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Online Estimation of Inertial Parameters with the use of an F/T sensor for objects carried in motion by robotic manipulator

Master Degree Project in Computer Science

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Abstract

Keywords

Inertial Parameters, Estimation, Least-Squares, Angular Mass-Spring-Damper System

Sammanfattning

Nyckelord

Acknowledgements

Write a short acknowledgements. Don't forget to give some credit to the examiner and supervisor.

Acronyms

Contents

Chapter 1

Introduction

Somewhat big picture describing where the work fits in. How far to zoom out depends on the topic. It should be one or two levels of abstraction up. Does not have to be all the way to “drones are important…”

Describe what the problem you study is using words that one can understand without having read the thesis first. It is good if this ends in a very explicit problem statement.

- Logistic robots increase in number in warehouse: 69,000 logistic systems were installed in 2017, 162 % more than in 2016 (26,294) accounting for 63 % of the total units and 36 % of the total sales (in value) of professional service robots [1].

- Interesting for economics, productivity reasons

1.1 Motivation

1.2 Formulation of the Problem

1.3 Scope

1.4 Report Outline

In text, describe what is presented in Chapters 2 and forward. Exclude the first chapter and references as well as appendix.

Chapter 2

<Theoretical Background>

In this chapter, a detailed description about background of the degree project is presented together with related work. Discuss what is found useful and what is less useful. Use valid arguments.

Explain what and how prior work / prior research will be applied on or used in the degree project /work (described in this thesis). Explain why and what is not used in the degree project and give valid reasons for rejecting the work/research.

Use references!

2.1 Use headings to break the text

Do not use subtitles after each other without text in between the sections.

2.1.1 Related Work

You should probably keep a heading about the related work here even though the entire chapter basically only contains related work.

Chapter 3

<Engineering-related content, Methodologies and Methods>

Describe the engineering-related contents (preferably with models) and the research methodology and methods that are used in the degree project.

Most likely it generally describes the method used in each step to make sure that you can answer the research question.

3.1 Engineering-related and scientific content:

Applying engineering related and scientific skills; modelling, analysing, developing, and evaluating engineering-related and scientific content; correct choice of methods based on problem formulation; consciousness of aspects relating to society and ethics (if applicable).

As mentioned earlier, give a theoretical description of methodologies and methods and how these are applied in the degree project.

Chapter 4

<The work>

Describe the degree project. What did you actually do? This is the practical description of how the method was applied.

Chapter 5

<Result>

Describe the results of the degree project.

Chapter 6

<Conclusions>

Describe the conclusions (reflect on the whole introduction given in Chapter 1).

Discuss the positive effects and the drawbacks.

Describe the evaluation of the results of the degree project.

Describe valid future work.

The sections below are optional but could be added here.

6.1 Discussion

6.1.1 Future Work

6.1.2 Final Words

Bibliography

[1] Robotics IFR, International Federation of. "Executive Summary World Robotics 2018 Service Robots". In: (2018), pp. 376–381. URL: https://ifr.org/downloads/press2018/Executive_Summary_WR_Service_Robots_2018.pdf.

Appendix - Contents

Appendix A

Mechanical model of the { tool + item } system

A.1 Quick reminder

A.1.1 Kinematics wrench

Considering A, a point of {2}, the motion of {2} with respect to {1} is described by the following wrench:

$$\{\mathcal{V}_{2/1}\} = \underset{A}{\left\{ \begin{array}{c} \overrightarrow{\Omega}_{2/1} \\ \overrightarrow{V}_{A,2/1} \end{array} \right\}} \tag{A.1}$$

- $\overrightarrow{\Omega}_{2/1}$ is the angular velocity of {2} with respect to {1},
- $\overrightarrow{V}_{A,2/1}$ is the linear velocity of A $\in$ {2} with respect to {1}.

A.1.2 Momentum wrench

Considering A, a point of {2}, m its mass and G its the center of mass, the momentum wrench describes the motion of {2} with respect to {1}:

$$\{\mathcal{C}_{2/1}\} = \underset{A}{\left\{ \begin{array}{ll} \overrightarrow{p}_{2/1} & = m\overrightarrow{V}_{G,2/1} \\ \overrightarrow{\sigma}_{A,2/1} & = I_{2,A} * \overrightarrow{\Omega}_{2/1} + m\overrightarrow{AG} \times \overrightarrow{V}_{A,2/1} \end{array} \right\}} \tag{A.2}$$

- $\overrightarrow{p}_{2/1}$ is the linear momentum of $\{2\}$ with respect to $\{1\}$,
- $\overrightarrow{V}_{G,2/1}$ is the linear velocity of G $\in \{2\}$ with respect to $\{1\}$,
- $\overrightarrow{\sigma}_{A,2/1}$ is the angular momentum of A $\in \{2\}$ with respect to $\{1\}$,
- $I_{2,A}$ is the tensor of inertia of $\{2\}$ about the point A.

