TS4

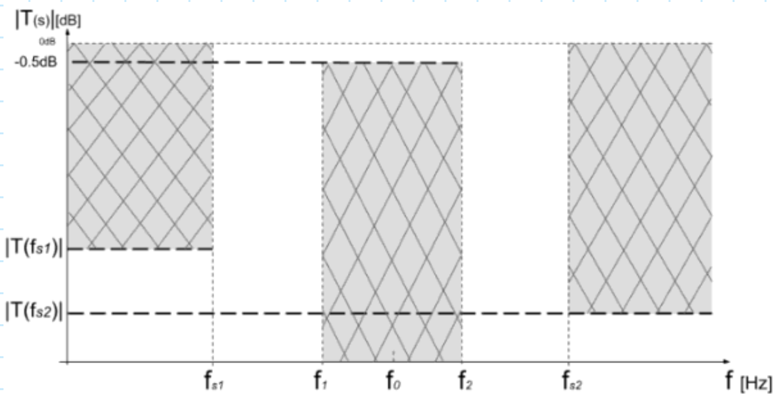
domingo, 29 de mayo de 2022 11:04 p. m.

Se pide diseñar un filtro pasabanda que cumpla con la siguiente plantilla:

- ω0=2π 22 kHz
- Q=5
- Aproximación Chebyshev con ripple de 0,5 dB

También se sabe que la transferencia del filtro debe ser:

- T(fS1)=-16 dB para fS1=17 kHz
- T(fS2)=-24 dB para fS2=36 kHz



Obtener la plantilla de diseño pasabanda normalizada

$F_o = 22\ KHz \longrightarrow \omega_o = 2\pi\, 22\ s^{-1} \longrightarrow \omega_{oN} = \omega_o/\omega_o \longrightarrow \boxed{\omega_{oN} = 1}$

$F_{s1} = 17\ KHz \longrightarrow \omega_{s1} = 2\pi\, 17\ s^{-1} \longrightarrow \omega_{s1N} = \omega_1/\omega_o \longrightarrow \boxed{\omega_{s1N} = 17/22}$

$F_{s2} = 36\ KHz \longrightarrow \omega_{s2} = 2\pi\, 36\ s^{-1} \longrightarrow \omega_{s2N} = \omega_2/\omega_o \longrightarrow \boxed{\omega_{s2N} = 18/11}$

NORMALIZO EN FRECUENCIA

Me dieron Q $\longrightarrow$ Averiguo F_1 y F_2

$B = (F_2 - F_1)\, 2\pi$

$Q = \frac{\omega_o}{B} = 5 \longrightarrow B = \frac{\omega_o}{5} \Rightarrow (F_2 - F_1)\cancel{2\pi} = \frac{F_o\, \cancel{2\pi}}{5}$

$F_2 = \frac{F_o}{5} + F_1$ (1)

$F_o^2 = F_1 . F_2$ (2)

(1) $\rightarrow$ (2)

$F_o^2 = F_1\left(\frac{F_o}{5} + F_1\right) = \frac{F_o}{5} F_1 + F_1^2 \longrightarrow F_1^2 + \frac{22}{5} K\, F_1 - (22\,K)^2 = 0$

22 K

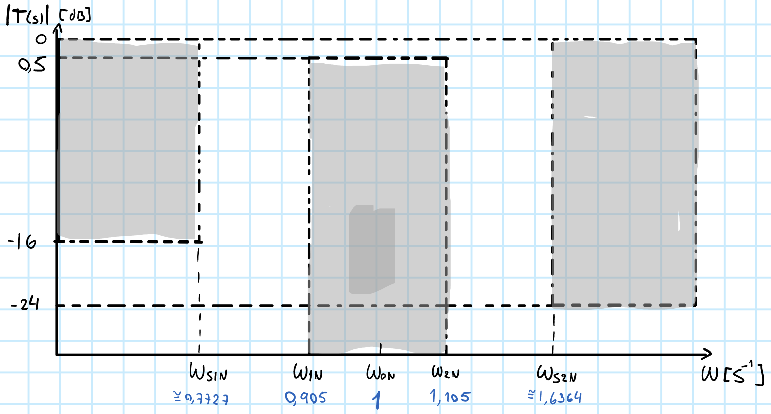
$F_1 \begin{cases} F_{1,1} \cong 19\,909,73 \\ \cancel{F_{1,2} = -24\,K} \end{cases} \rightarrow F_1 \cong 19\,910\ Hz$ (3)

