Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. 10011011_2 =
   $(2^0 * 1) + (2^1 *1) + (2^3 * 1) + (2^4 * 1) + (2^7 * 1)$ =
   $(1 * 1) + (2 * 1) + (8 * 1) + (16 * 1) + (128 * 1)$ =
   1 + 2 + 8 + 16 + 128 = **155**

2. $456_7 = (7^0 * 6) + (7^1 * 5) + (7^2 * 4) = 6 + 35 + 196 =$ **237**

3. $38A_{16} = (16^0 * 10) + (16^1 * 8) + (16^2 * 3) = 10 + 128 + 768 =$ **906**

4. $2214_5 = (5^0 * 4) + (5^1 * 1) + (5^2 * 2) + (5^3 * 2) = 4 + 5 + 50 + 250 =$ **309**

B. Convert the following numbers to their binary representation:

1. $69_{10} - ((2^6*0) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 69_{10}$
   $69_{10} - ((2^6*1) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 5_{10}$
   $69_{10} - ((2^6*1) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*1) + (2^1*0) + (2^0*0)) = 1_{10}$
   $69_{10} - ((2^6*1) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*1) + (2^1*0) + (2^0*1)) = 0$
   69_{10} = $\mathbf{1000101_2}$

2. $485_{10} - ((2^8*0) + (2^7*0) +(2^6*0) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 485_{10}$
   $485_{10} - ((2^8*1) + (2^7*0) +(2^6*0) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 229_{10}$
   $485_{10} - ((2^8*1) + (2^7*1) +(2^6*0) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 101_{10}$
   $485_{10} - ((2^8*1) + (2^7*1) +(2^6*1) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 37_{10}$
   $485_{10} - ((2^8*1) + (2^7*1) +(2^6*1) + (2^5*1) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 5_{10}$
   $485_{10} - ((2^8*1) + (2^7*1) +(2^6*1) + (2^5*1) + (2^4*0) + (2^3*0) + (2^2*1) + (2^1*0) + (2^0*0)) = 1_{10}$
   $485_{10} - ((2^8*1) + (2^7*1) +(2^6*1) + (2^5*1) + (2^4*0) + (2^3*0) + (2^2*1) + (2^1*0) + (2^0*1)) = 0$
   485_{10} = $\mathbf{111100101_2}$

3. $6D1A_{16}$ = $\mathbf{0110110100011010_2}$

$$6D1A_{16} = \underbrace{0\,1\,1\,0}_{6}\;\underbrace{1\,1\,0\,1}_{D}\;\underbrace{0\,0\,0\,1}_{1}\;\underbrace{1\,0\,1\,0}_{A}\,{}_2$$

C. Convert the following numbers to their hexadecimal representation:

1. 1101011_2 = $\mathbf{6B_{16}}$

$$1\,1\,0\,1\,0\,1\,1_2 =$$

$$= \underbrace{(1*2^0)+(1*2^1)+(0*2^2)+(1*2^3)}+\underbrace{(0*2^4)+(1*2^5)+(1*2^6)+(0*2^7)}$$

$$= \underbrace{2^0((1*2^0)+(1*2^1)+(0*2^2)+(1*2^3))}+\underbrace{2^4((0*2^0)+(1*2^1)+(1*2^2)+(0*2^3))}$$

$$= \underbrace{2^0(1+2+0+8)}+\underbrace{2^4(0+2+4+0)}$$

$$= \underbrace{2^0(11)} + \underbrace{2^4(6)}$$

$$= \underbrace{16^0*11}_{B_{16}} + \underbrace{16^1*6}_{6_{16}} = 6B_{16}$$

2. 895_{10} = $\mathbf{37F_{16}}$
   $895_{10}/16 = 55$ R $15_{10} \rightarrow F_{16}$
   $55_{10}/16 = 3$ R 7
   $3_{10}/16 = 0$ R 3

Question 2:

Solve the following, do all calculations in the given base. Show your work.