A.1.3 Dynamic wrench

Considering A, a point of $\{2\}$, its mass m and its center of mass G, the dynamic wrench describes the motion of $\{2\}$ with respect to $\{1\}$:

$$\{\mathcal{D}_{2/1}\} = \underset{A}{\left\{ \begin{array}{ll} \overrightarrow{\mathcal{A}}_{2/1} & = m\overrightarrow{\Gamma}_{G,2/1} \\ \overrightarrow{\delta}_{A,2/1} & = \frac{\partial}{\partial t}\overrightarrow{\sigma}_{A,2/1} + m\overrightarrow{V}_{A/1} \times \overrightarrow{V}_{G,2/1} \end{array} \right\}} \tag{A.3}$$

- $\overrightarrow{\mathcal{A}}_{2/1}$ is the linear dynamic momentum of $\{2\}$ with respect to $\{1\}$,
- $\overrightarrow{\Gamma}_{G,2/1}$ is the acceleration of G $\in \{2\}$ with respect to $\{1\}$,
- $\overrightarrow{\delta}_{A,2/1}$ is the angular dynamic momentum of A $\in \{2\}$ with respect to $\{1\}$,
- $\overrightarrow{V}_{A/1}$ is the linear velocity of A with respect to $\{1\}$ (note that A is not necessarily a point of $\{2\}$).

A.1.4 Parallel axis theorem

For an inertia tensor $I_{1,G}$ computed at the center of mass of a rigid body of mass m the tensor for a parallel axis is:

$$I_{1,A} = I_{1,G} - md^2 \tag{A.4}$$

with d the skew-symmetric matrix constructed from the vector $\overrightarrow{AG} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = d$.

$$-d^2 = -\begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}^2 = \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -yx & x^2+z^2 & -yz \\ -zx & -zy & x^2+y^2 \end{pmatrix} = (d^T d)\,\mathbb{I}_3 - (dd^T)$$

A.1.5 Fundamental Principle of Dynamics

Considering $\sum\{\mathcal{S}_{ext/1}\}$ the global external actions on the system $\{1\}$ and R_g a Galilean reference:

$$\sum\{\mathcal{S}_{ext/1}\} = \{\mathcal{D}_{1/R_G}\} \tag{A.5}$$

A.1.6 Notations for systems of linear equations

$$\theta \times c = \begin{pmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{pmatrix} \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} := [\theta\times]\, c$$

$$I * \theta = \begin{pmatrix} \theta_x & \theta_y & \theta_z & 0 & 0 & 0 \\ 0 & \theta_x & 0 & \theta_y & \theta_z & 0 \\ 0 & 0 & \theta_x & 0 & \theta_y & \theta_z \end{pmatrix} \begin{pmatrix} I_{xx} \\ I_{xy} \\ I_{xz} \\ I_{yy} \\ I_{yz} \\ I_{zz} \end{pmatrix} := [\bullet\theta] \begin{pmatrix} I_{xx} \\ I_{xy} \\ I_{xz} \\ I_{yy} \\ I_{yz} \\ I_{zz} \end{pmatrix}$$

with $I = I^T = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix}$.

A.1.7 Rotation matrix

Considering the orientation $\theta = \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}$, the corresponding rotation matrix is $R = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$ or:

$$R = \begin{pmatrix} \cos\theta_y\cos\theta_z & \sin\theta_x\sin\theta_y\cos\theta_z - \cos\theta_x\sin\theta_z & \cos\theta_x\sin\theta_y\cos\theta_z + \sin\theta_x\sin\theta_z \\ \cos\theta_y\sin\theta_z & \sin\theta_x\sin\theta_y\sin\theta_z + \cos\theta_x\cos\theta_z & \cos\theta_x\sin\theta_y\sin\theta_z - \sin\theta_x\cos\theta_z \\ -\sin\theta_y & \sin\theta_x\cos\theta_y & \cos\theta_x\cos\theta_y \end{pmatrix}$$

Considering the small-angle approximation ($\cos\alpha \approx 1$, $\sin\alpha \approx \alpha$ and $\alpha\beta \approx 0$ for small α and β):

$$R \approx \begin{pmatrix} 1 & \theta_x\theta_y - \theta_z & \theta_y + \theta_x\theta_z \\ \theta_z & \theta_x\theta_y\theta_z + 1 & \theta_y\theta_z - \theta_x \\ -\theta_y & \theta_x & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{pmatrix} = \mathbb{I}_3 + [\theta\times] \tag{A.6}$$

A.2 1-body rigid model of the {tool + item} system

The system considered is a single-body system {1} composed of the tool and the item rigidly linked to it. The mass of {1} is m, its global center of mass is G and the global inertia about the center of mass is $I_{1,G}$. The F/T sensor measures the force and the torque about the joint S.