(3) $\rightarrow$ (1)

$F_2 = \frac{F_o}{5} + F_1 = \frac{22\,K}{5} + 19\,910 \rightarrow F_2 = 24\,310$

$\omega_{2N} = \frac{\omega_2}{\omega_o} = \frac{F_2 . \cancel{2\pi}}{F_o . \cancel{2\pi}} = \frac{24\,310}{22\,K} \rightarrow \boxed{\omega_{2N} = 1,105}$

$\omega_{1N} = \frac{\omega_1}{\omega_o} = \frac{F_1 . 2\pi}{F_o . 2\pi} = \frac{19\,910}{22\,K} \rightarrow \boxed{\omega_{1N} = 0,905}$



$\boxed{\omega_{2N} = 1{,}105}$

$\boxed{\omega_{1N} = 0{,}905}$

$\boxed{\omega_{0N} = 1 \quad \omega_{S1N} = 17/22 \quad \omega_{S2N} = 18/11}$

👉 Obtener la función transferencia normalizada del prototipo pasabajo que satisfaga el requerimiento del filtro pasabanda.

Para encontrar las frecuencias equivalentes en Pasa Bajo

$$\boxed{\Omega_{LP} = Q\,\frac{(\omega_{BP}^2 - 1)}{\omega_{BP}}}$$

Uso la formula para encontrar las Ω equivalentes

ω_{BP}	Ω_{LP}
$\omega_{0N} = 1$	$\Omega_0 = 0$
$\omega_{1N} = 0{,}905$	$\Omega_1 \cong -0{,}999862 \cong -1$
$\omega_{2N} = 1{,}105$	$\Omega_2 \cong 1{,}000113 \cong 1$
$\omega_{S1N} = 17/22$	$\Omega_{S1} \cong -2{,}607$
$\omega_{S2N} = 18/11$	$\Omega_{S2} \cong 5{,}1263$

En atenuación, las exigencias que tengo son:

$\alpha_{S1} = 16\,dB$ y $\alpha_{S2} = 24\,dB$

De acuerdo al más exigente, elijo el n necesario

$\alpha_{min} = 10 \log(1 + C_n^2(\omega_0))$ donde $C_n(\omega_0) = \varepsilon \cosh^2[n \cosh^{-1}(\omega_0)]$

Caso Ω_{S1}

$n = 1 \rightarrow \alpha_{min} \cong 2{,}62$ ✗ , $n = 2 \rightarrow \alpha_{min} \cong 13{,}085$ ✗ ; $n = 3 \rightarrow \alpha_{min} \cong 26{,}87 > 16$ ✓

Caso Ω_{S2}

$n = 1 \rightarrow \alpha_{min} \cong 6{,}24$ ✗ $n = 2 \rightarrow \alpha_{min} \cong 25{,}1234 > 24$ ✓

$\varepsilon_1 = \sqrt{10^{\frac{\alpha_{max}}{10}} - 1}$

$\varepsilon = \sqrt{10^{\frac{0{,}5}{10}} - 1}$

$\varepsilon \cong 0{,}34931$

Elijo el n que cumpla ambas condiciones $\longrightarrow \therefore \boxed{n = 3}$

Armo la transferencia con el orden elegido

$$|T(j\Omega)|^2 = \frac{1}{1 + \varepsilon^2 C_3(\Omega)^2} \qquad C_{3\,Chebyshev}(\Omega) = 4\Omega^3 - 3\Omega$$

$$|T(j\Omega)|^2 = \frac{1}{1 + \varepsilon^2(4\Omega^3 - 3\Omega)^2} = \frac{1}{1 + \varepsilon^2(16\Omega^6 - 24\Omega^4 + 9\Omega^2)} \qquad A = \varepsilon^2\,16$$

$$|T(j\Omega)|^2 = \frac{1/A}{\Omega^6 - \frac{24}{A}\Omega^4 + \frac{9}{A}\Omega^2 + \frac{1}{A}}$$

