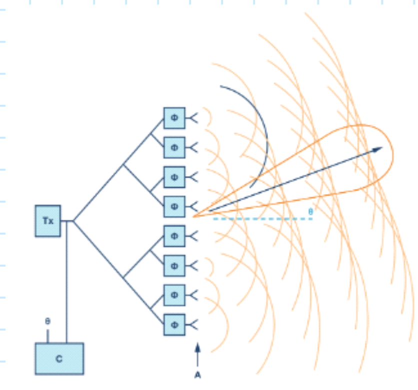
TS5

viernes, 1 de julio de 2022 4:08 p. m.

Se desea diseñar desfasadores pasivos para un sistema de este tipo que opera en banda ancha, buscándose que no alteren la respuesta de módulo de la señal.



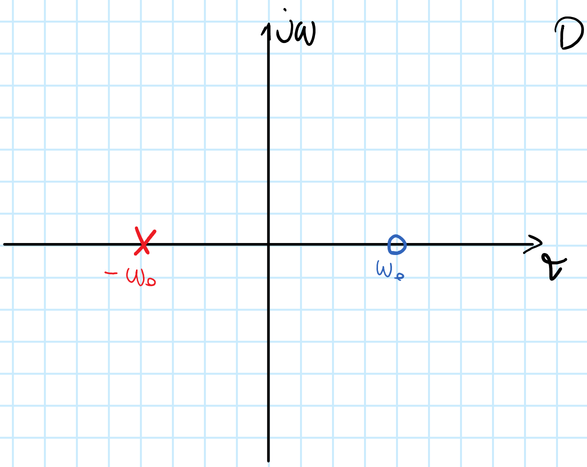
a) Proponga una función transferencia normalizada de primer orden que permita rotar la fase, sin alterar el módulo. Dibuje 1) el diagrama de polos y ceros, 2) la respuesta de fase en función de la frecuencia y 3) calcule el retardo de grupo.

b) Proponga una topología activa y una pasiva que implementen el diagrama de polos y ceros del punto anterior. Obtenga los valores de componentes pasivos (resistencias y capacitores) para lograr que la rotación de fase sea de 15º en ω=1 (medida respecto de la fase en ω=0).

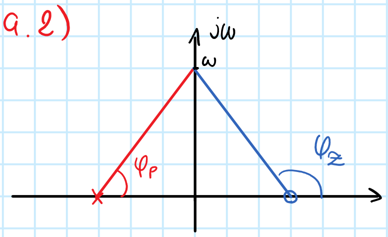
a) Para rotar la fase sin alterar el modulo, propongo una transferencia del tipo PASA TODO

$$\boxed{T(s) = \frac{s - \omega_0}{s + \omega_0}}$$

a.1) Diagrama Polos y Ceros



a.2)



$$\phi(\omega) = \phi_Z(\omega) - \phi_P(\omega) = \pi - \operatorname{arctg}\left(\frac{\omega}{\omega_0}\right) - \operatorname{arctg}\left(\frac{\omega}{\omega_0}\right)$$

$$\boxed{\phi(\omega) = \pi - 2\operatorname{arctg}\left(\frac{\omega}{\omega_0}\right)}$$

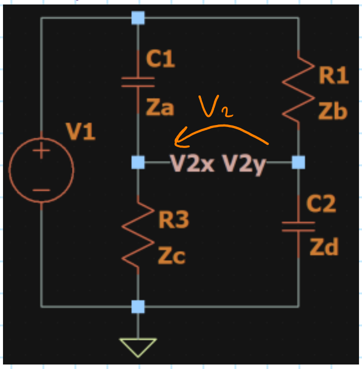
a.3) El retardo de grupo se lo define como:

$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega}$$

En nuestro caso

$$\boxed{\tau(\omega) = \frac{2 \cdot \omega_0}{\omega^2 + \omega_0^2}}$$

Topología Pasiva



$$V_{2x} = \frac{V_1 \cdot Z_c}{Z_a + Z_c} \qquad V_{2y} = \frac{V_1 \cdot Z_d}{Z_b + Z_d}$$

$$V_2 = V_{2x} - V_{2y} \longrightarrow V_2 = V_1\left(\frac{Z_c}{Z_a+Z_c} - \frac{Z_d}{Z_b+Z_d}\right)$$

$$\frac{V_2}{V_1} = \frac{Z_c(Z_b+Z_d) - Z_d(Z_a+Z_c)}{(Z_a+Z_c)(Z_b+Z_d)} = \frac{Z_c Z_b + \cancel{Z_c Z_d} - Z_a \cdot Z_d - \cancel{Z_c Z_d}}{(Z_a+Z_c)(Z_b+Z_d)}$$

