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EC 330 HW Assignment 1

1) a) $\sum_{i=1}^{330} \left(\frac{1}{3}\right)^i$ $\quad S_n = \frac{a_1(1-r^n)}{1-r}$ $\quad S_n = \frac{\frac{1}{3}\left(1-\frac{1}{3}^{330}\right)}{1-\frac{1}{3}} = \frac{1\left(1-\frac{1}{3}^{330}\right)}{\cancel{3}\left(\frac{2}{\cancel{3}}\right)}$

$a_1 = \left(\frac{1}{3}\right)^1 = \frac{1}{3} \quad r = \frac{1}{3}$ $\quad = \boxed{\frac{1}{2}}$

b) $\sum_{i=0}^{\infty} \left(\frac{2}{9}\right)^i$ $\quad r = \frac{2}{9}$ $\quad S_\infty = \frac{a_0}{1-r} = \frac{1}{1-\frac{2}{9}} = \frac{1}{\frac{7}{9}} = \boxed{\frac{9}{7}}$

$a = \left(\frac{2}{9}\right)^0 = 1$

c) $\sum_{i=1}^{N} (i^3 + 3i^2 - 5i + 7) = \sum_{i=1}^{N} i^3 + \sum_{i=1}^{N} 3i^2 - \sum_{i=1}^{N} 5i + \sum_{i=1}^{N} 7$

$3\sum_{i=1}^{N} i^2 \qquad 5\sum_{i=1}^{N} i$

$\frac{1}{4}N^2(N+1)^2 + \cancel{3}\cdot\frac{1}{\cancel{6}2}N(N+1)(2N+1) - \frac{5}{2}N(N+1) + 7N$

$\frac{N^2(N+1)^2}{4} + \frac{N(N+1)(2N+1)^2}{2} - \frac{10N(N+1)}{4} + \frac{28N}{4}$

$\boxed{\frac{1}{4}\left(N^2(N+1)^2 + N(N+1)(2N+1)^2 - 10N(N+1) + 7N\right)}$

2) a) $x^1 \cdot x^2 \cdot x^3 \cdots x^{330} = \prod_{i=1}^{330} x^i = x^{\sum_{i=1}^{330} i} = x^{\left(\frac{1}{2}\cdot 330(330+1)\right)} = \underline{\underline{x^{54615}}}$

b) $\log_x x^{330x} = ?$ $\quad x^? = x^{330x}$ $\quad ? = \underline{\underline{330x}}$

c) $\log_{330}(330^{330}\cdot 330) = \log_{330} 330^{330} + \log_{330} 330 = 330 + 1 = \underline{\underline{331}}$

3) a) 33-digit Hex $\_\,\_\,\_\,\_ \ldots 12 \ldots 12$ $\quad 12^{33} = 4.102\cdot 10^{35}$ hex decimal #'s

16 choices
− 4 choices
12 choices

~~b) $x_1 + x_2 + x_3 = 33$ $\quad x_1 \geq 5$ $\quad x_2 \geq 2$ $\quad x_3 \geq 3$ $\quad 5 \leq x_1 \leq 34$~~ $\quad -3 \leq x_3 \leq 26$

b) in the back

~~$33 - 5 - 2 - 3 = 29$~~ $\quad$ ~~$2 \leq x_2 \leq 31$~~

~~when $x_1 = 5$ x_2 could be anything from 2 to 31 which is 29 choices plus 2 additional options~~

~~$29C_3$~~ $\quad \binom{31}{2}$

3654

3) b) $x_1 + x_2 + x_3 = 33 \quad x_1 \geq 5, \; x_2 \geq 2, \; x_3 \geq -3$

$y_1 = x_1 - 5 \quad y_2 = x_2 - 2 \quad y_3 = x_3 + 3$

$y_1 + 5 = x_1 \quad y_2 + 2 = x_2 \quad y_3 - 3 = x_3$

$y_1 + 5 + y_2 + 2 + y_3 - 3 = 33$

$y_1 + y_2 + y_3 = 29 \quad N = 29 \quad K = 3$

$\binom{n+k-1}{k-1} = \binom{31}{2}$

$= \dfrac{31!}{(29!)(2)} = 465$

$\cancel{31!}$ $\dfrac{31!}{29!} = \dfrac{31 \cdot 30 \cdot \cancel{29} \cdots}{\cancel{29} \cdot \cancel{28} \cdots}$

