5.4 Working with Integrals

MATH 205

Symmetry

- Let a be a positive real number and let f be an integrable function on the interval $[-a, a]$.

I. If f is even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

II. If f is odd: $\int_{-a}^{a} f(x)dx = 0$

Even or Odd?

1. $\int_{-8}^{8}\left(x^5-18x^3+9x\right)dx$

2. $\int_{-\pi/4}^{\pi/4}\sec^2 x\,dx$

3. $\int_{-3}^{3}\left(8x^4+6x^2+\sin^5 x\right)dx$

Average value of a function:

- The average (mean) value of an integrable function on $[a, b]$ is $\overline{f} = \frac{1}{b-a}\int_a^b f(x)dx$
- Determine the average value of:

4. $f(x) = 2x$ on $[-1,3]$

5. $k(x) = |3x| - 5$ on $[-4,8]$

Mean Value Theorem for Integrals

- Recall the Mean Value Theorem, if the criteria is met, guarantees the existence of some real number, c, on $[a, b]$ where the average rate of change is equal to instantaneous rate of change.

- The Mean Value Theorem for Integrals is similar but now we are looking for a real number where the average value of the function is equal to the output of the function.

MVT for Integrals

- Let f be continuous on $[a, b]$. There exists a point c in $[a, b]$ such that $f(c) = \overline{f} = \frac{1}{b-a}\int_a^b f(t)dt$

- Find the point(s) at which the following function equals its average value on the given interval :

6. $k(x) = e^x$ on $[0, 3]$

MVT for Integrals

- Find the point(s) at which the following function equals its average value on the given interval :

7. $h(x) = 3x^2 - 4x + 7$ on $[1, 4]$