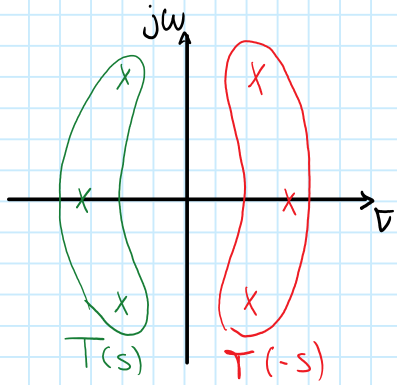
Pasamos a Laplace $\therefore \; \Omega \rightarrow \frac{\$}{j}$

$$|T(\$)|^2 = \frac{1/A}{-\$^6 - \frac{24}{A} S^4 - \frac{9}{A} S^2 + \frac{1}{A}}$$

Obtengo las siguientes Raices :

$\$_1$	(-0.31322824317013775+1.0219274910473608j)	✓
$\$_2$	(-0.31322824317013775-1.0219274910473608j)	✓
$\$_3$	(0.31322824317013714+1.02192749104736j)	✗
$\$_4$	(0.31322824317013714-1.02192749104736j)	✗
$\$_5$	(-0.6264564863402746+0j)	✓
$\$_6$	(0.6264564863402748+0j)	✗

Me quedo con las negativas



$$|T(s)|^2 = T(s) \cdot T(-s) = \frac{1/\sqrt{A}}{(\$+\$_1)(\$+\$_2)(\$+\$_5)} \cdot \cancel{\frac{1/\sqrt{A}}{(\$+\$_3)(\$+\$_4)(\$+S_6)}}$$

Transferencia del prototipo Pasa bajo

$$T(\$) = 1/\sqrt{A} \cdot \frac{1}{\$+\$_6} \cdot \frac{1}{(\$+\$_1)(\$+\$_2)}$$

Coeficientes del Polinomio de $\$_1$ y $\$_2$

A

B

C

$$\boxed{T_L(\$) = \frac{1}{4\xi} \cdot \frac{1}{\$+0{,}626456} \cdot \frac{1}{\$^2 + \$0{,}626456 + 1{,}14245}}$$

Obtener la transferencia pasabanda normalizada

0.626456	$= A$
1.14245	$= B$

$$\$ = Q\frac{(s^2+1)}{s}$$

$$\therefore \; T(s) = \frac{1}{4\xi} \cdot \frac{1}{Q\frac{(s^2+1)}{s} + A} \cdot \frac{1}{\left[Q\frac{(s^2+1)}{s}\right]^2 + A\left[Q\frac{(s^2+1)}{s}\right] + B}$$

$$T(s) = \frac{s \; 1/4\xi}{Qs^2 + As + Q} \cdot \frac{1}{\frac{Q^2(s^4+2s^2+1)}{s^2} + A\;QS + \frac{AQ}{S} + B}$$

$$T(s) = \frac{s \cdot 1/4\xi}{Qs^2 + As + Q} \cdot \frac{s^2}{Q^2s^4 + 2Q^2s^2 + Q^2 + AQs^3 + AQs + Bs^2}$$

$$T(s) = \frac{1/Q \;\; s \cdot 1/4\xi}{s^2 + \frac{A}{Q}s + 1} \cdot \frac{(1/Q^2)\, s^2}{s^4 + \frac{A}{Q}s^3 + \frac{(2Q^2+B)}{Q^2}s^2 + \frac{A}{Q}s + 1}$$

$$T(s) = \frac{1}{s^2 + \frac{A}{Q}s + 1} \cdot \frac{(1/(Q^3 4\xi))\; s^3}{s^4 + \frac{A}{Q}s^3 + \frac{(2Q^2+B)}{Q^2}s^2 + \frac{A}{Q}s + 1}$$

Divide mi transferencia en 3 secciones de 2do ORDEN

$$T_{BP}(S) = K \cdot \frac{S\cdot\frac{\omega_{01}}{Q_1}}{S^2 + S\cdot\frac{\omega_0}{Q_1}+\omega_{0_1}} \cdot \frac{S\cdot\frac{\omega_{02}}{Q_2}}{S^2+S\cdot\frac{\omega_0}{Q_2}+\omega_{0_2}}\cdot \frac{S\cdot\frac{\omega_{03}}{Q_3}}{S^2+S\cdot\frac{\omega_0}{Q_3}+\omega_{0_3}}$$

$$T_{BP}(S) = \underbrace{\frac{1}{Q^3\cdot 4\xi}}_{\beta}\cdot \overbrace{\frac{S}{S^2+S\cdot\underbrace{0{,}1253}_{\frac{A}{Q}}+1}}^{T_1}\cdot \overbrace{\frac{S}{S^2+S\cdot 0{,}069+1{,}2265}}^{T_2}\cdot\overbrace{\frac{S}{S^2+S\cdot 0{,}056+0{,}8154}}^{T_3}$$

