# An Improved Potential Field Method for Path Planning

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**Abstract:** In this paper, an improved potential field method for mobile robots path planning is proposed with modified attractive and repulsive functions.The attractive function decreases the attractive potential around a goal evidently to ensure that the goal position is the global minimum of the total potential field. The repulsive function builds repulsive potential for an arbitrarily shaped obstacle by discretizing outline of the obstacle with points. Therefore, the workspace of the robot is described more exactly. The convergence of this method is discussed to guarantee its reliability for most cases. The simulation has proved that the new potential method can solve the problem of goals nonreachable with obstacles nearby (GNRON), U-shaped trap, and oscillations. Moreover, this method has a great performance in dynamic path planning where a goal and obstacles are moving.

**Key Words:** artificial potential field (APF), path planning, mobile robot, obstacle avoidance

## 1 INTRODUCTION

The robot path planning problem can be described as follows [1]: given a robot and a description of its working environment, plan a collision free path between two specified locations that satisfies certain optimization criteria. Path planning for mobile robot has been studied and applied in the past decades. In the early time, much research focused on path planning in static environment and many methods were proposed, such as visibility graphs and Voronoi diagrams [2,3]. However, the workspace is dynamic in many real-life implementations. Not only obstacles are moving, but the goal is moving. Therefore, the latter studies focused on path planning in dynamic environment, such as artificial potential field (APF) method, genetic algorithm, neural network [4-8]. Among these approaches, the APF method is widely used for mobile robot path planning due to its elegant mathematical analysis and simplicity.

The APF method was first proposed by Oussama Khatib in 1986 [9]. It constructs a scalar field comprising artificial "hills" representing obstacles and "valleys" representing attractors. The negative gradient of the potential field generates appropriate repulsive and attractive forces to guarantee obstacle avoidance and facilitate target reaching [10]. However, the APF method often suffers traps due to local minima, oscillations and GNRON problem. Moreover, it is not suitable for arbitrarily shaped obstacles. All these drawbacks have limited its use in path planning for mobile robots.

To overcome these drawbacks, many researchers have attempted to improve the conventional APF method. Kim and Khosla proposed the harmonic potentials to eliminate the local minima problem [11]. Xiaoping Yun and Ko-Cheng Tan introduced a wall-following method for escaping local minima encountered [12]. This algorithm switches to a wall-following control mode when the robot falls into a local minimum. S.S.Ge and Y.J.Cui first described the GNRON problem and improved repulsive potential function to solve it [6,13]. Josu Agirrebeitia defined a new potential function based on establishing a potential density along the geometry of the obstacle [14]. This method can easily model for arbitrarily shaped obstacles and establish accurate analytical expression.

These methods all modify the conventional repulsive function to improve their performance. And their modified repulsive functions are complicated. If velocities of goal and obstacles were considered in dynamic workspace, repulsive functions would become more complicated and the time cost of computation would be longer. However, attractive functions in these methods are simple correspondingly and almost the same as the conventional one. Therefore, an improved attractive function is proposed in this paper, which increases the gradient of potential field near the goal and ensures the goal the minimum of total potential field. The repulsive function in this paper is an extension of Josu Agirrebeitia's one. Compared with conventional repulsive function, the modified one is able to deal with an arbitrarily shaped obstacle and overcome U-shaped trap, and oscillations. Moreover, the new method has a great performance in dynamic path planning.

## 2 IMPROVED ATTRACTIVE FUNCTION AND THE GNRON

The conventional attractive and repulsive functions are defined as

$$U_{att}\left(q\right)=k_a\rho^{m_1}\left(q,q_{goal}\right) \tag{1}$$

$$U_{rep}\left(q\right)=\frac{k_r}{\rho^{m_2}\left(q,q_{obs}\right)} \tag{2}$$

where $\rho(q,q_{goal})$ and $\rho(q,q_{obs})$ denote distances from the goal and obstacle to the robot, respectively. $k_a$, $k_r$, $m_2$ and $m_1$ are positive constants. The total potential function is the sum of attractive and repulsive ones as (3).

