`YalAA` - Yet Another Library for Affine Arithmetic

Stefan Kiel

University of Duisburg-Essen

Affine arithmetic (AA) is a model for verified computations proposed by Comba and Stolfi [1]. In contrast to interval arithmetic it tracks first-order correlations in variables during the computation and thus often leads to tighter range bounds for functions suffering from the dependency effect. An affine form $\hat{x} = x_0 + \sum_{i=1}^{n} \boldsymbol{x}_i \epsilon_i$ has a central value x_0 and error terms consisting of a partial deviation x_i and a symbolic noise variable $\epsilon_i \in [-1, 1]$ modeling linear correlations. Several improvements for the original model were proposed in [2]-[5]. However, existing publicly available implementations (e.g. `libaa`, `libaffa`) have a number of shortcomings and do not support these extensions.

In this talk we present `YalAA`, a newly developed object-oriented library for AA. Similarly to the `Boost.Interval` package, it uses a configurable base type for representation of the partial deviations. `YalAA`'s functionality is controlled by policy classes: `ErrorTerm`, `AffineCombination`, `ArithmeticKernel`, `ErrorPolicy`, `AffinePolicy`.

`ErrorTerm` defines the representation of the symbolic noise variables and the partial deviations, while `AffineCombination` models a combination of several `ErrorTerm` objects and supplies the basic affine operations: addition, scaling and translation. The kernel implements the actual mathematical operations. The library also contains a kernel suitable for standard floating-point types. If possible it implements the elementary functions as described in [6]. Otherwise we use a Chebyshev interpolation based approach. We plan to implement other kernels featuring the faster computation model proposed in a recent SCAN talk [3]. Handling such errors as domain violation or overflows is controlled by the `ErrorPolicy`. `AffinePolicy` controls the way new affine noise symbols are introduced. This allows us to implement the AF1 and AF2 forms described in [2].

Currently we are working on an implementation of generalized interval arithmetic [7] in `YalAA`. Similarly to AA, it tracks first order correlations but represents the partial deviations with tight intervals. A further goal is the support for higher order forms like those introduced in [4],[5]. However, a new arithmetic kernel which exploits the higher order noise symbols in the approximation of non-affine functions would be necessary in this case. Note that this topic is not entirely explored because the relevant publications focus on polynomial functions. Consequently they do not cover non-affine operations or elementary functions other than multiplication or the integer power function.

References:

[1] J. L. D. Comba and J. Stolfi *Affine Arithmetic and its Application to Computer Graphics.*

[2] F. Messine *Extensions of Affine Arithmetic: Applications to Unconstrained Global Optimization.*

[3] J. Ninin and F. Messine *Reliable Affine Arithmetics.*

[4] G. Bilotta *Self-Verified Extension of Affine Arithmetic to Arbitrary Order.*

[5] F. Messine and A. Touhami *A General Reliable Quadratic Form: An Extension of Affine Arithmetic.*

[6] L. H. de Figueiredo and J. Stolfi *Self-Validated Numerical Methods and Applications.*

[7] E. Hansen *A Generalized Interval Arithmetic.*