$$\text{Si } Z_a = Z_d \wedge Z_c = Z_b$$

$$\frac{V_2}{V_1} = \frac{Z_b^2 - Z_a^2}{(Z_b+Z_a)^2} = \frac{\cancel{(Z_b+Z_a)}(Z_b - Z_a)}{(Z_b+Z_a)^{\cancel{2}}} \longrightarrow \frac{V_2}{V_1} = \frac{Z_b - Z_a}{Z_b + Z_a}$$

$$Z_a = \frac{1}{sC} \wedge Z_b = R \longrightarrow T(s) = \frac{R - \frac{1}{sC}}{R + \frac{1}{sC}} \longrightarrow \boxed{T(s) = \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}}$$

$$\omega_0 = \frac{1}{RC} \longrightarrow \phi(\omega) = \pi - 2 \cdot \operatorname{artg}(\omega RC)$$

$$\text{En } \omega = 0 \longrightarrow \phi(0) = \pi$$

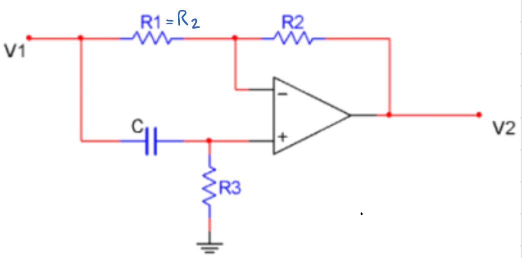
$$\text{En } \omega = 1 \longrightarrow \phi(1) = \pi - 15° \cdot \frac{\pi}{180°}$$

$$\therefore \cancel{\pi} \cancel{-} 15° \cdot \frac{\pi}{180°} = \cancel{\pi} \cancel{-} 2 \cdot \operatorname{artg}(RC)$$

$$RC = \operatorname{tg}\left(\frac{15° \, \pi}{2 \cdot 180°}\right)$$

$$RC = 0{,}1316525 \qquad \text{Si } R = 1 \longrightarrow C = 0{,}1316525$$

Topología Activa (TS1)



$$\boxed{T(s) = \frac{s - \frac{1}{R_3 C}}{s + \frac{1}{R_3 C}}} \qquad \omega_0 = \frac{1}{R_3 C}$$

De igual manera que antes

$$\phi(\omega=1) = \pi - \frac{15° \, \pi}{180°}$$

$$\therefore R_3 C \cong 0{,}1316525$$

$$\text{Si } R_3 = 1 \longrightarrow C \cong 0{,}1316525$$

2) Considere la siguiente expresión generalizada de una transferencia bicuadrática:

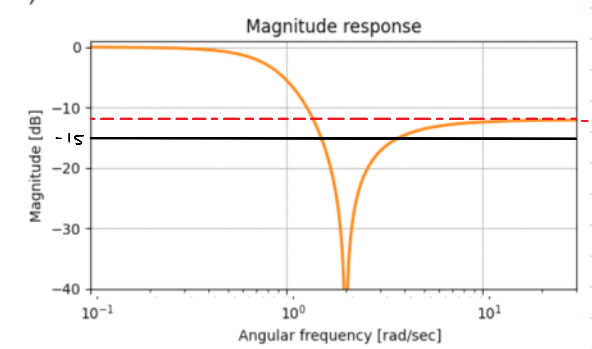
$$T(s) = k \cdot \frac{s^2 + s\cdot\frac{\omega_n}{Q_n} + \omega_n^2}{s^2 + s\cdot\frac{\omega_p}{Q_p} + \omega_p^2}$$

a) Considerando que el denominador de T(s) se corresponde con el de un filtro pasa-altos Butterworth de segundo orden, especifique las condiciones necesarias para los parámetros k, Q_n, ω_n, Q_p y ω_p de forma tal que la transferencia final resulte:

NOTCH PASA BAJO

$$10^{\frac{-12{,}0412}{20}} \cong 0{,}25 = K$$

a)



$$T(s) = K \cdot \frac{s^2 + \omega_m^2}{s^2 + s\sqrt{2} + 1}$$

Para $s = 0 \to T(0) = K\,\omega_m^2 = 1$ 0dB

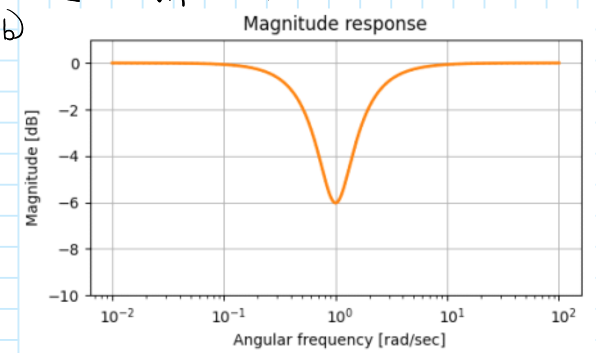
$$\omega_m^2 = \frac{1}{K} \to \omega_m = \frac{1}{\sqrt{K}} = \frac{1}{\frac{1}{\sqrt{4}}}$$

$$\therefore \omega_m = 2$$

$$\boxed{T(s) = \frac{1}{4}\,\frac{s^2 + 2^2}{s^2 + s\sqrt{2} + 1} \qquad K = \frac{1}{4}\ ;\ \omega_m = 2\ ;\ Q_m \to \infty}$$

