

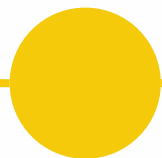
A high-magnification, grayscale photograph of a microchip's surface, showing a grid of square dies and intricate circuit patterns.

LOGIC DESIGN

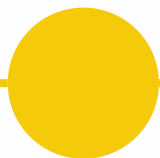
Chapter 3

Gate-Level Minimization

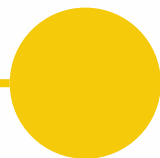
OUTLINE OF CHAPTER 3



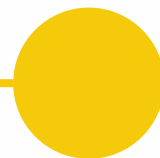
The Map
Method



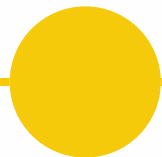
Four – Variable
K-Map



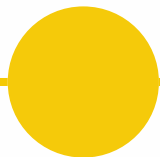
Product of
Sums
Simplification



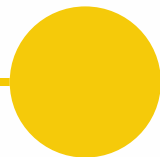
Don't Care
Conditions



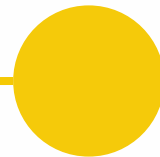
NAND and NOR
Implementations



Other Two
Level
Implementations



Exclusive-OR
Function



Hardware
Description
Language



3.1 THE MAP METHOD

THE MAP METHOD

- **Gate-level minimization** refers to the design task of finding an optimal gate-level implementation of Boolean functions describing a digital circuit.

THE MAP METHOD

- The complexity of the digital logic gates
 - The complexity of the algebraic expression
- Logic minimization
 - Algebraic approaches: lack of specific rules
 - **The Karnaugh map**
 - A simple straight forward procedure
 - A pictorial form of a truth table
 - Applicable if the # of variables < 7

THE MAP METHOD

- A diagram made up of squares
 - Each square represents one minterm
- The simplified expression produced by the map are always in one of the two standard forms:
 - Sum of products
 - Product of sums

THE MAP METHOD

- Boolean function
 - Sum of minterms
 - Sum of products (or product of sum) in the simplest form
 - A minimum number of terms
 - A minimum number of literals
 - The simplified expression may not be unique

THE MAP METHOD

- A two-variable map
 - **Four** minterms
 - $x' = \text{row } 0$; $x = \text{row } 1$
 - $y' = \text{column } 0$; $y = \text{column } 1$
 - A truth table in square diagram
 - xy

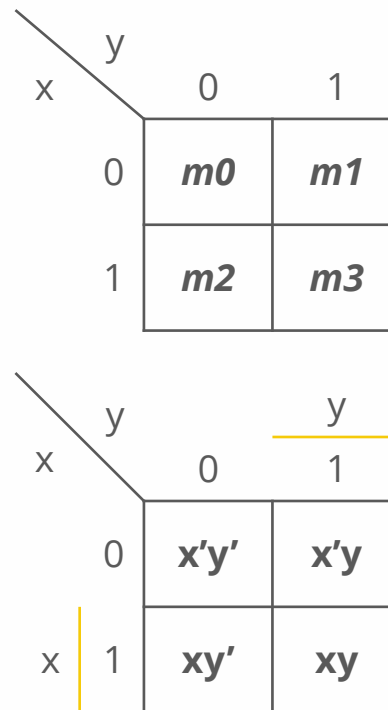


Figure 3.1 Two-variable Map

THE MAP METHOD

- Fig. 3.2(a):
 - $xy = m_3$
- Fig. 3.2(b):
 - $x+y = x'y+xy' +xy = m_1+m_2+m_3$

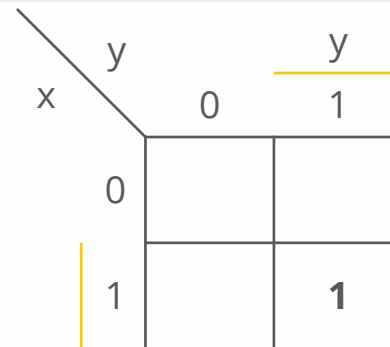
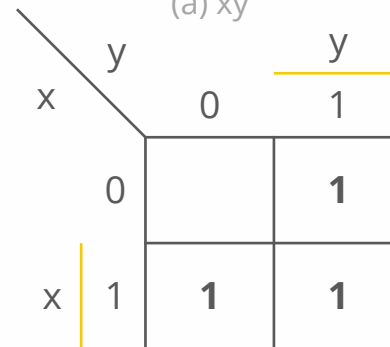
(a) xy (b) $x+y$

Figure 3.2 Two-variable Map

THE MAP METHOD

- Three-variable map:
 - For **3** binary variables.
 - **$2^n = 8$** minterms.
 - Map consists of **8** squares.
 - Minterms are not arranged in a binary sequence.
 - Only one bit changes in value from one adjacent column to the next.

		y		y	
X	x	0 0	0 1	1 1	1 0
	0	<i>m0</i>	<i>m1</i>	<i>m3</i>	<i>m2</i>
	1	<i>m4</i>	<i>m5</i>	<i>m7</i>	<i>m6</i>

		yz		y	
X	x	0 0	0 1	1 1	1 0
	0	<i>x'y'z'</i>	<i>x'y'z</i>	<i>x'yz</i>	<i>x'yz'</i>
	1	<i>xy'z'</i>	<i>xy'z</i>	<i>xyz</i>	<i>xyz'</i>

z

THE MAP METHOD

- Any two adjacent squares in the map differ by only one variable
 - Primed in one square and unprimed in the other
 - e.g., m_5 and m_7 can be simplified
 - $m_5 + m_7 = xy'z + xyz = xz(y' + y) = xz$

		y		y	
		0 0	0 1	1 1	1 0
x	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

		yz		y	
		0 0	0 1	1 1	1 0
x	0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
	1	$xy'z'$	$xy'z$	xyz	xyz'

z

THE MAP METHOD

- m_0 and m_2 (m_4 and m_6) are adjacent
- $m_0 + m_2 = x'y'z' + x'yz' = x'z'$
($y' + y$) = $x'z'$
- $m_4 + m_6 = xy'z' + xyz' = xz'(y' + y)$
= xz'

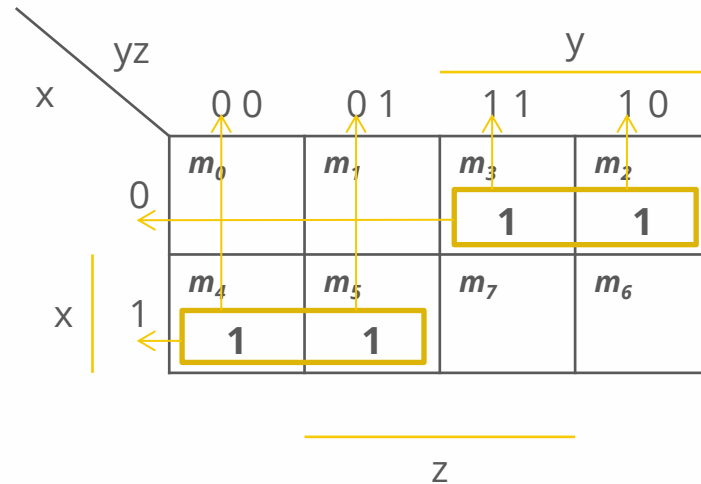
		y		y	
		0 0	0 1	1 1	1 0
x	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

		yz		y	
		0 0	0 1	1 1	1 0
x	0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
	1	$xy'z'$	$xy'z$	xyz	xyz'

z

THE MAP METHOD

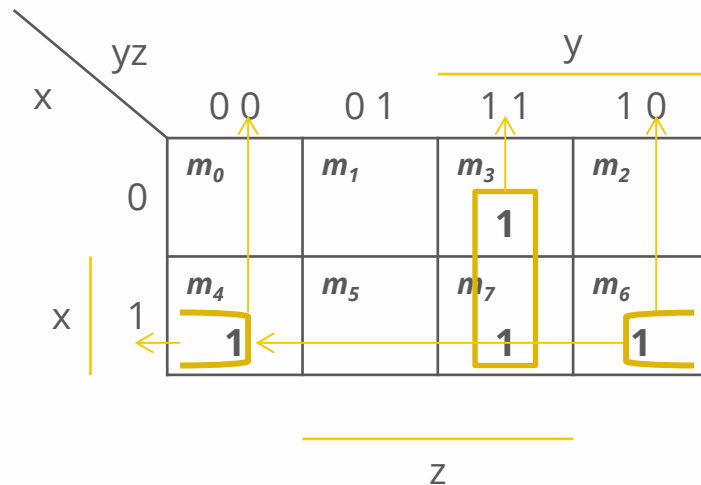
- Example 3.1: Simplify the Boolean function $F(x, y, z) = \Sigma(2, 3, 4, 5)$



$$F = \mathbf{x'y + xy'}$$

THE MAP METHOD

- Example 3.2: Simplify the Boolean function $F(x, y, z) = \Sigma(3, 4, 6, 7)$



$$F = yz + xz'$$

THE MAP METHOD

- Consider four adjacent squares in the three-variable map.
- Any such combination represents the logical sum of four minterms and results in an expression of only one literal.
- The number of adjacent squares that may be combined
 - power of two
 - 1,2,4 and 8.
- Larger number of adjacent squares
 - Product term with fewer literal

