Online Estimation of Inertial Parameters Using a Recursive Total Least-Squares Approach

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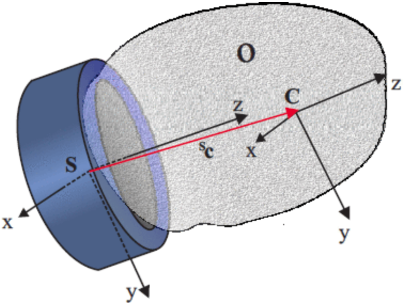
Overview

Definition of the problem

- F/T sensor Gamma measuring:
  - force ${}^Sf$,
  - torque ${}^S\tau$,
  - in the sensor frame S.
- Payload grasped by the suction tool:
  - mass m,
  - center of mass ${}^Sc$,
  - moment of inertia ${}^SI$,
  - linear acceleration ${}^Sa$,
  - angular acceleration ${}^S\alpha$,
  - angular velocity ${}^S\omega$,
  - gravity ${}^Sg$.



Definition of the problem

How to make a reliable online estimation of the weight of the item grasped?

Newton-Euler Approach

The motion of the body due to external forces is described by the two equations:

$$
\begin{aligned}
{}^S f &= m\,{}^S a - m\,{}^S g + {}^S\alpha \times m\,{}^S c + {}^S\omega \times ({}^S\omega \times m\,{}^S c) && (1)\\
{}^S \tau &= {}^S I\,{}^S\alpha + {}^S\omega \times ({}^S I\,{}^S\omega) + m\,{}^S c \times {}^S a - m\,{}^S c \times {}^S g && (2)
\end{aligned}
$$

also in matrix form:

$$
\begin{pmatrix} {}^S f \\ {}^S \tau \end{pmatrix} = {}^S A({}^S a, {}^S\alpha, {}^S\omega, {}^S g)\,{}^S\varphi \qquad (3)
$$

with ${}^S\varphi = (m, m\,{}^S c_x, m\,{}^S c_y, m\,{}^S c_z, {}^S I_{xx}, {}^S I_{xy}, {}^S I_{xz}, {}^S I_{yy}, {}^S I_{yz}, {}^S I_{zz})^T$

Optimisation problem

During the motion of the payload, $M\ {}^{S}A_{ext}$ matrices are compiled at subsequent instants of time:

$$ {}^{S}A_{\Xi} = [{}^{S}A_1^T\, {}^{S}A_2^T \ldots {}^{S}A_M^T]^T \tag{4} $$

$$ \begin{pmatrix} {}^{S}f \\ {}^{S}\tau \end{pmatrix}_{\Xi} = \left[\begin{pmatrix} {}^{S}f \\ {}^{S}\tau \end{pmatrix}_1^T \begin{pmatrix} {}^{S}f \\ {}^{S}\tau \end{pmatrix}_2^T \cdots \begin{pmatrix} {}^{S}f \\ {}^{S}\tau \end{pmatrix}_M^T \right]^T \tag{5} $$

The optimization problem is the following, for every instant of time (Ξ batch of size M):

$$ \begin{aligned} \underset{{}^{S}\varphi_M}{\text{minimize}} \quad & \left\| \begin{pmatrix} {}^{S}f \\ {}^{S}\tau \end{pmatrix}_{\Xi} - {}^{S}A_{\Xi}\, {}^{S}\varphi_M \right\| \\ \text{subject to} \quad & m_M \geq 0 \\ & c_{zM} \geq 0 \end{aligned} $$

Comparison of different methods

- Recursive Least-Squares methods
  - Error model: $\begin{pmatrix} {}^S f \\ {}^S \tau \end{pmatrix}_{\Xi} + e = {}^S A_{\Xi} {}^S \varphi$.
  - Errors in the data matrix ${}^S A_{\Xi}$ are not considered.
- Recursive Instrumental Variables Method
  - Yield unbiased estimates in the presence of correlated noise.
  - Errors in the data matrix ${}^S A_{\Xi}$ are not considered.
- Recursive Total Least-Squares (RTLS) Method
  - More appropriate error model: $\begin{pmatrix} {}^S f \\ {}^S \tau \end{pmatrix}_{\Xi} + e = ({}^S A_{\Xi} + E){}^S \varphi$.
  - Require update of the Singular Value Decomposition (SVD) at each estimation cycle with a complexity $O(mn \min(n, m))$.