ELIMINA BANDA

b)



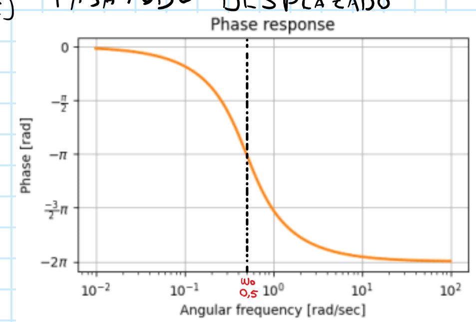
$$10^{\frac{-6}{20}} \cong 0{,}5$$

$$T(s) = \frac{s^2 - \rho\, s\,\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\,\frac{\omega_0}{Q} + \omega_0^2}$$

Segun SCHAUMANN pag 200, la profundidad de la atenuación esta dada por $|\rho| < 1$, Si $\rho = 0{,}5$, el valor de continua se reduce a la mitad (-6 dB) en ω_0

$$\therefore \boxed{T(s) = \frac{s^2 - 0{,}5\sqrt{2}\,s + 1}{s^2 + s\sqrt{2} + 1} \qquad K = 1\ ;\ \omega_m = \omega_p = 1\ ;\ Q_m = 2\,Q_p = \frac{2}{\sqrt{2}}}$$

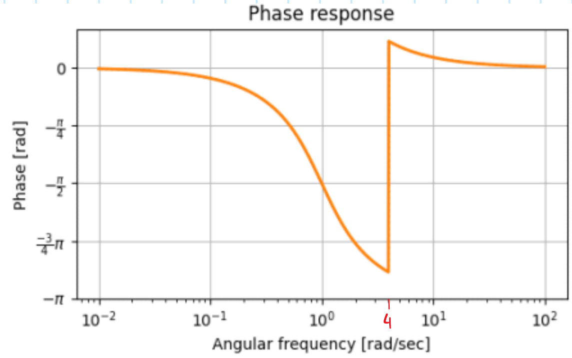
c) PASATODO DESPLAZADO



$$\omega_0 = 0{,}5 \to \omega_0^2 = 0{,}25 \qquad \frac{\omega_0}{Q} = \frac{0{,}5}{\frac{1}{\sqrt{2}}} \to \frac{\omega_0}{Q} = 0{,}5\sqrt{2}$$

$$\boxed{T(s) = \frac{s^2 - s\,0{,}5\sqrt{2} + (0{,}5)^2}{s^2 + s\,0{,}5\sqrt{2} + (0{,}5)^2} \qquad K = 1\ ;\ \omega_m = \omega_p = 0{,}5\ ;\ Q_m = Q_p = \frac{1}{\sqrt{2}}}$$

d) NOTCH DESPLAZADO (NUMERADOR)

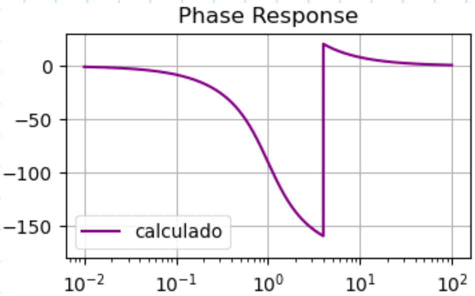


Se desplazó la ω_0 de notch hacia $\omega_0 = 4$. Si el DEN no cambia, lo debe hacer el NUM.

$$\boxed{\therefore \quad T(s) = \frac{s^2 + 4^2}{s^2 + s\sqrt{2} + 1} \qquad K = 1 \; ; \; Q \to \infty, \; \omega_m = 4}$$

NOTA

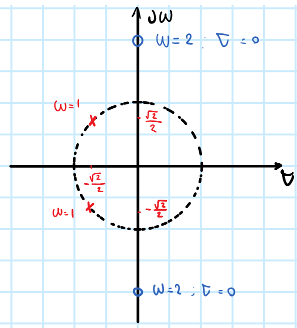
$K=1$ porque así obtenemos por simulación algo parecido al gráfico



a) En cada caso, grafique además el diagrama de polos y ceros, **detallando las coordenadas de todas las singularidades**.