A.2.1 External mechanical actions

- Action of the robotic arm on the last link about the joint S:

$$\{\mathcal{S}_{arm/1}\} = {}_S\left\{ \begin{array}{c} \overrightarrow{F}_{arm/1} = \overrightarrow{f} \\ \overrightarrow{T}_{S,arm/1} = \overrightarrow{\tau} \end{array} \right\}$$

- Action of the gravitational force:

$$\{\mathcal{S}_{grav/1}\} = {}_G\left\{ \begin{array}{c} m\overrightarrow{g} \\ \overrightarrow{0} \end{array} \right\} = {}_S\left\{ \begin{array}{c} m\overrightarrow{g} \\ m\overrightarrow{SG} \times \overrightarrow{g} \end{array} \right\}$$

A.2.2 Dynamics wrench

Considering the dynamic angular momentum of the system {1} about the joint S:

$$\overrightarrow{\delta}_{S,1/0} = \tfrac{\partial}{\partial t}\overrightarrow{\sigma}_{S,1/0} + m\overrightarrow{V}_{S/0} \times \overrightarrow{V}_{G,1/0}$$

The angular momentum of the system {1} about the joint S is:

$$\overrightarrow{\sigma}_{S,1/0} = I_{1,S} * \overrightarrow{\Omega}_{1/0} + m\overrightarrow{SG} \times \overrightarrow{V}_{S,1/0}$$

Its time derivative is:

$$\begin{aligned}
\left[\frac{\partial}{\partial t}\overrightarrow{\sigma}_{S,1/0}\right]_{R_0} &= \left[\frac{\partial}{\partial t}(I_{1,S} * \overrightarrow{\Omega}_{1/0})\right]_{R_1} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\frac{\partial}{\partial t}(\overrightarrow{SG} \times \overrightarrow{V}_{S,1/0}) \\
&= I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\frac{\partial}{\partial t}(\overrightarrow{SG}) \times \overrightarrow{V}_{S,1/0} + m\overrightarrow{SG} \times \frac{\partial}{\partial t}(\overrightarrow{V}_{S,1/0}) \\
&= I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\overrightarrow{V}_{G,1/0} \times \overrightarrow{V}_{S,1/0} + m\overrightarrow{SG} \times \overrightarrow{\Gamma}_{S,1/0}
\end{aligned}$$

Consequently, the dynamic rotational momentum is:

$$\begin{aligned}\overrightarrow{\delta}_{S,1/0} &= I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\overrightarrow{SG} \times \overrightarrow{\Gamma}_{S,1/0} \quad + m\overrightarrow{V}_{G,1/0} \times \overrightarrow{V}_{S,1/0} \\ &\qquad + m\overrightarrow{V}_{S/0} \times \overrightarrow{V}_{G,1/0} \\ &= I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\overrightarrow{SG} \times \overrightarrow{\Gamma}_{S,1/0}\end{aligned}$$

Thus the dynamics wrench of the system {1} in the in the Galilean reference {0} is:

$$\{\mathcal{D}_{1/0}\} = \underset{S}{\left\{\begin{array}{ll} \overrightarrow{\mathcal{A}}_{1/0} & = m\overrightarrow{\Gamma}_{G,1/0} \\ \overrightarrow{\delta}_{S,1/0} & = I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\overrightarrow{SG} \times \overrightarrow{\Gamma}_{S,1/0} \end{array}\right\}}$$

The position –hence, the acceleration– of the center of mass is unknown. The acceleration is computed with the position of the joint S.

$$\begin{aligned}\overrightarrow{\Gamma}_{G,1/0} = \frac{\partial}{\partial t}\overrightarrow{V}_{G,1/0} &= \frac{\partial}{\partial t}(\overrightarrow{V}_{S,1/0} + \overrightarrow{GS} \times \overrightarrow{\Omega}_{1/0}) \\ &= \overrightarrow{\Gamma}_{S,1/0} + \frac{\partial}{\partial t}(\overrightarrow{GS}) \times \overrightarrow{\Omega}_{1/0} + \overrightarrow{GS} \times \overrightarrow{\alpha}_{1/0} \\ &= \overrightarrow{\Gamma}_{S,1/0} + (\overrightarrow{\Omega}_{1/0} \times \overrightarrow{GS}) \times \overrightarrow{\Omega}_{1/0} + \overrightarrow{GS} \times \overrightarrow{\alpha}_{1/0} \\ &= \overrightarrow{\Gamma}_{S,1/0} + \overrightarrow{\Omega}_{1/0} \times (\overrightarrow{\Omega}_{1/0} \times \overrightarrow{SG}) + \overrightarrow{\alpha}_{1/0} \times \overrightarrow{SG}\end{aligned}$$

