KINEMATICS AND DYNAMICS OF PARTICLES

INTRODUCTION

KINEMATICS AND DYNAMICS OF PARTICLES

- Objects are in motion all around us. Planets moving around the sun, car moving along a road, blood flowing through veins, motion of computer cursor etc.
- Mechanics is the study of the physics of motions.
- It relates to the physical factors that affect them, like force, mass, momentum.
- divided into two branches: kinematics and Dynamics.
- Kinematics describes the motion of objects without reference to the causes of motion (i.e., forces).
- Dynamics is the study of motion and of physical concepts such as force and mass,

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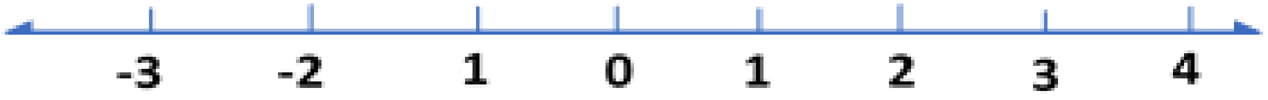
Displacement, velocity and Acceleration in 1D

- In *one-dimensional motion*, moving objects are restricted to motion along a straight line may i.e; vertical, horizontal, or slanted.
- motion where the object (race car, tectonic plate, blood cell, or any other object) moves along a single axis. Such motion is called *one-dimensional motion*.

Definition of Kinematical Quantities

- **Position:** - The location of an object with respect to a chosen reference point.

EX. : Position on an axis that is marked in units of length shown in Fig.below



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- **Distance (S or d):**- The length of the path followed by the object

  -is a *scalar* quantity and has SI unit m.

- **Displacement:** - The change in position of an object with respect to a given reference frame.

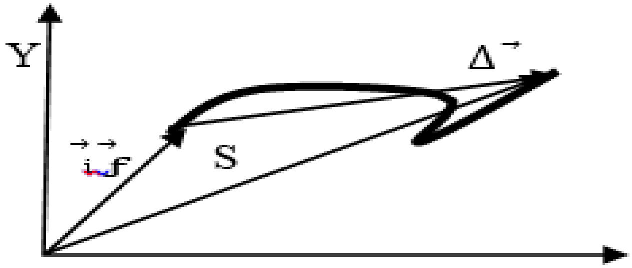
  -For 1D $\Delta x = x_f - x_i$

  $x_{tot} = x_f + x_i$

  - For 2D

  $S = \Delta\vec{r} = \vec{r}_f - \vec{r}_i$

  $S = r_{tot} = r_i + r_f$

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Position, displacement, velocity and acceleration in 2D

- At time t_i the particle is at point P, described by position vector $\vec{r}_i$

and some later time t_f it is at point Q,

described by position vector r_f.

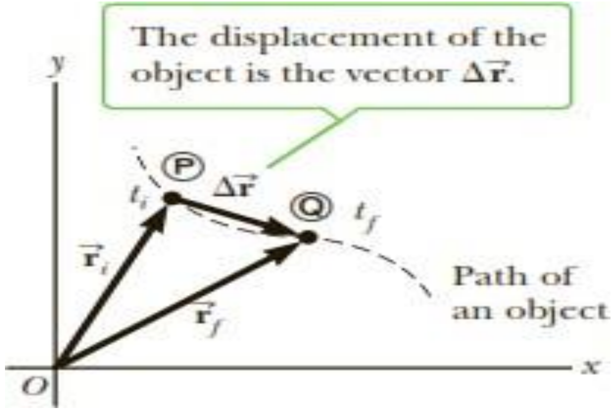
$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\Delta\bar{r} = r_f - \vec{r}_i$$

$$\Delta\bar{r} = \Delta x i + \Delta y j$$

$$V_a = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{a}_{av} = \frac{\Delta\vec{V}}{\Delta t}$$



