1 Introduction

The aim of this work is to estimate the parameters of the SIR model. In particular, I will focus on the first wave of contagion of the Coronavirus pandemic in Italy.

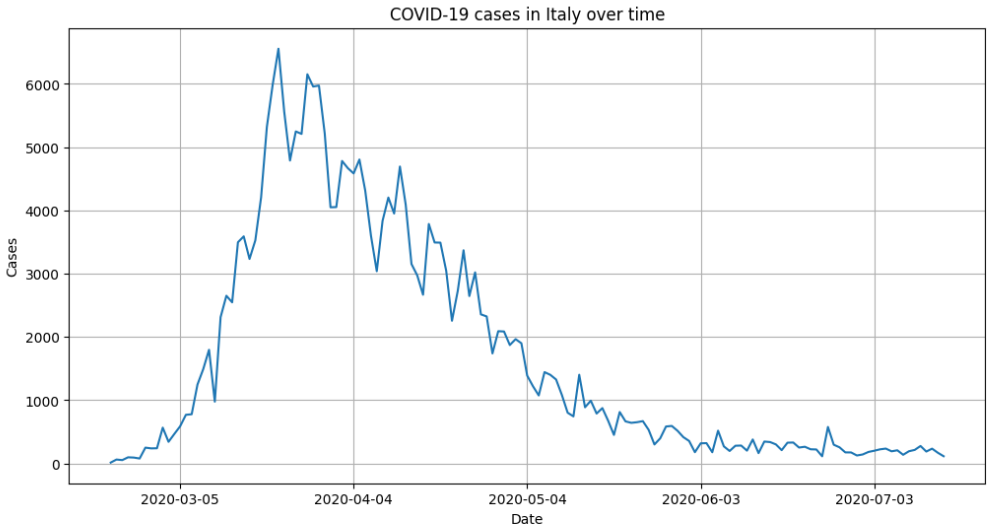

Figure 1: New infections per day in Italy

2 Model & Estimation

The model is as follows:

$$N_t = \gamma_1 N_{t-1} + \gamma_2 N_{t-1} \ln N_{t-1} + U_t, \tag{1}$$

where N_t is the total infected population at time t, and U_t is the error term.
The model has been estimated through GMM, using different combinations of the following instruments:

- N_{t-1}
- $N_{t-1} \ln N_{t-1}$
- Daily average temperature in Italy W_1
- Daily average mobility in Italy, in particular the percent change from baseline of occupation of:
  - Retail and recreation facilities W_2
  - Transit stations W_3

In particular:

Model	Instruments
1	N_{t-1} W_1
2	N_{t-1} W_1
3	N_{t-1} $N_{t-1} \ln N_{t-1}$ W_1 W_2 W_3
4	W_1 W_2 W_3

The results are as follows:

	Model 1	Model 2	Model 3	Model 4
γ_1	1.735***	1.722***	1.717***	1.726***
	(0.0217)	(0.0205)	(0.0201)	(0.0203)
γ_2	-0.0593***	-0.0583***	-0.0579***	-0.0586***
	(0.00177)	(0.00167)	(0.00163)	(0.00165)
Observations	129	129	129	129
J-stat	8.520	11.60	13.08	2.175
DoF	1	1	4	2

Table 1: GMM results: standard errors are in parentheses and $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

From Hansen J-test of model 4 and EHS test of model 3 and 4 we can see that N_{t-1} is endogenous with respect to U_t, and thus only model 4 is well-defined.

