Intro to Algorithm Analysis

- Algorithm analysis
  - worst case
  - big-O
  - $O(n)$, $O(1)$, $O(n^2)$
- Big-O of `merge` method
  - Merge algorithm example
  - finding big-O of Java library methods
- Big-O of `Sequence` class

Announcements

- Lab this week: bring your laptop with IntelliJ installed (see lab for details)
- PA2 due in 6 days

Algorithm analysis idea

- Compare efficiency one algorithm / data structure to another *before* implementation.
- For a given problem size ***n***, how long does it take?
  - **worst-case performance**.
  - best-case performance.
  - average-case performance.

Algorithm analysis idea (cont.)

- Ex: “search an array”:
  - does the value, *target*, appear in a given array of size ***n***, and if so, at what position?
- Want to know how long it takes as a function of ***n***.
- In the example below $n = 6$.

target = 12

	0	1	2	3	4	5
values	14	7	5	12	3	9

Big-O notation

asymptotic worst-case performance:

- behavior as *n* grows,
- on worst possible input of size *n*.
- Use *big-O* notation.
- units are program steps.

  e.g., 2 * 2 takes the same amount of time as 20000 * 20000

- big-O is also called the *time complexity*
- can also look at space complexity of algorithms (how much extra space to solve the problem)

O(n)

- Example 1: It takes *n* steps to print all the elements in an array with *n* elements.
  - We say this algorithm is "*order n*",
  - or *O(n)*, or
  - it takes "*linear time*".
- O(*n*) means number of steps is some linear function of *n*:

  $$c_1 * n + c_2$$

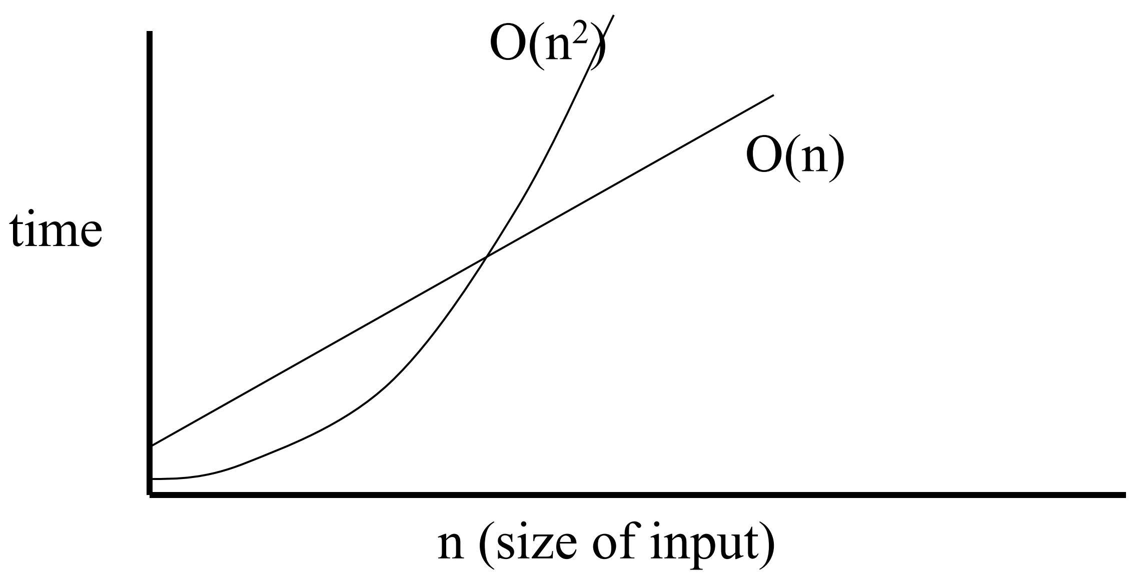
Counting steps

Example 2: compute the average of *n* numbers:

```
int sum = 0;
int n = in.nextInt();

for (int i = 1; i <= n; i++) {

  int value = in.nextInt();
  sum += value;

}

System.out.println("The average is: "
                   + sum / ((double) n);
```

Why ignore constants?

- constants fade away as n grows large.
- compare algorithm to another that may differ in order of magnitude, e.g., $O(n^2)$ or $O(2^n)$
- distinct from "tuning" a specific implementation



O(*1*)

Algorithm that takes the same amount of time no matter how big *n* is.

- called *constant time*
- or *order 1*
- or *O(1)*

O(1) examples

- Examples *:
  - assignment statements
  - arithmetic expressions
  - boolean expressions
  - println
  - simple-statement sequences
  - loops with constant bounds

* Warning: time taken inside method calls counts towards total. Exs above depends on *contents* of expressions and loops. E.g.,

```
System.out.println(Arrays.toString(myArr));
```

takes ???

Important example

- Input: an array of size n.
- Problem: find the kth element in the array.