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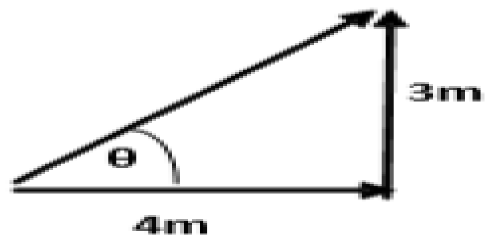
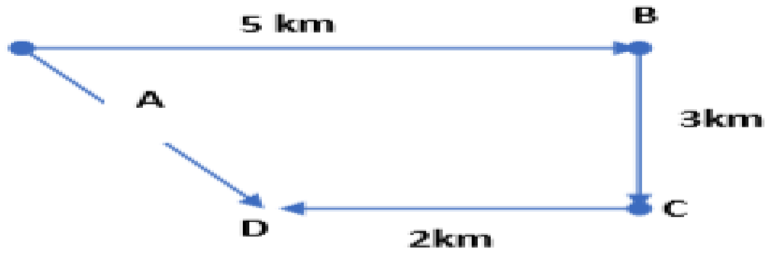
Examples:

1. Find the distance, displacement and the direction of the resultant displacement referring to the following diagram, if an object travels 4m to the positive x-axis and then 3m to the y-axis.



2. Compute the distance and displacement for the case of a particle moving from point A to point D shown below.



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Speed and Average Speed

- Speed (V):- is the ratio of the distance travelled by any object

$$V = \frac{S}{t}$$

:-involves both distance and time has SI system is *m/s or ms^{-1}*

- Average speed (V_{avg}):- the total distance covered divided by the total time taken.

$$Vav = \frac{S_{tot}}{t_{tot}} = \frac{\Delta x}{\Delta t}$$

locity and Average velocity

- **Velocity:-**the displacement travelled divided by time take

$$\vec{V} = \frac{\vec{S}}{t}$$

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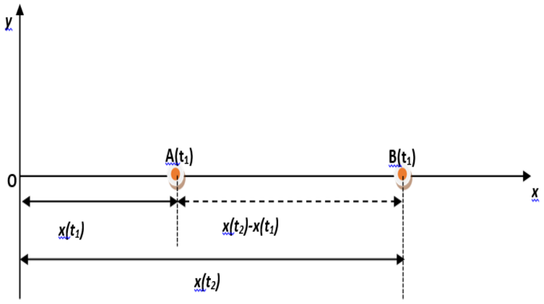
- **Average Velocity:-**the total displacement travelled divided by total time taken.

  -For 1D $\vec{V}_{av}=\frac{\Delta X}{\Delta t}=\frac{X_2-X_1}{t_2-t_1}=\frac{X_f-X_i}{t_f-t_i}$

  -For 2D shown position diagram

  above : $\vec{V}_{av}=\frac{\Delta \mathbf{r}}{\Delta t}$



- **Instantaneous velocity:-**refers to how fast a particle is moving at a given instant

$$\vec{\boldsymbol{V}}_{\boldsymbol{ins}} = \lim_{\Delta t \to 0} \boldsymbol{V}_{\boldsymbol{av}} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}} = \lim_{t \to 0} \frac{\boldsymbol{X}(\boldsymbol{t}_2) - \boldsymbol{X}(\boldsymbol{t}_1)}{\Delta \boldsymbol{t}}$$

  -For uniform motion $\vec{\boldsymbol{V}}_{\boldsymbol{ins}} = \vec{V}_{av}$

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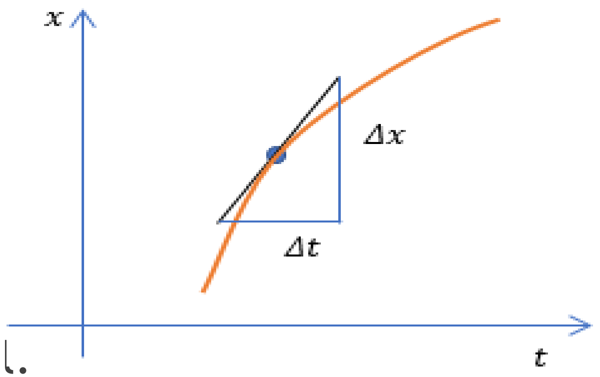
-*x*-*t* curve for non-uniform motion **shown below**



<u>Examples</u>

1. A car travelled 40km east in 1hr and then travelled 80km north in 2hrs. Calculate Its

(a) Average speed, (b) its average velocity And

(c) the direction of the velocity from the horizontal.