The the dynamics wrench of the system {1} becomes:

$$\{\mathcal{D}_{1/0}\} = \underset{S}{\left\{\begin{array}{ll} \overrightarrow{\mathcal{A}}_{1/0} & = m\overrightarrow{\Gamma}_{S,1/0} + \overrightarrow{\Omega}_{1/0} \times (\overrightarrow{\Omega}_{1/0} \times m\overrightarrow{SG}) + \overrightarrow{\alpha}_{1/0} \times m\overrightarrow{SG} \\ \overrightarrow{\delta}_{S,1/0} & = I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\overrightarrow{SG} \times \overrightarrow{\Gamma}_{S,1/0} \end{array}\right\}}$$

A.2.3 Fondamental Principle of Dynamics

From the equation of the FDP (A.5) on {1} at point S:

$$
{}_S\left\{ \begin{array}{c} \overrightarrow{f} \\ \overrightarrow{\tau} \end{array} \right\} + {}_S\left\{ \begin{array}{c} m\overrightarrow{g} \\ m\overrightarrow{SG} \times \overrightarrow{g} \end{array} \right\} = {}_S\left\{ \begin{array}{c} m\overrightarrow{\Gamma}_{S,1/0} + \overrightarrow{\Omega}_{1/0} \times (\overrightarrow{\Omega}_{1/0} \times m\overrightarrow{SG}) + \overrightarrow{\alpha}_{1/0} \times m\overrightarrow{SG} \\ I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) + m\overrightarrow{SG} \times \overrightarrow{\Gamma}_{S,1/0} \end{array} \right\}
$$

Force and torque at the joint are derived as follows:

$$
\overrightarrow{f} = m(\overrightarrow{\Gamma}_{S,1/0} - \overrightarrow{g}) + \overrightarrow{\Omega}_{1/0} \times (\overrightarrow{\Omega}_{1/0} \times m\overrightarrow{SG}) + \overrightarrow{\alpha}_{1/0} \times m\overrightarrow{SG} \tag{A.7}
$$

$$
\overrightarrow{\tau} = m\overrightarrow{SG} \times (\overrightarrow{\Gamma}_{S,1/0} - \overrightarrow{g}) + I_{1,S} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,S} * \overrightarrow{\Omega}_{1/0}) \tag{A.8}
$$

With the aim of shrinking the equation, the following notation are used:

$$
\dot{\theta} = \overrightarrow{\Omega}_{1/0} \qquad \ddot{\theta} = \overrightarrow{\alpha}_{1/0} \qquad a = \overrightarrow{\Gamma}_{S,1/0} \qquad g = \overrightarrow{g} \qquad c = \overrightarrow{SG} \qquad I = I_{1,S}
$$

$$
\left\{ \begin{array}{rl} f = & m(a-g) + \dot{\theta} \times (\dot{\theta} \times mc) + \ddot{\theta} \times mc \\ \tau = & mc \times (a-g) + I_{1,S} * \ddot{\theta} + \dot{\theta} \times (I_{1,S} * \dot{\theta}) \end{array} \right. \tag{A.9}
$$

A.2.4 Inertial parameters estimation

From the FDP equations (A.9) it is possible to estimate the inertial parameters of the item grasped by the tool using the following matrix equation.

$$\begin{pmatrix} f \\ \tau \end{pmatrix} = \begin{pmatrix} a-g & \left[\ddot{\theta}\times\right] + \left[\dot{\theta}\times\right]\left[\dot{\theta}\times\right] & 0 \\ 0 & [(g-a)\times] & \left[\bullet\ddot{\theta}\right] + \left[\dot{\theta}\times\right]\left[\bullet\dot{\theta}\right] \end{pmatrix} \begin{pmatrix} m \\ mc_x \\ mc_y \\ mc_z \\ I_{xx} \\ I_{xy} \\ I_{xz} \\ I_{yy} \\ I_{yz} \\ I_{zz} \end{pmatrix} \tag{A.10}$$

The idea is to optimize the 10 parameters vector φ such that $\begin{pmatrix} f \\ \tau \end{pmatrix} = A\varphi$ for every time t.

A.3 2-bodies rigid model of the {tool} + {item} system

The F/T sensor measures the force and the torque about the joint S. The system considered is a two-bodies system: {1} for the tool and {2} for the item grasped. The mass of {1} –respectively {2}– is m_1 –respectively m_2, its center of mass is G_1 –respectively G_2– and the inertia about the center of mass is I_{1,G_1} –respectively I_{2,G_2}. The mechanical link between the two systems is modeled as a spherical link (as known as ball joint) with a spring-damper system about the three axes of rotation.

