



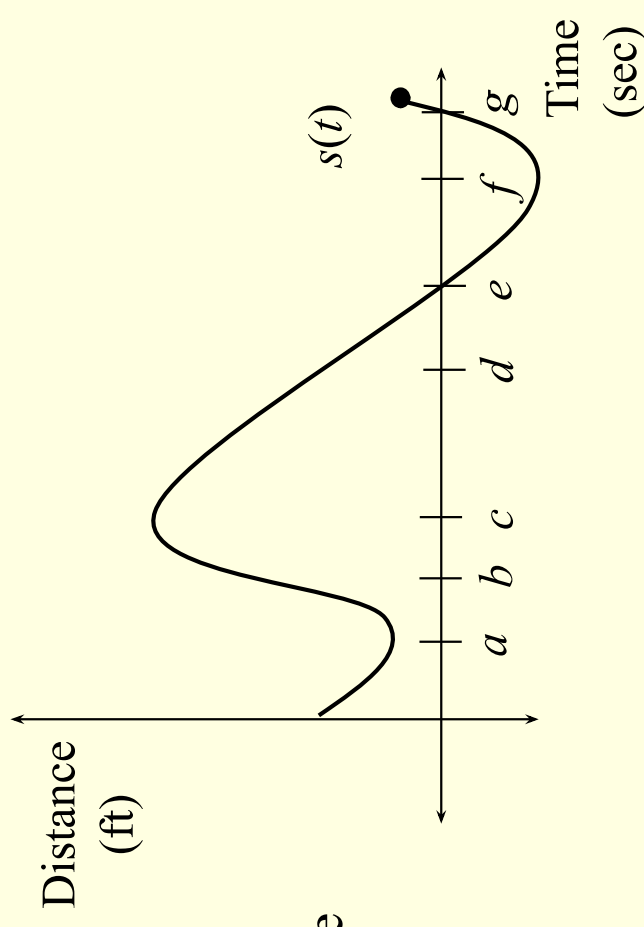
## 3.6 Derivatives as Rates of Change



MATH 205

# Queries

1. When does the vehicle have a negative velocity?
2. When is the vehicle traveling the fastest?
3. When does the vehicle change directions?
4. Sketch a graph of the derivative of  $s(t)$



# Definitions:

- Displacement:
  - The directed distance an object is from its starting position.
  - Net Distance traveled.
  - If  $s = f(t)$  is a position function, displacement is  $f(b) - f(a)$  on  $[a, b]$ .
- Average Velocity:
  - Displacement over time:  $\frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$
- (Instantaneous) Velocity: {the rate of change of position}
  - If an object's position at time  $t$  is  $s = f(t)$ , then

$$v(t) = s'(t) = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

# More about velocity

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- Velocity is a signed value!
  - Object is moving forward (right) when velocity is positive.
  - Object is moving backward (left) when velocity is negative
  - Object is at rest when velocity = 0.
- Speed is the absolute value of velocity!

# Acceleration and Jerk

- Acceleration:

- The rate of change of velocity

$$a(t) = v'(t) = s''(t)$$

- Object is speeding up when absolute value of the velocity graph has positive slope.
  - $v(t)$  is moving away from the x-axis.
- Object is slowing when absolute value of the velocity graph has negative slope.
  - $v(t)$  is moving towards the x-axis.
- Velocity is constant when acceleration is zero.

- Jerk:

- The rate of change of acceleration

$$j(t) = a'(t) = v''(t) = s'''(t)$$

# Motion in a line

- Suppose a particle is moving along a straight line (rectilinear motion) and its position at any time  $t$  is given by  $s(t) = t^2 - 9t + 18$
- 5. What is the particle's position at  $t = 0$ .
- 6. What is the particle's displacement on  $[2, 7]$ ?
- 7. When does the particle change direction?
- 8. When is the particle moving to the right?
- 9. When is the particle moving to the left?
- 10. When is the object speeding up?
- 11. What is the total distance traveled by the particle after 10 seconds?
- 12. What is the object's displacement?

# Practice

13. When is the object moving forward?

14. When is the object speeding up?

15. When is the object moving the fastest?

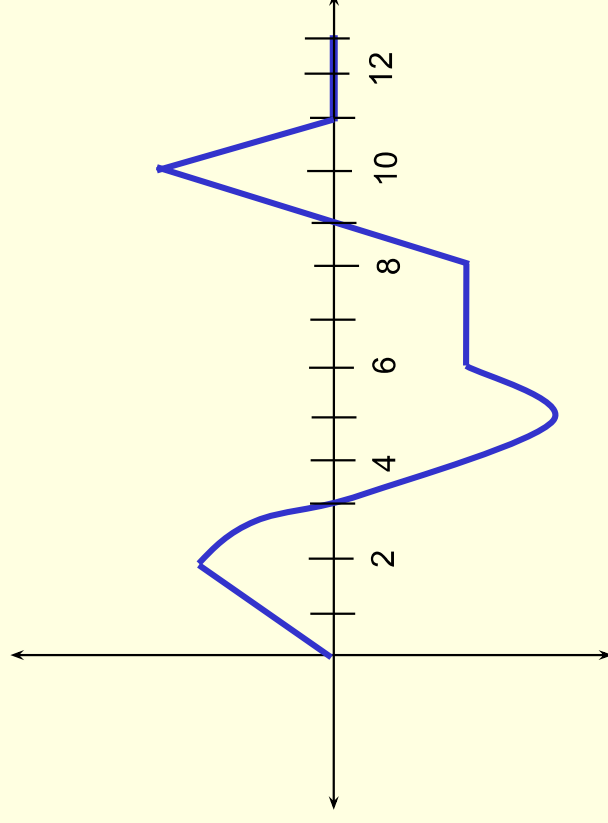
16. When is the object at rest?

17. When does the object change direction?

18. When is the object slowing down?

19. Will the object end up to the right or the left of its starting position?

■ The following is a velocity graph and each dash represents 1 unit.



# Marginal and Average Cost

- Let  $C(x)$  be a cost function which gives the cost to produce the first  $x$  items in a manufacturing process.
- The average cost to produce  $x$  items is  $\bar{C}(x) = \frac{C(x)}{x}$
- The marginal cost  $C'(x)$  is the approximate cost to produce one additional item after producing  $x$  items.
- Suppose  $C(x) = -.05x^2 + 60x + 200$ 
  20. Determine the average and marginal cost functions.
  21. Determine the average and marginal cost when  $x = 200$  and 1000. Interpret these values.



# Application

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22. Suppose that the distance an aircraft travels along a runway before takeoff is given by  $D = (10/9)t^2$ , where  $D$  is measured in meters from the starting point and  $t$  is measured in seconds from the time the brakes are released. The aircraft will become airborne when its speed reaches 200 km/h. How long will it take to become airborne, and what distance will it travel in that time?