THE MAP METHOD

- $m_0 + m_2 + m_4 + m_6 = \mathbf{x' y' z' + x' y z' + x y' z' + x y z'}$
 $= x'z'(y'+y) + xz'(y'+y) = x' z' + x z' = \mathbf{z'}$
- $m_1 + m_3 + m_5 + m_7 = \mathbf{x' y' z + x' y z + x y' z + x y z}$
 $= x' z (y' + y) + x z (y' + y) = x' z + x z = \mathbf{z}$

		yz					
				y			
x		00	01	11	10		
	0	m_0 $x'y'z'$	m_1 $x'y'z$	m_3 $x'yz$	m_2 $x'yz'$		
x	1	m_4 $xy'z'$	m_5 $xy'z$	m_7 xyz	m_6 xyz'		
				z			

THE MAP METHOD

- **1** square represents **1** minterm
 - A Term of **3** literals.
- **2** adjacent squares
 - A term of **2** literals.
- **4** adjacent squares
 - A term of **1** literal.
- **8** adjacent squares
 - Entire map
 - Function **$F = 1$**

THE MAP METHOD

- Example 3.2: Simplify the Boolean function $F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$

		yz			
		y			
		0 0	0 1	1 1	1 0
x	0	m_0 1	m_1	m_3	m_2 1
	1	m_4 1	m_5 1	m_7	m_6 1
		z			

$$F = z' + xy'$$

THE MAP METHOD

- If a function is not expressed in sum of minterms
 - Use the map to obtain the minterms
 - Simplify the function and find the minimum number of terms.
 - Make sure that the algebraic expression is in sum of products form.

THE MAP METHOD

- Example 3.2: Simplify the Boolean function $F = A' C + A' B + A B' C + B C$.
 - Express it in sum of minterms
 - And find the minimal sum of products expression.

		BC		B	
		0 0	0 1	1 1	1 0
A	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

C

$$F = C + A'B$$



3.2 FOUR – VARIABLE K-MAP

FOUR-VARIABLE MAP

- 16 minterms
- Combinations of 2, 4, 8, and 16 adjacent squares

		y			
		00	01	11	10
wx	00	m_0	m_1	m_3	m_2
	01	m_4	m_5	m_7	m_6
	11	m_{12}	m_{13}	m_{15}	m_{14}
	10	m_8	m_9	m_{11}	m_{10}

W

Z

X

		y			
		00	01	11	10
wx	00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
	01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
	11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
	10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wxyz'$

W

Z

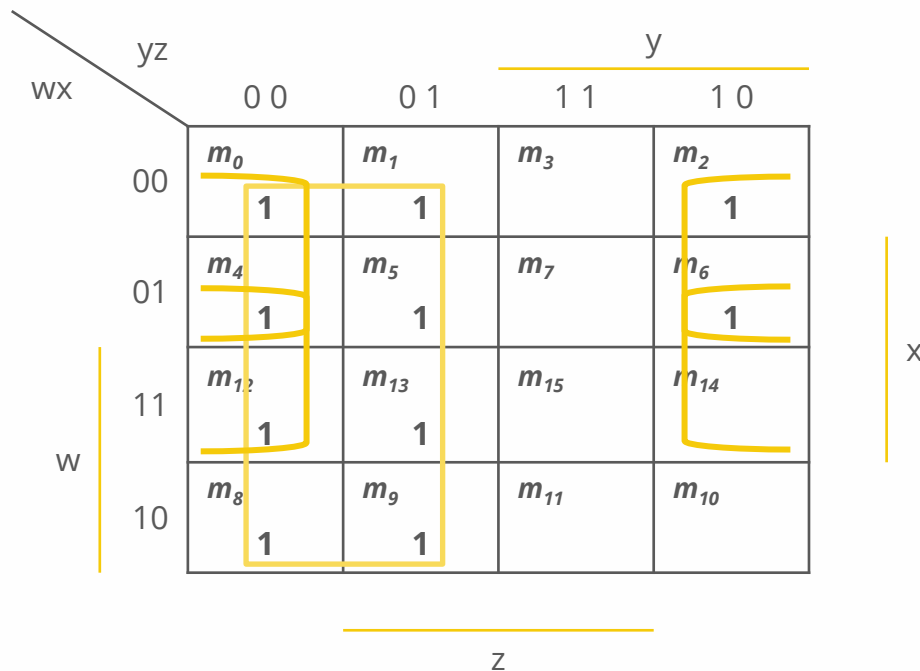
X

FOUR-VARIABLE MAP

- Minimization of four-variable Boolean function is similar to three-variable functions.
- Adjacent squares are defined to be squares next to each other.
 - Ex: m_0 and m_2 , m_3 and m_{11} .
- **1** square represents **1** minterm
 - A Term of **4** literals.
- **2** adjacent squares
 - A term of **3** literals.
- **4** adjacent squares
 - A term of **2** literal.
- **8** adjacent squares
 - A term of **1** literal.
- **16** adjacent squares
 - Entire map
 - Function **F = 1**

FOUR-VARIABLE MAP

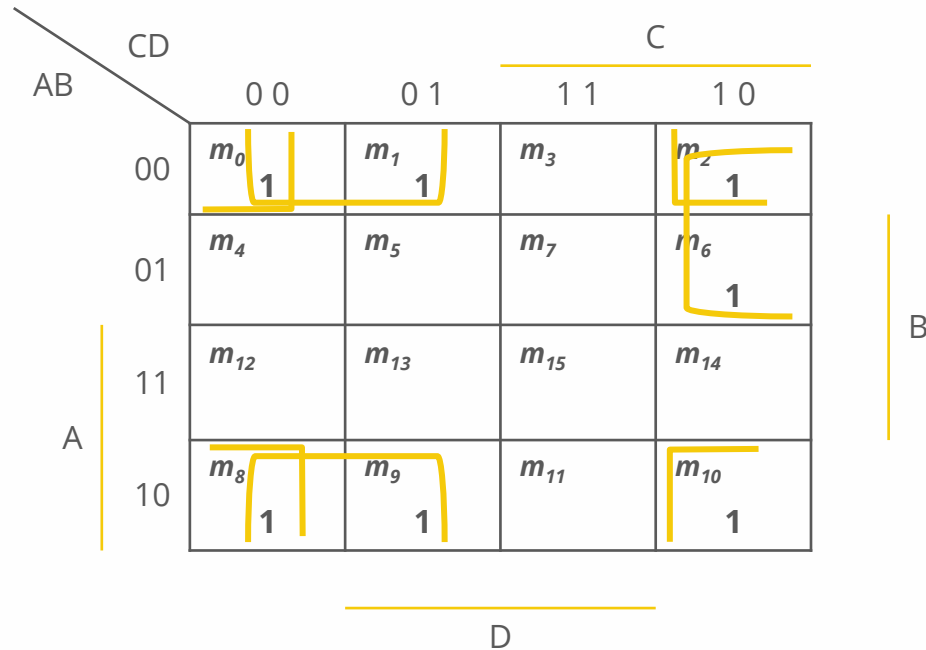
- Example 3.5: simplify $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$



$$F = y' + w'z' + xz'$$

FOUR-VARIABLE MAP

- Example 3-6: simplify $F = A' B' C' + B' C D' + A' B C D' + A B' C'$



$$F = \mathbf{B' C' + A' C D' + B' D'}$$



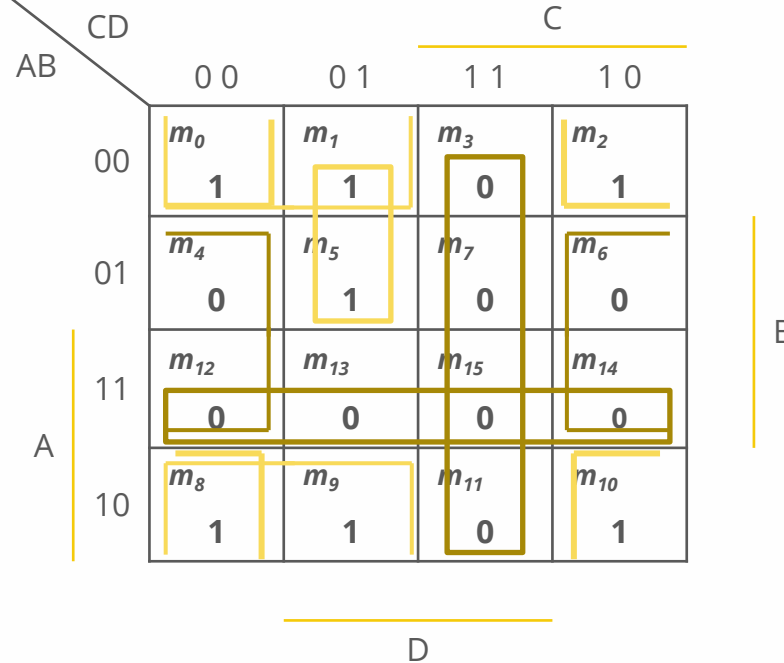
3.4 PRODUCT OF SUMS SIMPLIFICATION

PRODUCT OF SUMS SIMPLIFICATION

- The **1's** placed in the squares of the map represent the **minterms** of the function.
- The **minterms** not included in the function denote the complement of the function.
- Mark the empty squares by **0's**
- Combine them into valid adjacent squares
- Obtain simplified expression of the complement function **F'**.

