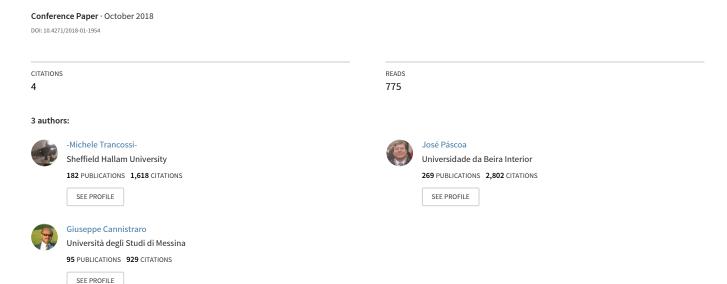
# Safety Analysis of an Airship Which Loses Lifting Gas from the Hull



2018-01-1954 Published 30 Oct 2018



# Safety Analysis of an Airship Which Loses Lifting Gas from the Hull

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Citation: Trancossi, M., Pascoa, J., and Cannistraro, G., "Safety Analysis of an Airship Which Loses Lifting Gas from the Hull," SAE Technical Paper 2018-01-1954, 2018, doi:10.4271/2018-01-1954.

#### **Abstract**

his study investigates the physical phenomena that affect a high-altitude airship in the presence of lifting gas losses from the hull. General atmospheric thermodynamics and basic physical principles are adopted to describe the behavior of an airship with envelope failures that generate buoyant gas dispersion or depressurisation phenomena. Overpressure that could grant to maintain some controllability during a large part of the descent is assessed by mean of the thermodynamic model of the envelope in the presence of gas losses. Optimisation of the inflation parameters is provided and the conditions for avoiding dangerous crashes on the ground and the potential recovery of a damaged vehicle, people and its payload. In particular, the requirements for a slow depressurisation is computed by the equilibrium with

the atmosphere and then how can it be possible to sustain controlled navigation are determined. A key factor for security relates directly to the capability of preserving some airship balloon overpressure for the longest time possible. This condition can extend much the range of control. Complete forfeit conditions will be determined to demonstrate that airship cannot be anymore controllable below 20% of the initial altitude at which the failure has started.

In some cases, specific manoeuvres could allow configuring the deflated balloon as a parachute, if coupled with adequate safety systems. This research about safety conditions will also be useful for designing safety systems. A general guideline for safety systems has been defined showing that airship if well created and well governed in emergency conditions will be much safer than any other aerial vehicle.

#### Introduction

his research work has started during the effort of studying a cruiser-feeder airship system such as the one defined by the MAAT (Multibody Advanced Airship for Transport) EU FP7 project [1, 2, 3, 4]. As some accidents occurred during the history of lighter-than-air aeronautics demonstrates, the primary safety problem of airships relates to its vulnerability because of significant dimensions of the hull, which has been gigantic sometimes [5, 6, 7]. The effort of this activity is to understand the behaviour of an airship when gas is released from the hull envelope causing the LTA vehicle to fail, sometimes with terrible effect for passengers and crew. This study must necessarily start from an accurate comprehension of the processes of buoyant gas release when the gas envelope is damaged. It is a necessary activity for a critical analysis of failure modes of an airship. It is evident that the loss of lifting gas affects buoyancy and produces a decrease in altitude. Otherwise, the mechanisms that govern airship descent are far from the effective comprehension.

An attempt of describing those mechanisms has been produced by Vogt [8]. He has defined a general model of lift capability of an airship with a damaged hull and lifting

gas envelope. This work is an essential reference with a high level of generality because it is based on fundamental physical laws. In particular, Vogt has focused his attention on the case of a near-space airship, but with some considerations, this model can be easily extended to any airship whatever is its operative altitude when damage is manifested. He has opened the road through a predictive method, which considers the evolution of airship's damages and consequently studies how the airship behaves as a function of the growth of damages.

Airship performances degrade gradually and then it is fundamental to understand how they affect the flight attitude of a lighter-than-air vehicle in different atmospheric conditions. Different architectures of LTA vehicles and structural concepts force also to consider how these influences the airship behaviour. Vogt has clearly stated that a free-floating system and propelled system could have different behaviour and survivability.

#### **Problem Overview**

First principles of physics allow performing a useful airship modelling. By using them, it is possible to understand the behaviour of an airship when the gas envelope is damaged. This study is slightly different from others before because it considers the hydrogen instead of helium. It can benefit of precedent studies but slightly differs because it must take into account the flammability conditions of hydrogen when it is mixed with air. It is then necessary to consider the gas buoyancy effects and their evolution when the sudden loss of lifting-gas compromises vehicle flight attitude.

Prime physical laws such as Newton's Laws of Motion, Archimedes' Principle, Bernoulli's Equation, and ideal gas law, ideal gas thermodynamics have been used to analyse how the performances degrade over time. The prediction of the behaviour of an LTA vehicle during descent has been produced. This activity aims to understand the consequences of damage to the controllability of the system. It means that after the accident the system keeps some buoyancy and allows to be controlled at least for an identifiable period.

According to Khoury and Gillett [9], the Lighter-than-air flight is achieved by exploiting the propensity of lighter fluid to rise to the point of equilibrium in a denser fluid. They are usually unstructured balloons and airships (which have been designed with a specific geometry depending on the mission and other operative parameters). An airship has a propulsion system on board to ensure the autonomy of motion rather than being passively moved by prevailing winds, such as it happens with Montgolfier. LTA vehicles can be classified in non-rigid, semi-rigid, and rigid:

- Non-rigid airships have an envelope that keeps the shape by mean of pressurised lifting gas. Buoyant gas has a higher pressure than external atmosphere, and it generates a tensile state on the envelope of the balloon. The payload is enclosed in a cabin attached to the hull often employing cables. Pressure is governed through some pressure relief system to prevent overexpansion as it rises. Non-rigid airships present inherent limits: slow horizontal speed 10 ÷ 20 m/s, little resistance to mechanical solicitations.
- Semi-rigid airships are similar to the non-rigid, but present reinforcing kneels, which help to maintain the shape and presents a higher mechanical strength to solicitations.
- Rigid airships have a rigid frame and gas envelop directly fixed to the frame. This frame provides structural advantages and higher capability of keeping the shape but presents higher mass than precedent architectures.

Lighter-than-air vehicle geometry strictly relates to system propulsion requirements. Aerodynamically efficient geometry is not necessarily a concern of LTA vehicles, because they need no energy for maintaining the altitude. As the need for navigation and control of a vehicle increases, propulsion systems can be added by motorised propellers. Propulsion must have enough power to ensure the independence from wind currents and steering capability. Geometry assumes particular importance for this type of vehicle. Typical Parsifal airships designs are tuned to an optimum fineness ratio to minimise drag forces on a propelled vehicle. The fineness ratio is merely a measure of the slenderness of the body and is a ratio of the vehicle's length to its maximum diameter [10].