1
0.0690175
1.22646

1
0.0562738
0.815356

Encuentro los Parametros de mi transferencia

T_1

$0{,}1253 = \frac{\omega_{01}}{Q_1} = \frac{1}{Q_1}$

$\therefore Q_1 = \frac{1}{0{,}1253} \cong 7{,}981$

T_2

$\omega_0^2 = 1{,}2265 \longrightarrow \omega_0 \cong 1{,}107$

$\frac{\omega_0}{Q_2} = 0{,}069 \longrightarrow Q_2 = \frac{1{,}107}{0{,}069} = 16{,}05$

T_3

$\omega_{03}^2 = 0{,}815356 \longrightarrow \omega_0 \cong 0{,}90297$

$\frac{\omega_{03}}{Q_3} = 0{,}0562738 \longrightarrow Q_3 = \frac{0{,}90297}{0{,}0562738} \cong 16{,}05$

Tengo que desglosar el valor de β en $\underbrace{K_1\cdot K_2\cdot K_3}_{K}\cdot\underbrace{\frac{\omega_{01}}{Q_1}\cdot\frac{\omega_{02}}{Q_2}\cdot\frac{\omega_0}{Q_3}}_{\alpha}$

$$\frac{\beta}{\alpha} = K \longrightarrow K = \frac{1}{Q^3\xi}\cdot\frac{Q_1}{\omega_{01}}\cdot\frac{Q_2}{\omega_{02}}\cdot\frac{Q_3}{\omega_{03}}$$

$$K = \frac{1}{5^3\cdot 4\cdot 0{,}34931}\cdot\frac{1}{\frac{0{,}626456}{5}}\cdot\frac{1}{0{,}0690175}\cdot\frac{1}{0{,}0562738}$$

$\frac{\omega_{01}}{Q_1} = \frac{A}{Q}$

$K \cong 11{,}766$

Si divido equitativamente las ganancias

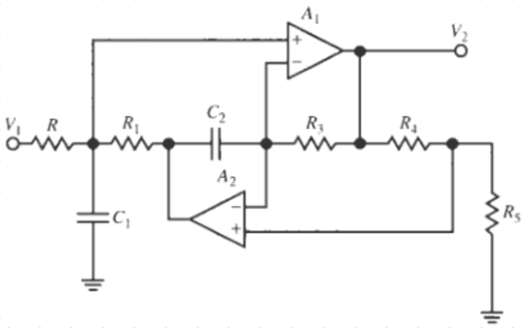
$K = K_1\cdot K_2\cdot K_3$ Si $K_1 = K_2 = K_3 = K_{eq}$

$\therefore K = K_{eq}^3 \longrightarrow K_{eq} = \sqrt[3]{K} = \sqrt[3]{11{,}766} \longrightarrow K_{eq} = 2{,}27445$

Mi TransferenciaNormalizada finalmente será:

$$\boxed{T_{BP}(S) = \underbrace{\frac{2.27445\cdot\frac{1}{7{,}981}S}{S^2+S\cdot\frac{1}{7{,}981}+1}}_{T_1}\cdot\underbrace{\frac{2.27445\cdot\frac{1{,}107}{16{,}05}S}{S^2+S\cdot\frac{1{,}107}{16{,}05}+1{,}107^2}}_{T_2}\cdot\underbrace{\frac{2.27445\cdot\frac{0{,}90297}{16{,}05}S}{S^2+S\cdot\frac{0{,}90297}{16{,}05}+0{,}90297^2}}_{T_3}}$$