PRODUCT OF SUMS SIMPLIFICATION

- Simplify the following function in (a) sum of product and (b) product of sums: $F(A, B, C, D) = \sum(0, 1, 2, 5, 8, 9, 10)$



$$F = B'D' + B'C' + A'C'D$$

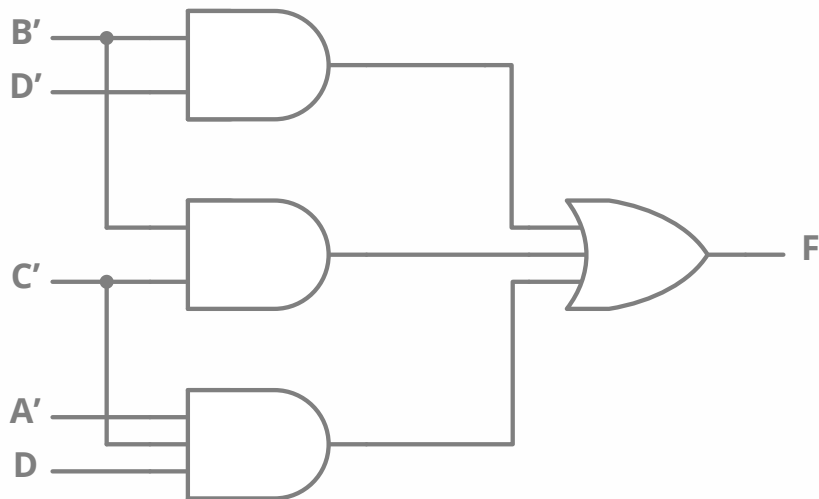
$$F' = AB + CD + BD'$$

Apply DeMorgan's theorem

$$(F' = (AB + CD + BD'))'$$

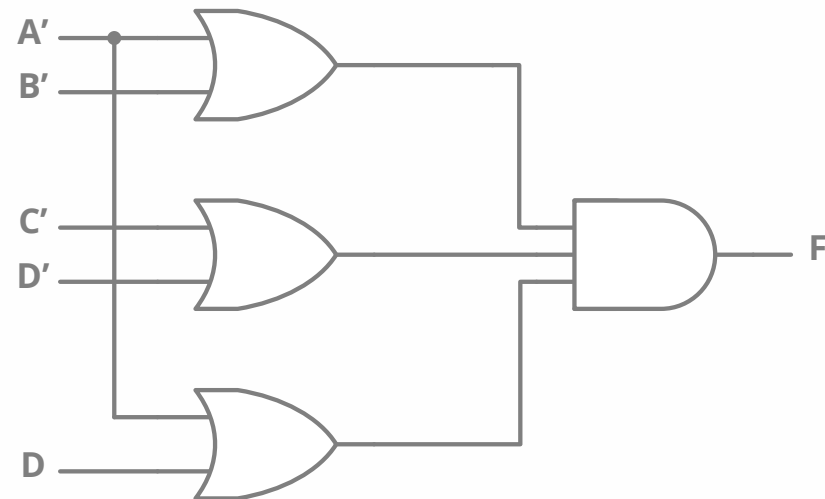
$$F = (A' + B') (C' + D') (B' + D)$$

PRODUCT OF SUMS SIMPLIFICATION



$$F = B'D' + B'C' + A'C'D$$

Sum of Products



$$F = (A' + B')(C' + D')(B' + D)$$

Products of Sum

PRODUCT OF SUMS SIMPLIFICATION

- Consider the function defined in Table 3.2.
- Sum of minterms:
 - $F(x, y, z) = \sum(1, 3, 4, 6)$
- Product of maxterms:
 - $F(x, y, z) = \prod(0, 2, 5, 7)$

		BC			
		00	01	11	10
A	0	m_0 0	m_1 1	m_3 1	m_2 0
	1	m_4 1	m_5 0	m_7 0	m_6 1

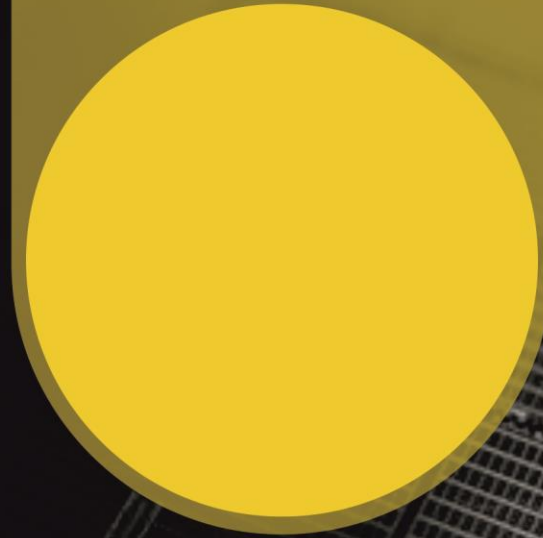
C

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

PRODUCT OF SUMS SIMPLIFICATION

- Sum of minterms:
 - $F(x, y, z) = \sum(1, 3, 4, 6)$
 - Sum of products: $x'z + xz'$
- Product of maxterms:
 - $F(x, y, z) = \prod(0, 2, 5, 7)$
 - Product of sums: $(x'+z')(x+z)$

		yz		y			
		0 0	0 1	1 1	1 0		
x	0	m_0 0	m_1 1	m_3 1	m_2 0		
	1	m_4 1	m_5 0	m_7 0	m_6 1		
						z	



3.5 DON'T CARE
CONDITIONS

DON'T CARE CONDITIONS

- The value of a function is not specified for certain combinations of variables
 - BCD; 1010-1111: don't care
- These don't care conditions can be used on a map to provide further simplification of the Boolean expression.
- Don't care minterm is a combination of variables whose logical value is not specified.

DON'T CARE CONDITIONS

- It cannot be marked with a 1 in the map
 - It would require that the function always be a 1 for such combination.
- It cannot be marked with a 0 in the map
 - It would require that the function always be a 0 for such combination.
- For don't care conditions an **X** is used.
- For adjacent squares in the map to simplify the function
 - The don't care minterms may be assumed to be either 0 or 1.

DON'T CARE CONDITIONS

- Simplify the Boolean function $F(w, x, y, z) = \sum(1, 3, 7, 11, 15)$
 - Don't care conditions $d(w, x, y, z) = \sum(0, 2, 5)$.

		y				
		00	01	11	10	
w	yz	m_0	m_1	m_3	m_2	x
	00	X	1	1	X	
	01	m_4	m_5	m_7	m_6	
	01	0	X	1	0	
11	m_{12}	m_{13}	m_{15}	m_{14}		
	11	0	0	1	0	
10	m_8	m_9	m_{11}	m_{10}		
	10	0	0	1	0	

$F = yz + w'x'$

		y				
		00	01	11	10	
w	yz	m_0	m_1	m_3	m_2	x
	00	X	1	1	X	
	01	m_4	m_5	m_7	m_6	
	01	0	X	1	0	
11	m_{12}	m_{13}	m_{15}	m_{14}		
	11	0	0	1	0	
10	m_8	m_9	m_{11}	m_{10}		
	10	0	0	1	0	

$F = yz + w'z$

DON'T CARE CONDITIONS

- Don't care minterms in the map are initially marked with **X's**.
- The choice between 0 and 1 is made depending on the way the incompletely specified function is simplified.
- Once the choice is made,
 - the simplified function obtained will consist of a sum of minterms
 - including those minterms that were initially marked with X's and
 - have been chosen to be included with **1's**.
- $F(w, x, y, z) = yz + w'x' = \sum (0, 1, 2, 3, 7, 11, 15)$
- $F(w, x, y, z) = yz + w'z = \sum (0, 1, 3, 5, 7, 11, 15)$



