Morphological Image Processing

Subject: FIP (181102)

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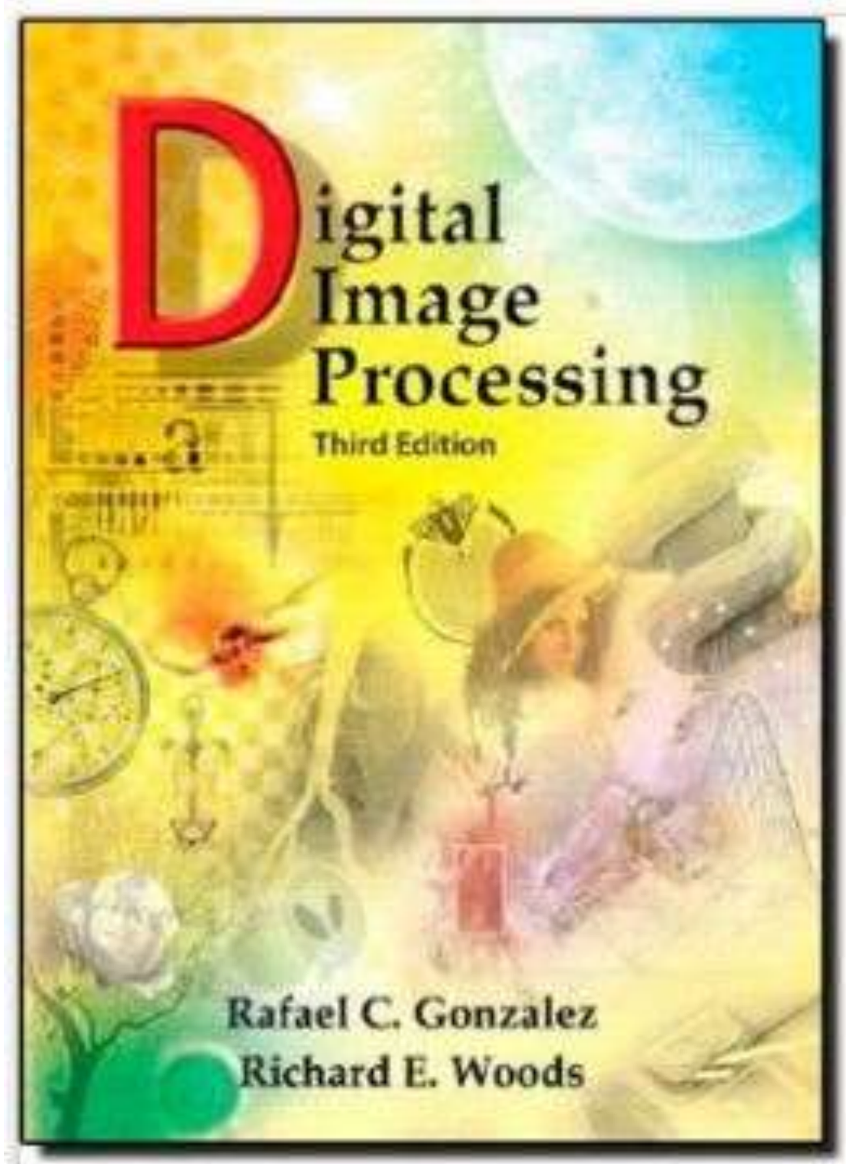
Digital Image Processing, 3rd edition by Gonzalez and Woods



Preview

- The word Morphology commonly denotes a branch of biology that deals with the form and structure of animals and plants
- We use mathematical morphological as a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries extraction, skeletons, convex hull, morphological filtering, thinning, pruning
- Binary images whose components are elements of Z^2 while in gray scale image elements belongs to Z^3

Preliminaries

- Reflections

The reflection of a set B, denoted $\hat{B}$, is defined as

$$\hat{B} = \{w \mid w = -b, \text{for } b \in B\}$$

- Translations

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_Z$, is defined as

$$(B)_Z = \{c \mid c = b + z, \text{for } b \in B\}$$

Preliminaries

Operators by examples:

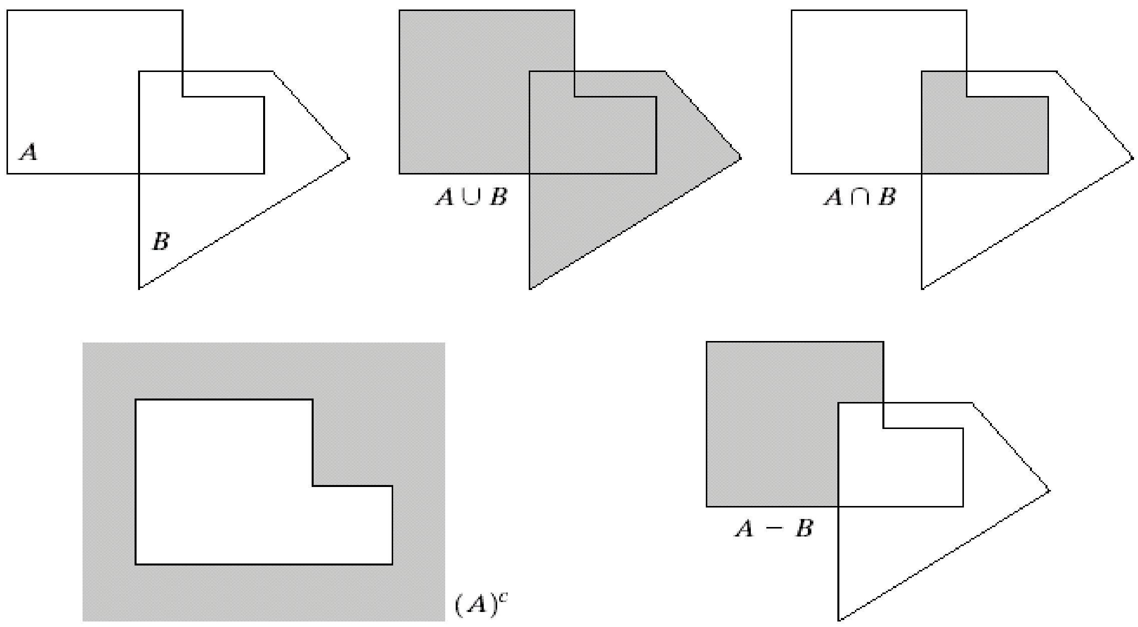


FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Preliminaries

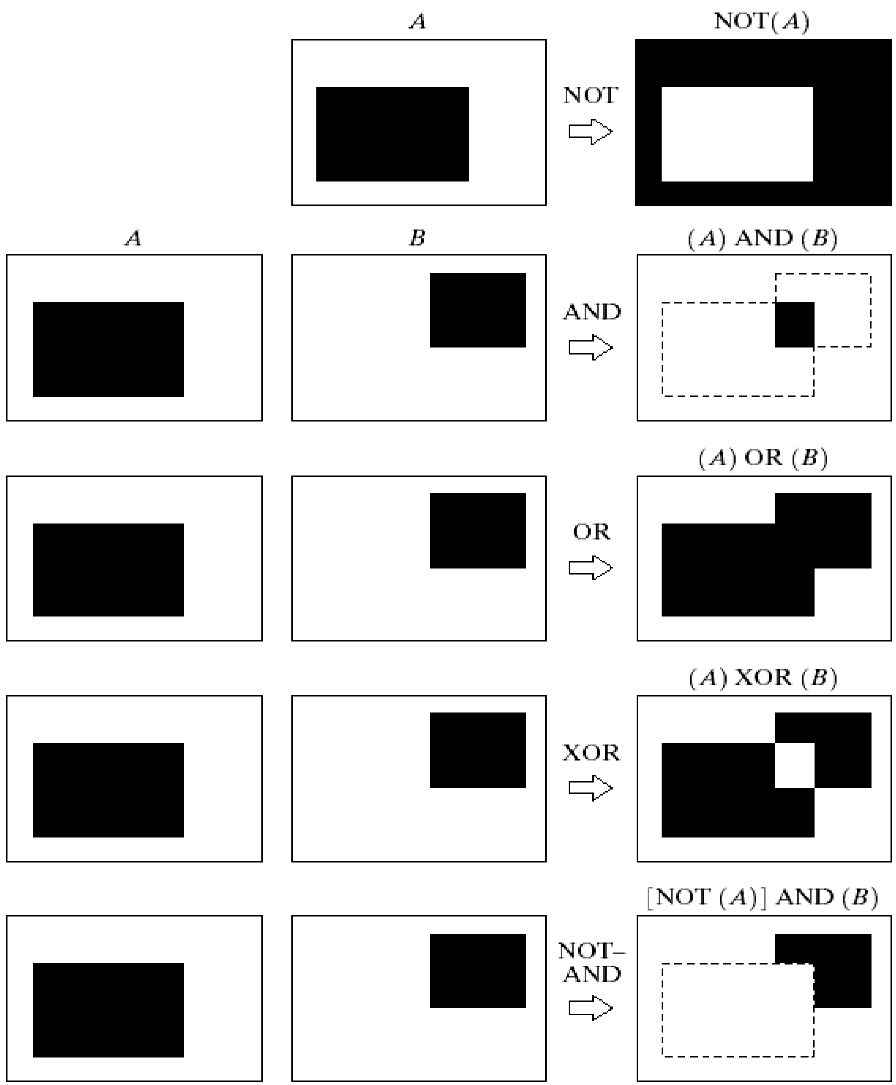
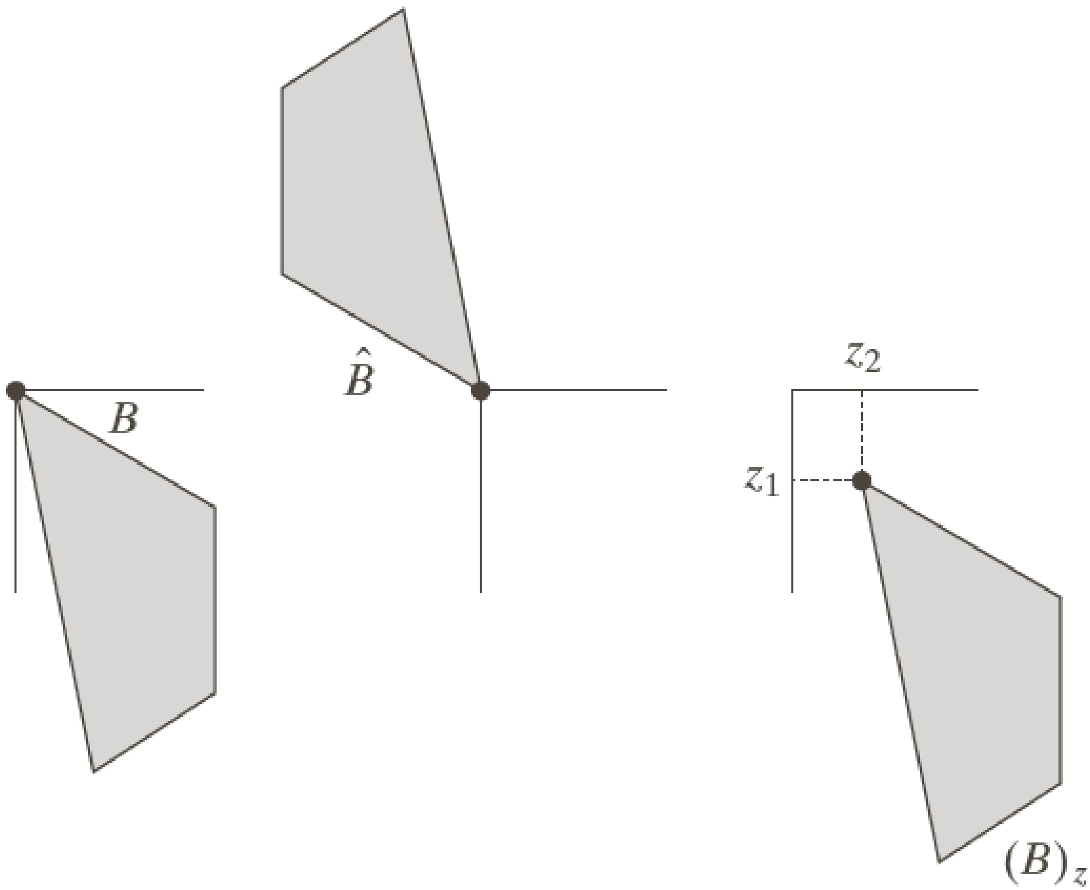