This paper aims to define the survivability models of a semi-rigid airship. Former models refer to non-rigid airships and free-floating research balloons. In order to produce comparable results, this model is based on the size of holes on the airship hull and analyses times and speeds for a safe and controllable descent. This analysis allows evaluating how long constant overpressure can be maintained. It also allows understanding when pressure decreases, how long the airship can respond to the control of the pilot.

#### **Method**

# Atmospheric Model and Ideal Gas Physics

The behaviour of LTA vehicles is influenced by the physical laws of an ideal gas because this assumption produces a negligible error [11]. In particular, both air and buoyant gasses are considered ideal gas [12]. Any gas in conditions, which are far from liquefaction point, can be reasonably described as an ideal gas [13]. They can be modelled by the ideal gas equation [14]:

$$\rho = \frac{p \cdot w}{R \cdot T} \tag{1}$$

where  $\rho$  is density, p is pressure, w is molar mass (for dry air is 28.9647 g/mol), T is absolute temperature, and R is the gas constant. The U.S. Standard Atmosphere 1976 (USSA1976) [15] assumes the gas constant R equal to  $8.31432 \times 10^3$  Nm kmol<sup>-1</sup> K<sup>-1</sup>. Consequently, equation (1) can be written as a function of  $R_{air}$ , which is the gas constant for air equal to 287.05 J/kg K

$$\rho = \frac{p}{R_{air} \cdot T}.$$
 (2)

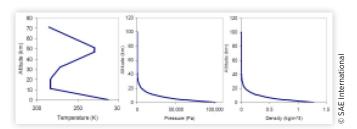
<u>Figure 1</u> shows the behaviour of temperature, pressure, and density in the terrestrial atmosphere. In the case of airship analysis, the maximum altitude can be limited with the upper limit of the stratosphere. US Standard Atmosphere 76 model can be adopted. With particular reference to the study of the behaviour of airships, it can be possible to consider only troposphere and lower stratosphere.

The atmospheric model for the two considered regions is:

1. Troposphere (0 to 11 km):

$$T = 288,19 - 0.00649 \cdot h$$
$$p = 101.29 \cdot \left(T/288.08\right)^{5.256}$$

**FIGURE 1** Atmospheric temperature, pressure, and density



#### 2. Lower stratosphere (11 to 25 km)

$$T = 216,69$$

$$p = 22.65 * e^{(1.73 - 0.000157 * h)}$$

In each zone, the density  $\rho$  is derived from the equation of state (2).

$$\rho = p/(0.28705 \cdot T).$$

In addition, pressure and density can be interpolated by simpler exponential functions of the operative altitude h:

$$\rho_{air} = \rho_{air,0} \cdot e^{-h^*} \tag{3}$$

$$p = p_{air,0} \cdot e^{-h^*} \tag{4}$$

where  $h^*$  is a scale height constant defined by

$$h^* = \frac{gh}{RT_{avg}} = h / \left(\frac{RT_{avg}}{g}\right) = h / H$$
 (5)

#### **Airship Physics**

The airship is the most energetically efficient solution, which can lift heavy payloads to high altitudes. They have the capability of sustaining a weight by Archimedes law. The airship hull is filled by a lighter than air gas. By assuming that air ballonets inside the hull are used to govern the lift and the pitch, it can be possible to express the volume of gas inside the hull (Figure 2).

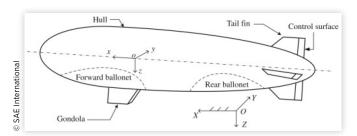
$$V_{gas} = V_{hull} - V_{ballonets} \tag{6}$$

It produces a vertical force directed upward by displacing heavy air with the less dense gas.

$$mg \le L = g \cdot V_{gas} \cdot (\rho_{air} - \rho_{gas}) \tag{7}$$

The airship will rise until its buoyant force L is equal to the weight of the airship. A constant overpressure must be ensured between the gas inside the airship hull and the air outside. The air in the ballonets is pumped by a ventilation system and released ay valves. In this way, it is ensured to keep the overpressure inside the hull. The Lift is generated by reducing the volume of the air ballonets that implies increasing the volume of the gas and decreasing the pressure (and density) inside the hull. Any reduction of the increasing the buoyant force and the altitude allows moving from an equilibrium point to the other. As the volume of the ballonets decreases, the buoyant force is reduced. When the volume of the ballonets reaches its minimum, the buoyant force will reach its maximum. In this

#### FIGURE 2 Airship drawing indicating ballonets



condition, the airship reaches the maximum admissible eight. At this point, the airship cannot increase anymore its height.

The maximum lift position can be determined by equation (8).

$$L = gV_{gas, max} \left( \rho_{air} - \rho_{gas} \right)_{h_{max}} \tag{8}$$

It can be assumed that the rate of change of the lifting gas density is equal to that of the air expelled from the hull. The density ratio  $\sigma$  can be consequently defined to relate the density of buoyant gas to the one of air at any altitude:

$$\frac{\rho_{air}}{\rho_{air,0}} = \sigma = \frac{\rho_{gas}}{\rho_{gas,0}} \tag{9}$$

Consequently, the volume of the buoyant gas present, the lift equation can be expressed as a function of the mass of lifting gas mass:

$$L = m_{gas}g\left(\frac{\rho_{air,0}}{\rho_{gas,0}} - 1\right) \tag{10}$$

Lifting gas mass should almost constant remain constant within the envelope during the airship mission, with the only exception of the buoyant gas losses by envelope permeability. If ballonet deflation is appropriately controlled, the airship will reach the maximum pressure height and maintain that altitude. If an airship rises above its pressure height, the lifting gas will be vented. Lift capacity will be reduced consequently. To prevent this from happening, some ballonet inflation must be maintained. In this way, the airship will not be able to reach the maximum altitude, but the risk of venting out some buoyant gas is prevented.

### Airship Volume

Pressure height of an airship is a function of the vehicle's hull size. Lifting gas must have sufficient volume to expand and achieve a similar density to the surrounding atmosphere, but keeping some overpressure. Correct design and sizing of the airship allow determining a service ceiling by considering that maximum lifting gas volume is reached when the density of the buoyant gas decreases to a minimum value that allows keeping the pressure differential in the hull and does not void the ballonets completely.

The mass of helium used remains constant during a mission, and the density of helium is a function of the density ratio 9and consequently of the altitude. The following equation can predict the airship size:

$$V_{\text{max}} = \frac{m_{\text{gas}}}{\rho_{\text{gas}}} = \frac{m_{\text{gas}}}{\sigma \cdot \rho_{\text{gas},0}} \tag{11}$$

where  $\sigma$  is a function of altitude:

$$\sigma = \frac{\rho_{air,0} \cdot e^{-h^*}}{\rho_{air,0}} = e^{-h^*} = e^{(-h/H)}$$
 (12)

The expression of  $V_{\rm max}$  and L allow determining the size of an airship as a function of ground densities of air and buoyant gas, and total weight to be transported.