1. $7566_8 + 4515_8 =$ **$14{,}303_8$**

```
  111
  7566
 +4515
 14303
```

2. $10110011_2 + 1101_2 =$**11000000_2**

```
  111111
 10110011
    +1101
 11000000
```

3. $7A66_{16} + 45C5_{16} =$ **$C02B_{16}$**

```
  11
  7A66
 +45C5
  C02B
```

4. $3022_5 - 2433_5 =$ **34_5**

$$\begin{array}{rcccc} & 2 & 4 & 11 & \\ & \cancel{3} & \cancel{0} & \cancel{2} & {}^{1}2 \\ - & 2 & 4 & 3 & 3 \\ \hline & 0 & 0 & 3 & 4 \end{array}$$

<u>Question 3:</u>

A. Convert the following numbers to their 8-bits two's complement representation. Show
your work.

1. 124_{10} - $((2^6*0) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 124_{10}$
   124_{10} - $((2^6*1) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 60_{10}$
   124_{10} - $((2^6*1) + (2^5*1) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 28_{10}$
   124_{10} - $((2^6*1) + (2^5*1) + (2^4*1) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 12_{10}$
   124_{10} - $((2^6*1) + (2^5*1) + (2^4*1) + (2^3*1) + (2^2*0) + (2^1*0) + (2^0*0)) = 4_{10}$
   124_{10} - $((2^6*1) + (2^5*1) + (2^4*1) + (2^3*1) + (2^2*1) + (2^1*0) + (2^0*0)) = 0_{10}$
   124_{10} = $\mathbf{01111100}_{\text{8 bit 2's comp}}$ (← Had to add the 0 in the LMB place to complete)

2. -124_{10} = $\mathbf{10000100}_{\text{8 bit 2's comp}}$

```
 [01111100] = 124 8 bit 2's comp

        11  <- Carry Over
  10000011 <- Inverse of 124
 +00000001 <- Addition of 1
  10000100 = -124 8 bit 2's comp
```

3. 109_{10} - $((2^6*0) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 109_{10}$
   109_{10} - $((2^6*1) + (2^5*0) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 45_{10}$
   109_{10} - $((2^6*1) + (2^5*1) + (2^4*0) + (2^3*0) + (2^2*0) + (2^1*0) + (2^0*0)) = 13_{10}$
   109_{10} - $((2^6*1) + (2^5*1) + (2^4*0) + (2^3*1) + (2^2*0) + (2^1*0) + (2^0*0)) = 5_{10}$
   109_{10} - $((2^6*1) + (2^5*1) + (2^4*0) + (2^3*1) + (2^2*1) + (2^1*0) + (2^0*0)) = 1_{10}$
   109_{10} - $((2^6*1) + (2^5*1) + (2^4*0) + (2^3*1) + (2^2*1) + (2^1*0) + (2^0*1)) = 0_{10}$
   109_{10} = $\mathbf{01101101}_{\text{8 bit 2's comp}}$ (← Had to add the 0 in the LMB place to complete)

4. -79_{10} = $\mathbf{10110001}_{\text{8 bit 2's comp}}$

```
 [01001111] = 79 8 bit 2's comp

            <- Carry Over
  10110000 <- Inverse of 79
 +00000001 <- Addition of 1
  10110001 = -79 8 bit 2's comp
```

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}} = (2^6*0) + (2^5*0) + (2^4*1) + (2^3*1) + (2^2*1) + (2^1*1) + (2^0*0)$
   $0 + 0 + 16 + 8 + 4 + 2 + 0 = \mathbf{30_{10}}$

2. $11100110_{8\text{ bit 2's comp}} = \mathbf{-26_{10}}$

   1111111 <- Carry Over

   11100110 <- 8 bit 2's Comp Starting Value

   +00011010 <- Amount added to sum $0_{8\text{ bit 2's Comp}}$

   100000000

   ---------- -------------------------

   00011010 = 26_{10}

   11100110 = -26_{10}

3. $00101101_{8\text{ bit 2's comp}} = (2^6*0) + (2^5*1) + (2^4*0) + (2^3*1) + (2^2*1) + (2^1*0) + (2^0*1)$
   $0 + 32 + 0 + 8 + 4 + 0 + 1 = \mathbf{45_{10}}$