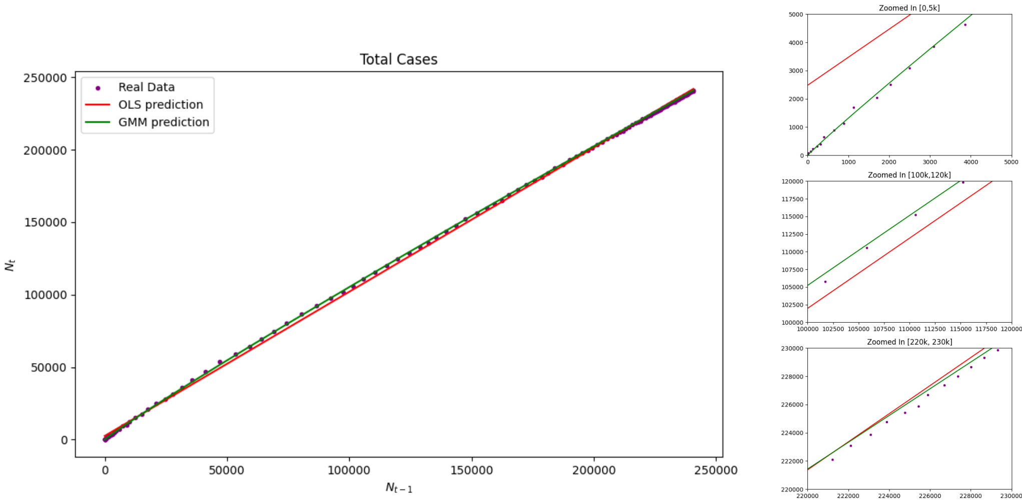

Figure 2: N_t vs N_{t-1} comparison between OLS and Model 4

3 Explicit Solution

In continuous time, the model equation (1) admits the following close form solution (i.e. total cases):

$$N(t) = \exp\left(\frac{Ke^{\gamma_2 t} - \gamma_1 + 1}{\gamma_2}\right), \qquad K = \gamma_1 + \gamma_2 \ln N_0, \tag{2}$$

And derivative (i.e. new cases):

$$\dot{N}(t) = K\, e^{\gamma_2 t}\, N(t). \tag{3}$$

K can therefore be estimated, in this case, I used grid search to find:

$$\hat{K} = \arg\min_K \left\| N_t - \hat{N}_K(t) \right\|, \qquad \hat{K} = 0.5175 \tag{4}$$

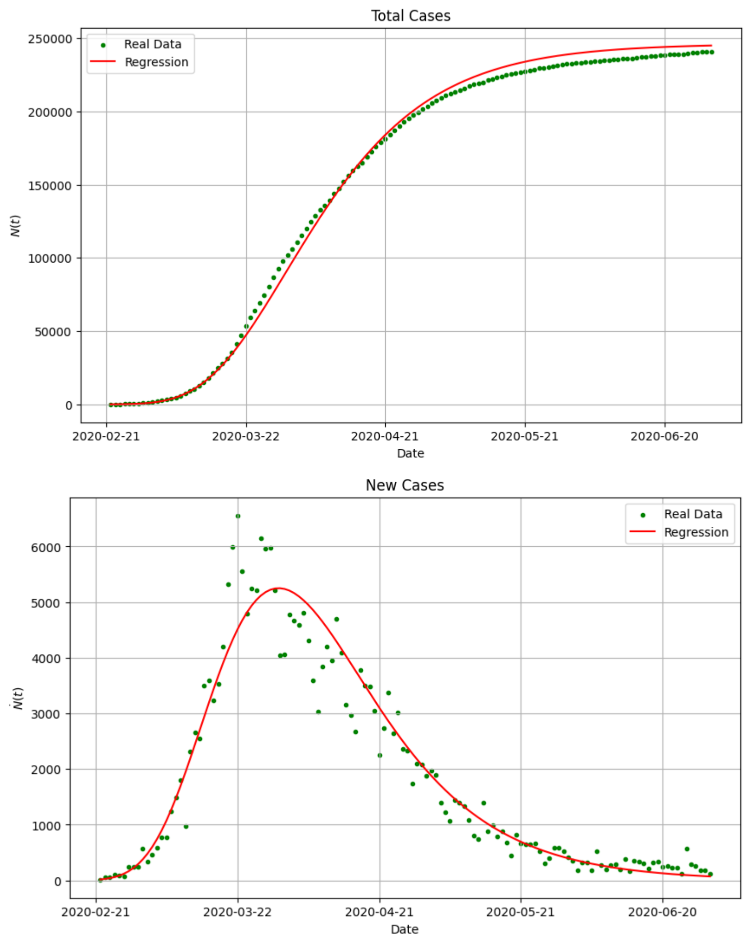

Figure 3: Comparison between the regressed equations and the real values

A Data Sources

- Covid infection data is available on the Covid section of the WHO website `https://data.who.int/dashboards/covid19/`
- Temperature data is available on the EU Copernicus Climate Data Store website `https://cds.climate.copernicus.eu/`
- Mobility data is available on the Google Covid19 Mobility Report website `https://www.google.com/covid19/mobility/`