#### **Methods**

Inside the airship's lifting gas envelope the pressure decreases uniformly with increasing altitude as the pressure which surrounds the envelope decreases. Since a fixed mass of gas exists inside the airship's hull, any changes in lifting gas density must result from pressure changes. The result is an increase in gas volume as the airship rises. Because very little elasticity is present in conventional envelope materials, the threat of rupture must be considered.

Airships safety requires an effective comprehension on how an airship performs after having its hull damaged by any external cause, including enemy attacks. The actual theory on airships is not sufficient to describe the airship failure in the case of any disruptive event that causes an uncontrolled escape of buoyant gas.

It is necessary to remark the differences between airship and balloons. Both systems use a buoyant gas to provide the lift, but they are distinct from each other. In particular, airships achieve their goals by mean of propulsion systems, aerodynamic shaping, and structural elements for both keeping the shape and adding some rigidity. This paper analyses the behaviour of an airship after the envelope of the hull, which contains lifting gas, has been punctured. The problem is a sufficient comprehension of what impact hull damages could have on an airship, and it allows the possibility of surviving or reaching as much safe as possible the ground.

Different model on the behaviour of airplanes in case of failure has been produced. Some samples have been considered [16, 17, 18, 19, 20]. In the case of airship operations, such literature has not been available. Consequently, potential operators must understand the failure modes and their potential effects on the airworthiness of airships and the possibility of minimising the effect of failures. In particular, a precise model of the different descent models will be produced. This analysis necessarily starts from the available mass of lifting gas contained in the hull envelope. When a hole allows the dispersion of the lifting gas, the lift gradually reduces, and the flight characteristics could degrade. The buoyant force decreases and ceases to maintain the equilibrium with the weight of the airship system. The descent rate will be related to the lifting gas mass flow rate, which depends on both the small overpressures, which is required to maintain the shape and the size of the puncture in its hull.

A typical airship typically has an ellipsoidal shape [21], because it allows minimizing the drag forces when moving horizontally or descending. In addition, this shape has demonstrated to be the fixed shape that achieves the best performances of the propulsion system. A balloon is almost spherical. It has no onboard propulsion and drifts with prevailing wind currents. More recently, Dumas [22] and Trancossi [23, 24, 25] has proposed a variable shape semi-rigid airship structure that allows reducing the aerodynamic drag by changing the volume of the hull and having no internal ballonets.

An airship is generally a controlled system that works to optimize the vehicle's flight. Often control fins are attached to the hull to help direct the vehicle's heading. As well, the previously discussed ballonets are built into the hull to help control ascent and descent as well as trim the vehicle's attitude during flight. The weather balloon lacks this sophistication.

Two different failure modes can be considered:

- buoyancy loss allow maintaining control of the airship during the descent;
- some buoyancy is preserved but can generate a loss of both control and ability to steer against the wind.

In both cases, the result is a descent in equilibrium condition of pressure with the atmosphere around. In some particular conditions the permanence in flight and airworthiness can be preserved.

#### **Gas Dispersion**

The airships hull is inflated with a certain overpressure with respect to the surrounding atmosphere. The internal overpressure allows maintaining the shape, reducing the effect of bending moments and nose deformations produced by the high stagnation pressures. Khoury and Gillett [9] assume that the overpressure, which is required for a non-rigid airship, can be estimated by the empirical formulation

$$\Delta p = 125 + 0.033 \cdot v_{\text{max}}^2 \tag{13}$$

where  $v_{max}$  is the airspeed in km/h. Airship speed v is the sum of both vehicle's inertial velocity and the speed of the headwind. In order to maintain the adequate hull rigidity, it is preferred to keep the overpressure  $\Delta p$  almost constant as altitude changes. In the case of hull failure, the best condition is realized when the structural rigidity can be conserved, and a controlled horizontal motion can be performed for a large part of the descent. As long as some overpressure can be maintained, the airship maintains its aerodynamic behavior. In this case, as the vehicle descends, it can be governed, and the pilot could attempt to recover the vehicle or terminate its flight.

The ideal gas law allows studying the vertical component of the velocity of a damaged airshipideal gas law. In particular, the descent can be described by a series of isothermal steps. In particular, the |buoyant gas volume is a function of lifting gas mass.

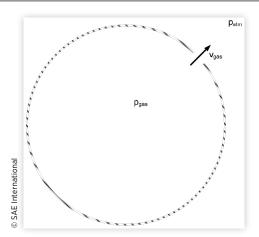
Bernoulli equation for a compressible fluid states that on a streamline of a fluid with incompressible and inviscid motion, the mechanical energy per unit mass is conserved. Because lifting gasses have a high compressibility ratio, Bernoulli's equation can be modified as follows.

$$RT\int \frac{dp}{p} + \frac{v^2}{2} + gz = const \tag{14}$$

Since the pressure inside the airship hull remains uniform, the potential term is neglected, and z is assumed zero.

The dispersion can be evaluated by the well-tested method by Dekker [26, 27]. If the orifice is large enough to allow neglecting the capillarity theory, it is possible to consider the buoyant gas release as an isentropic expansion.

When the gas is released from a closed vessel through a hole or any other opening such as a valve, it is possible to identify two different cases. They depend on the ratio between FIGURE 3 2D sample showing the gas exit from a hole in an airship



upstream pressure and downstream pressure and the isentropic expansion factor according to the following inequality:

$$\frac{P_{gas}}{P_{air}} \le \left[\frac{k+1}{2}\right]^{k/k-1},\tag{15}$$

that states a limiting condition at which the gas velocity has attained the critical condition of the speed of sound in the gas. For different gases, k ranges from 1.09 to 1.41. Therefore, equation (15) ranges from 1.7 to about 1.9. It means that sound speed occurs when the absolute upstream vessel pressure is  $1.7 \div 1.9$  times as high as the absolute downstream pressure. In the case of a leak to the ambient atmosphere, the downstream pressure is the atmospheric pressure. In the case of an airship, this condition cannot be reached because the overpressure inside the hull is quite small.

In this case, the equation for mass flow rate is:

$$\dot{m}_{gas} = CA \sqrt{2\rho_{gas}P_{gas} \left(\frac{k}{k-1}\right) \left[\left(\frac{P_{air}}{P_{gas}}\right)^{2/k} - \left(\frac{P_{air}}{P_{gas}}\right)^{\frac{k+1}{k}}\right]}$$
 (16)

where A is the area of the hole, k is the isentropic coefficient  $k = c_p/c_p$ , and C is a coefficient of discharge that depends on the geometry of the hole.