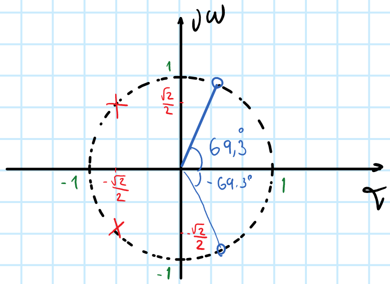
a)

$$\frac{1}{4} \; \frac{s^2 + 2^2}{s^2 + s\sqrt{2} + 1}$$



b)

$$T(s) = \frac{s^2 - 0.5\sqrt{2}\,s + 1}{s^2 + s\sqrt{2} + 1}$$

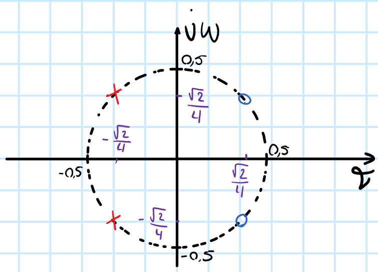


$$\frac{1}{2\cos\varphi} = \frac{2}{\sqrt{2}} \qquad \cos\varphi = \frac{\sqrt{2}}{4}$$

$$\varphi = \arccos\left(\frac{\sqrt{2}}{4}\right) = 69.3°$$

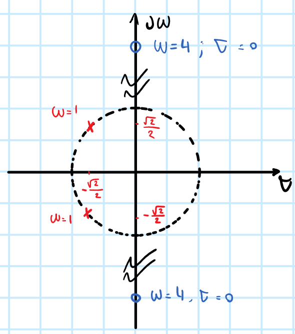
c)

$$T(s) = \frac{s^2 - s \cdot 0.5\sqrt{2} + 0.5^2}{s^2 + s \cdot 0.5\sqrt{2} + 0.5^2}$$

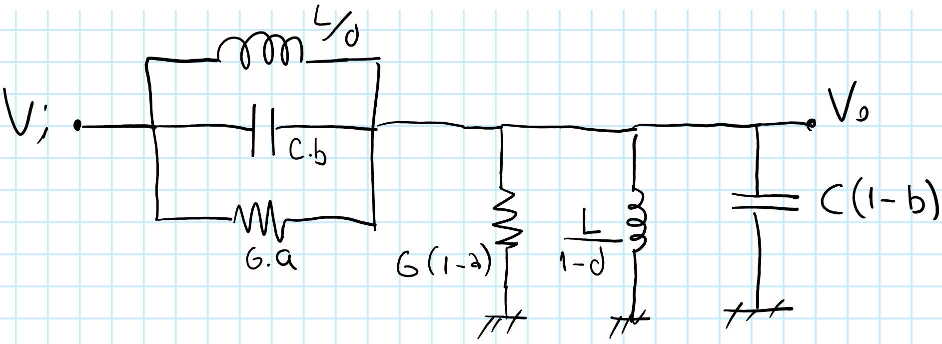


d)

$$T(s) = \frac{s^2 + 4^2}{s^2 + s\sqrt{2} + 1}$$



b) Proponga un circuito normalizado, de ser posible pasivo, que tenga la respuesta indicada.



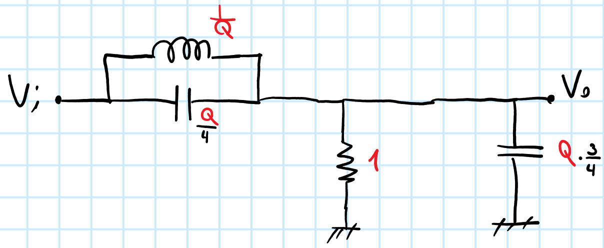
$$T(\$) = b \cdot \frac{\$^2 + \$ \frac{G.a}{C.b} + \frac{d}{CbL}}{\$^2 + \$ \frac{G}{C} + \frac{1}{LC}}$$

a)

