determine the local extrema values and where they occur.

$3x^2 - 4x - 15 = 0$

$\frac{4 \pm \sqrt{16 - 4(3)(-15)}}{6}$

$\frac{4 \pm \sqrt{196}}{6}$

$\frac{4 \pm 14}{6}$

$\frac{18}{6} \quad \frac{-10}{6}$

$3 \quad -5/3$

$f(-3) = 6 \quad f(-5/3) = \frac{562}{27} \approx 20.82$
$f(4) = -22 \quad f(3) = -30$

Local maximum of $\frac{562}{27}$ at $x = -5/3$

Local minimum of -30 at $x = 3$

$f(x) = x^3 - 2x^2 - 15x + 6 \quad [-3, 4] \qquad -3, -5/3, 3, 4$

$f'(x) = 3x^2 - 4x - 15$

$f''(x) = 6x - 4$

$f' =$ Inc/Dec - Max/min

$f'' =$ Concavity - Inf. Points

18. Determine the intervals on which $f(x)$ is:

x	-3	-2	-5/3	0	3	11/3	4
$f'(x)$		5		-15		$10\frac{2}{3}$	

a) Increasing:

$[-3, -5/3) \cup (3, 4]$

b) Decreasing:

$(-5/3, 3)$

19. Determine the location of any points of inflection:

$6x - 4 = 0$

$6x = 0$

$x = 2/3$

Point of Inflection at $x = 2/3$

20. Determine the intervals on which $f(x)$ is:

x	-3	0	2/3	2	4
$f''(x)$		-4		8	

a) Concave Up:

$[2/3, 4]$

b) Concave down:

$[-3, 2/3)$

21. Determine $\int_{-2}^{1} f(x)dx$.

$$\int_{-2}^{1}(x^3 - 2x^2 - 15x + 6)\,dx = \frac{x^4}{4} - \frac{2}{3}x^3 - \frac{15}{2}x^2 + 6x\Big|_{-2}^{1}$$

$$\left(\frac{1}{4} - \frac{2}{3} - \frac{15}{2} + 6\right) - \left(4 + \frac{16}{3} - 30 - 12\right)$$

$$-\frac{23}{12} - \left(-\frac{98}{3}\right) = \frac{123}{4}$$

22. Determine the average value of $f(x)$ on [-3, 4].

$$3 \quad \frac{1}{4+3}\int_{-3}^{4}(x^3 - 2x^2 - 15x + 6)\,dx = \frac{1}{7}\left(\frac{x^4}{4} - \frac{2}{3}x^3 - \frac{15}{2}x^2 + 6x\right)\Big|_{-3}^{4}$$

$$\frac{1}{7}\left[\left(64 - \frac{128}{3} - 120 + 24\right) - \left(\frac{81}{4} + 18 - \frac{135}{2} - 18\right)\right]$$

$$\frac{1}{7}\left[\frac{-224}{3} - \left(-\frac{189}{4}\right)\right]$$

$$\frac{-47}{12}$$

23. Use Newton's Method to determine the first negative root of $f(x) = \cos(x) + 3e^x$

a) Determine the explicit formula to find the $n + 1$ root $f'(x) = -\sin x + 3e^x$

$$x_{n+1} = x_n - \frac{\cos(x_n) + 3e^{x_n}}{\sin(x_n) + 3e^{x_n}}$$

b) Use an initial value of $x = -3$ and your calculator to approximate the sought after root. Show each new approximation. Terminate the process when successive iterations agree to 5 decimal places.

$x_0 = -3$
$x_1 = -2.89392$
$x_2 = -1.95370$
$x_3 = 0.038175$
$x_4 = 1.33699$
$x_5 = 1.11563$
$x_6 = 1.16212$
$x_7 = 1.15163$
$x_8 = 1.15396$
$x_9 = 1.15344$
$x_{10} = 1.15355$
$x_{11} = 1.15353$
$x_{12} = 1.15353$

Using Newtons method on the given function $f(x)$, 12 iterations are calculated to find an approximation of 1.15353.

24. a) Determine the linearization, $L(x)$, for $g(x) = x^2 + \ln(x)$ at $x = 1$.