$$U_{tot}(q)=U_{att}(q)+U_{rep}(q) \quad (3)$$

If an obstacle is very close to the goal, the repulsive potential near the goal is much larger than the attractive potential based on (1) and (2). When the robot moves here, it will be repulsed away rather reaching the goal. This is the problem of goals nonreachable with obstacles nearby (GNRON), which was first described by S.S.Ge in 2000. The GNRON problem arises because the goal position, which is near obstacles, is not the global minimum of the total potential field. After describing the GNRON problem, S.S.Ge introduced $\rho(q,q_{goal})$ in the repulsive function and succeeded in eliminating the problem. However, the repulsive function becomes complicated due to the modification while the attractive function is much simpler in this algorithm. If the velocities of robot and obstacles were considered in dynamic environment, the repulsive function would be more complicated and the time cost would be longer. So the attractive function is improved to overcome the GNRON problem in this paper.

A new attractive potential function is proposed as follows

$$U_{att}(q)=k_1\rho^{n_1}(q,q_{goal})+k_2\left(\frac{1}{\rho_0^{n_2}}-\frac{1}{\left(\rho_0+\rho(q,q_{goal})\right)^{n_2}}\right) \quad (4)$$

where $k_1,k_2$, $n_1$ and $n_2$ are positive constants. $\rho_0$ is a positive constant and $\rho_0<\rho(q_{goal},q_{obs})$. If $\rho(q,q_{goal})=0$, $U_{att}(q)=0$. When the robot is relatively far from the goal, $U_{att}(q)$ approximates $k_1\rho^{n_1}(q,q_{goal})+k_2/\rho_0^{n_2}$. Compared with conventional attractive function, the new one keeps strong attractive potential at far place and decreases the potential nearby the goal position evidently. Consequently, the gradient of the potential and the attractive force near the goal both increase.

Substituting (2) and (4) into (3), the total potential function is given by

$$U_{tot}(q)=k_1\rho^{n_1}(q,q_{goal})+k_2\left(\frac{1}{\rho_0^{n_2}}-\frac{1}{\left(\rho_0+\rho(q,q_{goal})\right)^{n_2}}\right)+\frac{k_3}{\rho^{n_3}(q,q_{obs})} \quad (5)$$

When $k_2=0$, equation (5) becomes the conventional total potential function. Then, we take two examples to illuminate the performance of total potential function.

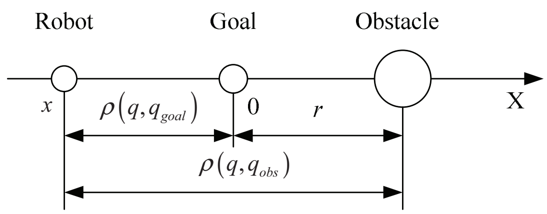
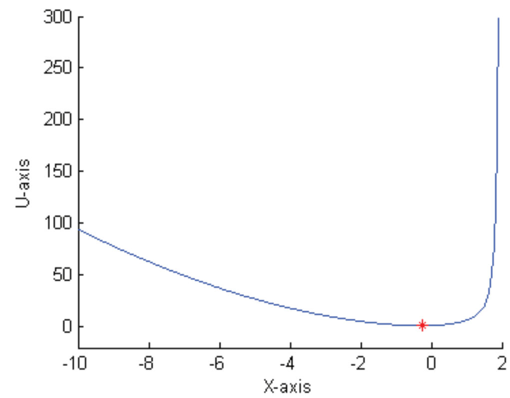
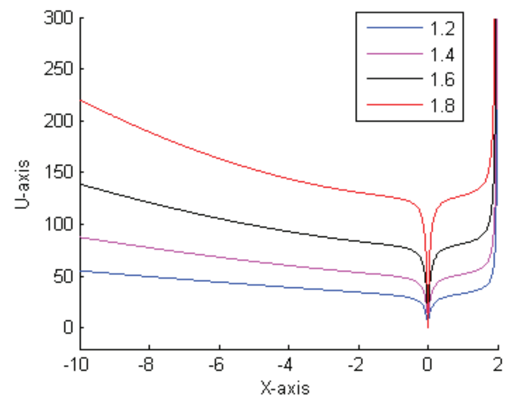