Recursive Total Least-Squares (RTLS) Method

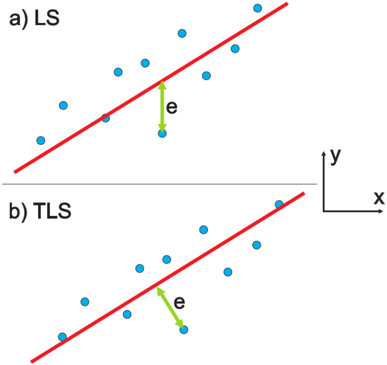


Figure: In contrast with LS approach, TLS approach considers errors in both x- and y-directions.

Relevance of a recursive least-square algorithm

- The *recursive* least-squares methods seems to be relevant for **online** estimation of inertial parameters.
- The payload can be estimated at the end of the placement move to the item recognition position.
- The optimisation problem become:

$$\begin{aligned} \underset{^S\varphi}{\text{minimize}} \quad & \left\| \begin{pmatrix} ^S f \\ ^S \tau \end{pmatrix}_{\Xi} - {}^S A_{\Xi} {}^S \varphi \right\| \\ \text{subject to} \quad & m \geq 0 \\ & c_z \geq 0 \end{aligned}$$

$\rightarrow$ Is it necessary to have online update of the inertial parameters?

My next steps

- Test a simplified version of the method:
    - Least-squares method only
    - Test on different items and differents trajectories
- Test of the recursive total methods:
    - Comparison of the running time of each method
- Setup performance improvement:
    - Estimation of the sensor offsets
    - Estimation of the sensitivity of ${}^{S}\varphi$ to error
    - (wide open for suggestions)

Estimation of the sensor offsets

Strain gage force/torque sensors typically show offsets that would deteriorate the estimation.

- Currently, the sensor is reset every 20/25 iterations (it takes a couple of second),
- Directly estimating the sensor offsets.

The number of parameters to estimate increase from 10 to 16:

$$ {}^{S}\varphi_{ext} = [f_{O_x}, f_{O_y}, f_{O_z}, {}^{S}\varphi]^T \tag{6} $$

$$ {}^{S}A_{ext} = [\mathbb{I}_{6\times 6} \, {}^{S}A] \tag{7} $$

Drift effects can be neglected since the estimation duration is quite small ($\sim 3\mathrm{s}$)

Estimation of the sensitivity of ${}^{S}\varphi$ to error

Considering the correlation matrix Υ consisting of M ${}^{S}A$ matrices[1]:

$$\Upsilon = {}^{S}A_{\equiv}^{T}\,{}^{S}A_{\equiv} \tag{8}$$

The sensitivity of ${}^{S}\varphi$ to error can be shown to increase with the condition number[2] $\kappa(\Upsilon)$.

[1]The correlation matrix should be computed with the experimental joint angle setpoints.

[2]The condition number of a function measures how much the output value of the function can change for a small change in the input argument: $\lim_{\varepsilon\to 0} \sup_{\|\delta x\| \leq \varepsilon} \|\delta f\| / \|\delta x\|$

Recursive Total Least-Squares (RTLS) Method

Basic steps:

1. Calculate an initial SVD with a standard SVD algorithm for a small number of data matrix.
2. Compile a new input matrix consisting of the current data matrix ${}^{S}A$ and force-torque vector $({}^{S}f, {}^{S}\tau)^T$ and perform an SVD update incorporating the new data.
3. Update estimate: If the deviation between the smallest singular values is less than ε, transform left singular vectors; compute TLS solution from the left singular vectors.
4. Continue with 2 or stop estimation.

References

2007 On-line rigid object recognition and pose estimation based on inertial parameters (D. Kubus, T. Kroger)

2008 On-Line Estimation of Inertial Parameters Using a Recursive Total Least-Squares Approach (D. Kubus, T. Kroger)

2014 Combining visual and inertial features for efficient grasping and bin-picking (D. Kubus, I. Weidauer)

2018 Real-Time Identification of Robot Payload using a Multirate Quaternion-based Kalman Filter and Recursive Total Least-Squares (S. Farsoni, C. Talignani Landi)