NORMALIZADO ↓

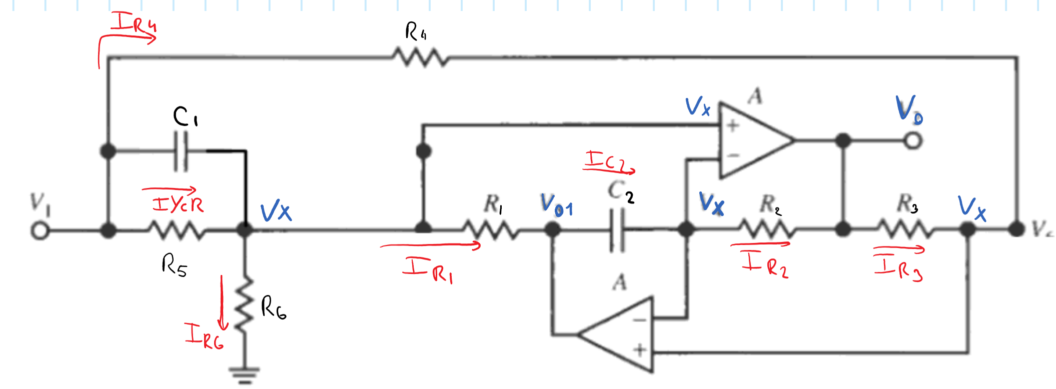
$$T(\$) = \frac{1}{4} \cdot \frac{\$^2 + 2^2}{\$^2 + \$\sqrt{2} + 1} \rightarrow b = \frac{1}{4} \; ; \; \frac{d}{b} = 4 \rightarrow d = 1 \; ; \; a = 0 \; ; \; G = 1$$

$$C = \frac{1}{\sqrt{2}} = Q \qquad L = \frac{1}{C} = \frac{1}{Q}$$



b) $T(\$) = \frac{\$^2 - 0.5\sqrt{2}\$ + 1}{\$^2 + \$\sqrt{2} + 1}$ ELIMINA BANDA SEGUNDO ORDEN

Usando la estructura bicuadratica



$$T(s) = \frac{V_2}{V_1} = \frac{s^2(2a-c) + s\,(\omega_0/Q)\,(2b-c) + c\omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}$$

$$R_5 = \frac{Q.R}{b} \qquad R_6 = \frac{Q\,R}{1-b}$$

$$T(\$) = \frac{\$^2 - 0.5\sqrt{2}\$ + 1}{\$^2 + \$\sqrt{2} + 1} \quad \therefore \; c = 1 \; ; \; a = 1 \qquad 2b - c = -0.5 \rightarrow b = \frac{0.5}{2} = \frac{1}{4}$$

$$R_5 = QR4 \qquad R_6 = \frac{4}{3}QR \qquad R_1 = R_2 = R_3 = R_4 = R \qquad C_1 = C_2 = C$$

$$\text{I)}\quad I_{Y_CR} = I_{RG} + I_{R_1} \qquad \text{I)}\quad (V_i - V_x)(Y_{C_1} + G_5) = V_x \cdot G_6 + (V_x - V_{01}) G_1$$

$$\text{II)}\quad I_{C_2} = I_{R_2} \qquad \text{II)}\quad (V_{01} - V_x) Y_{C_2} = (V_x - V_0) G_2$$

$$\text{III)}\quad I_{R_3} = -I_{R_4} \qquad \text{III)}\quad (V_0 - V_x)\cancel{G_3} = -(V_i - V_x)\cancel{G_4}$$

$$\text{III)}\quad V_0 - V_x = V_x - V_i \rightarrow V_x = \frac{V_0 + V_i}{2} \quad (A)$$

$$\text{II)}\quad V_{01} Y_{C_2} - V_x Y_{C_2} = V_x G_2 - V_0 G_2 \longrightarrow V_{01} = \frac{V_x (G_2 + Y_{C_2}) - V_0 G_2}{Y_{C_2}} \xrightarrow{A} V_{01} = \frac{(V_0 + V_i)(G_2 + Y_{C_2})}{2\, Y_{C_2}} - \frac{V_0 G_2}{Y_{C_2}} \quad (B)$$

$$\text{I)}\quad V_i(Y_{C_1} + G_5) - V_x(Y_{C_1} + G_5) = V_x G_6 + V_x G_1 - V_{01} G_1$$

$$V_i(Y_{C_1} + G_5) = \left(\frac{V_0 + V_i}{2}\right)(G_6 + G_1 + Y_{C_1} + G_5) - \left(\frac{(V_0 + V_i)(G_2 + Y_{C_2})}{2 Y_{C_2}} - \frac{V_0 G_2}{Y_{C_2}}\right) G_1$$

$$V_i\left(Y_{C_1} + G_5 - \left(\frac{G_6 + G_1 + Y_{C_1} + G_5}{2}\right) + \frac{G_1(G_2 + Y_{C_2})}{2 Y_{C_2}}\right) = V_0\left(\left(\frac{G_6 + G_1 + Y_{C_1} + G_5}{2}\right) - \frac{G_1(G_2 + Y_{C_2})}{2 Y_{C_2}} + \frac{G_1 G_2}{Y_{C_2}}\right)$$