Fig.1. Positions of the robot, goal and obstacle




(a)




(b)

Fig.2 Conventional and new total potential in a one-dimensional case

In a one-dimensional case, the robot is moving along the $x$-axis towards the goal in Fig.1 [13]. The goal lies on the origin and variable $x$ denotes the robot position. Fig.2(a) shows the total potential using traditional attractive and repulsive functions for the case in Fig.1, where $x_{obs}=2$, $k_2=0$, $k_3=5$, $\rho_0=0.1$ and $n_1=n_2=n_3=1.8$. The red star lies on $x=-0.26$. And it is the global minimum of the total potential rather $x=0$. So the robot is trapped at $x=-0.26$ and can't reach the goal. The total potential using the new attractive function in one-dimensional case is shown in Fig.2(b), where $n_1=n_2=n_3$ .The red, black, magenta and blue lines represent the total potential when $n_1=1.8$, $n_1=1.6$, $n_1=1.4$ and $n_1=1.2$, respectively. Other parameters are the same as Fig.2(a). Obviously, the total potential decreases sharply aroundthe goal and $x=0$ is the global minimum. Moreover, Fig.2(b) shows that the gradient of total potential is large when $n_1$, $n_2$ and $n_3$ are large. In fact, $n_1$, $n_2$, $n_3$ respectively affect the gradient of total potential at far place, around goal and obstacle nearby.

In a two-dimensional case, we compare the new method with traditional one in Fig.3. In this paper, blue and red blocks represent obstacles and the goal, respectively. A black block is the initial position of robot. A black line shows the track of robot. Red, yellow and blue