3.6 NAND AND NOR IMPLEMENTATIONS

NAND AND NOR IMPLEMENTATIONS

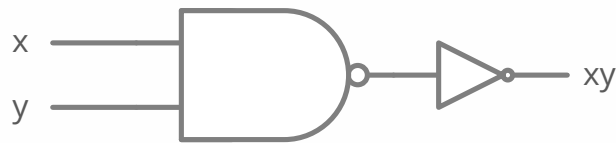
NAND Circuits:

- The NAND gate is a universal gate.
 - Can implement any digital system.
 - Complement operation is obtained from one-input NAND gate.
 - AND operation requires two NAND gates
 - OR operation is achieved through NAND gate with additional inverters in each input

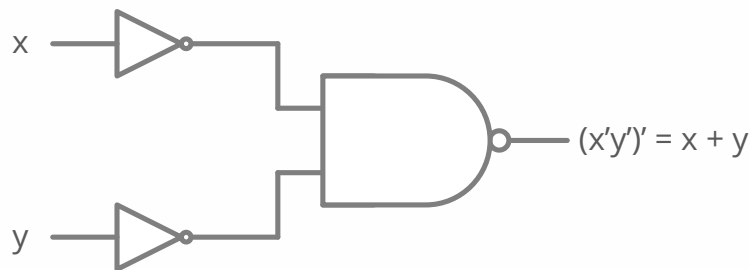
Inverter



AND



OR



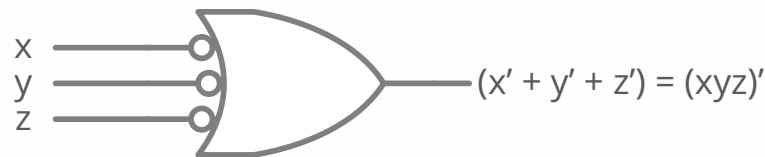
NAND AND NOR IMPLEMENTATIONS

NAND Circuits:

- Two graphic symbols for NAND gate



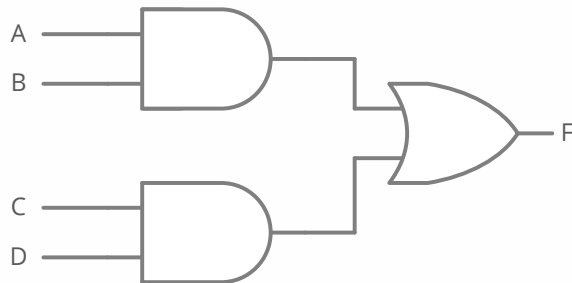
(a) AND-invert



(b) Invert-OR

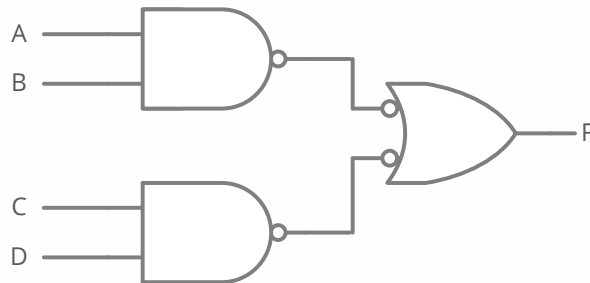
NAND AND NOR IMPLEMENTATIONS

- The implementation of Boolean functions with NAND gates
 - Require that the function be in sum of products form.
 - $F = A.B + C.D$



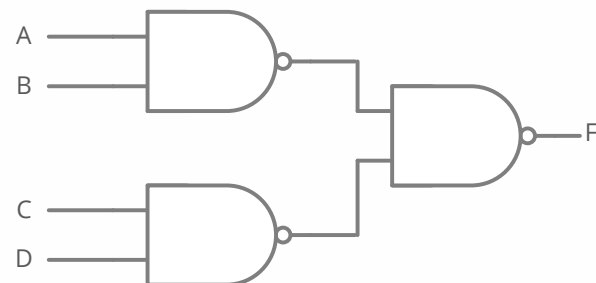
(a)

$$(a) F = A.B + C.D$$



(b)

$$\begin{aligned}(b) F &= ((A.B)')' + ((C.D)')' \\ &= (A+B)' + (C+D)' \\ &= A.B + C.D\end{aligned}$$

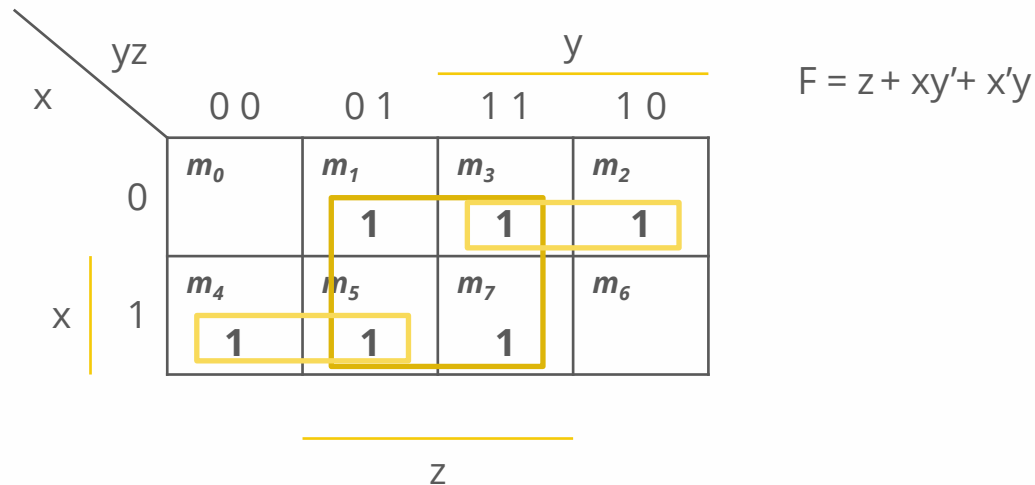


(c)

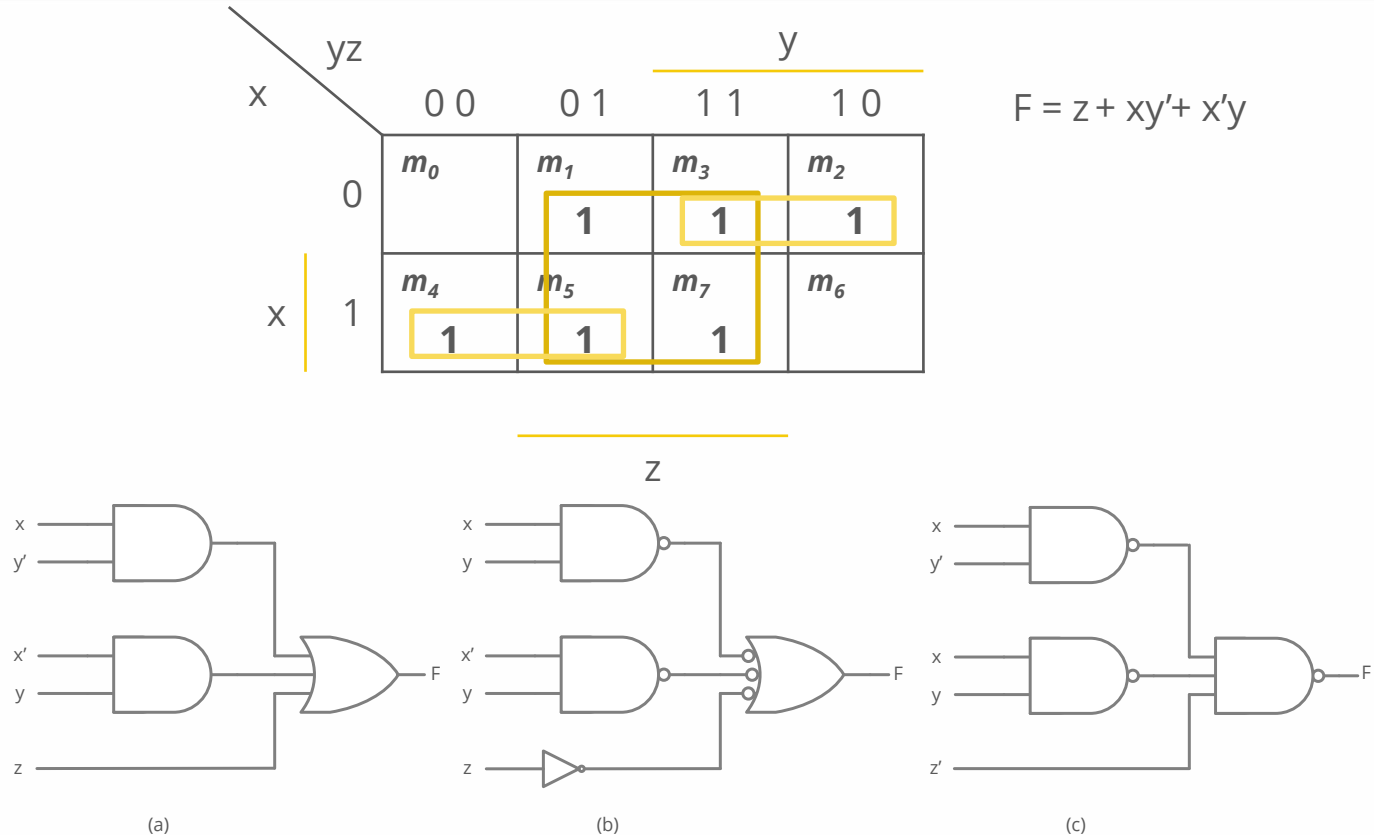
$$\begin{aligned}(c) F &= ((A.B)' (C.D)')' \\ &= ((A+B) (C + D))' \\ &= A.B + C.D\end{aligned}$$

NAND AND NOR IMPLEMENTATIONS

- Implement the following Boolean function $F(x, y, z) = (1, 2, 3, 4, 5, 7)$.
 - Simplify the function in sum of products using K-map.