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Examples of Reflection and Translation



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

Structuring Elements

- Small sets or sub-images used to probe an image under study for properties of interest is called SE

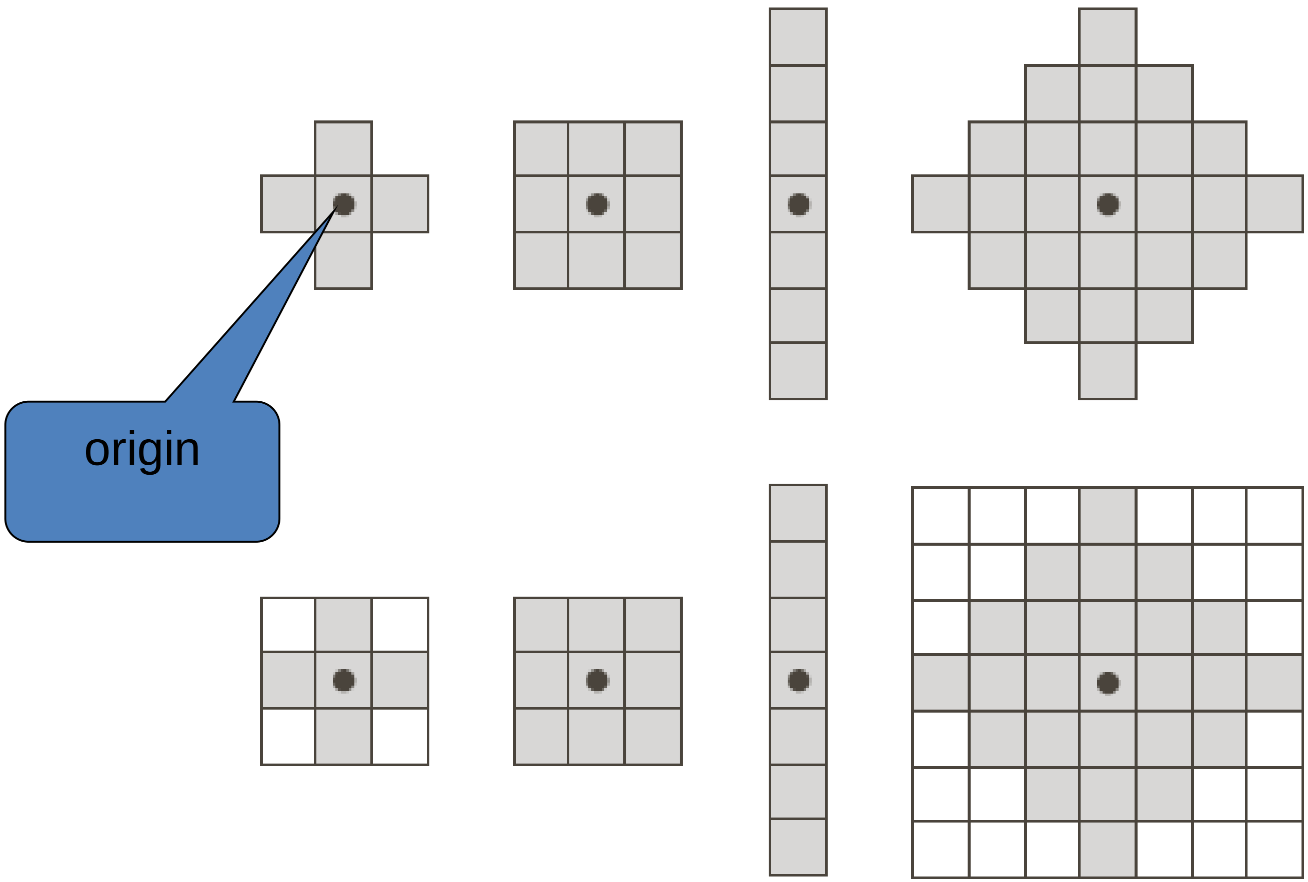
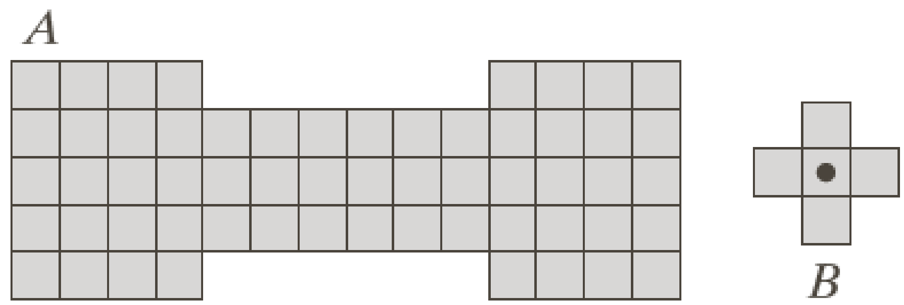
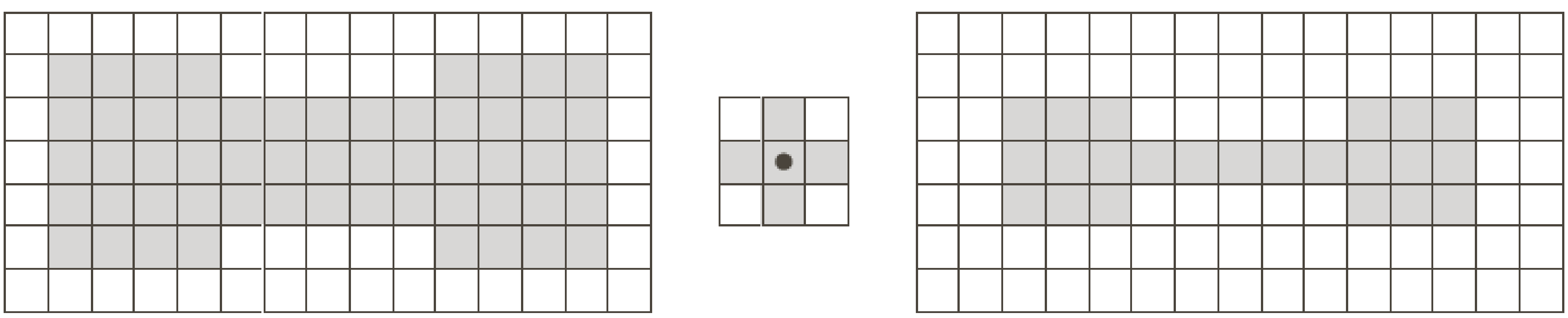


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

How Structuring Elements are used

- The Background border is made large enough to accommodate the entire structuring element when its origin is on the border of the original set (Padding)
- SE is of size 3×3 with the origin in the center, so as a one-element border that encompasses the entire set is sufficient





Erosion and Dilation

- Erosion:
  - With A and B as sets in Z^2 , the erosion of A by B is defined as

$$A \ominus B = \{ z \mid (B)_Z \subseteq A \}$$

  - Erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

$$A \ominus B = \{ z \mid (B)_Z \cap A^c = \varnothing \}$$

  - B has to be contained in A is equivalent to B not sharing any common elements with the background