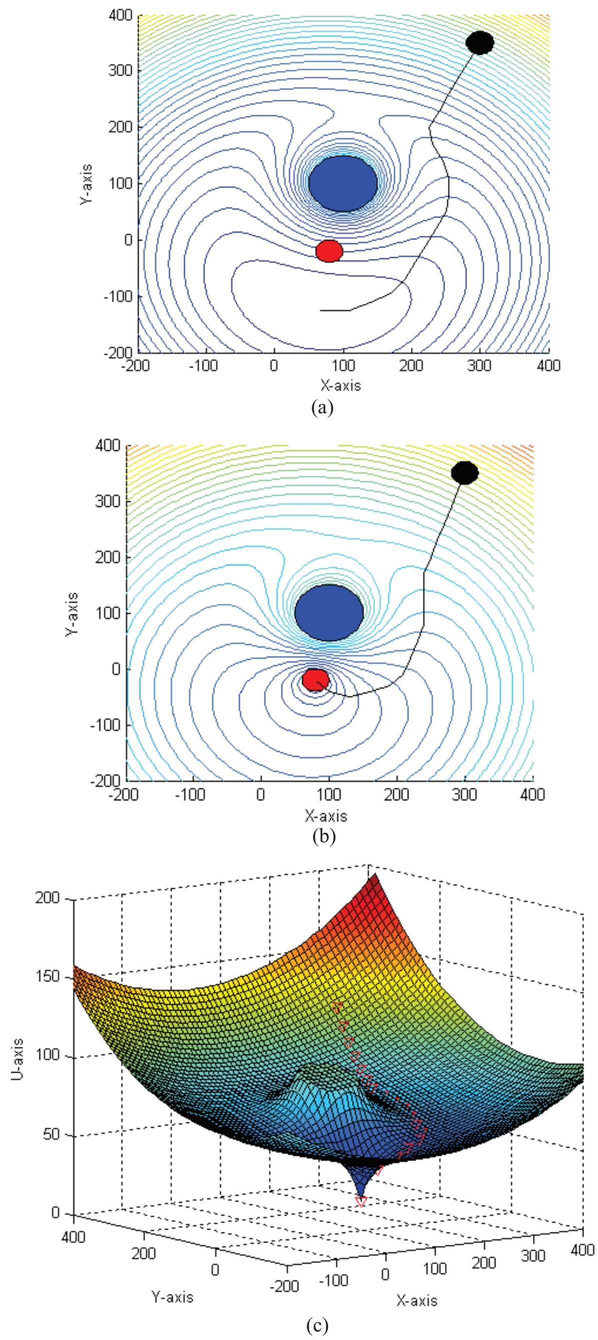



Fig.3 The total potential in a two-dimensional case

equipotential lines respectively denote high, middle and low potential. Fig.3(a) shows the total potential field using the conventional attractive function. The goal position isn't the center of red equipotential lines, so it isn't the global minimum. The location (65,-125) is the minimum and the robot is trapped there. The modified attractive function is applied in Fig.3(b). Because the new model decreases the potential near the goal evidently, the goal becomes the global minimum and is able to be reached. Fig.3(c) shows the three-dimensional potential field using the modified attractive function. X and Y coordinates describe locations and Z coordinate denotes the total potential. Red triangles represent the track of robot. In this figure, the position of obstacle sticks up like a hill while the goal descends like a valley. The robot advances from high potential positions to low ones and arrives at the goal finally.

## 3 THE CONVERGENCE

The modified attractive function succeeds in solving the GNRON problem in above two cases. Here, the convergence of improved algorithm is demonstrated in a general one-dimensional case. The locations of goal, robot and obstacle are shown in Fig.1. The total potential function is expressed in (5). To make sure that the goal is the minimum, the total potential function should decrease when $x \in (-\infty, 0)$ and increase when $x \in (0, r)$, monotonically. So we calculate its derivative with respect to $q$

$$U'_{tot}(q) = k_1 n_1 \rho\left(q, q_{goal}\right)^{n_1 - 1} + \frac{k_2 n_2}{\left(\rho_0 + \rho\left(q, q_{goal}\right)\right)^{n_2+1}} - \frac{k_3 n_3}{\rho^{n_3+1}\left(q, q_{obs}\right)} \tag{6}$$

When $x \in (-\infty, 0)$, we have $\rho(q, q_{goal}) = -x$, $\rho(q_{goal}, q_{obs}) = r$, $\rho(q, q_{obs}) = r - x$. Equation (6) can be written as

$$U'_{tot}(x) = -k_1 n_1 (-x)^{n_1 - 1} - \frac{k_2 n_2}{(\rho_0 - x)^{n_2+1}} + \frac{k_3 n_3}{(r - x)^{n_3+1}} \tag{7}$$

To ensure monotone decrease when $x \in (-\infty, 0)$, $U'_{tot}(x)$ should be less than zero. If (8) or (9) is satisfied, $U'_{tot}(x) < 0$.

$$k_1 n_1 (-x)^{n_1 - 1} > \frac{k_3 n_3}{(r - x)^{n_3+1}} \tag{8}$$

$$\frac{k_2 n_2}{(\rho_0 - x)^{n_2+1}} > \frac{k_3 n_3}{(r - x)^{n_3+1}} \tag{9}$$

When $x < -\left(\frac{k_3 n_3}{k_1 n_1}\right)^{\frac{1}{n_1 + n_3}}$, (10) is true.

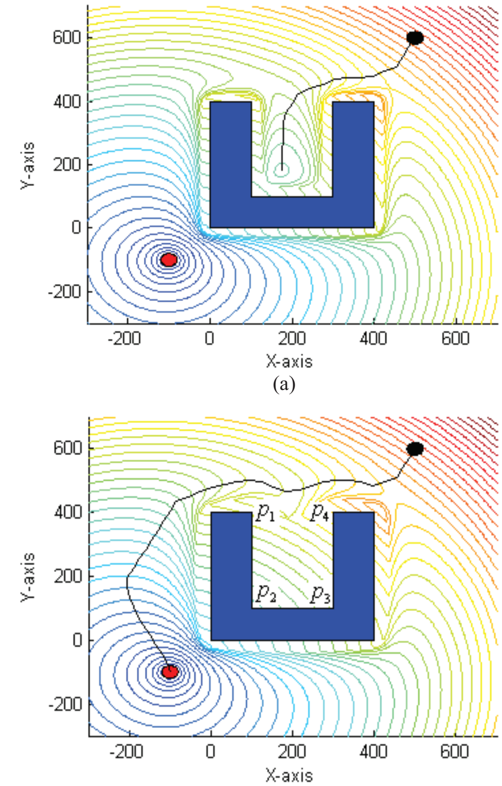
$$k_1 n_1 (-x)^{n_1 - 1} > \frac{k_3 n_3}{(-x)^{n_3+1}} \tag{10}$$