NAND AND NOR IMPLEMENTATIONS



NAND AND NOR IMPLEMENTATIONS

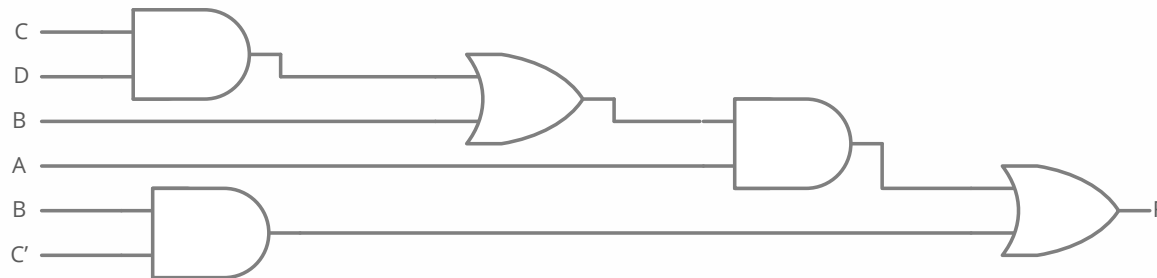
- The procedure
 - Simplify in the form of sum of products;
 - Draw a NAND gate for each product term;
 - The inputs to each NAND gate are the literals of the term (the first level);
 - A single NAND gate for the second sum term (the second level);
 - A term with a single literal requires an inverter in the first level if the single literal is not complemented. Otherwise it can be connected directly.

NAND AND NOR IMPLEMENTATIONS

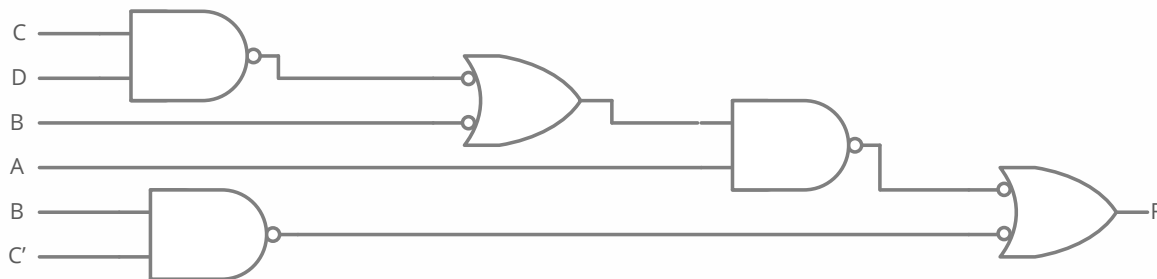
- General procedure for converting multilevel AND-OR diagram into all NAND diagram:
 - Convert all AND gates to NAND gates with AND-invert graphic symbol.
 - Convert all OR gates to NAND gates with invert-OR graphic symbol.
 - Check all the bubbles in the diagram.
 - For every bubble that is not compensated by another small circle along the same line,
 - insert an inverter (one-input NAND gate) or
 - complement the input literal

NAND AND NOR IMPLEMENTATIONS

- $F = A(CD + B) + BC'$



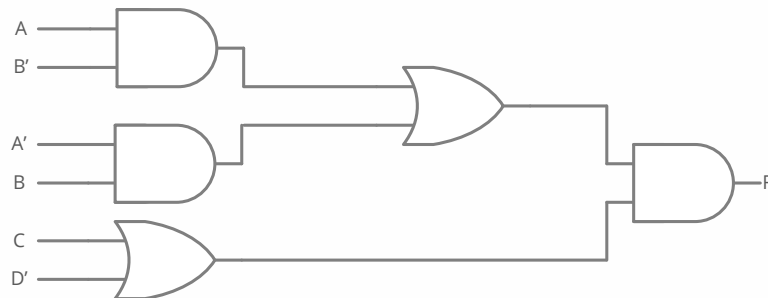
(a) AND-OR gates



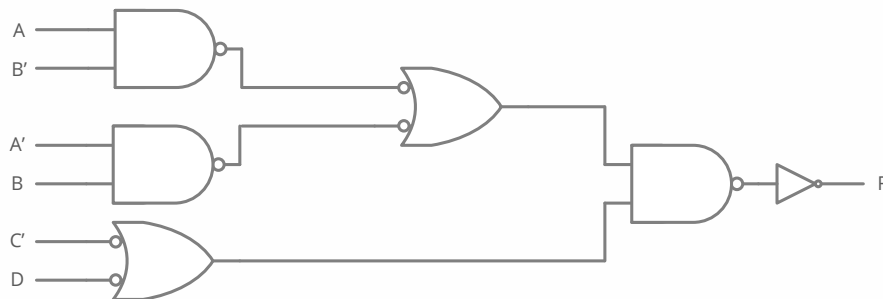
(b) NAND gates

NAND AND NOR IMPLEMENTATIONS

- $F = (AB' + A'B)(C + D')$



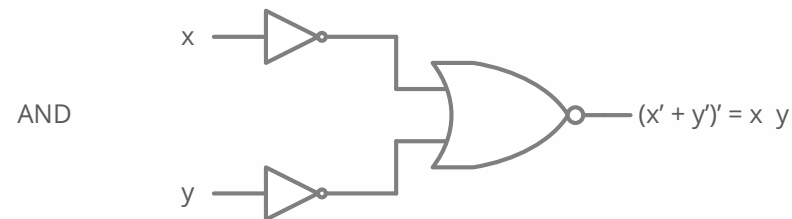
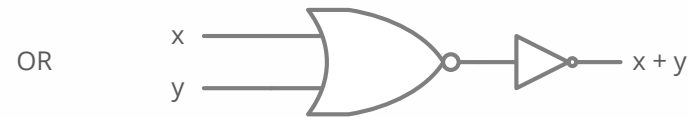
(a) AND-OR gates



(b) NAND gates

NAND AND NOR IMPLEMENTATIONS

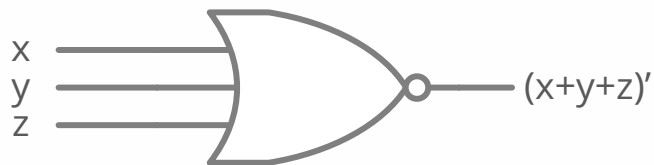
- NOR function is the dual of NAND function.
- The NOR gate is also universal.