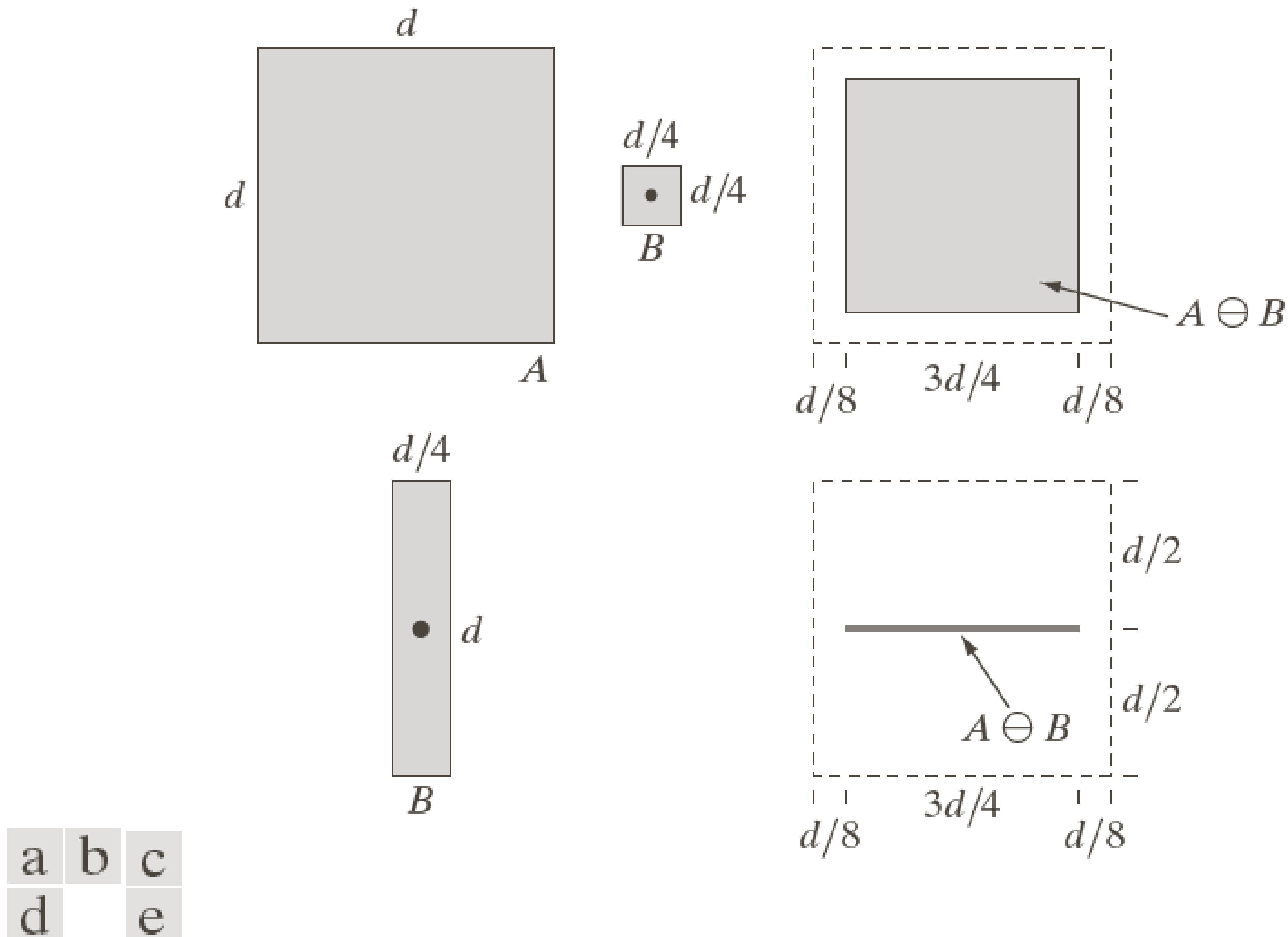
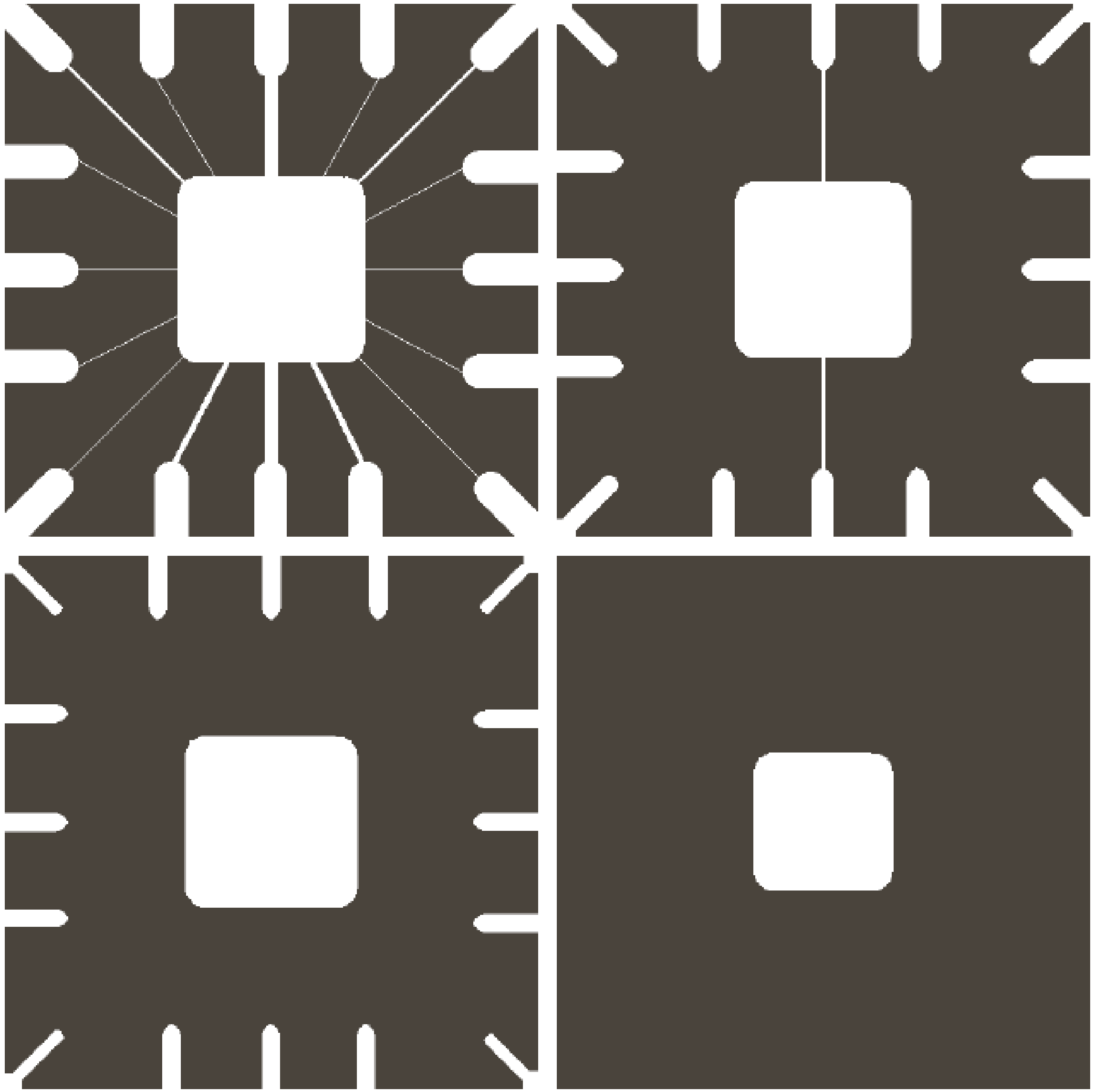


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Erosion and Dilation



a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

Erosion and Dilation

- Erosion shrinks or thins objects in a binary image
- Erosion as a morphological filtering operation in which image details smaller than the structuring elements are filtered from the image
- Erosion performed the function of a “line filter”

Erosion and Dilation

- Dilation:
  - With A and B as sets in Z^2 , the dilation of A by B is defined as $A \oplus B = \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \varnothing \right\}$
  - Reflecting B about its origin, and shifting this reflection by z
  - The dilation of A by B then is the set of all displacements, z, such that B^ and A overlap by at least one element

$$A \oplus B = \left\{ z \mid \left[\left(\hat{B}\right)_z \cap A \right] \subseteq A \right\}$$