$$\frac{V_0}{V_i} = \frac{\left(Y_{C_1} + G_5 - \left(\frac{G_6 + G_1 + Y_{C_1} + G_5}{2}\right) + \frac{G_1(G_2 + Y_{C_2})}{2 Y_{C_2}}\right)}{\left(\left(\frac{G_6 + G_1 + Y_{C_1} + G_5}{2}\right) - \frac{G_1(G_2 + Y_{C_2})}{2 Y_{C_2}} + \frac{G_1 G_2}{Y_{C_2}}\right)}$$

$$\frac{V_0}{V_i} = \frac{\cancel{2} Y_{C_1} Y_{C_2} + \cancel{2} Y_{C_2} G_5 - Y_{C_2} G_6 - \cancel{Y_{C_2} G_1} - \cancel{Y_{C_1} Y_{C_2}} - \cancel{Y_{C_2} G_5} + G_1 G_2 + \cancel{Y_{C_2} G_1}}{Y_{C_2} G_6 + \cancel{Y_{C_2} G_1} + Y_{C_1} Y_{C_2} + Y_{C_2} G_5 - \cancel{G_1 G_2} - \cancel{G_1 Y_{C_2}} + \cancel{2} G_1 G_2}$$

$$\frac{V_0}{V_1} = \frac{Y_{C_1} Y_{C_2} + Y_{C_2}(G_5 - G_6) + G_1 G_2}{Y_{C_1} Y_{C_2} + Y_{C_2}(G_5 + G_6) + G_1 G_2} \qquad C_1 = C_2 = C \qquad G_1 = G_2 = G$$

$$T(\$) = \frac{\$^2 C^2 + \$ C (G_5 - G_6) + G^2}{\$^2 C^2 + \$ C (G_5 + G_6) + G^2} \longrightarrow \boxed{T(\$) \quad \frac{\$^2 + \$\,\frac{(G_5 - G_6)}{C} + \left(\frac{G}{C}\right)^2}{\$^2 + \$\,\frac{(G_5 + G_6)}{C} + \left(\frac{G}{C}\right)^2}}$$

$$T(\$) = \frac{\$^2 + \$\left(\frac{R_6 - R_5}{C R_6 R_5}\right) + \left(\frac{1}{RC}\right)^2}{\$^2 + \$\left(\frac{R_6 + R_5}{C R_6 . R_5}\right) + \left(\frac{1}{RC}\right)^2}$$

$$\boxed{T(\$) = \frac{\$^2 - \$\left(\frac{0{,}5}{QR}\right) + \left(\frac{1}{RC}\right)^2}{\$^2 + \$\left(\frac{1}{QR}\right) + \left(\frac{1}{RC}\right)^2}}$$

$$T(\$) = \frac{\$^2 - 0.5\sqrt{2}\,\$ + 1}{\$^2 + \$\sqrt{2} + 1} \qquad Q = \frac{1}{\sqrt{2}}$$

C. A

$$\frac{1}{R_5} - \frac{1}{R_6} = \frac{R_6 - R_5}{R_6 . R_5}$$

$$R_5 = QR4 \qquad R_6 = \frac{4}{3} QR$$

$$\frac{\frac{4}{3} QR - 4QR}{Q^2 R^2 \frac{16}{3}} = \frac{\cancel{QR}\left(-\frac{8}{\cancel{3}}\right)}{Q^{\cancel{2}} R^{\cancel{2}}\left(\frac{16}{\cancel{3}}\right)} = \frac{-0{,}5}{QR}$$

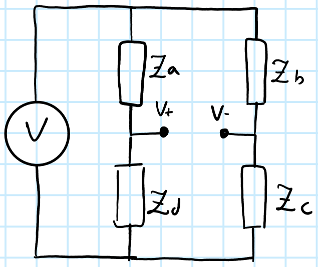
$$\frac{\frac{4}{3} QR + 4QR}{Q^2 R^2 \frac{16}{3}} = \frac{\cancel{QR} \cdot \cancel{\left(\frac{16}{3}\right)}}{Q^{\cancel{2}} R^{\cancel{2}} \cancel{\left(\frac{16}{3}\right)}} = \frac{1}{QR}$$

DISEÑO

$$\frac{1}{QR} = \sqrt{2} \longrightarrow R = \frac{1}{Q\sqrt{2}} \longrightarrow \boxed{R = 1}$$

$$\frac{1}{RC} = 1 \longrightarrow C = \frac{1}{R} \longrightarrow \boxed{C = 1}$$

c) Pasa todo Desplazado Pasivo



Si $Z_a = Z_d \;\wedge\; Z_c = Z_b$

$$\frac{V_2}{V_1} = \frac{Z_b - Z_a}{Z_b + Z_a}$$

Adopto

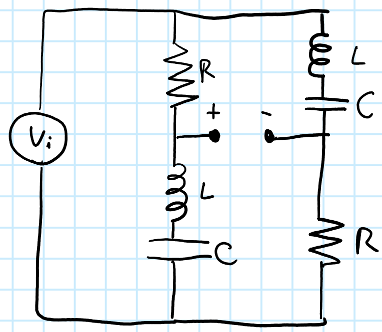
$$Z_b = \$L + \frac{1}{\$C} \longrightarrow Z_b = \frac{\$^2 LC + 1}{\$C} \quad ; \quad Z_a = R$$