As $r$>0, (8) is satisfied. Thus, the total potential function monotonically decreases when $x \in (-\infty, -\left(\frac{k_3 n_3}{k_1 n_1}\right)^{\frac{1}{n_1 + n_3}})$.

When $x > -\left(\frac{k_2 n_2}{k_3 n_3}\right)^{\frac{1}{n_2 - n_3}}$, (11) is true.

$$\frac{k_2 n_2}{(-x)^{n_2+1}} > \frac{k_3 n_3}{(-x)^{n_3+1}} \tag{11}$$

As $\rho_0 < r$ is satisfied, (9) can be obtained. So the total potential function monotonically decreases during




(a)

(b)

Fig.4 Dropping in and evading the U-shaped trap

$x \in (-\left(\frac{k_2 n_2}{k_3 n_3}\right)^{\frac{1}{n_2 - n_3}}, 0)$.

Therefore, $U'_{tot}(q) < 0$ is satisfied when $x \in (-\left(\frac{k_2 n_2}{k_3 n_3}\right)^{\frac{1}{n_2 - n_3}}, 0)$ or $x \in (-\infty, -\left(\frac{k_3 n_3}{k_1 n_1}\right)^{\frac{1}{n_1 + n_3}})$. If $k_1 n_1 > k_3 n_3$, $k_2 n_2 > k_3 n_3$, $n_2 > n_3$ and $\rho_0 < r$, (12) is true.

$$-\left(\frac{k_3 n_3}{k_1 n_1}\right)^{\frac{1}{n_1 + n_3}} > -\left(\frac{k_2 n_2}{k_3 n_3}\right)^{\frac{1}{n_2 - n_3}} \tag{12}$$

When (12) is obtained, $U'_{tot}(q) < 0$ is satisfied when $x \in (-\infty, 0)$.

when $x \in (0, r)$ ,we obtain $\rho(q, q_{goal}) = x$ , $\rho(q_{goal}, q_{obs}) = r$ , $\rho(q, q_{obs}) = r - x$ . Thus, (6) can be written as

$$U'_{tot}(x) = k_1 n_1 x^{n_1 - 1} + \frac{k_2 n_2}{(\rho_0 + x)^{n_2 + 1}} + \frac{k_3 n_3}{(r - x)^{n_3 + 1}} \tag{13}$$

Since $k_1, k_2, k_3, n_1, n_2, n_3$ are all positive constants, $U'_{tot}(q) > 0$ is always satisfied. Therefore, the total potential function monotonically increases during $x \in (0, r)$.

When $k_1 n_1 > k_3 n_3$, $k_2 n_2 > k_3 n_3$, $n_2 > n_3$, and $\rho_0 < r$, the total potential function decrease monotonically when $x \in (-\infty, 0)$ and increase monotonically when $x \in (0, r)$. It converges to the goal position, i.e., the goal position is the minimum of total potential.

## 4 THE REPULSIVE POTENTIAL FIELD

In section 2 and 3, the conventional repulsive potential function is used where obstacles are simplified into circles. It is simple and easy to control in real-time, but it is not exact for complicated shaped obstacles. Therefore, a new repulsive potential function extended from Josu Agirrebeitia's model is proposed. It can construct an accurate model for arbitrarily shaped obstacles and provide precise analytical representation.