Erosion and Dilation

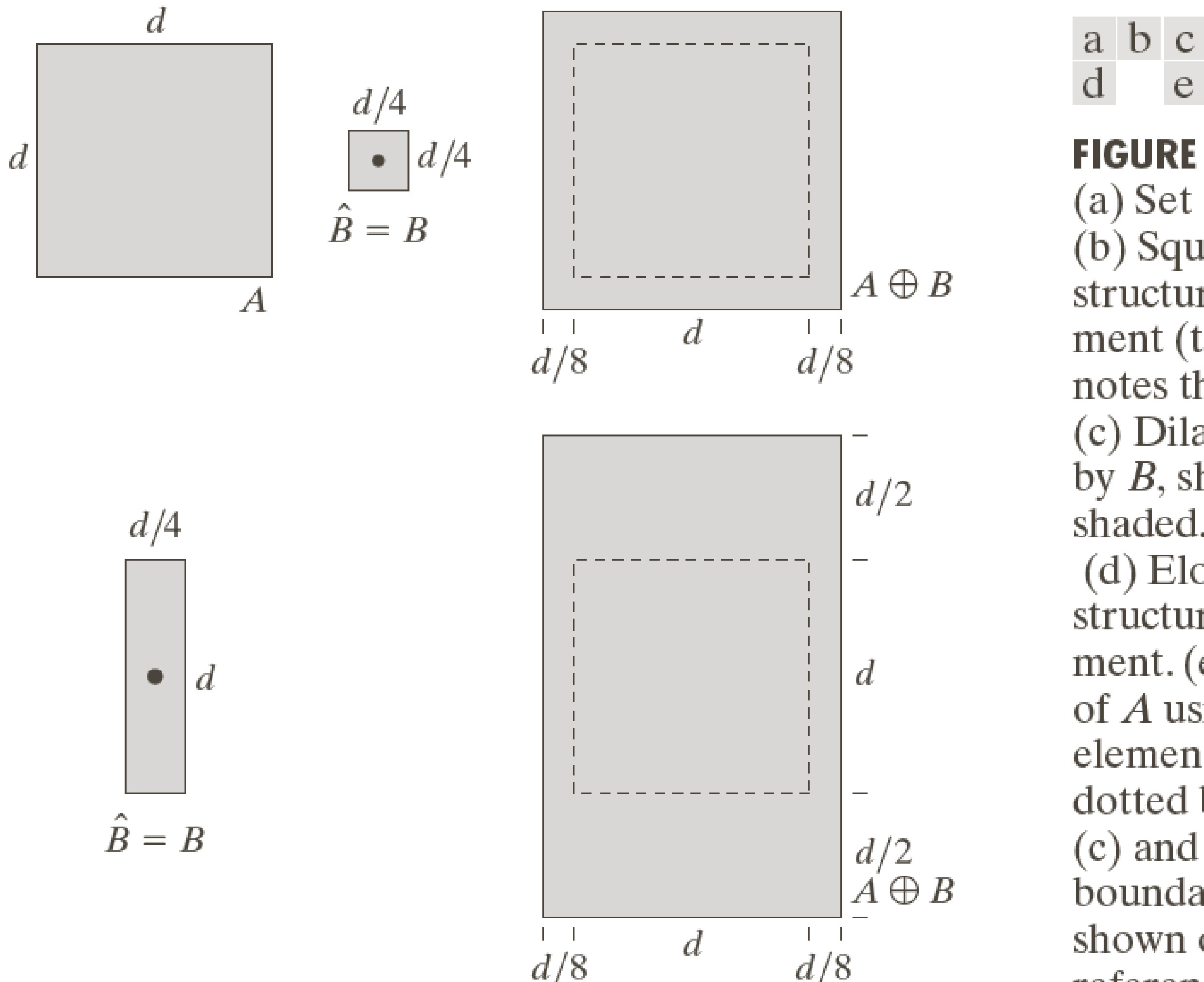
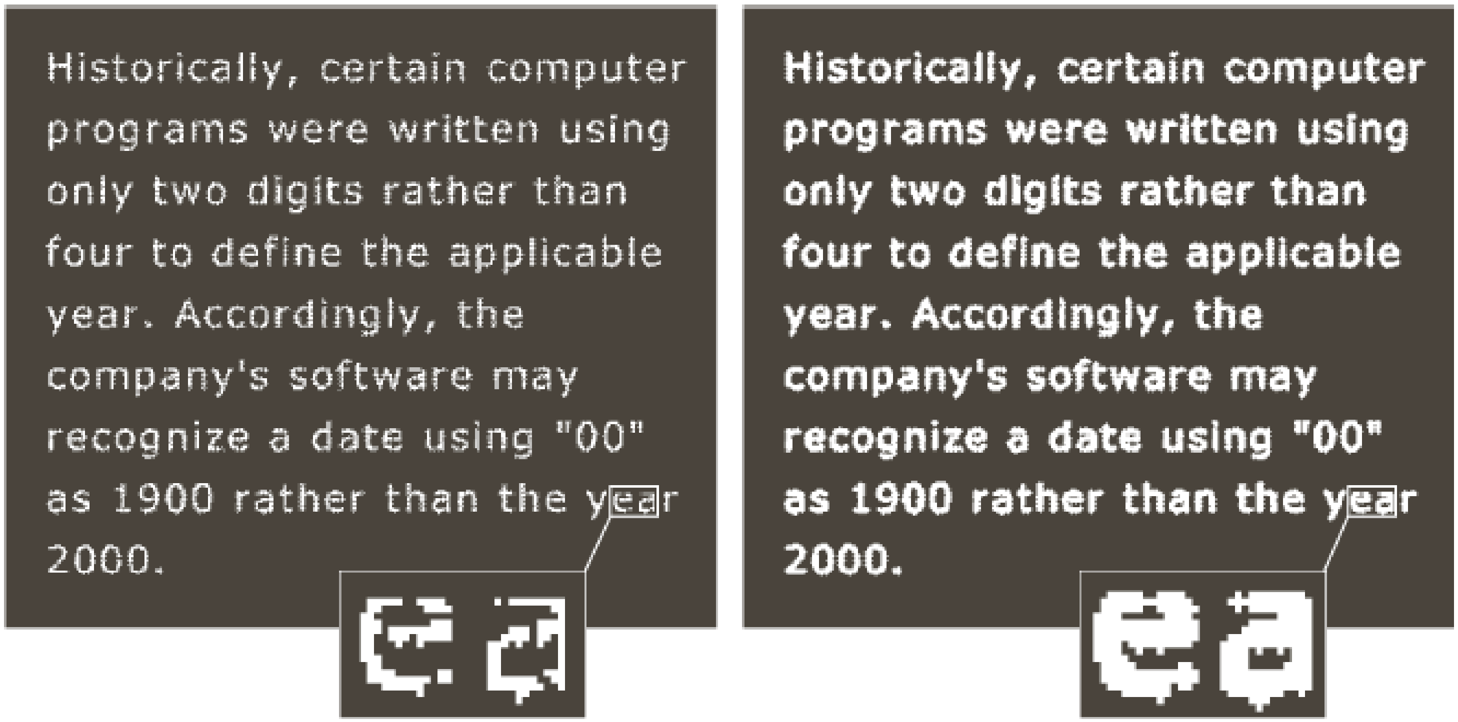


FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Erosion and Dilation

- Dilation:
  - Unlike erosion, dilation “grows” or “thickens” objects in a binary image
  - The specific manner and extent of this thickening is controlled by the shape of the structuring element used

Erosion and Dilation



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection
- That is,

$$\left(A \ominus B\right)^c = A^c \oplus \hat{B}$$

and

$$\left(A \oplus B\right)^c = A^c - \hat{B}$$

- First equation indicates that erosion of A by B is the complement of the dilation of A^c by B^, and vice versa
- Duality property is useful particularly when the structuring element is symmetric with respect to its origin so that B^=B
- Then, we can obtain the erosion of an image by B simply by dilating its background (i.e., dilating A^c) with the same

Duality

- Structuring element and complementing the result.
- Starting with the definition of erosion

$$
\begin{aligned}
\left(A \oplus B\right)^c &= \left\{ z \mid \left(\hat{B}\right)_Z \cap A \neq \varnothing \right\}^c \\
&= \left\{ z \mid \left(\hat{B}\right)_Z \cap A^c = \varnothing \right\} \\
&= A^c \ominus \hat{B}
\end{aligned}
$$

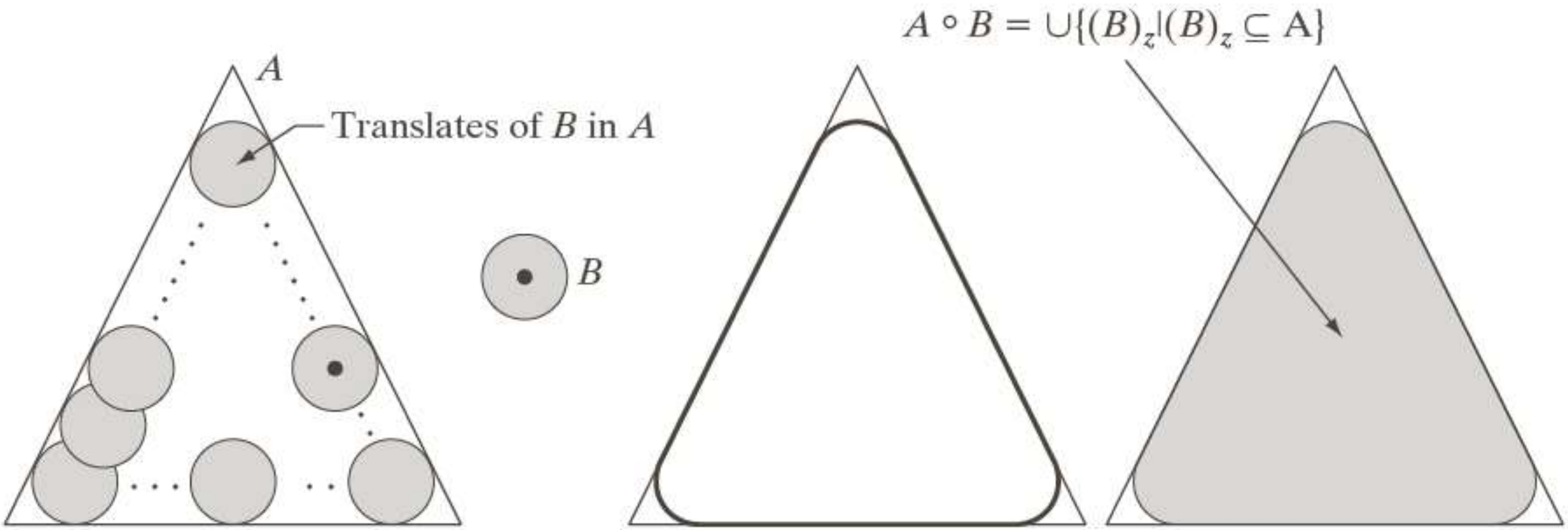
Opening and Closing

- Opening smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour
- Opening:
- Opening of set A by SE B is defined $A \circ B = (A \ominus B) \oplus B$
- Thus, opening A by B is the erosion of A by B, followed by a dilation of the result by B