Assuming for possible buoyant gas it results:

- *Helium:*  $c_p = 5.19 \text{ kJ/(kg K)}$ ;  $c_v = 3.12 \text{ kJ/(kg K)}$  and  $k = c_p/c_v = 1,6635$ ;
- Hydrogen: Helium:  $c_p = 14.32 \ kJ/(kg \ K)$ ;  $c_v = 10.16 \ kJ/(kg \ K)$  and  $k = c_p/c_v = 1,41$ ;

If we consider ideal gas law, it is possible to define the following equivalent form:

$$\dot{m}_{gas} = CAP_{gas} \sqrt{\frac{2M}{ZRT_{air}} \left(\frac{k}{k-1}\right) \left[ \left(\frac{P_{air}}{P_{gas}}\right)^{2/k} - \left(\frac{P_{air}}{P_{gas}}\right)^{\frac{k+1}{k}} \right]}$$
(17)

in addition, the lift becomes

$$L(t) = \left(m_{gas} - \dot{m}_{gas} \cdot t\right) \left(\frac{\rho_{air,0}}{\rho_{gas,0}} - 1\right) \cdot g. \tag{18}$$

Equation (18) can be also expressed as:

$$L(t) = L_{\text{max}} - \dot{m}_{\text{gas}} \cdot t \cdot \left(\frac{\rho_{\text{air},0}}{\rho_{\text{gas},0}} - 1\right) \cdot g = L_{\text{max}} - \frac{dL}{dt} \cdot t \quad (18')$$

Consequently, the maximum lift is

$$L_{\text{max}} = m_{\text{gas}} \cdot \left(\frac{\rho_{\text{air},0}}{\rho_{\text{gas},0}} - 1\right) \cdot g$$

moreover, the rate of change of the lift force is:

$$\frac{dL}{dt} = \dot{m}_{gas} \cdot \left(\frac{\rho_{air,0}}{\rho_{gas,0}} - 1\right) \cdot g \tag{19}$$

It is evident that the lift rate reaches its minimum and cannot be null in the presence of a hole

$$\frac{dL}{dt} = \dot{m}_{gas} \cdot \left(\frac{\rho_{air,0}}{\rho_{gas,0}} - 1\right) \cdot g = 0 \rightarrow \begin{cases} \dot{m}_{gas} = 0\\ \rho_{air,0} = \rho_{gas,0} \end{cases}$$

Equation (11) it can be written as follows

$$m_{gas} = \rho_{gas} \cdot V_{max} = \sigma \cdot \rho_{gas,0} \cdot V_0$$
.

Consequently, the lift components become

$$L_{\max} = \rho_{gas} \cdot V_{gas} \cdot \left(\frac{\rho_{air,0}}{\rho_{gas,0}} - 1\right) \cdot g = \sigma \cdot V_{gas} \cdot \left(\rho_{air,0} - \rho_{gas,0}\right) \cdot g$$

$$L(t) = L_{\max} - \frac{\dot{m}_{gas}}{\rho_{gas,0}} \cdot t \cdot \left(\frac{\rho_{air,0}}{\rho_{gas,0}} - 1\right) \cdot g = L_{\max} - \frac{dL}{dt} \cdot t$$

$$L(t) = L_{\max} - \dot{m}_{gas} \cdot t \cdot \left(\frac{\rho_{air,0}}{\rho_{gas,0}} - 1\right) \cdot g = L_{\max} - \frac{dL}{dt} \cdot t$$

#### Flight Model with Gas Losses

According to Koury and Gillett, it can be assumed that

- the volume of lifting gas is equivalent to the volume of displaced air;
- ballonets
   cannot deflate completely to maintain
   internal overpressure;
- ballonet size can extend to 40 % the overall hull volume when fully expanded at max altitude.

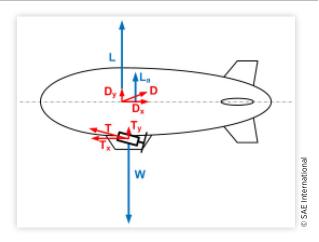
These principles will be applied to the analysis of vehicle motion.

Figure 4 shows the forces acting on an airship during the descent. The above system of force allows determining the cinematic parameters as a function of the isentropic dispersion of the aerostatic gas. Equation (18') describes the buoyant force)L. Aerodynamic drag and lift (if they exist) are proportional to both the atmospheric density and the airspeed u.

$$m \cdot a_x = \sum F_x = T_x - D_x \tag{20}$$

$$m \cdot a_v = \sum F_v = -W + T_v + D_v + L(t) + L_a$$
 (21)

**FIGURE 4** Forces acting on an airship.



where the following magnitudes have been used:

$$D_{x} = \frac{1}{2}C_{Dx}\rho_{air}A_{f}u_{x}^{2}; D_{x} = \frac{1}{2}C_{Dy}\rho_{air}A_{f}u_{y}^{2}; Ly = \frac{1}{2}C_{L}\rho_{air}A_{f}u_{x}^{2}$$

#### Case 1: Maintaining Hull Overpressure

It is evident that two different descent modes can be studied. They are namely vertical descent and oblique governed descent. The choice between the two modes depends on both the altitude and the dimensions of the hole. The most important parameter is a correct comprehension on how lifting gas volume changes, by a systematic methodology, which is defined by height and time intervals that allows keeping temperature almost constant in each step.

A mass balance will provide the descensional acceleration, and the vehicle will experience as lifting gas flows from the hull. In particular, by keeping into account <u>equation (21)</u>, it is possible to analyse the vertical motion of the airship.

The airship hull produces a drag force on the descending vehicle. For simplicity, the airship's  $C_D$  in a vertical direction is assumed to be that of a cylinder ( $C_D$  = 0.3). Vertical acceleration can be determined through an accurate estimation of the dispersion rate of the buoyant gas. It is also assumed the design condition that maximum gas volume does not exceed 60% of the total volume of the hull.

The analytical model assumes that overpressure is constant. In this case, there is only one unknown quantity. Once volume can be defined, the analysis proceeds quickly.

## **Case 2: Pressure Equalization**

Once this limit is reached, the vehicle can no longer maintain constant overpressure. The pressure inside the hull will begin to fall until it reaches atmospheric pressure. When pressure is equalized, it is necessary to develop a specific model for analyzing the descent and predicting the lifting gas loss. In this case, both hull pressure and volume changes in volume must be considered. This methodology is developed in the following case.