$$\therefore \quad T(\$) = \frac{\frac{\$^2 LC + 1}{\$C} - R}{\frac{\$^2 LC + 1}{\$C} + R} = \frac{\$^2 LC - \$RC + 1}{\$^2 LC + \$RC + 1}$$

$$\therefore \quad \boxed{T(\$) = \frac{\$^2 - s\frac{R}{L} + \frac{1}{LC}}{\$^2 + s\frac{R}{L} + \frac{1}{LC}}}$$

$$T(\$) = \frac{\$^2 - \$.0{,}5\sqrt{2} + 0{,}5^2}{\$^2 + \$.0{,}5\sqrt{2} + 0{,}5^2}$$

$R = R_v = 1$



$$\frac{1}{LC} = \frac{1}{4} \longrightarrow C = \frac{4}{L} \longrightarrow C = 2\sqrt{2}$$

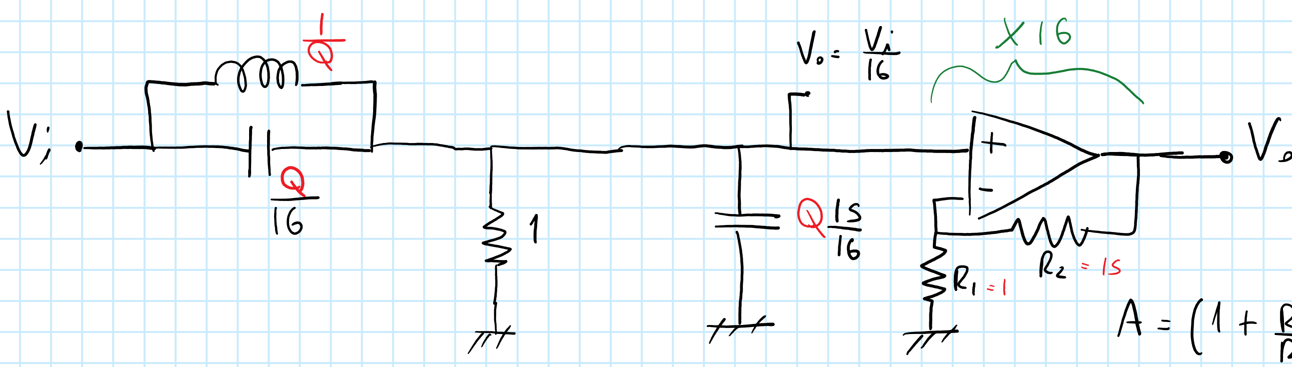
$$\overbrace{R}^{1}\!\!\big/ L = \frac{1}{2}\sqrt{2} \longrightarrow L = \frac{2}{\sqrt{2}}$$

d)

$$T(\$) = \frac{s^2 + 4^2}{\$^2 + \$\sqrt{2} + 1}$$

$$T(\$) = b \cdot \frac{\$^2 + \$\frac{G \cdot a}{C \cdot b} + \frac{d}{C\,b\,L}}{\$^2 + \$\frac{G}{C} + \frac{1}{LC}}$$

$b = \frac{1}{16}$ $\quad d = 1$ $\quad a = 0$ $\quad C = Q = \frac{1}{\sqrt{2}}$ $\quad L = \frac{1}{C} = \frac{1}{Q}$ $\quad G = 1$



$R_1 = 1$

$$A = \left(1 + \frac{R_2}{R_1}\right)$$

$R_2 = 15$

3) Dada la siguiente respuesta de fase de una transferencia:

$$\phi(\omega) = \frac{\pi}{2} - arctg\left(\frac{6\omega}{-\omega^2+4}\right)$$

a) Obtener la expresión de F(s)
b) Graficar el diagrama de polos y ceros, y con el mismo, verificar la respuesta de fase en extremos de banda
c) Obtener un circuito equivalente pasivo que implemente dicha respuesta

$$\phi(\omega) = \phi(\omega)_Z - \phi(\omega)_P \longrightarrow \phi_Z(\omega) = \frac{\pi}{2}$$

$$\phi(\omega)_P = \text{arctg}\left(\frac{\text{Im}_P}{\text{Real}_P}\right) \quad \therefore \quad \text{Im}_P = 6\omega \qquad \text{Real}_P = -\omega^2+4$$

$$\text{DENOMINADOR} = -\omega^2 + 4 + j6\omega \Rightarrow \quad \omega \rightarrow \frac{s}{j}$$

$$= -\frac{s^2}{\underbrace{j^2}_{-1}} + \cancel{j}\frac{6s}{\cancel{j}} + 4 \longrightarrow \text{DEN}(T(s)) = s^2 + s6 + 4$$