4. $10011110_{8\text{ bit 2's comp}} = \mathbf{-98_{10}}$

   1111111 <- Carry Over

   10011110 <- 8 bit 2's Comp Starting Value

   +01100010 <- Amount added to sum $0_{8\text{ bit 2's Comp}}$

   100000000

   ---------- -------------------------

   01100010 = 98_{10}

   11100110 = -98_{10}

<u>Question 4:</u>

Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.2.4, sections b, c**
   Write a truth table for each expression
   b. $\neg(p \lor q)$

p	q	$p \lor q$	$\neg(p \lor q)$
T	T	T	**F**
T	F	T	**F**
F	T	T	**F**
F	F	F	**T**

c. $r \lor (p \land \neg q)$

r	p	q	$\neg q$	$(p \land \neg q)$	$r \lor (p \land \neg q)$
T	T	T	F	F	**T**
T	T	F	T	T	**T**
T	F	T	F	F	**T**
T	F	F	T	F	**T**
F	T	T	F	F	**F**
F	T	F	T	T	**T**
F	F	T	F	F	**F**
F	F	F	T	F	**F**

2. **Exercise 1.3.4, sections b, d**
   Give a truth table for each expression

   b. $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	**T**
T	F	F	T	**T**
F	T	T	F	**F**
F	F	T	T	**T**

   d. $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	**T**
T	F	T	F	T	**T**
F	T	F	F	T	**T**
F	F	T	T	F	**T**

<u>Question 5:</u>

Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.2.7, sections b, c**

   Consider the following pieces of identification a person might have in order to apply for a credit card:

   B: Applicant presents a birth certificate.
   D: Applicant presents a driver's license.
   M: Applicant presents a marriage license.

   Write a logical expression for the requirements under the following conditions:

   b. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

      i. $(B \land D) \lor (D \land M) \lor (B \land M)$

   c. Applicant must present either a birth certificate or both a driver's license and a marriage license

      i. $B \lor (D \land M)$

2. **Exercise 1.3.7, sections b - e**

   Define the following propositions:

   s: a person is a senior
   y: a person is at least 17 years of age
   p: a person is allowed to park in the school parking lot

   Express each of the following English sentences with a logical expression:

   b. A person can park in the school parking lot if they are a senior or at least seventeen years of age.

      i. $(s \lor y) \to p$

   c. Being 17 years of age is a necessary condition for being able to park in the school parking lot.

      i. $p \to y$

   d. A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

      i. $(s \land y) \leftrightarrow p$

e. Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

i. $p \rightarrow (s \vee y)$

3. **Exercise 1.3.9, sections c, d**

Use the definitions of the variables below to translate each English statement into an equivalent logical expression.

y: the applicant is at least eighteen years old
p: the applicant has parental permission
c: the applicant can enroll in the course

c. The applicant can enroll in the course only if the applicant has parental permission

i. $c \rightarrow p$

d. Having parental permission is a necessary condition for enrolling in the course.

i. $c \rightarrow p$

<u>Question 6:</u>
Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.3.6, sections b - d**
   Give an English sentence in the form "if…then…" that is equivalent to each sentence.

   b. Maintaining a B average is necessary for Joe to be eligible for the honors program.
      i. **If Joe is eligible for the honors program, then he must be maintaining a B average.**

   c. Rajiv can go on the roller coaster only if he is at least four feet tall.
      i. **If Rajiv can go on the roller coaster, then he is at least four feet tall.**

   d. Rajiv can go on the roller coaster if he is at least four feet tall.
      i. **If Rajiv is at least four feet tall, then he can go on the roller coaster**

2. **Exercise 1.3.10, sections c - f**
   The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

   p = T
   q = F
   r = ?

   c. $(p \vee r) \leftrightarrow (q \wedge r)$
      i. **False. (p $\vee$ r) = T and (q $\wedge$ r) = F regardless of the value of r, therefore this can only be False.**

   d. $(p \wedge r) \leftrightarrow (q \wedge r)$
      i. **Unknown. If r = T, then this expression is True, but if r = F, this expression becomes False. Without knowing the value of r this truth value is unknown.**

   e. $p \rightarrow (r \vee q)$
      i. **Unknown. If r = T then the expression is True, if r = F then the expression is False.**

   f. $(p \wedge q) \rightarrow r$
      i. **True. Since (p $\wedge$ q) = F and all conditionals are True when the antecedent is False, this is True regardless of the value of r.**

Question 7:

Solve Exercise 1.4.5, sections b - d, from the Discrete Math zyBook:

Define the following propositions:

j: Sally got the job.
l: Sally was late for her interview
r: Sally updated her resume

Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

b. If Sally did not get the job, then she was late for her interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

i. $\neg j \rightarrow (l \vee \neg r)$
$(r \wedge \neg l) \rightarrow j$
ii. **These are also logically equivalent. Both evaluate to T in all scenarios except when $j = l =$ F and $r =$ T, in which case both evaluate to F.**

c. If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

i. $j \rightarrow \neg l$
$\neg j \rightarrow l$
ii. **These are not logically equivalent. If $j = l =$ T the top one evaluates to F, but that's a precise situation where the second will evaluate to T since $\neg j =$ F and anytime the antecedent is F the whole proposition is T.**

d. If Sally updated her resume or was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

i. $(r \vee \neg l) \rightarrow j$
$j \rightarrow (r \wedge \neg l)$
ii. **These are not logically equivalent. The first one can only evaluate to F in the event that $j =$ F, and all scenarios where $j =$ F means the second proposition evaluates to T.**

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.5.2, sections c, f, i**
   Use the laws of propositional logic to prove the following:

   c. $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r) \equiv p \to (q \land r)$	Starting Expression
$(\neg p \lor q) \land (\neg p \lor r) \equiv \neg p \lor (q \land r)$	Conditional Identities
$\mathbf{(\neg p \lor q) \land (\neg p \lor r) \equiv (\neg p \lor q) \land (\neg p \lor r)}$	Distributive Law

   f. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$	Starting Expression
$\neg p \land \neg(\neg p \land q) \equiv \neg p \land \neg q$	De Morgan's Law
$\neg p \land (\neg\neg p \lor \neg q) \equiv \neg p \land \neg q$	De Morgan's Law
$\neg p \land (p \lor \neg q) \equiv \neg p \land \neg q$	Double Negative
$(\neg p \land p) \lor (\neg p \land \neg q) \equiv \neg p \land \neg q$	Distribution Law
$F \lor (\neg p \land \neg q) \equiv \neg p \land \neg q$	Complement Law
$(\neg p \land \neg q) \lor F \equiv \neg p \land \neg q$	Commutative Law
$\mathbf{\neg p \land \neg q \equiv \neg p \land \neg q}$	Identity Law

   i. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r \equiv (p \land \neg r) \to \neg q$	Starting Expression
$\neg(p \land q) \lor r \equiv \neg(p \land \neg r) \lor \neg q$	Conditional Identity on Both Sides
$\neg p \lor \neg q \lor r \equiv \neg p \lor \neg\neg r \lor \neg q$	De Morgan's on Both Sides
$\neg p \lor \neg q \lor r \equiv \neg p \lor r \lor \neg q$	Double Negation Law
$\mathbf{\neg p \lor \neg q \lor r \equiv \neg p \lor \neg q \lor r}$	Associative Law

2. **Exercise 1.5.3, sections c, d**
Use the laws of propositional logic to prove that each statement is a tautology.

c. $\neg r \lor (\neg r \to p)$

$\neg r \lor (\neg r \to p)$	Starting Expression
$\neg r \lor (\neg\neg r \lor p)$	Conditional Identity
$\neg r \lor (r \lor p)$	Double Negation
$(\neg r \lor r) \lor p$	Associative
$T \lor p$	Complement Law
$\mathbf{p} \lor \mathbf{T} \equiv \mathbf{T}$	Commutative Law, Domination Law

d. $\neg(p \to q) \to \neg q$

$\neg(p \to q) \to \neg q$	Starting Expression
$\neg\neg(p \to q) \lor \neg q$	Conditional Identity
$(p \to q) \lor \neg q$	Double Negative
$\neg p \lor q \lor \neg q$	Conditional Identity
$\neg\mathbf{p} \lor \mathbf{T} \equiv \mathbf{T}$	Complement Law

<u>Question 9:</u>
Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.6.3, sections c, d**
   Consider the following statements in English. Write a logical expression with the same meaning. The domain is the set of all real numbers.

   c. There is a number that is equal to its square.
      i. $\exists x(x = x^2)$

   d. Every number is less than or equal to its square plus 1.
      i. $\forall x(x \leq (x^2 + 1))$

2. **Exercise 1.7.4, sections b - d**
   In the following question, the domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

   S(x): x was sick yesterday
   W(x): x went to work yesterday
   V(x): x was on vacation yesterday

   Translate the following English statements into a logical expression with the same meaning.

   b. Everyone was well and went to work yesterday.
      i. $\forall x(\neg S(x) \land W(x))$
   c. Everyone who was sick yesterday did not go to work.
      i. $\forall x(S(x) \rightarrow \neg W(x))$
   d. Yesterday someone was sick and went to work.
      i. $\exists x(S(x) \land W(x))$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.7.9, sections c - i**

   The domain for this question is the set {a, b, c, d, e}. The following table gives the value of predicates P, Q, and R for each element in the domain. For example, Q(c) = T because the truth value in the row labeled c and the column Q is T. Using these values, determine whether each quantified expression evaluates to true or false.