While overpressure decreases until it reaches the same pressure of the surrounding atmosphere, the loss rate of lifting gas is lower than in the case of constant overpressure. Some increased buoyancy is maintained, and despite some reduction in structure effectiveness, the vehicle may remain at a higher altitude where favourable wind conditions exist to allow some down-range navigation to a suitable recovery/termination location. This case presents some similitude with an atmospheric balloon. The main difference is that a hull overpressure still exists with uniform pressure distributed across the envelope. Hence, the airship is still capable of some sustained navigation. The buoyant gas mass flow is represented by both the ideal gas equation and summing the contribution of the volume change at constant pressure and the pressure change at constant volume.

$$\dot{m} = \frac{dp}{dt} \frac{V}{R_{gas}T} + \frac{dV}{dt} \rho_{gas}$$
 (22)

According to Vogt, <u>Equation (22)</u> is not rigorous but describes how pressure and volume change as the airship depressurises and descends with a reasonable error. In reality, mass flow can be determined precisely by <u>equation (17)</u>, and the mass flow rate <u>equation (22)</u> can be used to determine how the mass flow rate of buoyant gas determines the rates of change for both hull pressure and volume as the airship reduces the altitude.

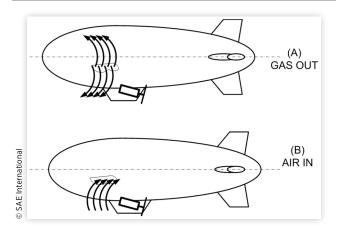
The model requires a second equation that states the different contribution of the differential terms of volume and pressure to the total mass loss rate. We assume (for simplicity) that the contribution of both pressure and volume is equal. Vogt claims that sensitivity tests allow establishing a weight factor of 0.33 for the pressure change rate and 0.67 for the volume change rate.

#### **Isobaric Behaviour**

The cases, which have been previously considered, assume that the airship presents during the entire descent some overpressure. This section considers which the case in which the overpressure cannot be maintained with the consequent total loss of any overpressure.

In this case, as Lewitt [28] remarks the airship maintains a mass of lifting gas creating a buoyant force, but it losses its structural rigidity to neither adequately support its payload nor maintain any aerodynamic shaping and losses any horizontal flight capability. The pressures of lifting gas and atmosphere are almost identical, and the density of the lifting gas depends solely on atmospheric pressure. As the airship descends, it can be possible to describe the buoyant gas release process as diffusive and assume the lifting gas and atmospheric gasses are stratified in the hull. At a specific moment of the descent, the lifting gas is no longer flowing out of the hull, and the mass does not change, the occupied volume decreases and its density increases. It behaves such as a bubble that occupies the top portion of the unpressurized hull is subject to "sloshing" such as a bubble of air trapped underwater. In the analysis, it can be assumed that only descent can be modelled with any confidence and horizontal motion is FIGURE 5 Isobaric gas losses and pressure equalisation:
(A) Atmospheric gasses cannot enter hull because of lifting gas overpressure and lifting gas flow out of the hull.

(B) Atmospheric gasses begin to enter the airship hull after pressure has equalized resulting in stratified layers of lifting and atmospheric gasses.



assumed to be controlled mostly by prevailing atmospheric currents.

Descent analysis is straightforward. The ideal gas relationship is used to predict lifting gas density changes as the hull pressure changes. Because the hull pressure is essentially equal to the atmospheric pressure, an atmospheric scale height prediction of pressure provides the necessary insight to complete the analysis. The mass flow is not required for this portion of the descent, as we have shown that lifting gas is no longer expelled from the airship as it descends. Since no additional gas is lost, the vehicle will maintain a constant buoyant force for the remainder of its descent up to the airship eventually falls to the ground.

Some difficulty relates to the structural inconsistency of the hull, which creates a substantial uncertainty about an adequate determination of the coefficient of drag because of the lack of rigidity in the hull complicates the vehicle's geometry. In the previous cases, the CD of a cylinder has been assumed during the descent. It is acceptable if a sufficient overpressure existed within the hull to maintain its shape roughly similar to a cylinder. The sloshing of the lifting gas bubble inside the hull makes it hard to predict a constant drag coefficient. For the sake of simplicity, in this case, a drag coefficient similar to that used previously can be used again in this analysis. In this way, we are neglecting impacts of the sloshing gas bubble has on vehicle's center of gravity and vehicle pitch angle during descent. The only force applied horizontally is assumed to the prevailing atmospheric winds.

Another consideration must focus on the gas volume, which is characterized by a hydrostatic pressure gradient, which was negligible in the other cases. In particular, the behaviour of the airship, which drifts as a balloon, is difficult to be predicted. It can be concluded that the system behaviour is challenging to be predicted, even if some modelling can be produced by roughly considering it as an atmospheric balloon.

#### **Jump Risk Unloading Payload**

After the descent, another risk must be accounted. It relates to the possible aerostatic jump that may occur after disembarking people and payload from the airship. A tragic sample of this behaviour has been the Goodyear airship disaster in 2011 [29, 30].

The weight of the airship is reduced, and if the buoyant gas is not expelled in an adequate quantity, uncontrollable events such as aerostatic jumps may happen. If the reduction of payload brings to

$$W - \Delta W m \le L(t) \tag{23}$$

it is evident that the vehicle can jump. Those phenomena may lead to a structural failure, which is caused by the vertical acceleration. Usually, height is gained suddenly up to the structural break, and then an accelerate fall makes the airship crash on the ground. Those phenomena can take place only when the airship reaches the ground. They are governed by equation (21), modified as follows:

$$m \cdot a_v = -(W - \Delta W) - D_v + L(t) \tag{24}$$

It is consequently evident that the risk can be avoided or at least minimized by mean of a sufficient preliminary rest on the ground to allow releasing an adequate amount of gas.

#### **Test Case**

The objective of this research is an accurate study of the forced descent modes of a damaged airship. The time and trajectories are estimated in different scenarios in order to allow an effective and rapid estimation of how a damaged airship could reach a recovery/termination location safely.

In particular, different accident altitudes are assumed and the two scenarios in which it is possible to keep some hull overpressure for the entire descent and to have a fast equalization of pressure are estimated. In particular, the conditions for keeping as long as possible the overpressure are estimated.

### **Analysis of the Results**

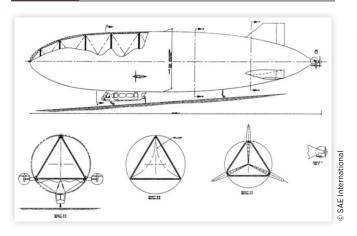
#### Reference Airship

A first step has been producing an accurate conceptual design of the airship, which is parameterized with maximum altitude. The design has been performed according to the methodology by Pant [31].

A preliminary, even if basic operation and design synthesis of an airship are based on a well-identified set of parameters that affect the operation and configuration of airships and performance requirements.

Pressure altitude and atmospheric properties have a direct influence on the volume and surface of the airship and the payload capacity. The volume of the hull and one of the internal ballonets are determined by assuming the pressure altitude and the minimum operating altitude.