2. A person walks first at a constant speed of 5m/s along the straight line from point A to point B, and then back along the same line from B to A at a constant speed of 3m/s. What is his average speed and average velocity over the entire trip.

3. A hiker walks 2.00 km north and then 3.00 km east, all in 2.50 hours. Calculate the magnitude and direction of the hiker’s (a) displacement (in km) and (b) average velocity (in km/h) during those 2.50 hours. (c) What was her average speed during the same time interval?

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3. The position of a particle at any time t in the x-axis is described by the equation $X(t) = 4t^2 - 7(t) + 7$ where x is in m and t is in s. Compute the

(a). Average velocity b/n 1s and 2s.

(b). Instantaneous velocity at t=1s and t=2s.

Average and Instantaneous Accelerations

- If the velocity of a particle changes with time, then the particle is said to be accelerating.
- **Average acceleration:-** is the change in velocity ($\Delta\vec{V}$) of an object divided by the time interval during which that change occurs.

$$\vec{a}_{av}=\frac{\Delta\vec{V}}{\Delta t}=\frac{\vec{V}_f-\vec{V}_i}{t_f-t_i}$$

-if t_i=0 and t_f=t, hence $V_i = u \; and \; V_f = v$:

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- The **Average acceleration is given by:**

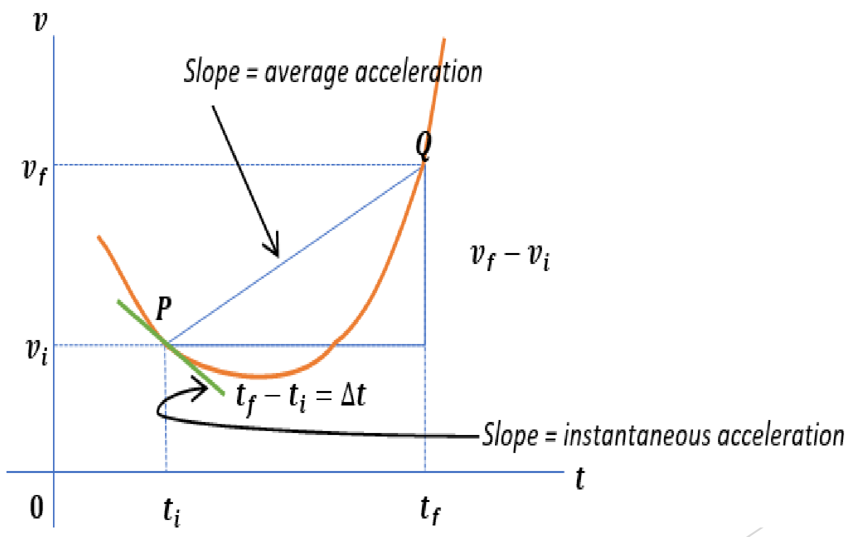
$$\vec{a}_{\text{avg}} = \mathrm{a} = \frac{\mathrm{v} - \mathrm{u}}{\mathrm{t}} = \frac{v - v_0}{t}$$

- **Instantaneous acceleration:** -The limit of average acceleration as Δt approaches zero.

$$\vec{\boldsymbol{a}}_{ins} = \lim_{\Delta t \to 0} \boldsymbol{a}_{av} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \lim_{t \to 0} \frac{V(t_2) - V(t_1)}{\Delta t}$$

Or

$$v = v_0 + at$$



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Motion with Constant Acceleration

- For motion with constant acceleration:

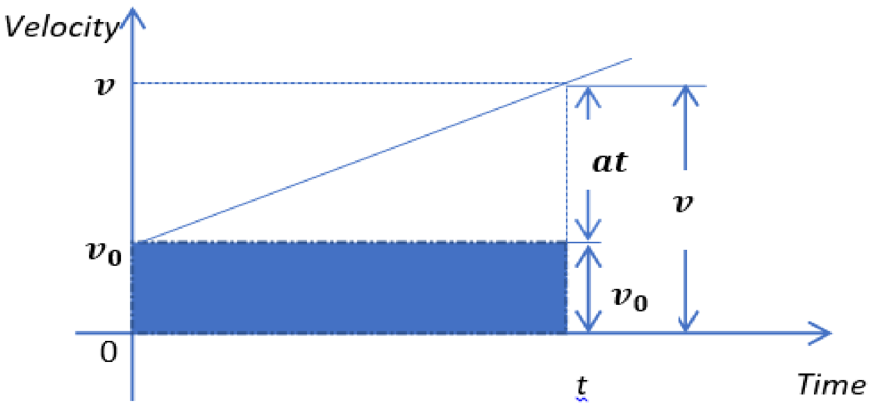
  -The velocity changes at the same rate throughout the motion