Opening and Closing

- Closing:
- Closing of set A by SE B is defined $A \bullet B = \left(A \oplus B\right) \ominus B$
- Closing of set A by B is simply the dilation of A by B, followed by the erosion of the result by B

Examples: Opening



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Examples: Closing

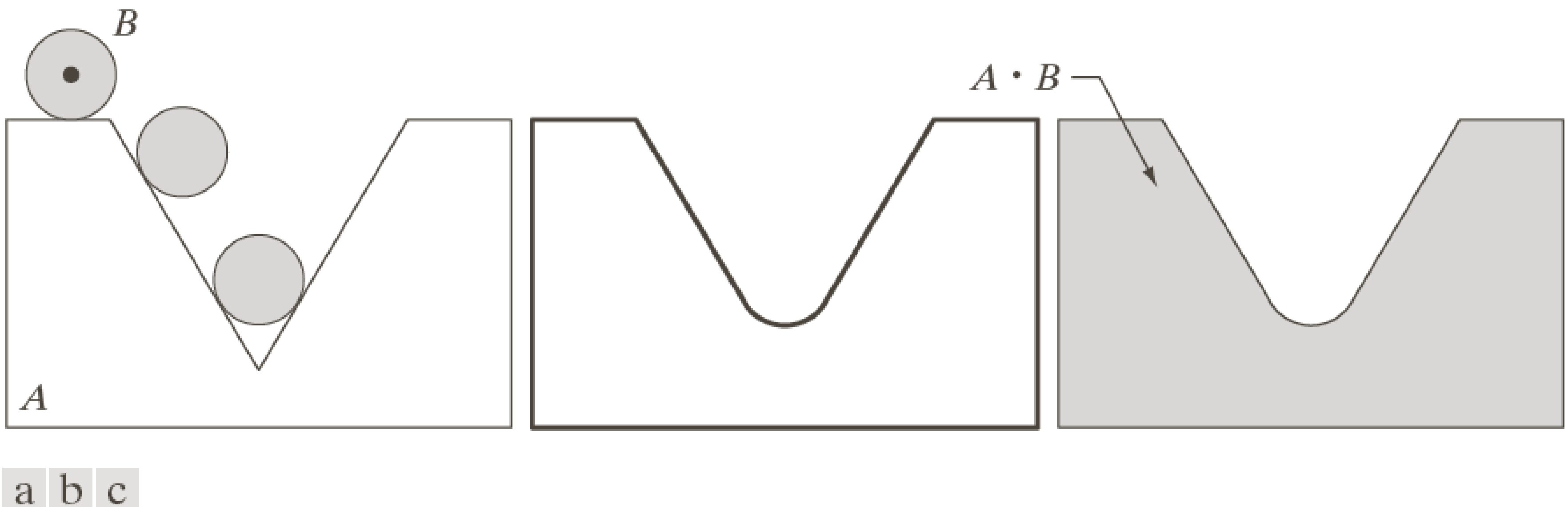
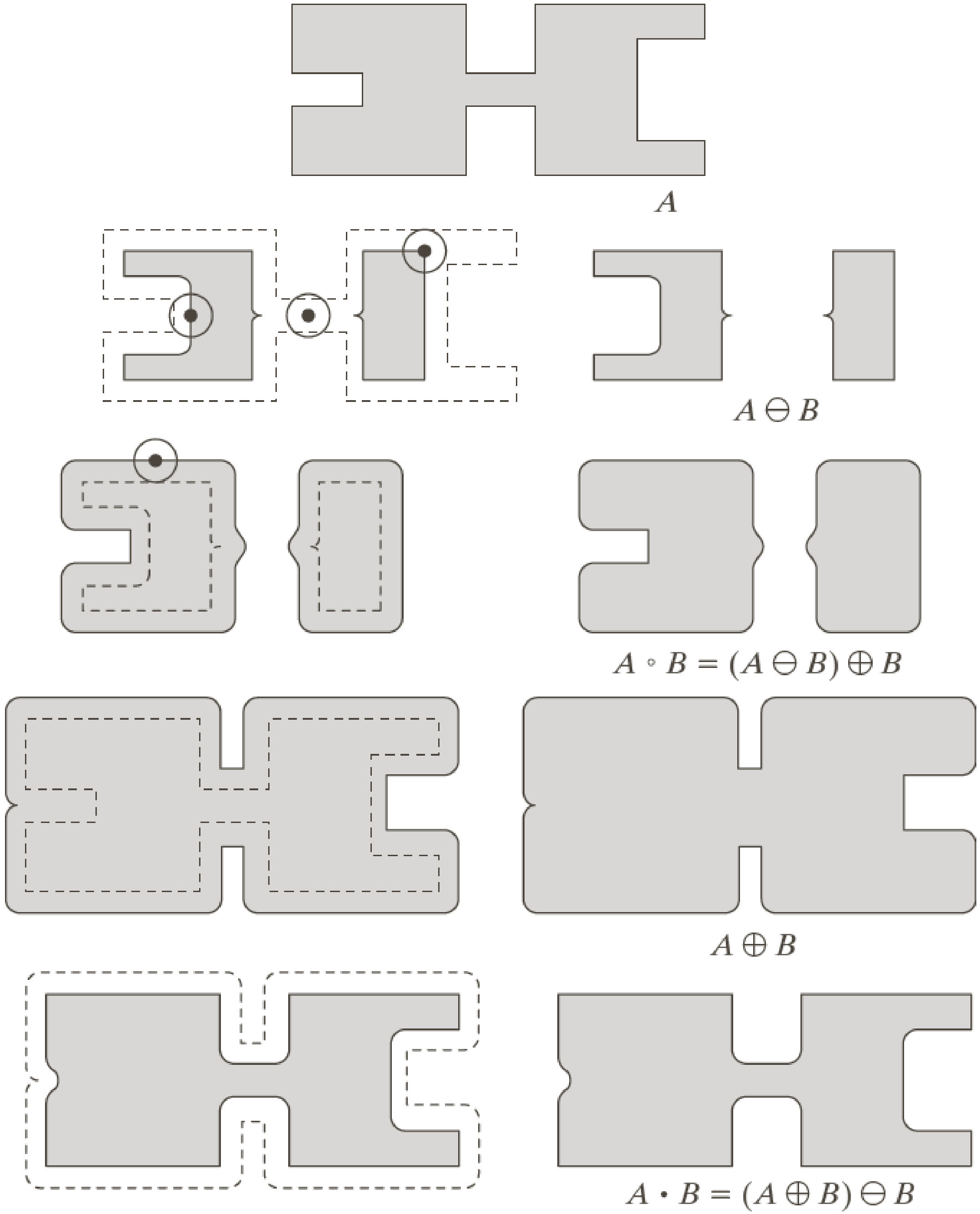


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.



a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Opening and Closing

- Opening and closing are duals of each other with respect to set complementation and reflections

$$\left(A \bullet B\right)^c = (A^c \circ \hat{B})$$

$$\left(A \circ B\right)^c = (A^c \bullet \hat{B})$$

- Opening Properties:
- A o B is a subset of A
- If C is a subset of D, then C o B is a subset of D o B
- (A o B) o B = A o B

Opening and Closing

- Closing properties:
- A is a subset of $A g B$
- If C is a subset of D, then $C g B$ is a subset of $D g B$
- $(A g B) g B = A g B$

Opening and Closing

a b
d c
e f

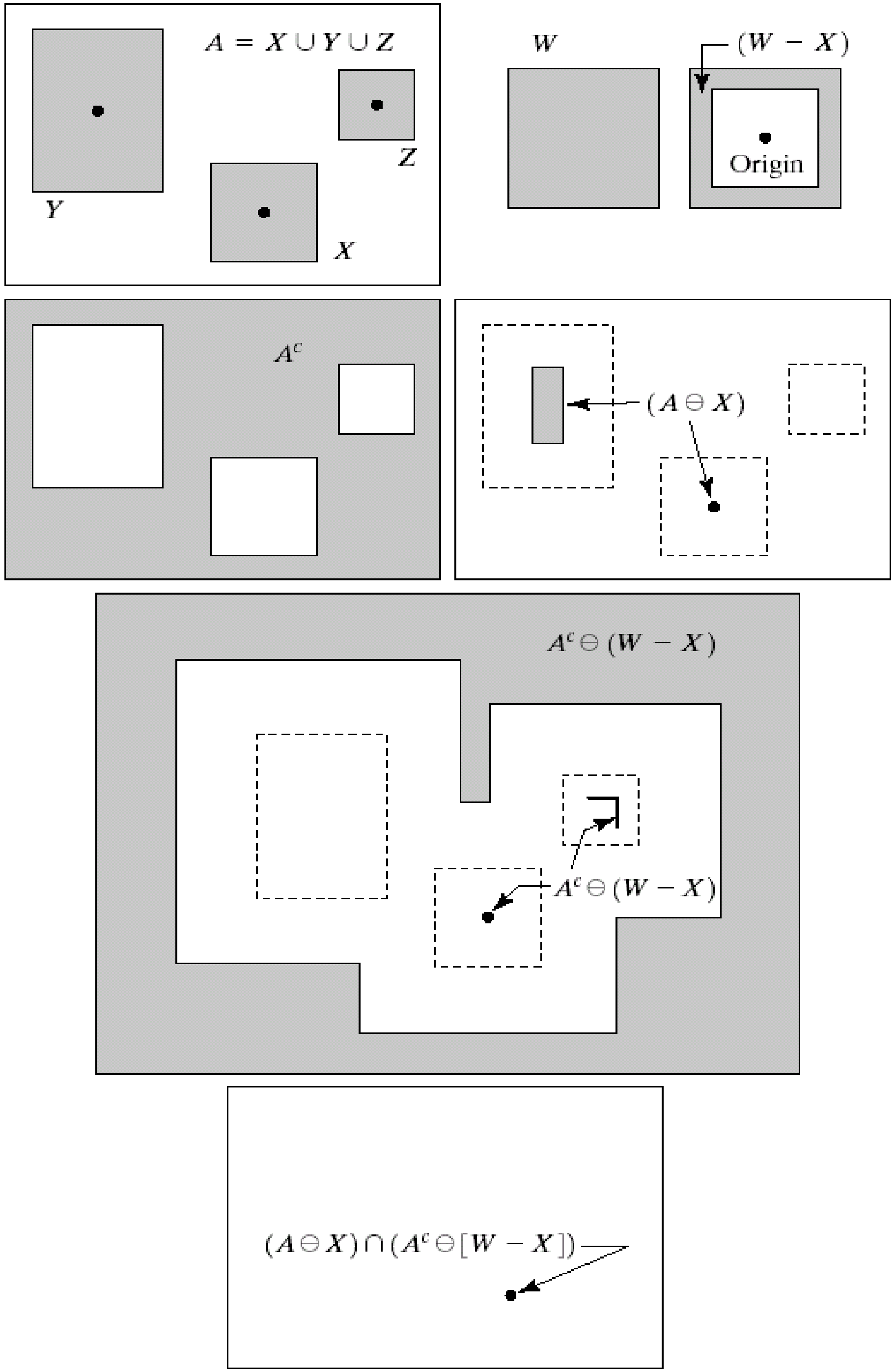
FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Hit-or-Miss Transformation