FIGURE 6 Zeppelin NT drawing (Zeppelin GmbH)



It has been considered as a reference airship the Zeppelin NT because of the broad availability of data. Reference data have been reassumed in <u>Table 1</u>, and a drawing is reported in <u>Figure 7</u>.

Ballonet volume ratio from different airships has been studied by Pant. The results have been reported as a function of pressure altitude of the airship (<u>Figure 6</u>).

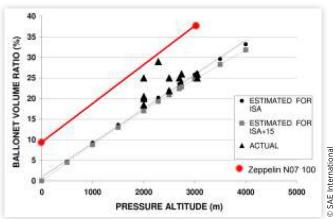
According to Figure 6, allows understanding that the initial hypothesis by Khoury and Gillett of a volume of ballonets can be justified in the phases of preliminary design when some uncertainty about weights exist but are much lower in case of real airships. Into real operations, according to Pant, an average value of around 25% for an airship with a pressure altitude of 3 km can be assumed.

In the case on Zeppelin NT, the volume of ballonets is in the range from 9.5% to 37.5%. The net volume of the buoyant gas varies between about 5280 and 7650 m<sup>3</sup>. The ballonet volume ratio has been evaluated for Zeppelin NT, and it has been integrated into <u>Figure 6</u>. The overpressure inside the hull varies between 300 and 600 Pa.

**TABLE 1** Zeppelin NT Relevant Data for the problem [33]

Magnitude	Value	Unit
Hull volume	8450	m³
Length	70.1	m
Diameter	14.2	m
max Width	19.5	m
Height	19.4	m
Max ballonet volume	1065-1600	m³
Min ballonet volume	400-600	$m^3$
Ballonets	2	-
Max permissible lifting gas pressure	600	Pa
Min lifting gas pressure	300	Pa
Airspeeds Maneuverings Speed	83	km/h
Maximum altitude	3048	m
Never Exceed Speed VNE	130	km/h
Max payload	1900	kg
Maximum Takeoff Weight	10690	kg

FIGURE 7 Ballonet volume ratio for different airships including Zeppelin NT (adapted from Pant [31])



### **Times of Operations**

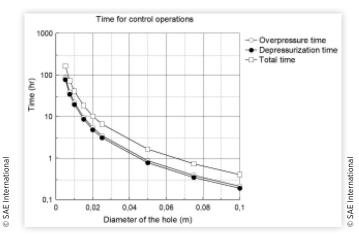
The modelling will provide data on speed, range, and altitude for an airship while varying the size of the hole in the lifting gas envelope. In particular, it has been estimated the time for hull depressurisation, which provides ample time for manoeuvring the airship and navigating it down range.

The results illustrated in <u>Figure 8</u> provide the model's estimations of the airship's ability to be successfully navigated for multiple diameters hull puncture and attempts by operators to maintain a constant overpressure in the hull for as long as possible.

<u>Figure 8</u> presents model results for the minimum safe time during which overpressure can be maintained. A variety of hole sizes from 5 to 100 mm have been considered. It is assumed that vertical velocity for the airship remains constant during after hull damage for the time, during which constant overpressure can be maintained.

After overpressure cannot be maintained, the model indicates the airship would have another period nearly as long

**FIGURE 8** Estimation of times in which overpressure clan be kept, times of depressurisation and the overall time during which control can be achieved for different diameters of the hole.



in which its hull overpressure would decrease until it reaches atmospheric pressure.

The pressure begins to fall when the ballonets have reached the maximum volume. The descent speed starts to reduce. During the depressurisation phase, the available times for safe manoeuvring operations have been evaluated.

It was stated empirically that an airship with a small hole in its hull would have some time to manoeuvre before it is lost and the model bears this out.

It must be remarked that after losing the overpressure and the start of the depressurisation stage, the airship would be capable of maintaining a decreasing velocity related to its instantaneous overpressure, further extending its possible range.

For a hole with a diameter of 20 millimetres, the airship is expected to be controllable for around 10 hours. Indications are the airship would have more than 10 hours of controllable flight, at least 5 of which would be with the hull at designed overpressure. It would provide operators with the potentiality of travelling not less than 750 kilometers in conditions that allows at least essential manoeuvring and reach the final recovery base.

This result allows the airship to reach the recovery location in a reasonable time at a constant even if continually reducing speed (during the depressurisation period).

When airship loses pressure and equals the atmospheric one the airship can continue its descent with minimal control. In this condition, the hull loses its aerodynamic integrity. It starts to deform with the only support of the three structural composite beams that constitute the semi-rigid structure (Figure 6). For a blimp, the effects of the pressure equalisation are less impacting because the aerodynamic forces have lower effects on its forward edge and keep some capability of maintaining some fineness ratio.

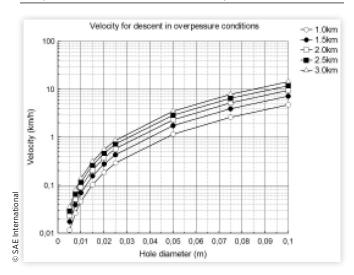
It is necessary to remark that further analyses beyond this point depend on unpredictable factors, the vehicle losses most of its capability of controlled flight and drifts with prevailing winds, which effects can be considered detrimental regarding controlled flight and recovery of the damaged vehicle.

#### **Vertical Speed Analysis**

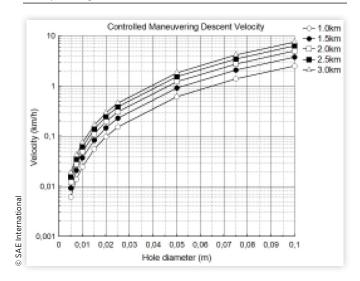
A consequence of the above results is the possibility of determining the ideal average speed component during the descent, which can be governed with the support of propulsion. Optimal vertical component of speed has been consequently evaluated for different dimensions of the holes. For different altitudes, vertical speed for a controllable descent has been estimated.

The vertical speed has been evaluated under two different conditions: the first allows keeping constant overpressure; the second considers both overpressure and pressure reduction phases. Minimum velocities that allow keeping some overpressure have been plotted in <u>Figure 9</u>. The ones that allow performing a controllable descent are presented in <u>Figure 10</u>. Different initial altitudes have been considered, and they are namely 3 km, 2,5 km, 2 km, 1,5 km, 1 km.

**FIGURE 9** Descent velocity that allows keeping overpressure inside the hull of the airship



**FIGURE 10** Descent velocity that allows controlling the airship during the descent.



#### Effects of Buoyant Gas Impurities

It must be remarked that buoyant gas is never a pure gas. It contains different gaseous impurities, which are caused by both the original gas furniture and the effects of permeability through the envelope. In particular, it can be assumed that the density is the averaged value of the density of buoyant gas and the ones of the impurities.

It is evident, also considering the results by Trancossi [35] and verifying from the effective furniture conditions for both Helium and Hydrogen it can be assumed that effective lift can be assumed around 10% lower than the calculated one.