$g'(x) = 2x + 1/x$ $\quad g(1) = 1 + 0 = 1 \quad$ Pt(1,1)
$g'(1) = 2 + 1 = 3$
$L(x) = 1 + 3(x-1)$
$L(x) = 1 + 3x - 3$
$L(x) = 3x - 2$

b) Use $L(x)$ to approximate $g(x)$ at $x = 1.03$

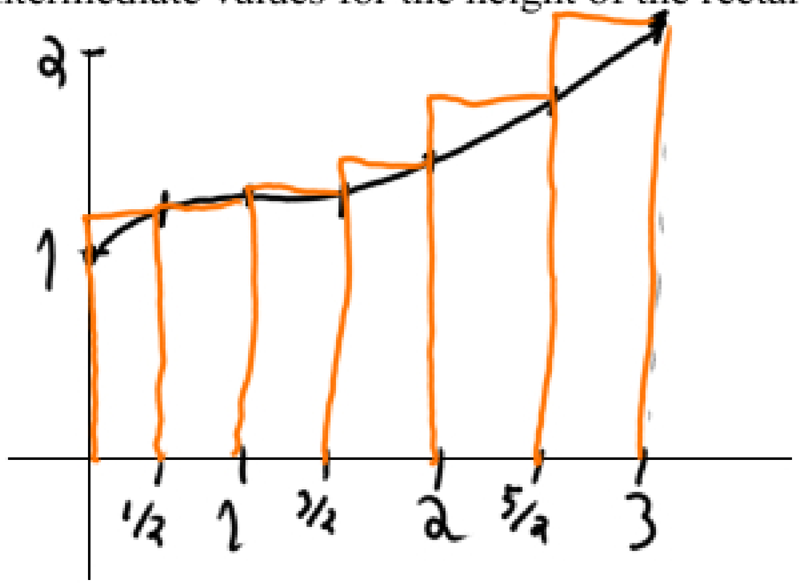
$L(1.03) = 3(1.03) - 2 = 1.09$

$\Delta x = \frac{3}{6} = \frac{1}{2}$
$x_k = a + (k-0)\Delta x$
$x_k = k/2$

$$\frac{1}{2}\sum_{k=1}^{6} = \frac{e^{k/2}}{(k/2)^2 + 1}$$

25. Evaluate $\int_0^3 \frac{e^x}{x^2+1}dx$ using a Right Riemann sum with 6 equal length sub-intervals.

Show all work including a graph of the function, the rectangles you create, and the intermediate values for the height of the rectangles.

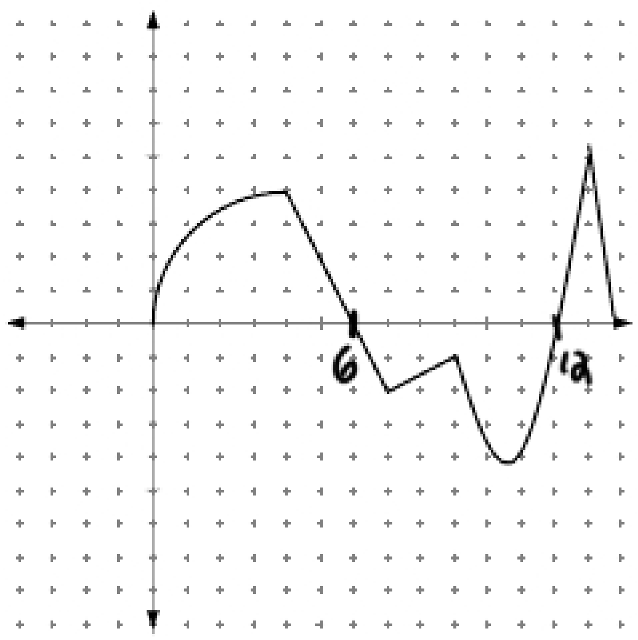


26. Use the Fundamental Theorem of Calculus to determine $\frac{d}{dx}\int_x^7 \left(8^{\cos t} + \sin t\right)dt$

$-\int_7^x \left(8^{\cos t} + \sin t\right)dt$

$-\left[8^{\cos x} + \sin x\right]$

For questions 27 through 29, use the graph below that gives a particle's ***VELOCITY*** (ft/sec) at time *t* (secs). {Scale is 1 tick mark = 1 unit}



This is the VELOCITY GRAPH!

It is not, I repeat, IS NOT, the position graph!

neg - Left
pos - Right

27. When is the particle moving to the left?

(6,12)