A.3.1 External mechanical actions

- Action of the robotic arm on the tool about the grasping point A:

$$\{\mathcal{S}_{arm/1}\} = {}_S\left\{ \begin{array}{c} \overrightarrow{F}_{arm/1} = \overrightarrow{f} \\ \overrightarrow{T}_{S,arm/1} = \overrightarrow{\tau} \end{array} \right\} = {}_A\left\{ \begin{array}{c} \overrightarrow{f} \\ \overrightarrow{\tau} + \overrightarrow{AS} \times \overrightarrow{f} \end{array} \right\}$$

- Action of the tool {2} on the item {1} about the grasp point A:

$$\{\mathcal{S}_{2/1}\} = {}_A\left\{ \begin{array}{c} \overrightarrow{F}_{2/1} \\ \overrightarrow{T}_{A,2/1} = \overrightarrow{0} \end{array} \right\} \text{ and respectively } \{\mathcal{S}_{1/2}\} = {}_A\left\{ \begin{array}{c} \overrightarrow{F}_{1/2} = -\overrightarrow{F}_{2/1} \\ \overrightarrow{T}_{A,1/2} = \overrightarrow{0} \end{array} \right\}$$

- Action of the gravitational force on {1} and on {2}:

$$\{\mathcal{S}_{grav/1}\} = {}_{G_1}\left\{ \begin{array}{c} m_1\overrightarrow{g} \\ \overrightarrow{0} \end{array} \right\} = {}_A\left\{ \begin{array}{c} m_1\overrightarrow{g} \\ m_1\overrightarrow{AG_1} \times \overrightarrow{g} \end{array} \right\}$$

$$\{\mathcal{S}_{grav/2}\} = {}_{G_2}\left\{ \begin{array}{c} m_2\overrightarrow{g} \\ \overrightarrow{0} \end{array} \right\} = {}_A\left\{ \begin{array}{c} m_2\overrightarrow{g} \\ m_2\overrightarrow{AG_2} \times \overrightarrow{g} \end{array} \right\}$$

- Action of spring-damper system on the item {1} and {2} about the grasp point A:

$$\{\mathcal{S}_{spring/1}\} + \{\mathcal{S}_{damper/1}\} = {}_A\left\{ \begin{array}{c} \overrightarrow{0} \\ K * \overrightarrow{\Theta}_{2/1} \end{array} \right\} + {}_A\left\{ \begin{array}{c} \overrightarrow{0} \\ \Lambda * \overrightarrow{\Omega}_{2/1} \end{array} \right\} = {}_A\left\{ \begin{array}{c} \overrightarrow{0} \\ K * \overrightarrow{\Theta}_{2/1} + \Lambda * \overrightarrow{\Omega}_{2/1} \end{array} \right\}$$

$$\{\mathcal{S}_{spring/2}\} + \{\mathcal{S}_{damper/2}\} = {}_A\left\{ \begin{array}{c} \overrightarrow{0} \\ K * \overrightarrow{\Theta}_{1/2} + \Lambda * \overrightarrow{\Omega}_{1/2} \end{array} \right\}$$

$$\text{with } K = K^T = \begin{pmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{xy} & k_{yy} & k_{yz} \\ k_{xz} & k_{yz} & k_{zz} \end{pmatrix} \text{ and } \Lambda = \Lambda^T = \begin{pmatrix} \lambda_{xx} & \lambda_{xy} & \lambda_{xz} \\ \lambda_{xy} & \lambda_{yy} & \lambda_{yz} \\ \lambda_{xz} & \lambda_{yz} & \lambda_{zz} \end{pmatrix}.$$

A.3.2 Dynamics wrench

By using the equations derived in A.2.2, the dynamic wrenches of {1} and {2} are:

$$\{\mathcal{D}_{1/0}\} = {}_{A}\left\{ \begin{array}{ll} \overrightarrow{\mathcal{A}}_{1/0} & = m_1 \overrightarrow{\Gamma}_{A,1/0} + \overrightarrow{\Omega}_{1/0} \times (\overrightarrow{\Omega}_{1/0} \times m_1 \overrightarrow{AG_1}) + \overrightarrow{\alpha}_{1/0} \times m_1 \overrightarrow{AG_1} \\ \overrightarrow{\delta}_{A,1/0} & = I_{1,A} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,A} * \overrightarrow{\Omega}_{1/0}) + m_1 \overrightarrow{AG_1} \times \overrightarrow{\Gamma}_{A,1/0} \end{array} \right\}$$

$$\{\mathcal{D}_{2/0}\} = {}_{A}\left\{ \begin{array}{ll} \overrightarrow{\mathcal{A}}_{2/0} & = m_2 \overrightarrow{\Gamma}_{A,2/0} + \overrightarrow{\Omega}_{2/0} \times (\overrightarrow{\Omega}_{2/0} \times m_2 \overrightarrow{AG_2}) + \overrightarrow{\alpha}_{2/0} \times m_2 \overrightarrow{AG_2} \\ \overrightarrow{\delta}_{A,2/0} & = I_{2,A} * \overrightarrow{\alpha}_{2/0} + \overrightarrow{\Omega}_{2/0} \times (I_{2,A} * \overrightarrow{\Omega}_{2/0}) + m_2 \overrightarrow{AG_2} \times \overrightarrow{\Gamma}_{A,2/0} \end{array} \right\}$$