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

c. $\exists x((x = c) \rightarrow P(x))$
   i. **False**

d. $\exists x(Q(x) \wedge R(x))$
   i. **True**

e. $Q(a) \wedge P(d)$
   i.**True**

f. $\forall x((x \neq b) \rightarrow Q(x))$
   i. **True**

g. $\forall x(P(x) \vee R(x))$
   i. **False**

h. $\forall x(R(x) \rightarrow P(x))$
   i. **True**

i. $\exists x(Q(x) \vee R(x))$
   i. **True**

2. **Exercise 1.9.2, sections b - i**
The tables below show the values of predicates $P(x, y)$, $Q(x, y)$, and $S(x, y)$ for every possible combination of values of the variables x and y. The row number indicates the value of x and the column number indicates the value for y. The domain for x and y is $\{1, 2, 3\}$.

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

Indicate whether each of the quantified statements is true or false.

b. $\exists x \forall y Q(x,y)$
   i. **True**
c. $\exists y \forall x P(x,y)$
   i. **True**
d. $\exists x \exists y S(x,y)$
   i. **False**
e. $\forall x \exists y Q(x,y)$
   i. **False**
f. $\forall x \exists y P(x,y)$
   i. **True**
g. $\forall x \forall y P(x,y)$
   i. **False**
h. $\exists x \exists y Q(x,y)$
   i. **True**
i. $\forall x \forall y \neg S(x,y)$
   i. **True**

<u>Question 11:</u>
Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.10.4, sections c - g**
   Translate each of the following English statements into logical expressions. The domain is the set of all real numbers.

   c. There are two numbers whose sum is equal to their product.
      i. $\exists x \exists y((x+y) = xy)$

   d. The ratio of every two positive numbers is also positive.
      i. $\forall x \forall y\ ((x > 0) \land (y > 0) \rightarrow (x{:}y > 0))$

   e. The reciprocal of every positive number less than one is greater than one.
      i. $\forall x(((x > 0) \land (x < 1)) \rightarrow (1/x > 1))$

   f. There is no smallest number.
      i. $\forall x \exists y(y < x)$

   g. Every number other than 0 has a multiplicative inverse.
      i. $\forall x \exists y((x \neq 0) \rightarrow (xy = 1))$

2. **Exercise 1.10.7, sections c - f**
   The domain is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates:

   P(x,y): x knows y's phone number. (A person may or may not know their own phone number.)
   D(x): x missed the deadline
   N(x): x is a new employee

   Give a logical expression for each of the following sentences.

   c. There is at least one new employee who missed the deadline.
      i. $\exists x(N(x) \land (D(x))$
   d. Sam knows the phone number of everyone who missed the deadline.
      i. $\forall x(D(x) \rightarrow P(Sam, x))$
   e. There is a new employee who knows everyone's phone number.
      i. $\exists x \forall y(N(x) \land P(x,y))$
   f. Exactly one new employee missed the deadline.
      i. $\exists x \forall y(N(x) \land D(x) \land ((N(y) \land y \neq x) \rightarrow \neg D(y)))$

3. **Exercise 1.10.10, sections c -f**
The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of math classes offered at that university. The predicate T(x,y) indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

T(x,y) : student x has taken class y

c. Every student has taken at least one class other than Math 101
   i. $\forall x \exists y(T(x, y) \land (y \neq \text{Math } 101))$

d. There is a student who has taken every math class other than Math 101.
   i. $\exists x \forall y(y \neq \text{Math } 101 \rightarrow T(x,y))$

e. Everyone other than Sam has taken at least two different math classes.
   i. $\forall x \exists y \exists z(x \neq \text{Sam} \rightarrow ((T(x,y) \land T(x,z) \land (y \neq z))))$

f. Sam has taken exactly two math classes.
   i. $\exists y \exists z \forall a(T(\text{Sam},y) \land T(\text{Sam},z) \land z \neq y \land ((a \neq y \land a \neq z) \rightarrow \neg T(\text{Sam},a))$

