Multivariate Time Series Phase Coherence Classifier

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INTRODUCTION

Phase coherence is a measure of phase synchronization, a phenomenon known to occur in nonlinear systems whereby the phases but not necessarily the amplitudes of multiple signals are time-synchronized. A novel multivariate phase coherence classifier based on *persistent homology* from Topological Data Analysis and *ordinal partition transition networks* from symbolic nonlinear dynamics was developed then evaluated using simulated and electroencephalogram (EEG) seizure data.

CLASSIFIER COMPUTATION

Time Series Symbolization

Given multivariate time series data (A1), each of the N constituent series (A2) are symbolized (B1) using a sliding window into $n!$ symbols based on ordinal rank as illustrated for $n = 3$ (B2). At each point in time in the symbolized time series (B1) there are $(n!)^N$ possible states, each of which can be given a symbol to produce a univariate symbolized time series as illustrated for $n = 2$, $N = 3$, and $(n!)^N = 8$ (C1).

Periodicity in the Symbolized Data

A Rossler nonlinear time series ($N = 3$) symbolized with $n = 2$ partitions the state space into $(n!)^N = 8$ symbols (C2). In the phase coherent case, interdependence between variables and periodic time series patterns generate state space orbits that oscillate around an anulus (C2), corresponding to a periodic symbolic sequence (C1). The relationship between phase coherence and periodic symbolic sequences holds in general for $n > 2$, despite the fact that the state space cannot be interpreted as being partitioned by symbols in this case.

Ordinal Partition Transition Networks

An ordinal partition transition network is a graph representation (D1) of a symbolic sequence (C1) with nodes as symbols, edges as time successions, and cycles as state space orbits (C2). Where in Fourier analysis periodic time patterns correspond to frequency domain points, in our analysis they correspond to cycles in the graph of the symbolized data.

Computing Persistent Homology

The number of graph cycles and the quantity of nodes in each cycle is estimated through persistent homology: A *clique rank filtration* (D2) is performed by successively adding edges to the graph in an ordered determined by a shortest path distance matrix (E1). A persistence diagram (F1) is produced by recording when holes are created, and when they are covered by *cliques*, which are shaded below in blue (D2). The differences between hole death and birth times in the persistence diagram (F1) points are the computed classifier.

ANALYSIS METHODOLOGY

Breakspear and Terry's [1] statistical methods were applied to phase coherent and non-phase coherent Rossler systems (C2) in addition to seizure epochs, which are known to be phase coherent [1], from artefact (e.g. eye blinks and heartbeats) corrected bandpass filtered downsampled EEG recordings. They write:

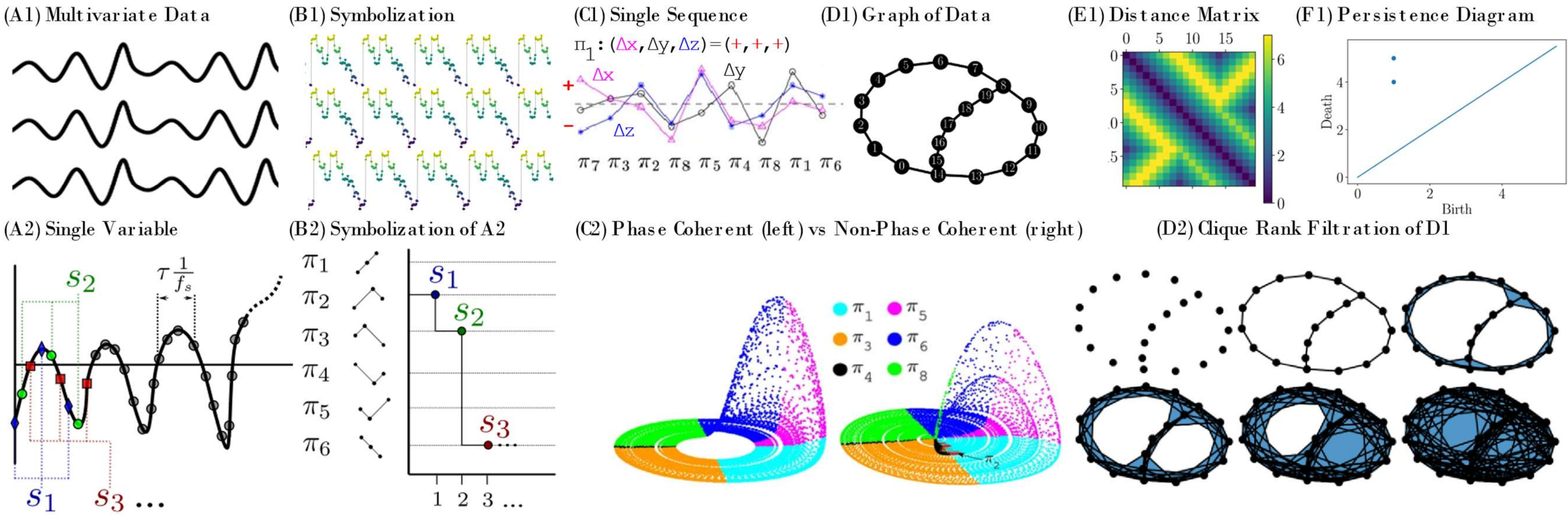
"[Artefacts] such as ... sampling error [and] filtering can lead to the false identification of non-linearity … In order to ensure that [classifier values] are not artefacts caused by these properties of the data … surrogate data … was employed. [Surrogate data computation involves] preserving all the original linear properties but destroying any nonlinear structure present by phase randomising the Fourier components. This allows testing of the null hypothesis that the time series are exclusively linear, with purely linear coherence."

KEY RESULTS

The classifier ($n = 5$) was evaluated using a phase coherent (C2) Rossler system ($N = 3$) window: Using $m = 99$ surrogate data realizations and a 5% significance level Šidák corrected to α, a one tailed z-test on the classifier values yielded $p = 5.7 \times 10^{-41} < \alpha$. A similar analysis was applied using $m = 19$ with 40 seizure epochs (2.5 s) randomly selected from each of 3 patients: *Permutation entropy* selected the $N = 5$ most phase coherent channels per sliding window to evaluate the classifier ($n = 4$) on. The z-test null hypothesis was rejected for 22%, 16%, and 14% of all epochs for Šidák corrections on the level of an epoch, a patient, and all patients respectively.

REFERENCES

[1] M. Breakspear and J. Terry, Clinical Neurophysiology **113**, 735 (2002).



References: (C1) and (C2) were adapted from DOI: 10.1038/s41598-017-08245-x while the remaining figures were adapted from DOI: 10.1103/PhysRevE.100.022314