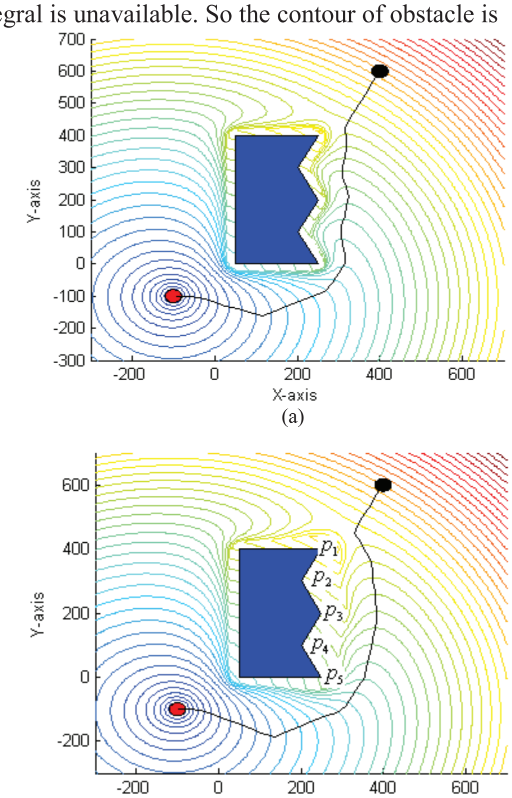
The repulsive potential function in Josu Agirrebeitia's algorithm is defined as

$$U_{rep}(X, Y) = \oint_C D(X, Y, x, y) dl \tag{14}$$

where $D(X, Y, x, y)$ is the density function. It is given as:

$$D(X, Y, x, y) = \frac{K_R}{d(X, Y, x, y)^{n_r}} \tag{15}$$

where $K_R$ and $n_r$ are positive constant values. $P(X, Y) \in \Phi$, $\Phi$ is the space of repulsive potential field. $p(x, y) \in C$, $C$ is the outline of obstacle. $d(X, Y, x, y)$ is the distance from $P(X, Y)$ to $p(x, y)$. The repulsive potential at the point $P(X, Y)$ equals the integral of the density function along the contour of obstacle. However, the function of contour is usually unknown and direct integral is unavailable. So the contour of obstacle is




(a)

(b)

Fig.5 Suffering and avoiding oscillations scattered into discrete points. The curve integral is changed into the integral of lines. The contour of obstacle is redefined as:

$$C=\left\{p_i\left(x,y\right)\middle|\, i\le n, n\in N\right\} \tag{16}$$

where $n$ is the number of discrete points. The repulsive potential function is given as

$$U_{rep}\left(X,Y\right)=\sum_{i=0}^{n-1}K_{ri}\int_{p_i}^{p_{i+1}}\frac{1}{d\left(X,Y,x,y\right)^{n_1}}dl \tag{17}$$

where $p_0=p_n$ . The repulsive potential model can be applied in arbitrarily shaped obstacles. $K_{ri}$ is a positive constant and indicates the repulsive ability for $i$th edge of the obstacle. When $K_{ri}$ is large, the repulsive force of $i$ th edge is strong. Otherwise the repulsive force is weak. If we change the value of $K_{ri}$ , the repulsive potential field near the $i$th edge is altered. Therefore, we can adjust $K_{ri}$ to avoid unfavorable workspace. For example, U-shaped traps and oscillations are often encountered in a mobile robot path planning. By adjusting $K_{ri}$ , these problems can be solved.

There is a U-shaped obstacle represented by a blue block in Fig.4. Equation (17) is used to build repulsive potential field. $K_{ri}$ for eight edges of the obstacle are the same. The robot locates at the point (500,600) initially and moves towards the goal at (-100,-100). In Fig.4(a), the goal position is the global minimum of total potential due to the improved attractive function. But a local minimum point appears at (175,180) within the U-shaped obstacle. As a consequent, the robot drops in the U-shaped trap. To avoid the trap, the repulsive force within the U-shaped obstacle should be increased. Therefore, we enhance the values of $K_{ri}$ for $p_1p_2$ , $p_2p_3$ , $p_3p_4$ edges in Fig.4(b). Accordingly, the local minimum disappears. Finally, the robot avoids the U-shaped obstacle and reaches the goal.

