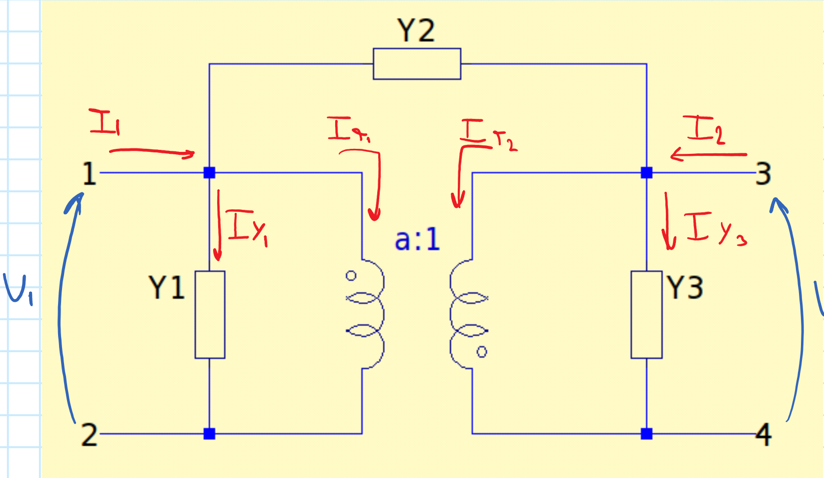
TS7

martes, 12 de julio de 2022 5:04 p. m.

2) Para el mismo cuadripolo, determine las admitancias **Y1, Y2, Y3** y el parámetro **a** que hacen que la red sea simétrica y recíproca.



$$I_1 = V_1 \cdot Y_1 + I_{T_1} + (V_1 - V_2) Y_2 \quad (A)$$
$$I_2 = V_2 \, Y_3 + I_{T_2} + (V_2 - V_1) Y_2 \quad (B)$$

Ec T.I. No HOMOLOGO

$$V_1 = (-a) V_2 = -a V_2$$
$$I_{T_1} = \left(-\frac{1}{a}\right)\left(-I_{T_2}\right) = \frac{I_{T_2}}{a}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

si es SIMETRICO y RECIPROCO $\longrightarrow$ $Z_{11} = Z_{22}$, $Z_{12} = Z_{21}$

Evaluo en $I_2 = 0$ para averiguar Z_{11} y Z_{21}

$I_2 = 0$

(B) $0 = V_2\, Y_3 + I_{T_2} + (V_2 - V_1)\, Y_2$

$I_{T_2} = (V_1 - V_2)\, Y_2 - V_2\, Y_3$

$I_{T_1} = \frac{I_2}{a} \rightarrow I_{T_1} = \frac{(V_1 - V_2)\, Y_2 - V_2\, Y_3}{a}$

(A) $I_1 = V_1 \cdot Y_1 + \frac{(V_1 - V_2)\, Y_2 - V_2\, Y_3}{a} + (V_1 - V_2)\, Y_2$

$V_1 = -a\, V_2$

$I_1 = V_1 \cdot Y_1 + \frac{(V_1 + \frac{V_1}{a})\, Y_2 + \frac{V_1}{a}\, Y_3}{a} + \left(V_1 + \frac{V_1}{a}\right) Y_2$

$I_1 = V_1\, Y_1 + \frac{V_1\, Y_2}{a} + \frac{V_1\, Y_2}{a^2} + \frac{V_1}{a^2}\, Y_3 + V_1\, Y_2 + \frac{V_1\, Y_2}{a}$

$I_1 = V_1 \left(Y_1 + 2\frac{Y_2}{a} + \frac{Y_2}{a^2} + \frac{Y_3}{a^2} + Y_2 \right)$

$Z_{11} = \frac{V_1}{I_1} = \left(\frac{a^2}{Y_1 a^2 + 2 Y_2 a + Y_2 + Y_3 + Y_2 a^2} \right) \Rightarrow \boxed{Z_{11} = \frac{a^2}{K}}$ donde $K = Y_1 a^2 + 2 Y_2 a + Y_2 + Y_3 + Y_2 a^2$

$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2 = 0}$ Como Z_{11} esta medido para la misma condición de $I_2 = 0$