  - $a_{ins} = \vec{a}_{av}$

- The equations of motion wit

Constant a are:

1. $v_{av} = \frac{v_o + v}{t}$
2. $v = v_0 + at$
3. $\Delta x = x - x_o = \left(\frac{v + v_0}{t}\right)t$
4. $x - x_o = v_0 t + \frac{1}{2}at^2$
5. $v^2 - {v_o}^2 = 2a(x - x_o)$



velocity – time graph for rectilinear motion

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Examples

1. A track covers 800m in 10s while smoothly slowing down to a final speed of 4m/s. Find: (a). its acceleration and (b). Its original speed

2. An object moves along the x – axis with constant acceleration $a = 5\ m.\ s^{-2}$. At time $t = 0$ it is at $x = 6m$ and has velocity $v = 3m/s$.

(a). Find the position and velocity at time *t* = 2*s*.

(b). Where is the body when its velocity is *6m/s*?

3. A jet plane lands with a speed of 100m/s and slows down at a rate of $5m/s^2$ as it comes to rest.

(a). What is the time interval needed by the jet to come to rest?

(b). Can this jet land on an airport where the runway is 0.8km long?

4. A certain automobile manufacturer claims that its super-deluxe sport car will accelerate uniformly from rest to a speed of 87 mi/h in 8s. Determine the acceleration of the car.

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Free Fall Motion

- The motion of an object near the surface of the Earth under the only control of the force of gravity.
- At or near the earth's surface the magnitude of g is approximately 9.8 m/s^2, or 980$c\ m/s^2$, or 32 ft/s^2.
- For freely falling bodies the motion is vertical along *y*- axis so that a is replaced by g and *x* is replaced by *y* in the equations of motion for rectilinear motion.

1. $\boldsymbol{v_y = v_{0y} + gt}$ final velocity at any time $\boldsymbol{t}$
2. $\boldsymbol{y = v_{0y}t + \frac{1}{2}gt^2}$ vertical position at any time $\boldsymbol{t}$
3. $\boldsymbol{v_y{}^2 = v_{0y}{}^2 + 2gy}$
4. For freely falling object $\boldsymbol{v_0 = 0}$.
5. When an object reaches a maximum height $\boldsymbol{v = 0}$.

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Example

1. A girl throws a ball upwards, giving it an initial speed 40m/s. (i) How long does the ball take to return to the girl's hand? (b) What will be its velocity then?