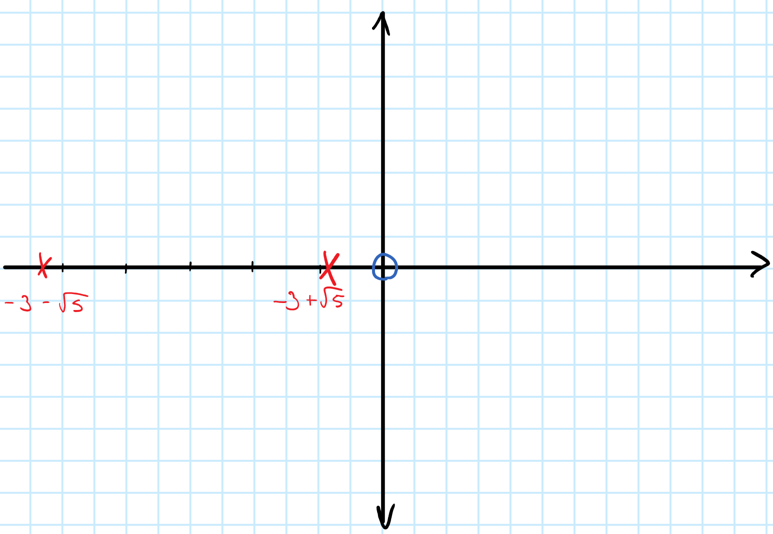
$1 + 4K$ con $K = 1, 2, \ldots$ Son la cantidad de 0 en el origen que aportan una fase de $\frac{\pi}{2}$. Como el grado del DEN es 2, elegimos el orden de Z, tal que el grado del NUM $\leq$ DEN $\therefore$ DEN $= s$

$$T(s) = \frac{s\,K}{s^2 + s\,6 + 4}$$

Para 0 dB de ganancia $K = \frac{\omega_0}{Q} = 6$

$$\therefore \quad \boxed{T(s) = \frac{s\,6}{s^2 + s\,6 + 4}}$$

b) Raices Polos: $R_1 = -3 - \sqrt{5} \cong 5{,}236$ $\quad R_2: -3 + \sqrt{5} \cong 0{,}764$

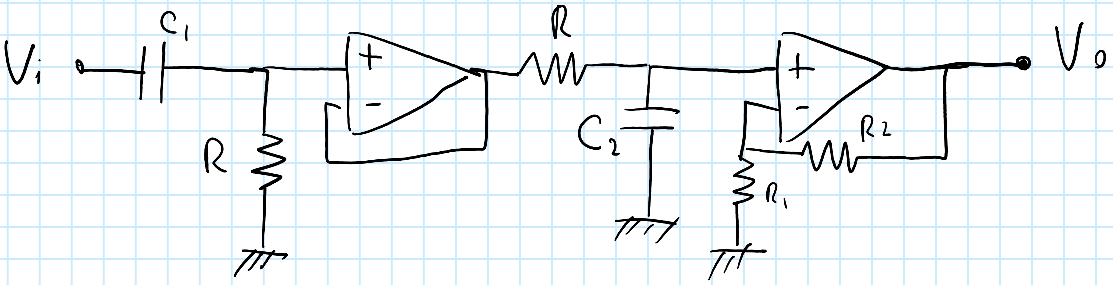


Para $\omega = 0 \longrightarrow \frac{\pi}{2}$ Fase del cero

Para $\omega \rightarrow \infty \rightarrow -\frac{\pi}{2}$

$$\phi_T = \phi_Z - \phi_P = \frac{\pi}{2} - \pi = -\frac{\pi}{2}$$

c) $\underbrace{6}_{\text{OPAMP}} \cdot \underbrace{\frac{s}{(s+5.236)}}_{\text{Pasa Alto}} \cdot \underbrace{\frac{1}{(s+0.764)}}_{\text{Pasa Bajo}}$



H.P $R=1$

$\omega_0 = \frac{1}{RC_1} \rightarrow C_1 = \frac{1}{5{,}236}$

LP $\rightarrow R=1$

$\omega_0 = \frac{1}{RC_2} \rightarrow C_2 = \frac{1}{0{,}764}$

OPAMP $\rightarrow R_1 = R = 1$

$K = \left(1 + \frac{R_2}{R_1}\right) = 6$

$\therefore R_2 = 5$

SIMULACION EN JUPYTER