# Avoidance of the Aerostatic Jump

The last numerical analysis presented regards the necessity of avoiding the aerostatic jump when passengers, crew freight, or instrumental payload are released on the ground.

In this case, it is necessary to make a precise evaluation of the system weights. The maximum payload that can be landed is around 1900 kg. The remaining mass of the vehicle can be estimated around 8 tons and remaining fuel.

These values show clearly that Zeppelin NT, if compared to non-rigid blimps, has a very low risk of an aerostatic jump, also because it appears evident that the aerostatic system is designed to lift the empty system weight and not the payload.

## **Analysis of the Results**

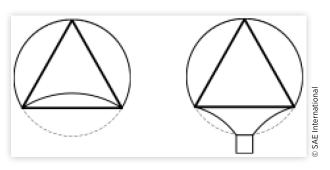
The obtained results by thermodynamic transformations appear congruent with the ones, which have been computed by Vogt [8], who used a gas dispersion model based on Bernoulli's law. In any case, it must be remarked that the obtained results are reasonable, even if affected by large uncertainty areas:

- the effects of leading-edge pressure,
- the variability of atmospheric wind direction and intensity.

The model has been evaluated for different sizes of the holes. An effective reduction of the times for airship recovery has been evidenced as the size of the hole in the hull increases. It must be remarked that the size of the holes must be observed. The range capability and the necessary time for safely reaching the ground are reduced by increasing the size of the hull. It is consequently necessary that the operators can decide effectively in reduced times about the airship descent in case of any problem.

If referred to the analysis by Vogt, it must be remarked that the positive effect of altitude during the pressure equalization phase, which is appreciable for airships operating in the stratosphere are almost null. It is also necessary to observe that the differences in the airship architecture for the

FIGURE 11 Samples of possible deformation of the lower surface of the hull as limited by the internal structure in the Zeppelin NT



non-rigid blimp, which has been considered by Vogt, create a different behaviour that is not accounted for now.

It must be remarked that the internal structure during uncontrollable vertical descent will ensure some "parachute-like" shaping of the lower surface of the hull, which has not been evaluated in this paper. Those deformations cause an increase of the vertical aerodynamic drag component during the descent.

#### **Conclusions**

This paper presents a useful estimation of the capability of a semi-rigid airship to reach the ground and to land passengers, freight, and crew in case of a failure of the hull.

Maximum time for reaching the ground and the recovery site has been estimated, and the optimal average vertical speed has been calculated. It has not been possible to determine the horizontal speed and necessary thrust because they depend on random variables such as wind intensity and directions. It can be concluded that a question about optimal trajectories during controlled flight is still opened. It is not a fundamental problem while a constant hull pressure could be maintained because the airship can move with a traditional movement up to the overpressure can be maintained. Then, during the depressurisation phases, decreasing speeds are necessary. During this period, the speed decreases in a way that is nearly proportional to the reduction of the overpressure up to it fall to zero. In the last part, some ability to not governable descent is ensured.

With respect to previous literature, this paper describes the gas according to the isentropic expansion model that is expected to produce much more accurate results. The uncertainty still affects the hole coefficient which has been considered to have the reasonable value of 0.71, even if a much more accurate analysis of the hole shaping can ensure a much higher precision. Final considerations regard the behaviour of the airship after pressure equalisation analysing the differences between a semi-rigid airship and a non-rigid blimp such as the ones, which have been considered by previous literature. It is expected that this analysis can support future research about increasing the general safety of airship operations.

#### References

- CORDIS, Multibody Advanced Airship for Transport, CORDIS (Community Research and Development Information Service), European Commission, Brussels, 2011 <a href="http://cordis.europa.eu/search/index.cfm?fuseaction=proj.document&PJ\_LANG=EN&PJ\_RCN=12149444&pid=8&q=1DBC26FEDA78A901E7C4E7F31040F180&type=adv.">http://cordis.europa.eu/search/index.cfm?fuseaction=proj.document&PJ\_LANG=EN&PJ\_RCN=12149444&pid=8&q=1DBC26FEDA78A901E7C4E7F31040F180&type=adv.</a>
- CORDIS, MAAT Project Multibody Advanced Airship for Transport - Has Started, CORDIS Wire, CORDIS (Community Research and Development Information Service), European Commission, Brussels, 2011, <a href="http://cordis.europa.eu/wire/index.cfm?fuseaction=article.">http://cordis.europa.eu/wire/index.cfm?fuseaction=article.</a>
   Detail&rcn=28667.

- 3. Dumas, A., Trancossi, M., Madonia, M., and Giuliani, I., "Multibody Advanced Airship for Transport," SAE Technical Paper 2011-01-2786, 2011, doi:10.4271/2011-01-2786.
- Dumas, A., Madonia, M., Giuliani, I., and Trancossi, M., "MAAT Cruiser/Feeder Project: Criticalities and Solution Guidelines," SAE Technical Paper 2011-01-2784, 2011, doi:10.4271/2011-01-2784.
- 5. Topping, D. In: Eric Brothers, editor. When Giants Roamed the Sky. (Akron, OH, University of Akron Press, 2001).
- 6. Whale, G., *British Airships, Past/Present/Future* (London: Blackmask Online, 2002).
- 7. Ridley-Kitts, D.G., Military, Naval and Civil Airships since 1783: The History and Development of the Dirigible Airship in Peace and War (UK: The History Press, 2012).
- 8. Vogt, C.W., "Performance Capability of a Damaged Lighter-Than-Air Vehicle," MS thesis, Department of Aeronautics and Astronautics, Graduate School of Engineering and Management, Air Force Institute of Technology, 2006.
- 9. Khoury, G.A. and Gillett, J.D., *Air Ship Technology* (Cambridge University Press, 1999).
- Mueller, J.B., Paluszek, M.A., and Zhao, Y., "Development of an Aerodynamic Model and Control Law Design for a High Altitude Airship," "Unmanned Unlimited" Technical Conference Workshop and Exhibit, AIAA-2004-6479, Chicago IL, Sept. 20-23, 2005.
- Kreider, J.F., "Mathematical Modeling of High Altitude Balloon Performance," AIAA 5th Aerodynamic Deceleration Systems Conference, AIAA-75-1385, Albuquerque, NM, Nov 17-19, 1975.
- 12. Zucker, R.D. and Biblarz, O., *Fundamentals of Gas Dynamics* Second Edition (Wiley Books, 2002).
- Bohren, C.F. and Albrecht, B.A., Atmospheric Thermodynamics (New York: Cambridge University Press, 1998).
- 14. Moran, M. and Shapiro, H., Fundamentals of Engineering Thermodynamics Fourth Edition (Wiley, 2000).
- VV.AA., "U.S. Standard Atmosphere," OAA, NASA, USAF, U.S. Government Printing Office, 1976, Washington, DC NOAA-S/T 76-1562. Part 1, 1976.
- Campbell, G.S. and Lahey, R., "A Survey of Serious Aircraft Accidents Involving Fatigue Fracture," *International Journal* of Fatigue 6(1):25-30, 1984.
- Bigün, E.S., "Risk Analysis of Catastrophes Using experts' Judgements: An Empirical Study on Risk Analysis of Major Civil Aircraft Accidents in Europe," *European Journal of Operational Research* 87(3):599-612, 1995.
- 18. Bristow, J.W. and Irving, P.E., "Safety Factors in Civil Aircraft Design Requirements," *Engineering Failure Analysis* 14(3):459-470, 2010.
- 19. Ren, Y. and Xiang, J., "A Comparative Study of the Crashworthiness of Civil Aircraft with Different Strut Configurations," *International Journal of Crashworthiness* 15(3):321-330, 2010.
- Tiffany, C.F., Gallagher, J.P., Babish, I.V., and Charles, A., "Threats to Aircraft Structural Safety Including a Compendium of Selected Structural Accidents/Incidents," No. ASC-TR-2010-5002, 2010.