Sequential search

- Ex 3: Big-O to search in an unordered array of size *n*.
  - does the value, *target*, appear in a given array of size ***n***, and if so, at what position?
  - time depends on values in array and what target is
  - we're interested in the *worst* case.

target = 12

	0	1	2	3	4	5
values	14	7	5	12	3	9

Sequential search on *sorted* array

- Ex 3 (variation): Big-O to search in an ordered array of size n using **linear search**
- Worst case?
- best case, average case?

$target = 8$

	0	1	2	3	4	5
values	3	5	7	9	12	14

$O(n^2)$

- Ex 4: print out a multiplication table for the integers 1 to n
- *Quadratic time* ($O(n^2)$) is any quadratic function of n:

  $an^2 + bn + c$

Ex: merge two ordered lists

- problem: create one large ordered ArrayList out of two ordered ArrayLists (no duplicates).
- Example:
  - list1: 3 7 9 12 15
  - list2: 2 5 6 8 9 20
  - merged list: 2 3 5 6 7 8 9 12 15 20

slow **`merge`** method

- Idea: for **`merge(list1, list2)`**:
  - copy arraylist in **`list1`** to **`result`** arraylist (copy constructor)
  - for each element of arraylist in **`list2`**:
    - find its location in **`result`**
    - insert the element at that location in result

      (use **`ArrayList`** method **`add(index, elmt)`**)
  - return **`result`**

list1: 3 7 9 12 15

list2: 2 5 6 8 9 20

- big-O? (size of **`list1`** is m, size of **`list2`** is n)
- How to find big-O of Java methods?

Better-performing `merge` method

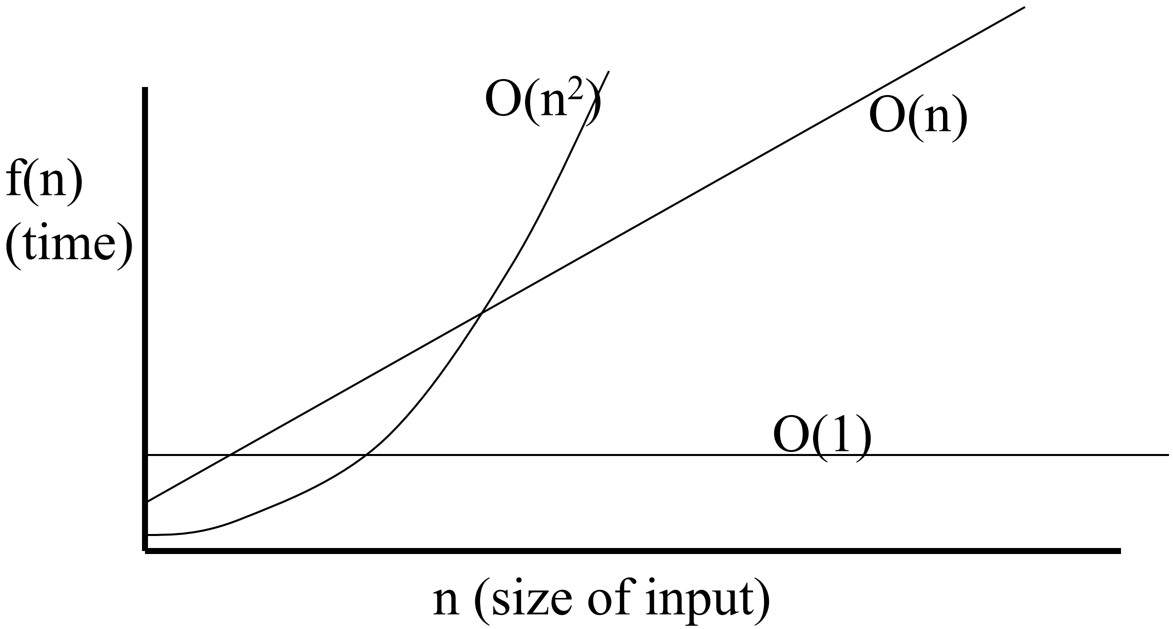
- take advantage of the fact that both arrays are already sorted
- traverse both arrays in one loop:
  - take the smaller element of the two arrays and add it to the result array, and update index of the one moved.
- every loop iteration get closer to the end of one of the arrays
- always adding new values at the *end* of result
- *merge algorithm*

Merge example

List1:	**9**	**11**	**16**	**20**	
List2:	**2**	**5**	**16**	**17**	**18**

Comparing different time bounds

- Sometimes there exist fast and slow algorithms to solve the same problem.
- Here's an idea of what some of these time bounds look like when plotted.



big-O practice

- **`Sequence`** class:
  - to represent a sequence (list) of numbers
- Internal representation:
  - values are stored in an array (beginning and end of array corresponds to beginning and end of the sequence)
- Operations next page . . .

`Sequence` class operations

Operation	big-O (for array rep)
`Sequence s = new Sequence();`	
`s.getValAt(loc)` → `val`	
`s.contains(val)` → `t/f`	
`s.removeValAt(loc)` → `success`	
`s.insertAtEnd(val)`	
`s.insertInFront(val)`	
`s.numVals()` → `length`	