<u>Question 12:</u>
Solve the following questions from the Discrete Math zyBook:

1. **Exercise 1.8.2, sections b - e**

   In the following question, the domain is a set of male patients in a clinical study. Define the following predicates:

   P(x): x was given the placebo
   D(x): x was given the medication
   M(x): x had migraines

   Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

   b. Every patient was given the medication or the placebo or both.
   - **$\forall x(P(x) \vee D(x))$**
   - **$\neg\forall x(P(x) \vee D(x))$**
   - **$\exists x\neg(P(x) \vee D(x))$**
   - **$\exists x(\neg P(x) \wedge \neg D(x))$**
   - **There exists a patient who was not given the placebo and was not given the medication.**

   c. There is a patient who took the medication and had migraines.
   - **$\exists x(D(x) \wedge M(x))$**
   - **$\neg\exists x(D(x) \wedge M(x))$**
   - **$\forall x\neg(D(x) \wedge M(x))$**
   - **$\forall x(\neg D(x) \vee \neg M(x))$**
   - **Every patient was either not given the medication or they did not have migraines.**

   d. Every patient who took the placebo had migraines.
   - **$\forall x(P(x) \rightarrow M(x))$**
   - **$\neg\forall x(\neg P(x) \vee M(x))$**
   - **$\exists x\neg(\neg P(x) \vee M(x))$**
   - **$\exists x(\neg\neg P(x) \wedge \neg M(x))$**
   - **$\exists x(P(x) \wedge \neg M(x))$**
   - **There exists a patient who was given the placebo and did not have migraines.**

e. There is a patient who had migraines and was given the placebo.
- **$\exists x(M(x) \wedge P(x))$**
- **$\neg\exists x(M(x) \wedge P(x))$**
- **$\forall x\neg(M(x) \wedge P(x))$**
- **$\forall x(\neg M(x) \vee \neg P(x))$**
- **Every patient either did not have migraines or they were not given the placebo.**

2. **Exercise 1.9.4, sections c - e**
Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

c. $\exists x\forall y(P(x,y) \rightarrow Q(x,y))$

$\exists x\forall y(P(x,y) \rightarrow Q(x,y))$	Starting Expression
$\neg\exists x\forall y(P(x,y) \rightarrow Q(x,y))$	Negation of Starting Expression
$\forall x\exists y\neg(P(x,y) \rightarrow Q(x,y))$	De Morgan's Applied Twice
$\forall x\exists y\neg(\neg P(x,y) \vee Q(x,y))$	Conditional Identity
$\forall x\exists y(\neg\neg P(x,y) \wedge \neg Q(x,y))$	De Morgan's Law
$\forall x\exists y(P(x,y) \wedge \neg Q(x,y))$	Double Negation

d. $\exists x\forall x(P(x,y) \leftrightarrow P(y,x))$

$\exists x\forall y(P(x,y) \leftrightarrow P(y,x))$	Starting Expression
$\neg\exists x\forall y(P(x,y) \leftrightarrow P(y,x))$	Negation of Starting Expression
$\forall x\exists y\neg(P(x,y) \leftrightarrow P(y,x))$	De Morgan's Applied Twice
$\forall x\exists y\neg((P(x,y) \rightarrow P(y,x)) \wedge (P(y,x) \rightarrow P(x,y)))$	Conditional Identity
$\forall x\exists y\neg((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y))$	Conditional Identity
$\forall x\exists y(\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y))$	De Morgan's Law
$\forall x\exists y((\neg\neg P(x,y) \wedge \neg P(y,x)) \vee (\neg\neg P(y,x) \wedge \neg P(x,y))$	De Morgan's Law to both sides
$\forall x\exists y((P(x,y) \wedge \neg P(y,x)) \vee (\neg P(x,y) \wedge P(y,x))$	Double Negative, Commutative

e. $\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$

$\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$	Starting Expression
$\neg(\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y))$	Negation of Starting Expression
$\neg \exists x \exists y P(x,y) \vee \neg \forall x \forall y Q(x,y)$	De Morgan's Law
$\forall \mathbf{x} \forall \mathbf{y} \neg \mathbf{P(x,y)} \vee \exists \mathbf{x} \exists \mathbf{y} \neg \mathbf{Q(x,y)}$	De Morgan's Law