A.3.3 Fondamental Principle of Dynamics on {2}

From the equation of the FDP (A.5) on {2} at point A:

$$\begin{aligned} & {}_{A}\left\{ \begin{array}{c} -\overrightarrow{F}_{2/1} \\ \overrightarrow{0} \end{array} \right\} + {}_{A}\left\{ \begin{array}{c} m_2 \overrightarrow{g} \\ m_2 \overrightarrow{AG_2} \times \overrightarrow{g} \end{array} \right\} + {}_{A}\left\{ \begin{array}{c} \overrightarrow{0} \\ K * \overrightarrow{\Theta}_{1/2} + \Lambda * \overrightarrow{\Omega}_{1/2} \end{array} \right\} \\ = & {}_{A}\left\{ \begin{array}{l} m_2 \overrightarrow{\Gamma}_{A,2/0} + \overrightarrow{\Omega}_{2/0} \times (\overrightarrow{\Omega}_{2/0} \times m_2 \overrightarrow{AG_2}) + \overrightarrow{\alpha}_{2/0} \times m_2 \overrightarrow{AG_2} \\ I_{2,A} * \overrightarrow{\alpha}_{2/0} + \overrightarrow{\Omega}_{2/0} \times (I_{2,A} * \overrightarrow{\Omega}_{2/0}) + m_2 \overrightarrow{AG_2} \times \overrightarrow{\Gamma}_{A,2/0} \end{array} \right\} \end{aligned}$$

Considering only the equation on torques:

$$m_2 \overrightarrow{AG_2} \times \overrightarrow{g} + K * \overrightarrow{\Theta}_{1/2} + \Lambda * \overrightarrow{\Omega}_{1/2} = I_{2,A} * \overrightarrow{\alpha}_{2/0} + \overrightarrow{\Omega}_{2/0} \times (I_{2,A} * \overrightarrow{\Omega}_{2/0}) + m_2 \overrightarrow{AG_2} \times \overrightarrow{\Gamma}_{A,2/0} \tag{A.11}$$

With the aim of shrinking the equation, the following notation are used:

$$\theta_1 = \overrightarrow{\Theta}_{1/0} \qquad \theta_2 = \overrightarrow{\Theta}_{2/0} \qquad \dot{\theta}_1 = \overrightarrow{\Omega}_{1/0} \qquad \dot{\theta}_2 = \overrightarrow{\Omega}_{2/0} \qquad \ddot{\theta}_1 = \overrightarrow{\alpha}_{1/0} \qquad \ddot{\theta}_2 = \overrightarrow{\alpha}_{2/0}$$

$$a = \overrightarrow{\Gamma}_{A,1/0} = \overrightarrow{\Gamma}_{A,2/0} \qquad g = \overrightarrow{g} \qquad c_1 = \overrightarrow{AG_1} \qquad c_2 = \overrightarrow{AG_2} \qquad I_1 = I_{1,A} \qquad I_2 = I_{2,A}$$

By rearranging (A.11):

$$I_2 * \ddot{\theta}_2 + \dot{\theta}_2 \times (I_2 * \dot{\theta}_2) + \Lambda * \dot{\theta}_2 + K * \theta_2 + m_2 c_2 \times (a - g) = \Lambda * \dot{\theta}_1 + K * \theta_1$$

The vector from the grasp point and the center of mass of the item $\overrightarrow{AG_2}$ can be computed as $\overrightarrow{AG_2} = R(\theta_{2/1}) * \overrightarrow{AG_{2init}}$ with $R(\theta_{2/1})$ the rotation matrix from $\{2\}$ to $\{1\}$ and $\overrightarrow{AG_{2init}}$ the initial vector from A to the center of mass G_2 (at the moment the tool move to contact). With the approximation of the sec. A.1.7: $c_2 = R(\theta_{2/1}) * c_{2,init} \approx c_{2,init} + \theta_{2/1} \times c_{2,init}$, the equation (A.11) is rearranged:

$$I_2 * \ddot{\theta}_2 + \dot{\theta}_2 \times (I_2 * \dot{\theta}_2) + \Lambda * \dot{\theta}_2 + K * \theta_2 + m_2(\theta_2 \times c_{2,init}) \times (a - g) = \Lambda * \dot{\theta}_1 + K * \theta_1 + m_2(\theta_1 \times c_{2,init} - c_{2,init}) \times (a - g) \tag{A.12}$$

In order to estimate the inertial parameters of the item (m_2, $c_{2,init}$ and I_2), the idea is to compute the orientation of the item as a function of the F/T sensor measurement ($\theta_2(f, \tau)$) by using the FDP on the tool {1}. Then derive a matrix equation from a known-variables-only equation as in the sec. A.2.4 to estimate the inertial parameters. Hence, the parameters of the spring-damper system has to be estimated. A method to estimate those parameters is introduced later in the next sections.