2. A body is released from rest and falls freely. Compute its position and velocity after 1 and 2s.

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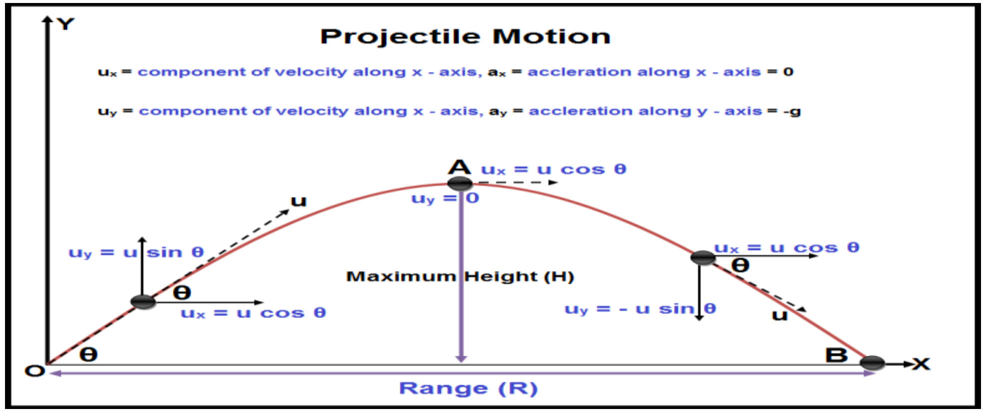
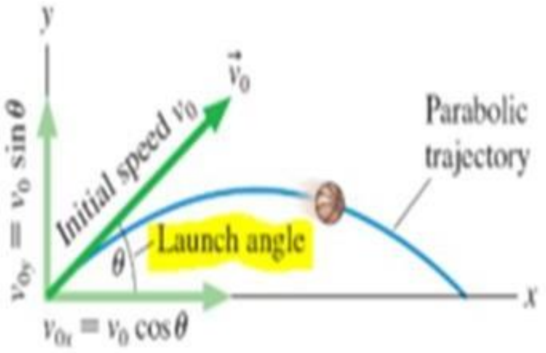
Projectile Motion /Projectile Motion with Constant Acceleration/

- Projectile is any object thrown obliquely into the space.
- The object which is given an initial velocity and afterwards follows a path determined by the gravitational force acting on it is called projectile.
- The motion projectile is called projectile motion.

EX. A stone projected at an angle, a bomb released from an aero plane, a shot fired from a gun, a shot put or javelin thrown by the athlete etc.





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Basic assumptions in projectile motion

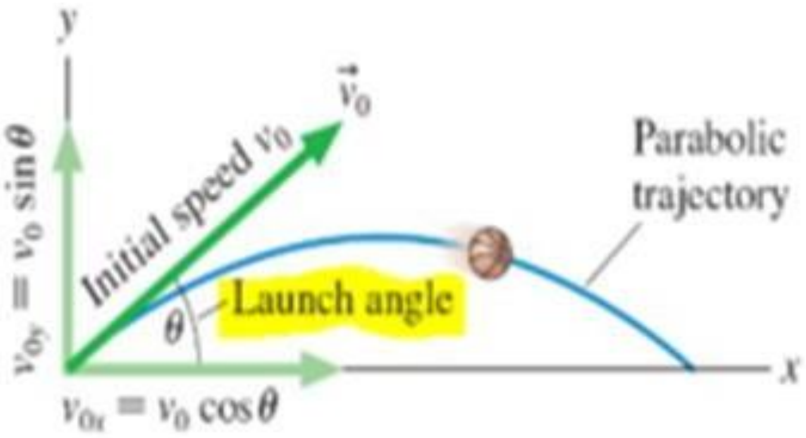
- The free fall acceleration (g) is constant over the range of motion and it is directed downward.
- The effect of air resistance is negligible.

The mathematics and goals of projectile motion

- Consider a body projected from a point 'O' with velocity 'V_O or u' at an angle θ as shown below.

Maximum height y_{max} or H

$$H = \frac{{V_O}^2 \, (\sin\theta)^2}{2g}$$



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Total time of flight, T

$$T = \frac{2V_O \sin\theta}{g}$$

Horizontal Range Or range of projectile, R

$$R = \frac{{V_O}^2 2\sin\theta\cos\theta}{g}$$

$$R = \frac{{V_O}^2 2\sin 2\theta}{g}$$

- These are the main achievement in projectile motion based on its mathematics.

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Examples

1.A ball is thrown upward from the top of a building at an angle of 30.0° above the horizontal and with an initial speed of 28.0 m/s. The point of release is 55.0 m above the ground. (a) How long does it take for the ball to hit the ground? (b) Find the ball's speed at impact. Find the horizontal range of the ball.

2. A rocket is fired with an initial velocity of 100m/s at an angle of 55^0 above the horizontal. It explodes on the mountain side 12s after its firing. What is the x-and y- coordinates of the rocket relative to its firing point?

3. A plane drops a package to a party of explorer. If the plane is travelling horizontally at 40m/s and is 100m above the ground, where does the package the ground relative to the point at which it is released?

4. A projectile is thrown with an initial velocity $x\,i + y\,j$. The range of the projectile is twice the maximum height of the projectile calculate the ratio of y/x.