- The hit-or-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image.

- The hit-and-miss transform is a basic tool for shape detection.

- Concept: To detect a shape:
  Hit object
  Miss background



a	b
c	d
e	
f	

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

Hit-or-Miss Transformation

- The structural elements used for Hit-or-miss transforms are an extension to the ones used with dilation, erosion etc.
- The structural elements can contain both foreground and background pixels, rather than just foreground pixels, i.e. both ones and zeros.
- The structuring element is superimposed over each pixel in the input image, and if an exact match is found between the foreground and background pixels in the structuring element and the image, the input pixel lying below the origin of the structuring element is set to the foreground pixel value. If it does not match, the input pixel is replaced by the boundary pixel value.

Hit-or-Miss Transformation

- The hit-or-miss transform is defined as:

  Let B ={B1, B2}, where B1 is the set formed from elements of B associated with an object and B2 is the set of elements of B associated with the corresponding background, where B1 and B2 are disjoint.

$$A * B = (A \Theta X) \cap [A^c \Theta (W - X)]$$

$$A * B = (A \Theta B_1) \cap [A^c \Theta B_2]$$

B1: Object related

B2: Background related

Some Basic Algorithms

- Boundary Extraction
- Hole Filling
- Extraction of Connected Components
- Thinning and Thickening
- Skeletons
- Convex Hull
- Prunning

Hole Filling

- A hole may be defined as a background region surrounded by a connected border of foreground pixels
- Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole). Given a point in each hole, the objective is to fill all the holes with 1s

1. Forming an array X0 of 0s (the same size as the array containing A), except the locations in X0 corresponding to the given point in each hole, which we set to 1.

2. $X_k = (X_{k-1} \oplus B) \cap A^c \quad k=1,2,3,\ldots$

Stop the iteration if $X_k = X_{k-1}$

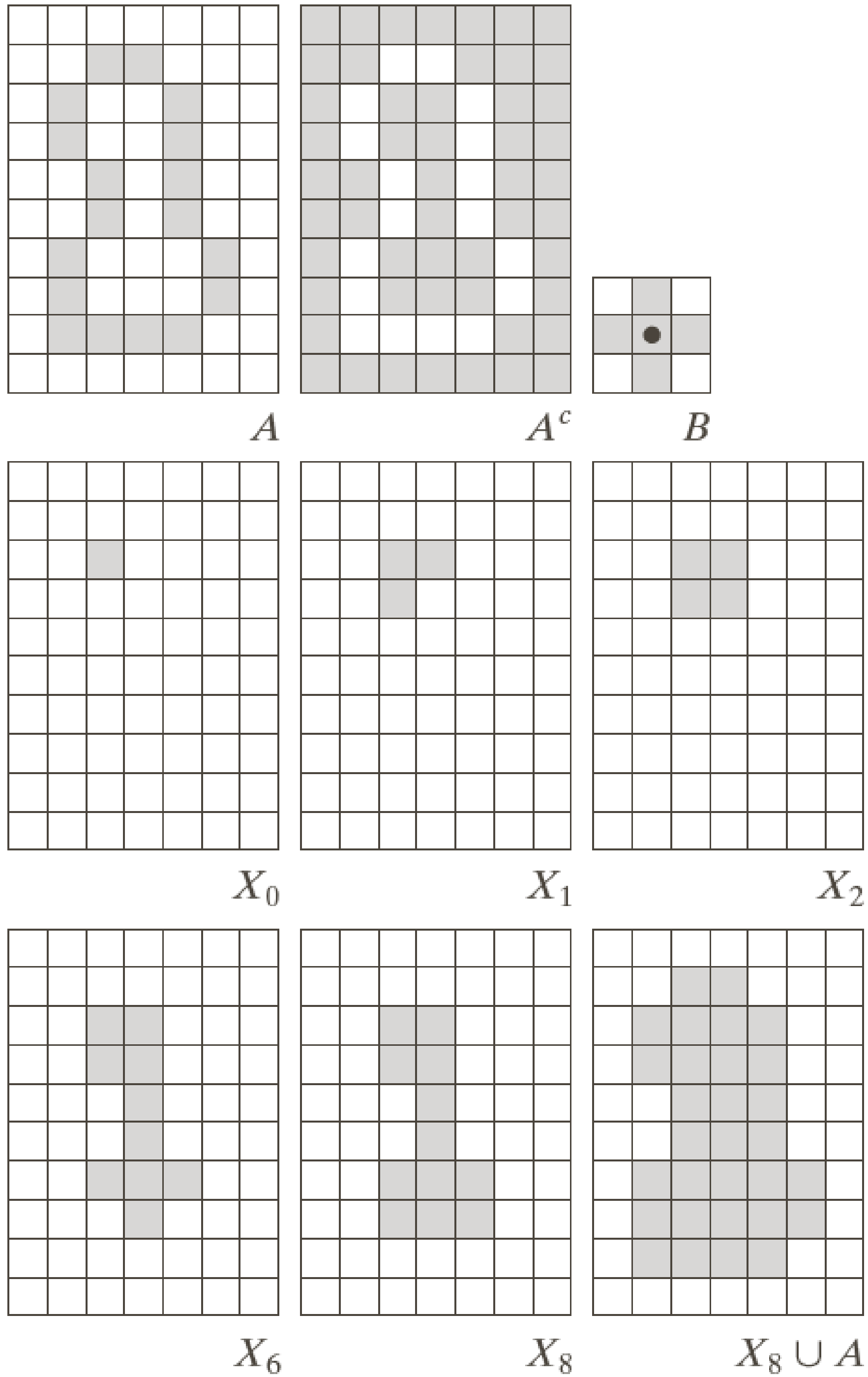
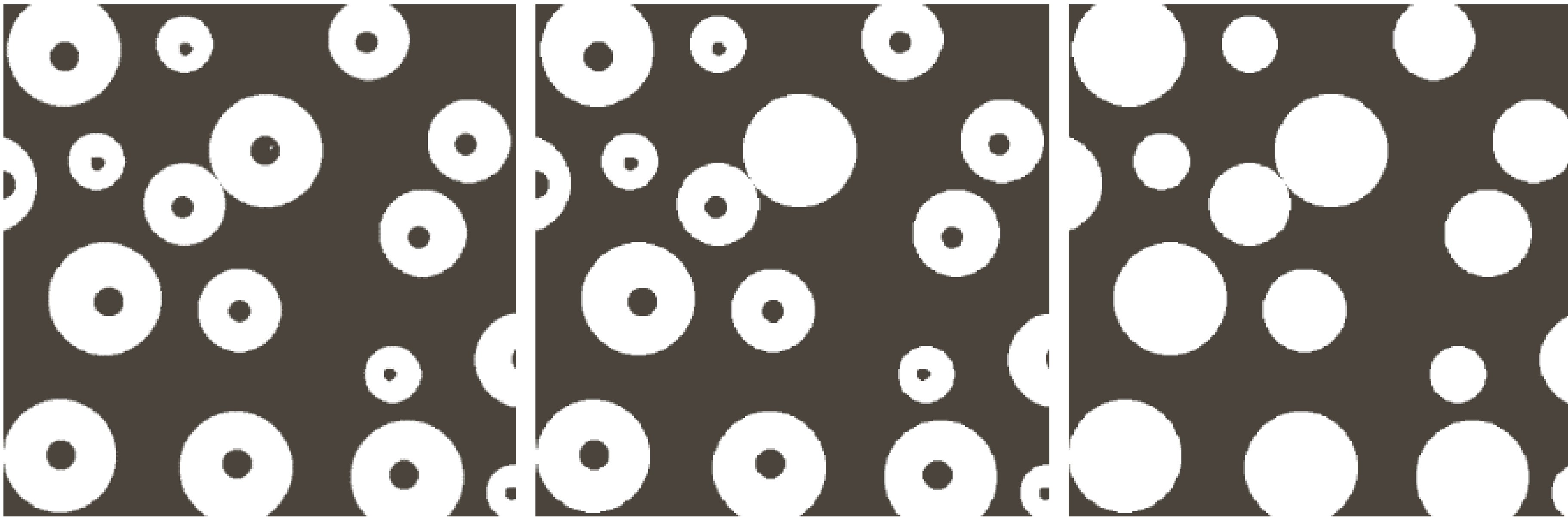


FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Extraction of Connected Components

- Extraction of connected components from a binary image is central to many automated image analysis applications
- Here we introduce connectivity and connected components
- Let A be a set containing one or more connected components , and form an array X_0 (of the same size as the array containing A) whose elements are 0s (background values), except at each location known to correspond to a point in each connected component in A, which we set to 1 (foreground value)

Extraction of Connected Components

$$X_k = (X_{k-1} + B) \cap A$$

$$B: \text{structuring element}$$

$$\text{until } X_k = X_{k\text{-}1}$$

- SE used is based on 8-connectivity between pixels

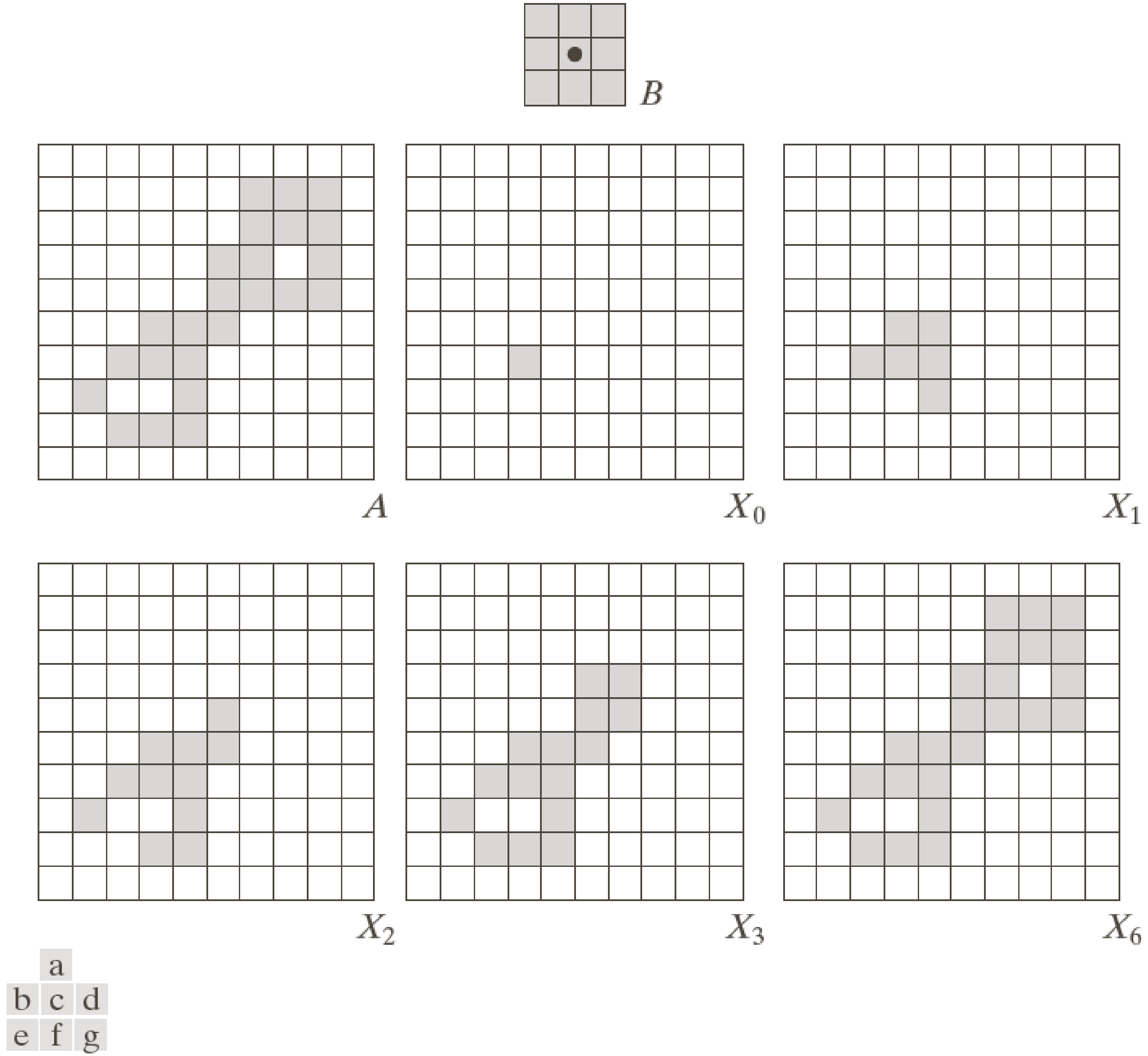
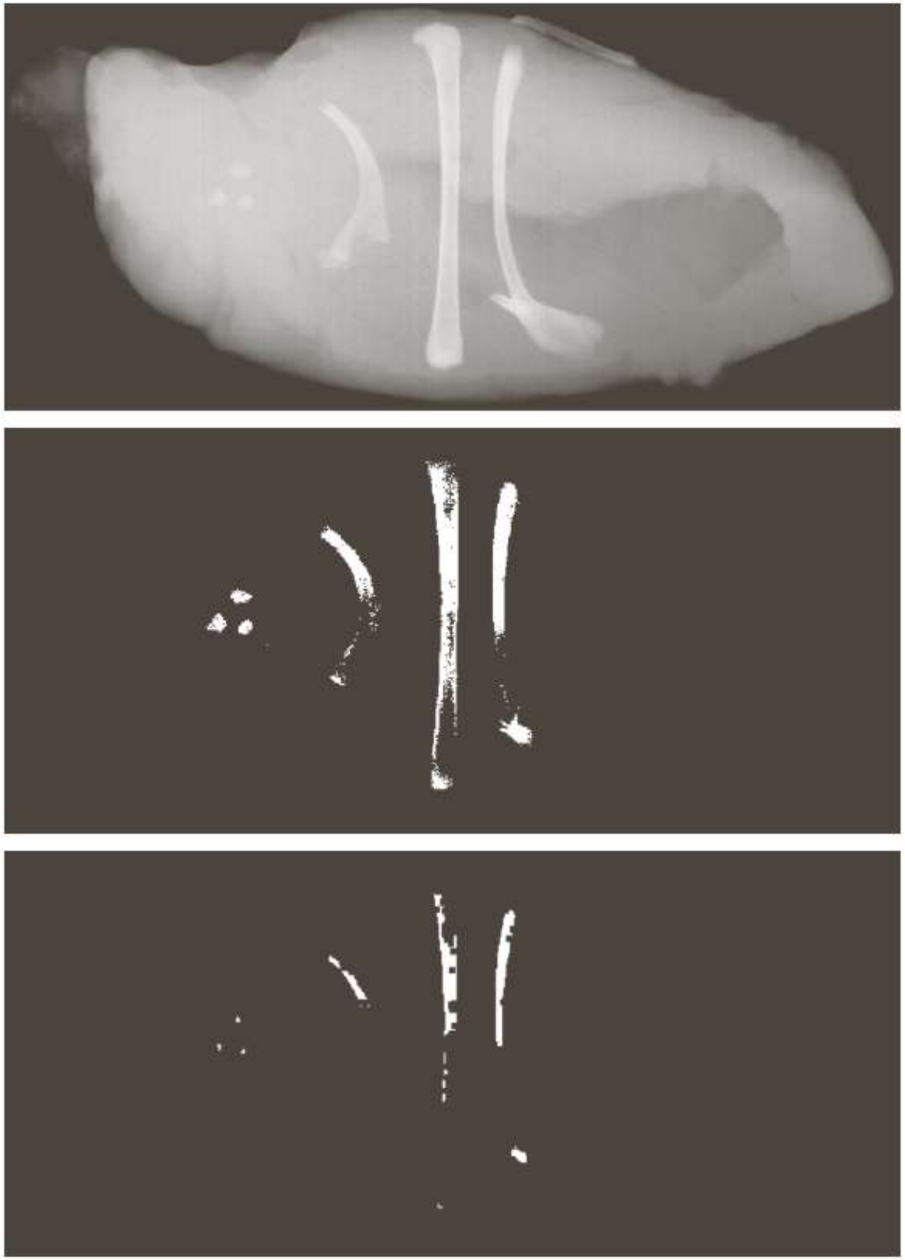