A.3.4 Fondamental Principle of Dynamics on {1}

From the equation of the FDP (A.5) on {1} at point A:

$$\left\{ \begin{array}{c} \overrightarrow{f} \\ \overrightarrow{\tau} + \overrightarrow{AS} \times \overrightarrow{f} \end{array} \right\}_A + \left\{ \begin{array}{c} \overrightarrow{F}_{2/1} \\ \overrightarrow{0} \end{array} \right\}_A + \left\{ \begin{array}{c} m_1 \overrightarrow{g} \\ m_1 \overrightarrow{AG_1} \times \overrightarrow{g} \end{array} \right\}_A + \left\{ \begin{array}{c} \overrightarrow{0} \\ K * \overrightarrow{\Theta}_{2/1} + \Lambda * \overrightarrow{\Omega}_{2/1} \end{array} \right\}_A$$
$$= \left\{ \begin{array}{c} m_1 \overrightarrow{\Gamma}_{A,1/0} + \overrightarrow{\Omega}_{1/0} \times (\overrightarrow{\Omega}_{1/0} \times m_1 \overrightarrow{AG_1}) + \overrightarrow{\alpha}_{1/0} \times m_1 \overrightarrow{AG_1} \\ I_{1,A} * \overrightarrow{\alpha}_{1/0} + \overrightarrow{\Omega}_{1/0} \times (I_{1,A} * \overrightarrow{\Omega}_{1/0}) + m_1 \overrightarrow{AG_1} \times \overrightarrow{\Gamma}_{A,1/0} \end{array} \right\}_A$$

The equation on force is:

$$\overrightarrow{f} + \overrightarrow{F}_{2/1} + m_1\overrightarrow{g} = m_1\overrightarrow{\Gamma}_{A,1/0} + \overrightarrow{\Omega}_{1/0} \times (\overrightarrow{\Omega}_{1/0} \times m_1\overrightarrow{AG_1}) + \overrightarrow{\alpha}_{1/0} \times m_1\overrightarrow{AG_1} \quad \text{(A.13)}$$

A.3.5 Spring-damper parameters estimation

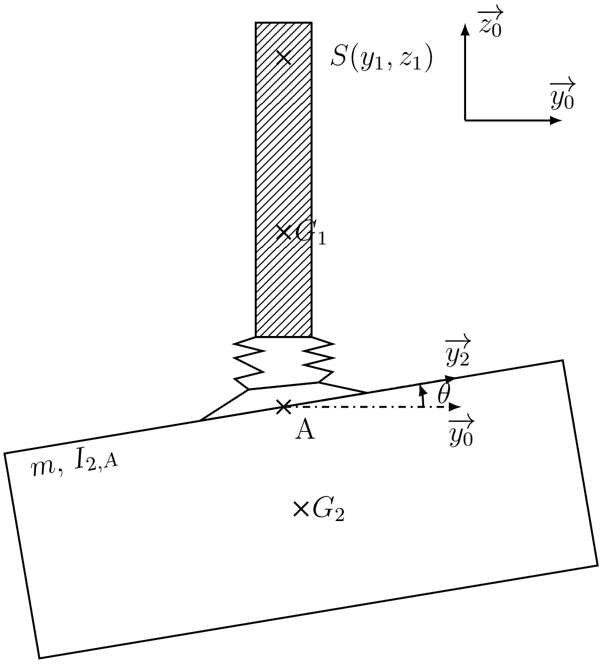

Figure A.3.1: 2-bodies with 2-dimensions assumptions system.

Considering the following assumptions (see Figure A.3.1 for details):

- A 2-dimensions problem is considered: $\overrightarrow{\Omega}_{2/0} = \begin{pmatrix} \dot{\theta} \\ 0 \\ 0 \end{pmatrix}$ and $\overrightarrow{V}_{G_2,2/0} = \begin{pmatrix} 0 \\ v_{2y} \\ v_{2z} \end{pmatrix}$.
- The spring-damper has a circular symmetry about the z-axis:

$$K = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k' \end{pmatrix} \text{ and } \Lambda = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda' \end{pmatrix}$$

- The excitation is in the yz-plane: $\overrightarrow{\Omega}_{1/0} = \overrightarrow{0}$ and $\overrightarrow{V}_{A,2/0} = \begin{pmatrix} 0 \\ v_{1y} \\ v_{1z} \end{pmatrix}$ and $\overrightarrow{\Gamma}_{A,2/0} =$

$$\begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix}.$$

- The initial position is: $\theta_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $\theta_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $c_{2,init} = \begin{pmatrix} 0 \\ 0 \\ -c \end{pmatrix}$ with $c > 0$, $\overrightarrow{AS} = \begin{pmatrix} 0 \\ 0 \\ s \end{pmatrix}$ with $s > 0$ and $\overrightarrow{AG_1} = \begin{pmatrix} 0 \\ 0 \\ q \end{pmatrix}$ with $q > 0$.