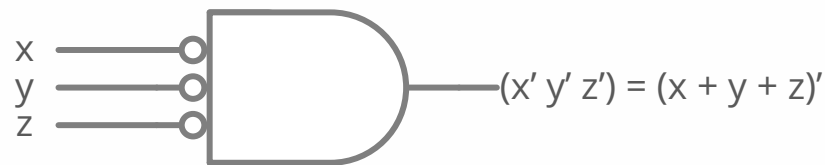
NAND AND NOR IMPLEMENTATIONS

NAND Circuits:

- Two graphic symbols for NAND gate



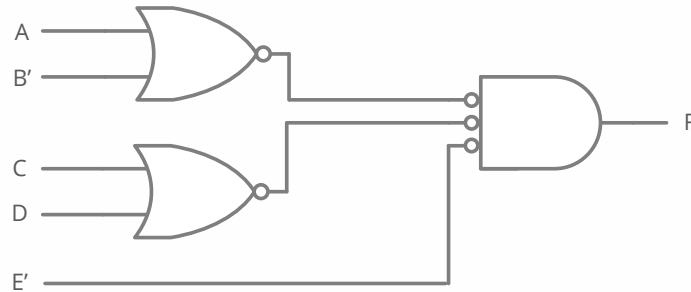
(a) OR-invert



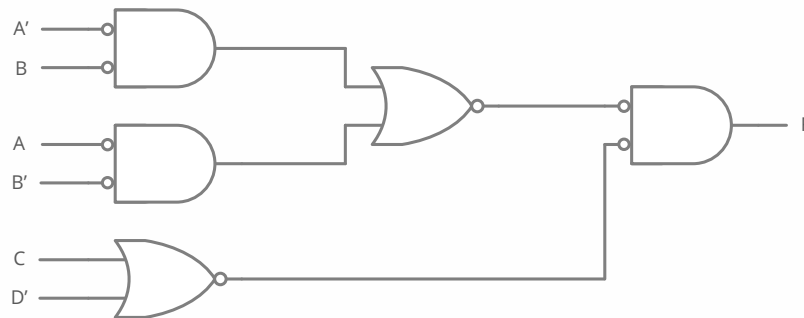
(b) Invert-AND

NAND AND NOR IMPLEMENTATIONS

- Example: Implementing $F = (A + B)(C + D)E$



- Example: Implementing $F = (A B' + A' B)(C + D')$





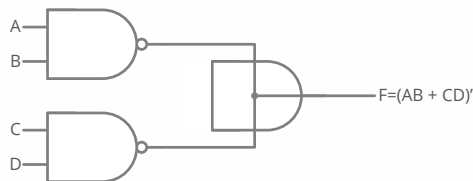
3.7 OTHER TWO LEVEL IMPLEMENTATIONS

OTHER TWO LEVEL IMPLEMENTATIONS

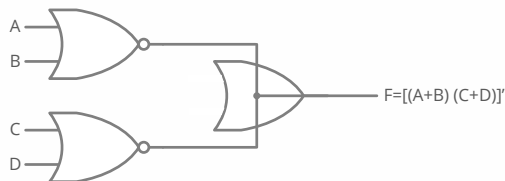
- NAND and NOR gates most often found in integrated circuits.
- NAND and NOR are the most important gates from a practical point of view.
- Some NAND or NOR gates allow the possibility of a wire connection between the outputs of two gates to provide a specific logic function.
- This type of logic is called **wired logic**.
- Example:
 - Open-collector TTL NAND gates when tied together performs wired-AND logic.

OTHER TWO LEVEL IMPLEMENTATIONS

- Example:
 - Open-collector TTL NAND gates when tied together performs wired-AND logic.
 - The wired – AND gate is not a physical gate, but only a symbol to designated the function obtained from the indicated wired connection.
 - $F = (AB)' \cdot (CD)' = (AB + CD)'$



(a) Wired-AND in open collector TTL
NAND gates
AND-OR-INVERT



(b) Wired-OR in ECL gates
OR-AND-INVERT

OTHER TWO LEVEL IMPLEMENTATIONS

Nondegenerate Forms

- Consider four types of gate: **AND**, **OR**, **NAND** and **NOR**.
- Assign one type of gate for the first level and one type of gate for the second level.
- There are 16 possible combination of two-level forms.
 - Eight of them: **degenerate forms = a single operation**
 - AND-AND, AND-NAND, OR-OR, OR-NOR, NAND-OR, NAND-NOR, NOR-AND, NOR-NAND.

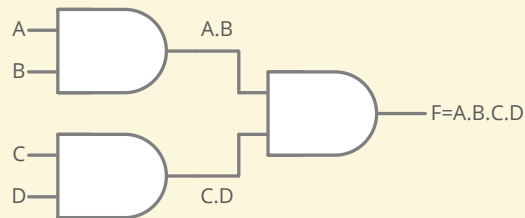
OTHER TWO LEVEL IMPLEMENTATIONS

degenerate forms = a single operation

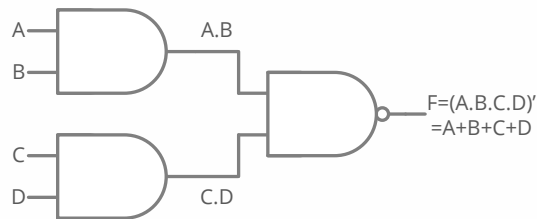
Logic Operation

Circuit Diagram

AND-AND



AND-NAND



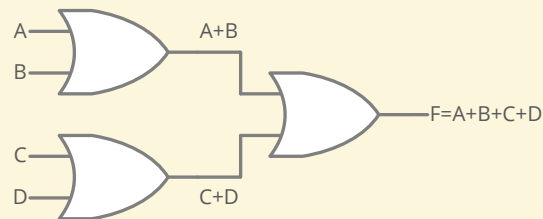
OTHER TWO LEVEL IMPLEMENTATIONS

degenerate forms = a single operation

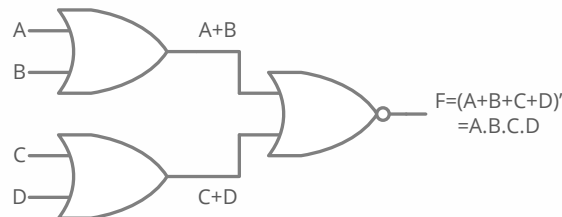
Logic Operation

Circuit Diagram

OR-OR



OR-NOR



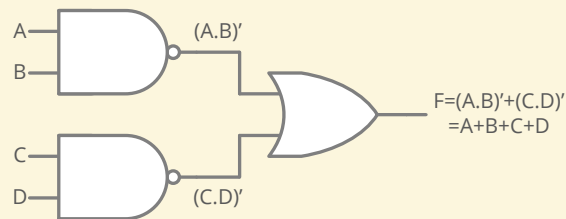
OTHER TWO LEVEL IMPLEMENTATIONS

degenerate forms = a single operation

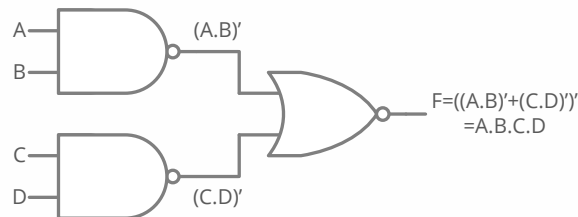
Logic Operation

Circuit Diagram

NAND-OR



NAND-NOR



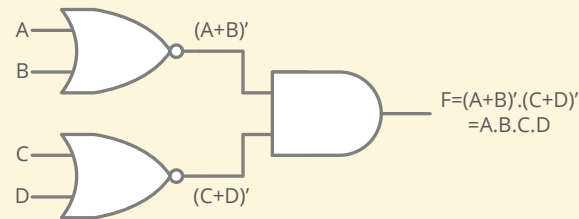
OTHER TWO LEVEL IMPLEMENTATIONS

degenerate forms = a single operation

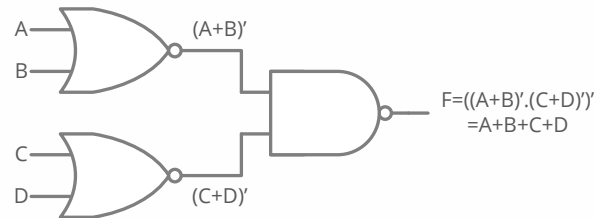
Logic Operation

Circuit Diagram

NOR-AND



NOR-NAND



OTHER TWO LEVEL IMPLEMENTATIONS

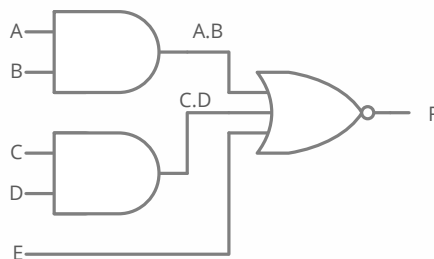
- The eight **non-degenerate forms**

- AND-OR, OR-AND, NAND-NAND, NOR-NOR, NOR-OR, NAND-AND, OR-NAND, AND-NOR.
- **AND-OR** and **NAND-NAND** = **sum of products**.
- **OR-AND** and **NOR-NOR** = **product of sums**.
- NOR-OR, NAND-AND, **OR-NAND**, **AND-NOR** = ?