$Z_{11} = \frac{V_1}{I_1} \Rightarrow \frac{-a\, V_2}{I_1} = Z_{11} \rightarrow \frac{V_2}{I_1} = \frac{Z_{11}}{-a}$

$\boxed{Z_{21} = -\left(\frac{a}{K}\right)}$

Para $I_1 = 0 \rightarrow Z_{22} = \frac{V_2}{I_2}$

(A) $V_1 \cdot Y_1 + I_{T_1} + (V_1 - V_2)\, Y_2 = 0$

$I_{T_1} = (V_2 - V_1)\, Y_2 - V_1\, Y_1$

$\begin{cases} V_1 = -a\, V_2 \\ I_{T_1} = \frac{I_{T_2}}{a} \end{cases} \rightarrow I_{T_2} = (V_2 + aV_2)\, Y_2\, a + a V_2\, Y_1 \cdot a$ (C)

(B) $I_2 = V_2\, Y_3 + I_{T_2} + (V_2 + aV_2)\, Y_2$

(C) → (B) $I_2 = V_2\, Y_3 + (V_2 + aV_2)\, Y_2\, a + a^2 V_2\, Y_1 + (V_2 + aV_2)\, Y_2$

$I_2 = V_2\, Y_3 + V_2\, Y_2\, a + a^2 V_2\, Y_2 + a^2 V_2\, Y_1 + V_2\, Y_2 + a V_2\, Y_2$

$I_2 = V_2 (Y_3 + Y_2 a + a^2\, Y_2 + a^2 Y_1 + Y_2 + a\, Y_2) = V_2 (Y_1 a^2 + 2 Y_2 a + Y_2 + Y_3 + Y_2 a^2) = V_2 \cdot K$

$\boxed{Z_{22} = \frac{V_2}{I_2} = \frac{1}{K}}$

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ → Medido en mismas condiciones que Z_{22}

$$Z_{22} = \frac{V_2}{I_2} = -\frac{1}{a} \cdot \frac{V_1}{I_2} = -\frac{1}{a} Z_{12} \rightarrow Z_{12} = -a\, Z_{22}$$

$$\therefore \boxed{Z_{12} = -\frac{a}{K}}$$

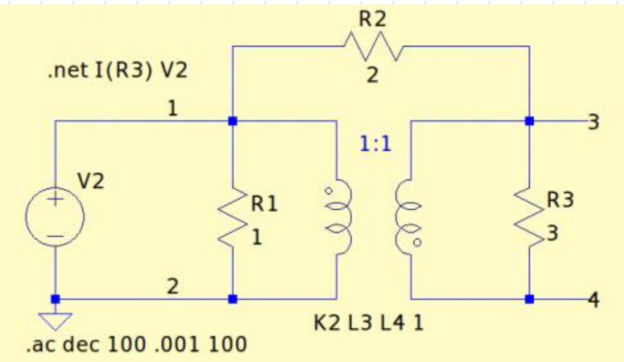
NOS QUEDA QUE LA MATRIZ Z ES:

$$Z = \begin{pmatrix} \frac{a^2}{K} & -\frac{a}{K} \\ -\frac{a}{K} & \frac{1}{K} \end{pmatrix}$$

INDEPENDIENTEMENTE DE LOS VALORES DE Y, LA MATRIZ ES RECIPROCA, algo que esperabamos por ser PASIVA

Para ser SIMETRICA → $a = 1$

1) Para el siguiente cuadripolo se pide calcular los parámetros Z.



$$Y_1 = \frac{1}{R_1} = 1$$

$$Y_2 = \frac{1}{R_2} = \frac{1}{2}$$

$$Y_3 = \frac{1}{R_3} = \frac{1}{3}$$

$$a = 1$$

$$K = Y_1 a^2 + 2 Y_2 a + Y_2 + Y_3 + Y_2 a^2 = 1 + 2 \cdot \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{2}$$

$$K = \frac{10}{3}$$

$$Z = \begin{pmatrix} \frac{a^2}{K} & -\frac{a}{K} \\ -\frac{a}{K} & \frac{1}{K} \end{pmatrix} \quad \therefore \quad \boxed{Z = \begin{pmatrix} 0{,}3 & -0{,}3 \\ -0{,}3 & 0{,}3 \end{pmatrix}}$$