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Thinning

- Thinning

  The thinning of a set A by a structuring element B, defined

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

Thinning

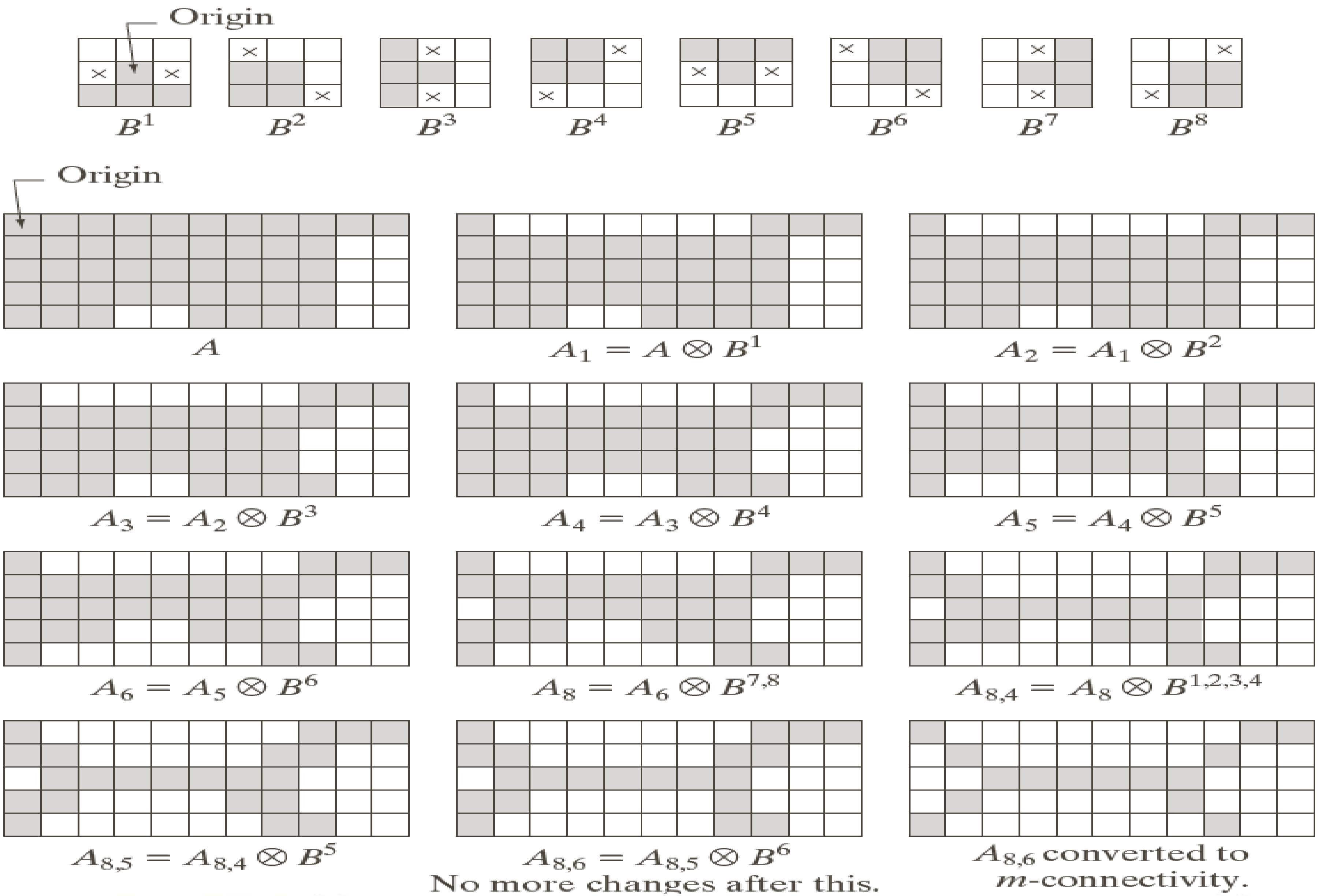
- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

where B^i is a rotated version of $B^{i\text{-}1}$

The thinning of A by a sequence of structuring element $\{B\}$

$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$



a
b c d
e f g
h i j
k l m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

Skeletons

- Skeletons

A skeleton, $S(A)$ of a set A has the following properties

a. if z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk containing $(D)_z$ and included in A. The disk $(D)_z$ is called a maximum disk.

b. The disk $(D)_z$ touches the boundary of A at two or more different places.

Skeletons

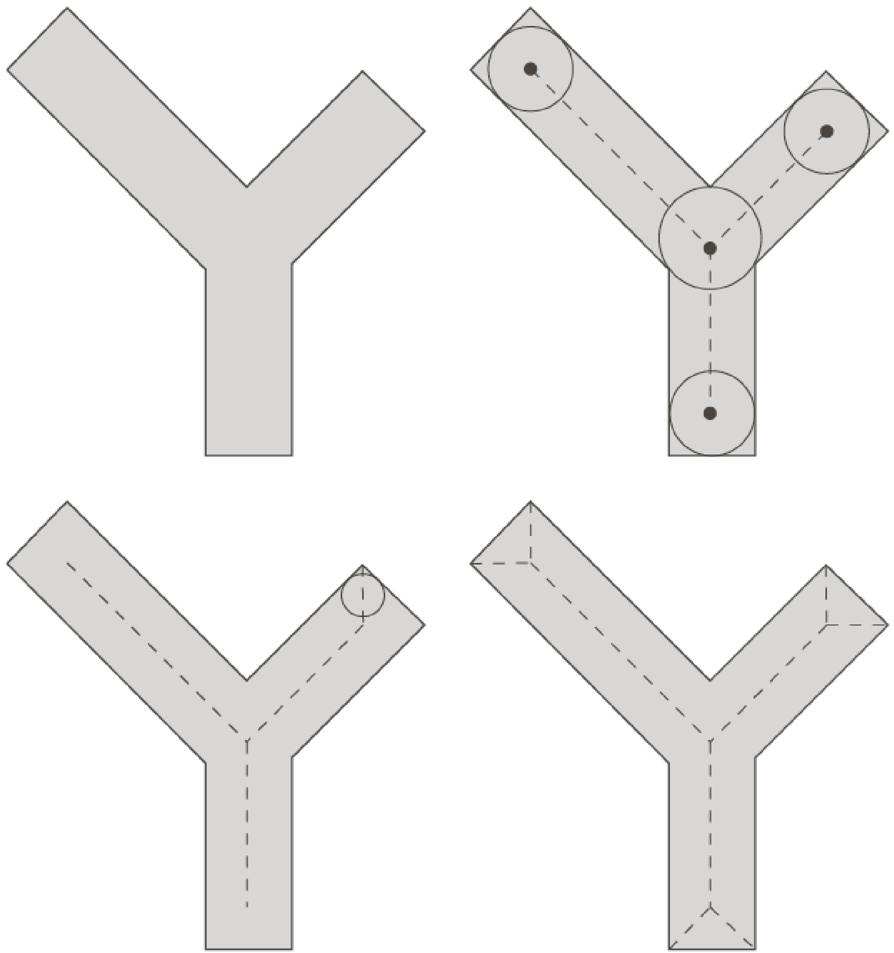


FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

Skeletons

The skeleton of A can be expressed in terms of erosion and openings.

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $K = \max\{k \mid A \ominus kB \neq \phi\};$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where B is a structuring element, and

$$(A \ominus kB) = ((..((A \ominus B) \ominus B) \ominus ...) \ominus B)$$

k successive erosions of A.

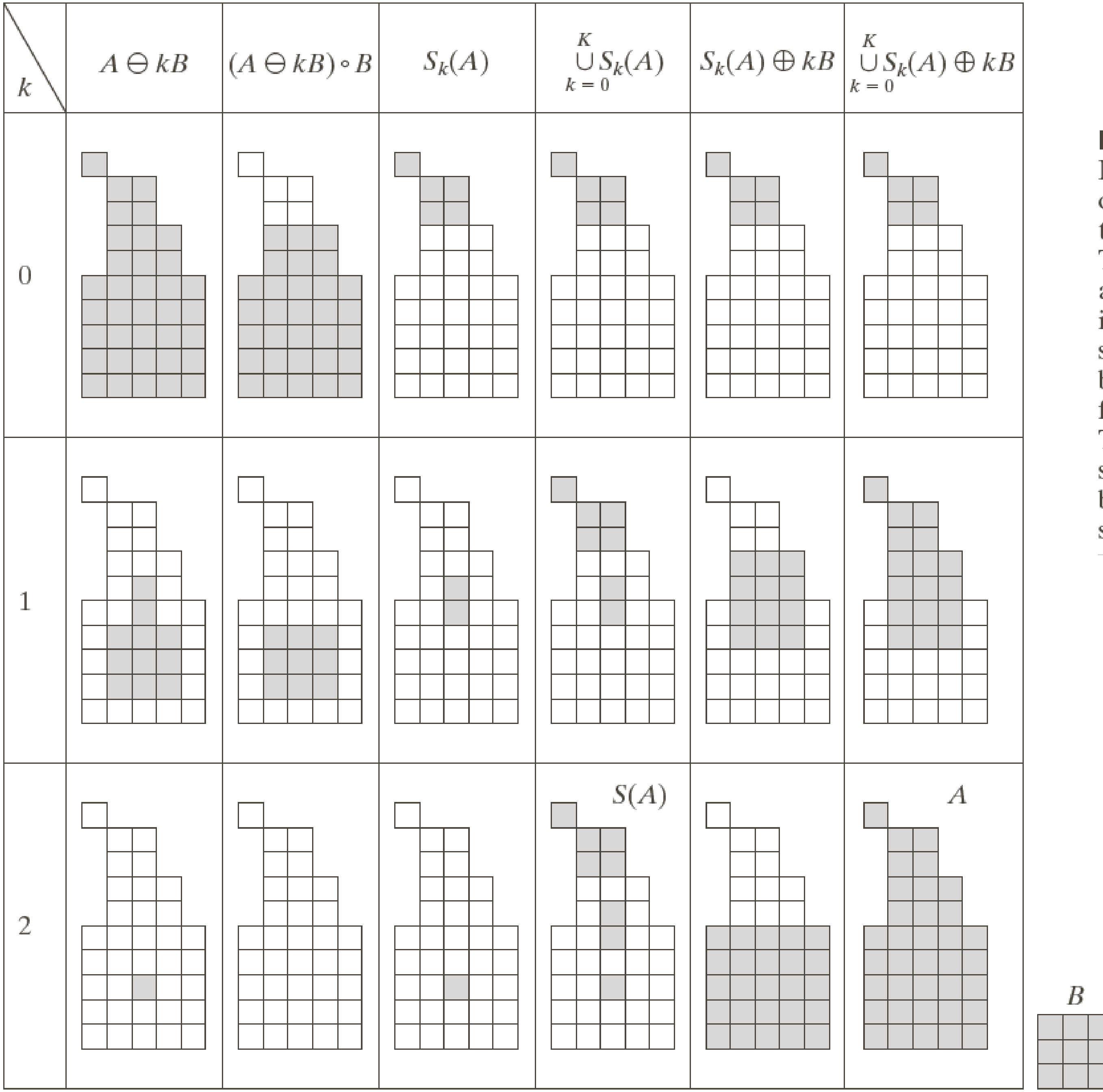


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Skeletons

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes k successive dilations of A.

$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B)\ldots \oplus B)$$