Project One Template

MAT350: Applied Linear Algebra

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Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution:

Output = Input

A:

$x_1+2x_2=100$

B:

$x_3+x_5=x_1+x_2$

$-x_1-x_2+x_3+x_5=0$

C:

$x_3+x_5=x_2+50$

$-x_2+x_3+x_5=50$

D:

$x_2+120=x_4+x_5$

$-x_2+x_4+x_5=120$

E:

$x_4=x_2+x_3+x_5$

$x_2+x_3-x_4+x_5=0$

Solving Systems of Linear Equations

Receiver = Sender

$x_3+120=100+50$

$x_3+120=150$

$x_3=30$

B=C

$x_1+x_2=x_2+50$

$x_1=50$

A

$x_1+2x_2=100$

$(50)+2x_2=100$

$2x_2=50$

$x_2=25$

C

$x_3+x_5=x_2+50$

$(30)+x_5=(25)+50$

$30+x_5=75$

$x_5=45$

D

$x_4=x_2+x_3+x_5$

$x_4=(25)+(30)+(45)$

$x_4=100$

x_1=50

x_2=25

x_3=30

x_4=100

x_5=45

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

```
%Matrix A
A = [1 2 0 0 0; -1 -1 1 0 1; 0 -1 1 0 1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
%Column matrix of constants
b = [100; 0; 50; 120; 0]
```

```
b = 5×1
   100
     0
    50
   120
     0
```

```
%Augmented matrix
Ab = A
```

```
Ab = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
Ab(:,6) = b
```

```
Ab = 5×6
     1     2     0     0     0   100
    -1    -1     1     0     1     0
     0    -1     1     0     1    50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

```
%Reduced augmented matrix
R = rref(Ab)
```

```
R = 5×6
     1     0     0     0     0    50
     0     1     0     0     0    25
     0     0     1     0     0    30
     0     0     0     1     0   100
     0     0     0     0     1    45
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

```
%Decomposition of A
[L, U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
   -1.0000    1.0000         0         0         0
         0   -1.0000    1.0000         0         0
         0   -1.0000    0.5000    1.0000         0
         0    1.0000         0   -1.0000    1.0000
U = 5×5
     1     2     0     0     0
     0     1     1     0     1
     0     0     2     0     2
     0     0     0     1     1
     0     0     0     0     1
```

```
%Unkown vector
y = L \ b
```

```
y = 5×1
   100
   100
   150
   145
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%Inverse of U
invU = inv(U)
```

```
invU = 5×5
    1.0000   -2.0000    1.0000         0         0
         0    1.0000   -0.5000         0         0
         0         0    0.5000         0   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

```
%Systeom of Equations Computation
x = invU * y
```

```
x = 5×1
    50
    25
    30
   100
    45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
%Cramer's Rule - Create individual matrices for each column
A1 = A
```

```
A1 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A2 = A
```

```
A2 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A3 = A
```

```
A3 = 5×5
```

```
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A4 = A
```

```
A4 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A5 = A
```

```
A5 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
%Replace each column with column of constants
A1(:,1) = b
```

```
A1 = 5×5
   100     2     0     0     0
     0    -1     1     0     1
    50    -1     1     0     1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
A2(:,2) = b
```

```
A2 = 5×5
     1   100     0     0     0
    -1     0     1     0     1
     0    50     1     0     1
     0   120     0     1     1
     0     0     1    -1     1
```

```
A3(:,3) = b
```

```
A3 = 5×5
     1     2   100     0     0
    -1    -1     0     0     1
     0    -1    50     0     1
     0    -1   120     1     1
     0     1     0    -1     1
```

```
A4(:,4) = b
```

```
A4 = 5×5
     1     2     0   100     0
    -1    -1     1     0     1
     0    -1     1    50     1
     0    -1     0   120     1
     0     1     1     0     1
```

```
A5(:,5) = b
```

```
A5 = 5×5
     1     2     0     0   100
    -1    -1     1     0     0
     0    -1     1     0    50
     0    -1     0     1   120
     0     1     1    -1     0
```

```
%Solve for system of equations
x1 = det(A1) / det(A)
```

```
x1 = 50
```

```
x2 = det(A2) / det(A)
```

```
x2 = 25
```

```
x3 = det(A3) / det(A)
```

```
x3 = 30.0000
```

```
x4 = det(A4) / det(A)
```

```
x4 = 100
```

```
x5 = det(A5) / det(A)
```

```
x5 = 45
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

*Fill out the table in the original project document and export your table as an image. Then, use the **Insert** tab in the MATLAB editor to insert your table as an image.*

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Nathan Hallam MAT 350 Project One Table

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	50	Upgrade	This link is near its capacity taking half the burden of Router A to Router B. The likelihood that this link will be overburdened if the senders output increases is high
x_2	50	25	Nothing	Link x_2 is used by Routers A and D with the bandwidth output of each router on other links being fairly high. On Router A link x_2 is used twice to send data to Routers C and E while Router D has a static output of 120mbs in this scenario leaving the burden on link x_2 to have its capacity.
x_3	100	30	Downgrade / Remove	Link is underutilized as the load of the other links is higher. In both cases of Router B's and D's output the other links can take the majority of the load leaving little for this link.
x_4	100	100	Upgrade	This link is the sole output for Router E, while having the most input links in the network at 3. This link is overused and may run over the current recommended bandwidth.
x_5	50	45	Upgrade	This link is nearly overburdened at its current capacity. The link is used from Routers C to D and Routers B to E and shares its output with link x_3. As of currently x_3 takes a smaller portion of the networks burden while x_5 takes the majority, leaving it overutilized.