- Estructura a utilizar :



donde $$T(s) = \frac{V_2}{V_1} = \frac{s\left(1+\frac{G_5}{G_4}\right)\frac{G}{C_1}}{s^2 + s\frac{G}{C_1} + \frac{G_1G_3G_5}{C_1C_2G_4}}$$

$$T(s) = \frac{s\left(1+\frac{R_4}{R_5}\right)\frac{G}{C_1}}{s^2 + s\frac{G}{C_1} + \frac{R_4}{R_1R_3R_5C_1C_2}}$$

Adopto $R = R_1 = R_3 = R_4 \wedge C = C_1 C_2$

$$T(s) = \frac{s\left(1+\frac{R}{R_5}\right)\frac{G}{C}}{s^2 + s\frac{G}{C} + \frac{1}{R.R_5C^2}}$$

CASO $T_1 = \frac{2.27445 \cdot \frac{1}{7,981}\, s}{s^2 + s.\frac{1}{7,981} + 1}$

$R_v = 1 \rightarrow \Omega_z = R \qquad K = 2.27445$

$\frac{\omega_1}{Q_1} = \frac{G}{C} = \rightarrow G = \frac{C}{Q_1} \rightarrow R_G = \frac{1}{G} = \frac{Q_1}{C} \rightarrow R_G = \frac{7,981}{1,12891541} \rightarrow R_G = 7,0696174$

$\left(1+\frac{1}{R_5}\right) = K \rightarrow R_5 = \frac{1}{K-1} \rightarrow R_5 = \frac{1}{1,27445} \rightarrow R_5 = 0,7846522$

$\frac{1}{R_5C^2} = 1 \rightarrow C = \sqrt{K-1} \rightarrow C = \sqrt{1,27445} \rightarrow C_v = 1,12891541$

CASO $T_2 = \frac{2.27445 \cdot \frac{1,107}{16,05}\, S}{S^2 + S.\frac{1,107}{16,05} + 1,107^2}$

Adopto $R = R_1 = R_3 = R_4 \wedge C = C_1 C_2$

$$T(s) = \frac{s\left(1+\frac{R}{R_5}\right)\frac{G}{C}}{s^2 + s\frac{G}{C} + \frac{1}{R.R_5C^2}}$$

$R_v = 1 \rightarrow \Omega_z = R \qquad K = 2.27445$

SIEMPRE

$\frac{\omega_2}{Q_2} = \frac{G}{C} = \rightarrow G = \frac{C\,\omega_2}{Q_2} \rightarrow R_G = \frac{1}{G} = \frac{Q_2}{C.\,\omega_2} \xrightarrow{\text{Usando } *} R_G = \frac{Q_2}{\frac{1}{\cancel{\omega_0}}\sqrt{K-1}\,.\,\omega_0} \rightarrow R_{Gv} = \frac{16,05}{\sqrt{1,27445}} \rightarrow R_{Gv} \cong 14,2172$

$\left(1+\frac{1}{R_5}\right) = K \rightarrow R_5 = \frac{1}{K-1} \rightarrow R_{5v} = \frac{1}{1,27445} \rightarrow R_{5v} = 0,7846522$

$\frac{1}{R_5C^2} = \omega_0^2 \rightarrow C = \frac{1}{\omega_0}.\sqrt{K-1}^{\,*} \rightarrow C_v = \frac{1}{1,107}\sqrt{1,27445} \rightarrow C_v \cong 1,0198$

CASO $T_3 = \frac{2.27445 \cdot \frac{0.90297}{16,05}\, S}{S^2 + S.\frac{0.90297}{16,05} + 0,90297^2}$

Adopto $R = R_1 = R_3 = R_4 \wedge C = C_1 C_2$

$$T(s) = \frac{s\left(1+\frac{R}{R_5}\right)\frac{G}{C}}{s^2 + s\frac{G}{C} + \frac{1}{R.R_5C^2}}$$

USANDO LO VISTO EN T_2

$R_G = \frac{Q_3}{\sqrt{K-1}} \rightarrow R_{GN} = \frac{16,05}{\sqrt{1,27445}} \rightarrow R_{SN} = 14,2172$

$R_5 = \frac{1}{K-1} \rightarrow R_{5N} = \frac{1}{1,27445} \rightarrow R_{5N} = 0,7846522$

$C = \frac{1}{\omega_3}.\sqrt{K-1} \rightarrow C_N = \frac{1}{0.90297}\sqrt{1,27445} \rightarrow C_N = 1,250225$