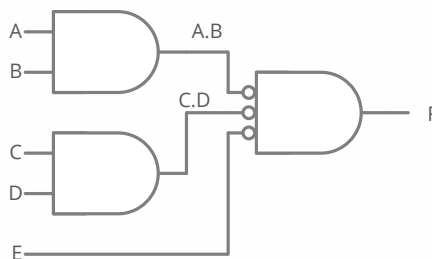
OTHER TWO LEVEL IMPLEMENTATIONS

AND – OR – INVERT Implementation

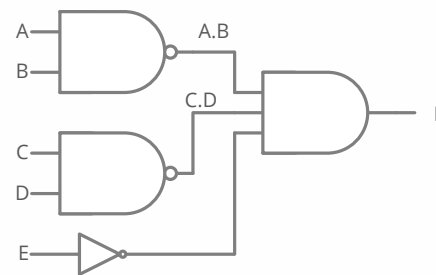
- The two forms **NAND – AND = AND – NOR**
- Both perform the **AND – OR – INVERT** function.
- $F = (AB + CD + E)'$
- $F' = AB + CD + E$ (sum of product)



(a) AND-NOR



(b) AND-NOR

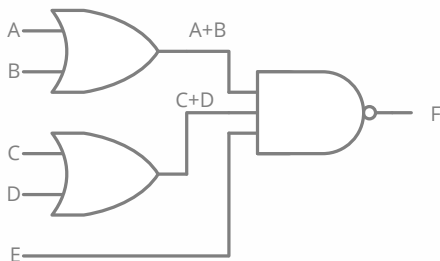


(c) NAND-AND

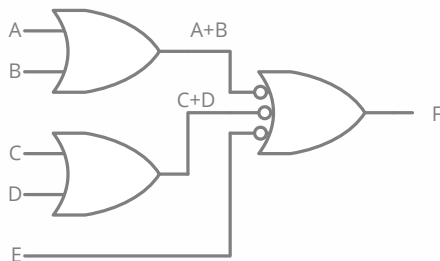
OTHER TWO LEVEL IMPLEMENTATIONS

OR – AND – INVERT Implementation

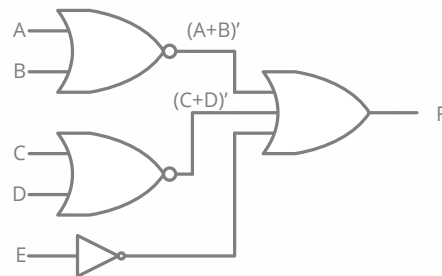
- The two forms **OR – NAND = NOR – OR**
- Both perform the **OR – AND – INVERT** function.
- $F = [(A+B) (C+D) E]'$
- $F' = (A+B) (C+D) E$ (product of sum)



(a) OR-NAND



(b) OR-NAND



(c) NOR-OR

OTHER TWO LEVEL IMPLEMENTATIONS

Equivalent Nondegenerate Form		Implements the Function	Simplify F' in	To get an output of
(a)	(b)*			
AND- NOR	NAND-AND	AND-OR-INVERT	Sum of products by combining 0's in the map	F
OR- NAND	NOR-OR	OR-AND-INVERT	Product of sums by combining 1's in the map and then complementing	F

*Form (b) requires and inverter for a single literal term.

OTHER TWO LEVEL IMPLEMENTATIONS

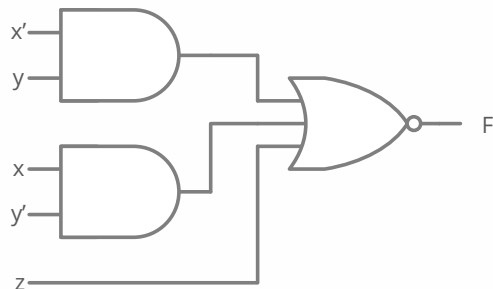
- Example 3-11: $F' = x'y + xy' + z$
(F' : sum of products)
 - Step1: $x'y(z + z') = x'yz + x'yz'$
 - Step2: $xy'(z + z') = xy'z + xy'z'$
 - Step3: $z(x + x') = xz + x'z$
 - Step4: $xz + x'z(y + y') = xyz + x'yz + xy'z + x'y'z$
 - Step5: $x'yz + x'yz' + xy'z + xy'z' + xyz + x'y'z$

		y			
	yz	00	01	11	10
x	0	m_0 1	m_1 0	m_3 0	m_2 0
	1	m_4 0	m_5 0	m_7 0	m_6 1
		z			

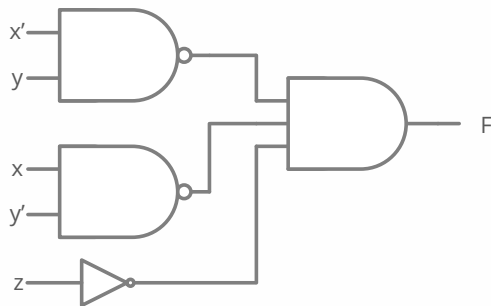
$$F' = z + xy' + x'y$$

OTHER TWO LEVEL IMPLEMENTATIONS

- Example 3-11: $F' = x'y + xy' + z$ (F' : sum of products)



AND-NOR



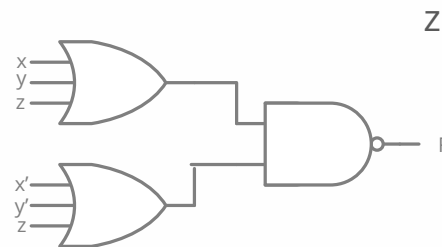
NAND-NAND

- AND-OR-INVERT

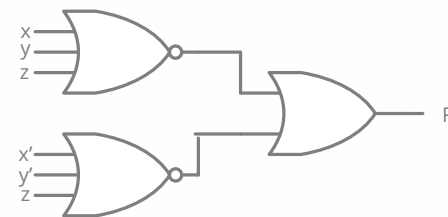
OTHER TWO LEVEL IMPLEMENTATIONS

- AND-OR-INVERT form require a simplified expression of the complement of the function in product of sums.
 - Combine the 1's in the map
 - $F = x'y'z' + xyz'$
 - Take the complement
 - $F' = (x+y+z)(x'+y'+z)$
 - $F = [(x+y+z)(x'+y'+z)]'$

		y			
		yz		1 1	1 0
x	0	m_0 1	m_1 0	m_3 0	m_2 0
	1	m_4 0	m_5 0	m_7 0	m_6 1



(a) OR-NAND



(c) NOR-OR

OTHER TWO LEVEL IMPLEMENTATIONS

- Summary
 - $F' = x'y + xy' + z$ (F': sum of products)
 - $F = (x'y + xy' + z)'$ (F: AOI implementation)
 - $F = x'y'z' + xyz'$ (F: sum of products)
 - $F' = (x+y+z)(x'+y'+z)$ (F': product of sums)
 - $F = ((x+y+z)(x'+y'+z))'$ (F: OAI)



3.8 EXCLUSIVE-OR FUNCTION

EXCLUSIVE-OR FUNCTION

- Exclusive-OR (XOR)
 - $x \oplus y = x y' + x' y$
 - $F = 1$, if only $x = 1$ or $y = 1$, but not when x and $y = 1$.
- Exclusive-NOR (XNOR) (equivalence)
 - $(x \oplus y)' = x y + x' y'$
 - $F = 1$, if both x and $y = 1$ or both $= 0$.
 - $(x \oplus y)' = (x y' + x' y)' = (x' + y) (x + y') = x'x + x'y' + xy = xy + x'y'$

EXCLUSIVE-OR FUNCTION

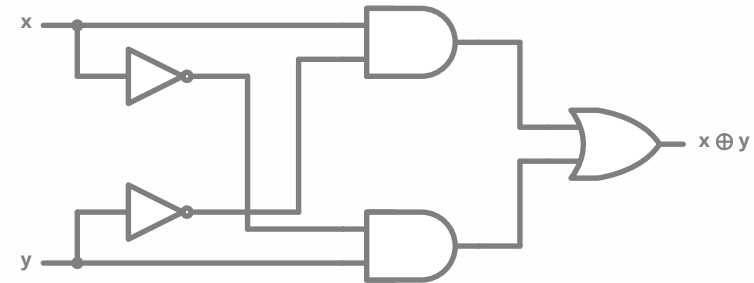
- Some identities
 - $x \oplus 0 = x$
 - $x \oplus 1 = x'$
 - $x \oplus x = 0$
 - $x \oplus x' = 1$
 - $x \oplus y' = (x \oplus y)'$
 - $x' \oplus y = x' \oplus y = (x \oplus y)'$
- Commutative and associative
 - $A \oplus B = B \oplus A$
 - $(A \oplus B) \oplus C = A \oplus (B \oplus C)$
 $= A \oplus B \oplus C$

EXCLUSIVE-OR FUNCTION

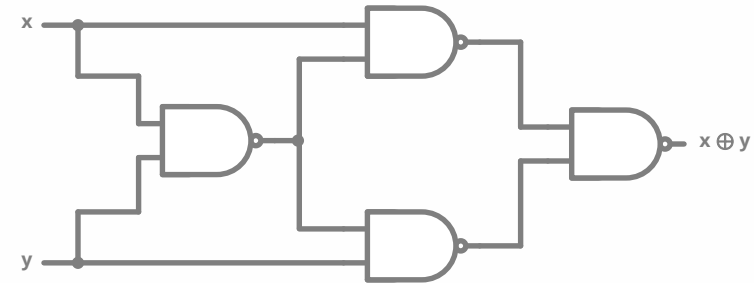
- Implementations

$$= (x' + y')x + (x' + y')y$$

$$= xy' + x'y = x \oplus y$$



(a) with AND - OR - NOT gates



(b) with NAND gates

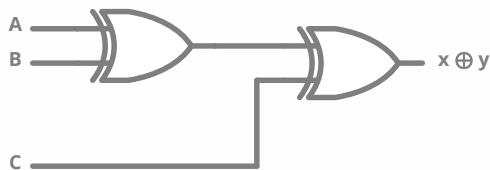
EXCLUSIVE-OR FUNCTION

- The XOR operation with three or more variables can be converted into an ordinary Boolean function by replacing \oplus symbol with its equivalent Boolean expression.
 - $A \oplus B \oplus C = (AB' + A'B)C' + (AB + A'B')C$
 $= AB'C' + A'BC' + ABC + A'B'C$
 $= \Sigma(1, 2, 4, 7)$
 - XOR is a odd function \rightarrow an odd number of 1's, then $F = 1$.
 - XNOR is a even function \rightarrow an even number of 1's, then $F = 1$.