The robot encounters a zigzag obstacle in Fig.5(a). Positions of robot, goal and obstacles are shown there. $K_{ri}$ for each edge is the same and equipotential lines near the obstacle are similar with their contours. Because the robot moves along the negative gradient of potential field, oscillations of robot appear subsequently. So we increase the value of $K_{ri}$ for $p_1p_2$ , $p_2p_3$ , $p_3p_4$ , $p_4p_5$ edges and the new potential field is shown in Fig.5(b). Oscillations eliminate and the robot advances towards the goal smoothly.

The modified repulsive potential function can be combined with the conventional one. For round or regular obstacles, the conventional function is enough. Moreover, the model is built simply and quickly. However, a complicated shaped obstacle needs the modified function as well as the conventional one. When the obstacle is far and unclear, its contour isn't available. The obstacle can be treated as a circle using the conventional function. When it is near and clear, the modified function is adopted to build an exact repulsive model. This combination improves the efficiency of algorithm and keeps its accuracy for arbitrarily shaped obstacles.

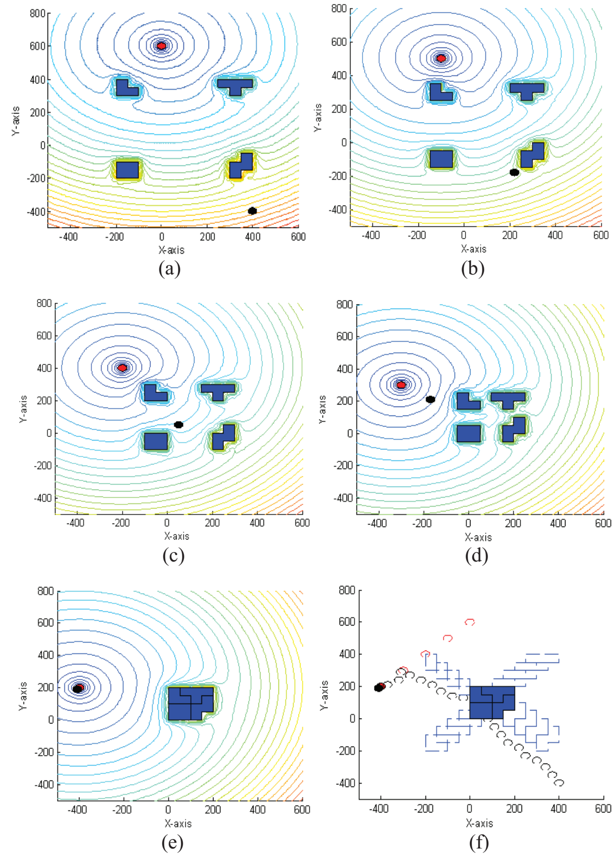



Fig.6 Dynamic path planning using the new algorithm

## 5 DYNAMIC ENVIRONMENT

Using the modified attractive and repulsive functions, the new potential model has a great performance in dynamic environment. The simulation in Fig.6 illustrates its validity when obstacles and goal are moving.

There are four moving obstacles and their initial positions are illustrated in Fig.6(a). Then they move towards the point (100,100) while the goal moves along a line. The motion process and the potential field are shown in (b) (c) (d) (e).The trajectories of robot, goal and obstacles are illustrated in (f). When the goal arrives at (-300,300), the robot catches it and then follows it to move. From these figures, the robot can avoid the moving obstacles and catch the moving goal even passagesbetween obstacles are narrow.

## 6 CONCLUSION

The proposed potential method here enhances the precision of algorithm and overcomes some drawbacks of traditional potential one. Based on the modified potential methodology, the GNRON problem is eliminated by increasing the gradient of attractive potential function around the goal. Moreover, the repulsive potential function can deal with arbitrarily shaped obstacles, keep away U-shaped trap and evade oscillations. From the simulation, we can see that the proposed method has a great performance in dynamic environment. Nevertheless, the modified repulsive potential model isn't available if the contour of obstacle is not clear. Therefore, the modified and the traditional repulsive functions are combined to overcome this drawback and improve its efficiency.

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