- 21. Wang, X.L. and Shan, X.X., "Shape Optimization of Stratosphere Airship," *Journal of Aircraft* 43(1):283-286, 2006
- Dumas, A., Madonia, M., Trancossi, M., and Vucinic, D., "Propulsion of Photovoltaic Cruiser-Feeder Airships Dimensioning by Constructal Design for Efficiency Method," SAE Int. J. Aerosp. 6:273-285, 2013, doi:10.4271/2013-01-2303.
- Trancossi, M., Dumas, A., Madonia, M., Pascoa, J. et al., "Fire-Safe Airship System Design," SAE Int. J. Aerosp. 5(1):11-21, 2012, doi:10.4271/2012-01-1512.
- 24. Trancossi, M., Dumas, A., and Madonia, M. "Energy and Mission Optimization of an Airship by Constructal Design for Efficiency Method," ASME International Mechanical Engineering Congress and Exposition, American Society of Mechanical Engineers, 2013, 2013, V001T01A021-V001T01A021.
- Trancossi, M., Dumas, A., and Madonia, M., "Optimization of Airships with Constructal Design for Efficiency Method," SAE Technical Paper <u>2013-01-2168</u>, 2013, doi:<u>10.4271/2013-</u> 01-2168.
- Deckker, B.E.L., "Compressible Flow through Square Edge Rectangular Orifices," Proceedings of the Institution of Mechanical Engineers 192(1):277-288, 1978.
- 27. Deckker, B.E.L. and Chang, Y.F., "Paper 19: Slow Transient Compressible Flow through Orifices," *Proceedings of the Institution of Mechanical Engineers, Conference Proceedings* 182(8):175-183, 1967.
- 28. Lewitt, E.H., The Rigid Airship; a Treatise on the Design and Performance (London: New York Sir I. Pitman & Sons, 1925).
- 29. Allen, H., "Pilot Killed Goodyear Blimp Plunges Ground Flames in Germany," *Daily Mail*, 2011 <a href="http://www.dailymail.co.uk/news/article-2002935/Pilot-killed-Goodyear-blimp-plunges-ground-flames-Germany.html">http://www.dailymail.co.uk/news/article-2002935/Pilot-killed-Goodyear-blimp-plunges-ground-flames-Germany.html</a>.
- 30. Croft, J., "Goodyear Blimp 'Overloaded' in Germany Crash," *Aviation Week and Space Technology*, 2013 <a href="http://aviationweek.com/awin/goodyear-blimp-overloaded-germany-crash">http://aviationweek.com/awin/goodyear-blimp-overloaded-germany-crash</a>.
- 31. Pant, R., "A Methodology for Determination of Baseline Specifications of a Non-rigid Airship," *AIAA's 3rd Annual Aviation Technology, Integration, and Operations (ATIO) Forum*, 2003, 6830.
- 32. VV.AA., "EASA Type Certificate EASA.AS.001", EASA European Aviation Safety Agency, 5, 2014. https://www.easa.europa.eu/sites/default/files/dfu/EASA-TCDS-AS.001
  Zeppelin LZ N07--100-05-05082014.pdf
- 33. Dumas, A., Trancossi, M., and Madonia, M., "Effect of Altitude and Temperature on Volume Control of an Hydrogen Airship," ASME International Mechanical Engineering Congress and Exposition: Fluids and Heat Transfer, Parts A, B, C, and D 7:499-512, 2012, doi:10.1115/IMECE2012-87575.

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## **Acknowledgments**

This paper is a follow-up of the research activity, which has been performed during the MAAT (Multibody Advanced Airship for Transport) Project, which was founded by EC FP7 Transport with Project ID: 285602.

### **Definitions/Abbreviations**

 $\Delta p$  - internal hull overpressure [Pa]

 $\rho$  - density [kg/m<sup>2</sup>]

 $\sigma$  - rate of change of gas density [-]

 $\rho_{air}$  - density of air [kg/m<sup>3</sup>]

 $\rho_{air,0}$  - density of air at sea level [kg/m<sup>3</sup>]

 $\rho_{gas}$  - density of buoyant gas [kg/m<sup>3</sup>]

 $\rho_{ga,0}$  - density of buoyant gas at sea level [kg/m<sup>3</sup>]

 $c_D$  - drag coefficient [-]

c<sub>L</sub> - lift coefficient [-]

**h** - height [m]

 $h_{max}$  - maximum altitude [m]

h\* - height constant [-]

M - mass of the airship [kg]

 $m_{gas}$  - mass of buoyant gas [kg]

**p** - pressure [Pa]

 $p_{air}$  - pressure of air [kg/m<sup>3</sup>]

 $p_{air,0}$  - pressure of air at sea level [kg/m<sup>3</sup>]

 $p_{gas}$  - pressure of buoyant gas [kg/m<sup>3</sup>]

 $p_{ga,0}$  - pressure of buoyant gas at sea level [kg/m<sup>3</sup>]

W - molar mass [g/mol]

D - drag force [N]

L - lift force [N]

 $L_a$  - Aerodynamic lift [N]

**R** - gas constant  $[8.31432 \times 10^3 \ N \ m \ kmol^{-1} \ K^{-1}]$ 

 $R_{air}$  - gas constant for air [287.05 J/kg K]

*T* - absolute temperature [K]

 $V_{air}$  - Volume of air ballonets [m<sup>3</sup>]

 $V_{air}$  - Volume of air ballonets [m<sup>3</sup>]

 $V_{gas}$  - Volume of buoyant gas in the hull [m<sup>3</sup>]

 $V_{gas,max}$  - Maximum volume of buoyant gas in the hull [m<sup>3</sup>]