EXCLUSIVE-OR FUNCTION

		y			
		00	01	11	10
x	0	m_0	m_1 1	m_3	m_2 1
	1	m_4 1	m_5	m_7 1	m_6

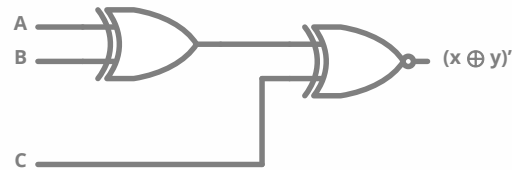
(a) Odd Function
 $F = A \oplus B \oplus C$



(a) 3-input odd function

		y			
		00	01	11	10
x	0	m_0 1	m_1	m_3 1	m_2
	1	m_4	m_5 1	m_7	m_6 1

(b) Even Function
 $F = (A \oplus B \oplus C)'$



(b) 3-input even function

EXCLUSIVE-OR FUNCTION

- Four-variable Exclusive-OR function

$$A \oplus B \oplus C \oplus D = (AB' + A'B) \oplus (CD' + C'D) = (AB' + A'B)(CD + C'D') + (AB + A'B')(CD' + C'D)$$

yz		y			
		00	01	11	10
wx	00	m_0	m_1	m_3	m_2
	01	m_4	m_5	m_7	m_6
	11	m_{12}	m_{13}	m_{15}	m_{14}
	10	m_8	m_9	m_{11}	m_{10}
w		z			

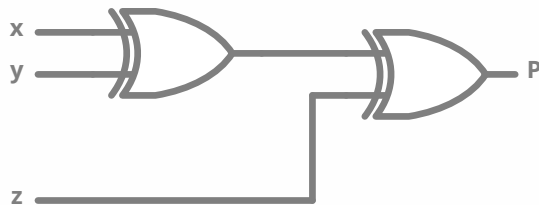
$F = A \oplus B \oplus C \oplus D$

yz		y			
		00	01	11	10
wx	00	m_0	m_1	m_3	m_2
	01	m_4	m_5	m_7	m_6
	11	m_{12}	m_{13}	m_{15}	m_{14}
	10	m_8	m_9	m_{11}	m_{10}
w		z			

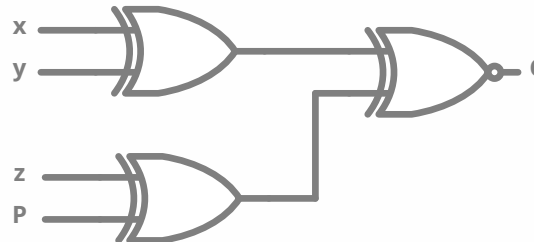
$F = (A \oplus B \oplus C \oplus D)'$

EXCLUSIVE-OR FUNCTION

- Parity Generation and Checking
 - A parity bit: $P = x \oplus y \oplus z$
 - Parity check: $C = x \oplus y \oplus z \oplus P$
 - $C=1$: one bit error or an odd number of data bit error
 - $C=0$: correct or an even # of data bit error



(a) 3-input odd function



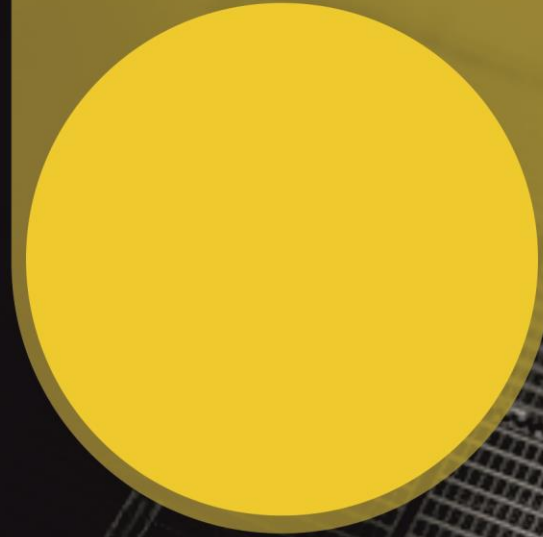
(b) 3-input even function

EXCLUSIVE-OR FUNCTION

Three-Bit Message			Parity Bit
x	y	z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

EXCLUSIVE-OR FUNCTION

Three-Bit Message			Parity Error Check	
x	y	z	P	C
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0



3.9 HARDWARE DESCRIPTION LANGUAGE

HARDWARE DESCRIPTION LANGUAGE

- Describe the design of digital systems in a textual form
 - Hardware structure
 - Function/behavior
 - Timing
- VHDL and Verilog HDL

HARDWARE DESCRIPTION LANGUAGE

Specification

RTL design and Simulation

Logic Synthesis

Gate Level Simulation

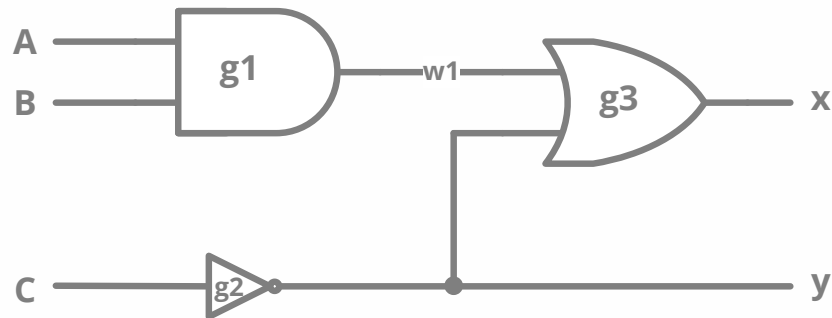
ASIC Layout

**FPGA
Implementation**

HARDWARE DESCRIPTION LANGUAGE

- Documentation language
- HDL is used to represent and document digital systems in a form that can be read by both humans and computers
- Examples of keywords:
 - **module, end-module, input, output, wire, and, or, and not**

HARDWARE DESCRIPTION LANGUAGE



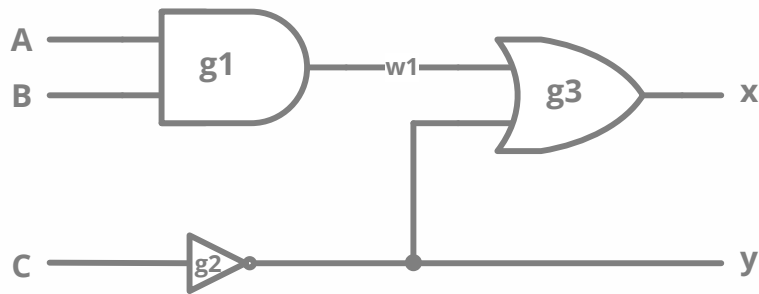
//Description of Simple circuit

```
module smpl_circuit (A, B, C, x, y);
    input A, B, C;
    output x, y;
    wire w1;
    and g1 (e, A, B);
    not g2 (y, C);
    or g3 (x, e, y);
endmodule
```

HARDWARE DESCRIPTION LANGUAGE

- Boolean expressions are specified in Verilog HDL with a
 - continuous assignment statement consisting of the keyword **assign** followed by a Boolean expression.
- To distinguish the arithmetic plus from logical OR, Verilog HDL uses the symbols
 - (&) for AND,
 - (|) for OR,
 - (~) for NOT

HARDWARE DESCRIPTION LANGUAGE



//Description of Simple circuit

module smpl_circuit (A, B, C, x, y);

input A, B, C;

output x, y;

assign x = (A & B) | ~C

assing y = ~C

endmodule

- Boolean expression:

- $x = A.B + C'$

- $y = C'$