4) a) $F_0, F_1, F_2, \ldots$

$F_0 = 1, \; F_1 = 1, \; F_n = F_{n-1} + F_{n-2}$

$F_n \geq 2^{\frac{1}{2}n}$ for $n \geq 2$

Base Case: $F_2 = \phi + 1 = 2$ $\cancel{F_3 = 2 + 1 = 3}$

$2 \geq 2^{\frac{1}{2} \cdot 2} \quad 2 \geq 2$ ✓

induction step: $F_k \geq 2^{0.5k}$

$\cancel{F_{k+1} \geq 2^{0.5}}$ $F_{k+1} = F_{k+1-1} + F_{k+1-2}$

$F_k \Rightarrow F_{k+1}$

$F_{k+1} = F_k + F_{k-1}$

assume true for F_k

$\geq 2^{0.5k}$

$F_{k-1} \geq 2^{0.5(k-1)}$

$F_{k-1} \geq 2^{0.5k - 0.5}$

$F_{k+1} \geq 2^{0.5(k+1)}$ $\cancel{= 2^{0.5k+0.5}}$

$F_{k+1} \geq 2^{0.5k} + 2^{0.5k-0.5}$

$2^{0.5k} + 2^{0.5k-0.5} \geq 2^{0.5k+0.5}$

$2^{0.5k} + \dfrac{2^{0.5k}}{2^{0.5}} \geq 2^{0.5k+0.5}$

$\dfrac{2^{0.5}(2^{0.5k}) + 2^{0.5k}}{2^{0.5}} \geq 2^{0.5k}(2^{0.5})$

$2^{0.5}(2^{0.5k}) + 2^{0.5k} \geq 2^{0.5k}(2)$

$2^{k} + 2^{0.5k} \geq 2^{0.5k+1}$ ✓

$\cancel{2^{0.5k+0.5} = 2^{0.5k} + 2^{0.5k-2}}$

$\cancel{2^{0.5k} \cdot 2^{0.5} = (2^{0.5k}) \; 2^{0.5k} + \dfrac{2^{0.5k}}{2^2}}$

$\cancel{2^{0.5k} \cdot 2^{0.5} = \dfrac{4 \cdot 2^{0.5k} + 2^{0.5k}}{4} = \dfrac{2^{0.5k}(4+1)}{4}}$

$\cancel{2^{0.5k} = \dfrac{5}{4}}$

4 b) $F_n \leq 2^{cn}$ $c<1$ $F_0 = 1$ $F_1 = F$ $F_{n+2} = F_{n+1} + F_n$ or $F_n = F_{n-1} + F_{n-2}$

Base case: $F_{0+2} = F_1 + F_0 = 2$ $2 \leq 2^{cn}$ and $c < 1$ ✓ if $n \geq 2$ $\frac{1}{2}$

$1 \leq cn$ $1 \leq c \cdot 2$ $\frac{1}{2} \leq c < 1$ ✓

induction step:

assume for $k \geq 2$ $F_k \leq 2^{ck}$ $F_{k+1} \leq 2^{ck+c}$ $F_{k+2} = 2^{ck+2c}$

$F_{k+2} = F_k + F_{k+1} \leq 2^{ck} + 2^{ck+c} = 2^{ck}(1+2^{c}) = \frac{(1+2^{c})\,2^{c(k+2)}}{2^{2c}} = 2^{c(k+2)}$

$\frac{1+2^c}{2^{2c}} = 1$ $= \frac{(1+2^c)}{2^{2c}} = \frac{2^{c(k+2)}}{2^{c(k+2)}}$ $\frac{(1+2^c)2^{ck}\cancel{(2^{2c})}}{\cancel{2^{2c}}}$

$1+2^c = 2^{2c}$ $(1+2^c)2^{-2c} = 1$

$1 = 2^{2c} - 2^c$ $(1+2^c)(2^c)^{-2} = 1$ $2^c = u$

$1 = 2^c(2^c) - 2^c$ No solution

$1 = 2^c(2^c - 1)$ $(1+u)(u^{-2}) = 1$ $2^c = \frac{1+\sqrt{5}}{2}$ or $\cancel{2^c = \frac{1-\sqrt{5}}{2}}$ for $c \in \mathbb{R}$

$u = \frac{1+\sqrt{5}}{2}$ $u = \frac{1-\sqrt{5}}{2}$ $c = \frac{\ln\left(\frac{1+\sqrt{5}}{2}\right)}{\ln(2)} < 1$ ✓

5 what is the value of sum?

```
int sum=0;                                    sum=0
for (int i=n; i>0; i--){                      iterates n times
    for (int j=n-i; j<n; j++{                 starts at 0 goes up to n-i iterates n times
        sum= sum + 1
    }
}
```

say $n = 10$

Sum = ~~38~~

~~Sum = n(n-1)+n-2+ ... 0~~

when $i = 10$

int $j = n - i = 0$ $j = 0 \to 9$ when $i = 10$ 1st time sum=0

$j{+}{+} \Rightarrow j = 1$ $j = 1 \to 9$ when $i = 9$ Sum = 1 + 0

$j < n$ ✓ $j = 2 \to 9$ when $i = 8$ sum=4+1+0 when i=9 ... i=1

sum= 1 ⋮ j=0→8

10→1 9 times + 8 times + 7 times ...

$\boxed{\sum_{i=1}^{n} n}$

32 10 times

times times i^2

9^2 ... 9^2

1 ... 2^2 ... 9^2

+ 19^2