28. When does the particle change directions?

at $t=6$ & $t=12$

29. When is the particle slowing down?

$(4,6) \cup (7,9) \cup (\sim 11,12) \cup (13,14]$

30. Given the graph of *f*(*x*) shown below, determine $A(x) = \int_3^x f(t)\,dt$ for each of the following:

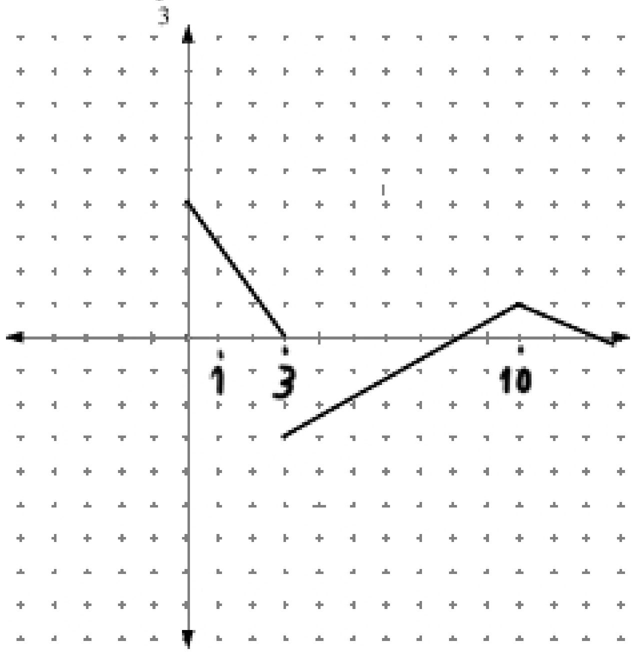
a) $A(10) = \frac{1}{2}(3\cdot 5) + \frac{3}{2}$ $= -6$

$-\frac{15}{2} + \frac{3}{2}$

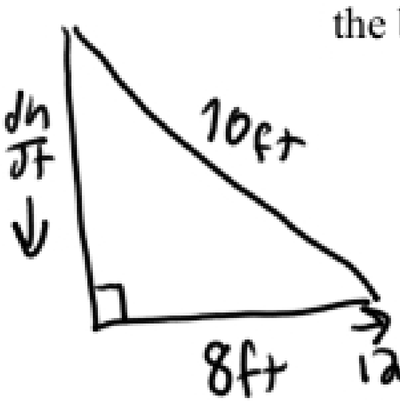
b) $A(1) = \frac{1}{2}(3\cdot 4) = \frac{12}{2} = -6$

c) $A(3)$

$A(3)=0$



A ten foot ladder is leaning against a house when its base starts to slide away. By the time the base is 8 feet from the house, the base is moving away at the rate of 12 ft/sec.



31. How fast is the top of the ladder sliding down the wall then?

have
L - Ladder - 10ft const
b - base - 8ft ATM
12ft/s $\frac{db}{dt}$ - 12 ft/s ATM

$\frac{dL}{dt} = 0$ ft/s - unchanging

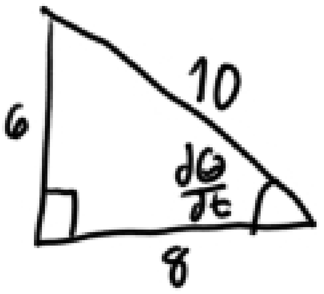
want
$\frac{dh}{dt}$

$a^2+b^2=c^2$
$2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$
$2(6)\frac{dh}{dt}+16(12)=20(0)$
$12\frac{dh}{dt}+192=0$
$12\frac{dh}{dt}=-192$
$\frac{dh}{dt}=-16$ ft/s

The Ladder is sliding down the wall at a rate of -16ft/s.

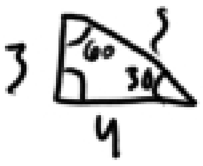
$a^2+8^2=10^2$
$a^2+64=100$
$a^2=36$
$a=6$ ATM

32. At what rate is the angle between the ladder and the ground changing then?

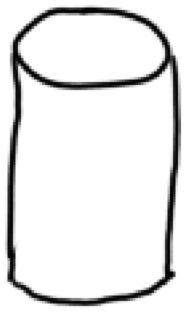


$\tan\left(\frac{o}{a}\right)=\theta$
$\tan\left(\frac{do}{da}\right)=d\theta$
$\tan\left(\frac{-16}{12}\right)=d\theta$
$\tan\left(\frac{-4}{3}\right)=d\theta$
$d\theta=-4.13$

The angle between the ladder and the ground is changing at $-4.13°$/sec when the base of the ladder is 8ft away from the wall.



33. A right circular cylinder is to be designed to hold 40 cubic inches of a power drink. The cost for the material for the top and bottom is \$2.00 per square inch. The cost for the sides is \$0.50 per square inch. What should the dimensions of the can be in order to minimize its cost? {Round to the nearest hundredth of an inch}



S \$0.5
L - \$2

$V = 40 \text{ in}^3$

$V_{cyl} = \pi r^2 h$

$A = \pi r^2$

$40 = 0.5\pi r^2 h + 4\pi r^2$
$20 = \pi r^2 h + \pi r^2$
$20 = \pi r^2(1+h)$
$20/\pi = r^2(1+h)$

$S = 0.5\pi r^2 h$
$L = 4\pi r^2$
$\frac{20}{\pi r^2} - 1 = h$

X