Hence, the equation (A.12) is projected on the x-axis:

$$\ddot{\theta} + \frac{\lambda}{I_{2xx}}\dot{\theta} + \frac{k + m_2 c(a_z + g)}{I_{2xx}}\theta = -\frac{m_2 c}{I_{2xx}} a_y$$

The solution parameters of θ can be computed with the measured force f. From the FDP on {1} (A.3.4), the force equation within the assumptions becomes:

$$\overrightarrow{f} = m_1(\overrightarrow{a} - \overrightarrow{g}) - \overrightarrow{F_{2/1}} \tag{A.14}$$

$$\overrightarrow{f} = m_1 \begin{pmatrix} 0 \\ a_y \\ a_z + g \end{pmatrix} - \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} -F_x \\ m_1 a_y - F_y \\ m_1(a_z + g) - F_z \end{pmatrix} \tag{A.15}$$

Then using the force equation of the FDP on {2}:

$$\begin{pmatrix} -F_x \\ -F_y \\ -F_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -m_2 g \end{pmatrix} = m_2 \left[\begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix} + \begin{pmatrix} \dot{\theta} \\ 0 \\ 0 \end{pmatrix} \times \left(\begin{pmatrix} \dot{\theta} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -\theta c \\ c \end{pmatrix} \right) + \begin{pmatrix} \ddot{\theta} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -\theta c \\ c \end{pmatrix} \right]$$

$$\begin{pmatrix} -F_x \\ -F_y \\ -F_z \end{pmatrix} = \begin{pmatrix} 0 \\ m_2[a_y + c(\dot{\theta}^2\theta - \ddot{\theta})] \\ m_2[a_z + g + c(\dot{\theta}^2 + \ddot{\theta}\theta)] \end{pmatrix}$$

F_x, F_y and F_z are substituted in (A.14):

$$\overrightarrow{f} = \begin{pmatrix} 0 \\ m_1 a_y + m_2[a_y + c(\dot{\theta}^2\theta - \ddot{\theta})] \\ m_1(a_z + g) + m_2[a_z + g + c(\dot{\theta}^2 + \ddot{\theta}\theta)] \end{pmatrix} \tag{A.16}$$

In order to estimate λ and k, the motion of the point A is sinusoidal: $\overrightarrow{OA} = \alpha \sin(\omega t) \cdot \overrightarrow{y_0}$

$$\ddot{\theta}(t) + \frac{\omega_0}{Q}\dot{\theta}(t) + \omega_0^2\theta(t) = B\sin(\omega t) \tag{A.17}$$

with $\omega_0 = \sqrt{\frac{k+m_2cg}{I_{2xx}}}$, $Q = \frac{\omega_0 I_{xx}}{\lambda}$ and $B = -\frac{m_2 c\omega^2\alpha}{I_{2xx}}$.

The solution of (A.17) is then:

$$\theta(t) = \theta_m \sin(\omega t + \phi) \tag{A.18}$$

$$\text{with:} \quad \theta_m = \frac{B}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{\omega\omega_0}{Q}\right)^2}} \qquad \phi = -\frac{\pi}{2} - \arctan\left[Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

Finally, the equation (A.18) is substituted into the previous (A.16) with $a_z = 0$:

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} 0 \\ (m_1+m_2)a_y + m_2c\theta_m\omega^2\sin(\omega t+\phi)(\theta_m^2\cos^2(\omega t+\phi)+1) \\ (m_1+m_2)g + m_2c\theta_m^2\omega^2 \end{pmatrix}$$

Considering small-angle approximation, $\theta_m^2\cos^2(\omega t+\phi)+1 \approx 1$ leads to:

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} 0 \\ (m_1+m_2)a_y + m_2c\theta_m\omega^2\sin(\omega t+\phi) \\ (m_1+m_2)g + m_2c\theta_m^2\omega^2 \end{pmatrix} \tag{A.19}$$

The parameters θ_m and ϕ can be derived and spring-damper parameters (k and λ) are further computed with equations (A.17) and (A.18):

$$\theta_m = \frac{\sqrt{f_x + (m1 + m2)g}}{\omega\sqrt{m_2 c}} \qquad \phi = -\omega t + \arcsin\left(\frac{f_y - a_y(m1 - m2)}{c m_2 \omega^2 \theta_m}\right) \tag{A.20}$$

Appendix B

Second Appendix

this is the information