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```
clear all;  
close all;  
clc;  
warning('off')
```

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**This code is to compute the primal game LP of player 1. This code will**

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**give the game value, player 1's optimal strategy and the initial vector**

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**payoff over player 2's state**

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**Initialization**

---

```
P{1,1}=[.8 .1 .1; .1 .4 .5; .2 .7 .1 ]; %Player 1's Payoff matrix if a=1 and b=1  
P{1,2}=[.4 .5 .1; .2 .3 .5; .4 .4 .2 ]; %Player 1's Payoff matrix if a=1 and b=2  
P{2,1}=[.2 .2 .6; .5 .2 .3; .2 .2 .6 ]; %Player 1's Payoff matrix if a=2 and b=1  
P{2,2}=[.3 .3 .4; .1 .8 .1; .1 .1 .8 ]; %Player 1's Payoff matrix if a=2 and b=2  
save P.mat;  
Q{1,1}=[.8 .2;.5 .5]; %Player 2's Payoff matrix if a=1 and b=1  
Q{1,2}=[.2 .8;.1 .9]; %Player 2's Payoff matrix if a=1 and b=2  
Q{2,1}=[.6 .4;.5 .5]; %Player 2's Payoff matrix if a=2 and b=1  
Q{2,2}=[.7 .3;.1 .9]; %Player 2's Payoff matrix if a=2 and b=2  
save Q.mat;  
T=2; %Number of stages in a game  
A=2; %Number of player 1's actions  
B=2; %Number of player 2's actions  
k=3; %Number of states of player 1  
l=2; %Number of states of player 2  
lm=0.3; %discounted value  
p=[0.5 0.3 0.2]; %player 1's initial probability for state  
q=[0.5 0.5]; %player 2's initial probability for state  
load M.mat; %payoff matrix  
G=M;
```

```
%information set of player 1 and 2
[is1,n_is1]=info_I(T,A,B,k);
[is2,n_is2]=info_J(T,A,B,l);
```

## Constraint $\sum_{a_t} R_{\{I_t\}}(a_t) = P_{\{a_{t-1}, b_{t-1}\}}(k_{t-1}, k_t) R_{\{I_{t-1}\}}(a_{t-1})$ ;

```
%\forall t=1,...,n, \forall \mathcal{I}_t
%Creating the equation as matrix multiplication [Aeq]*[variable]=[beq]. In
%this equation the variable is R_{I_t}

Aeq=zeros(sum(n_is1),sum(n_is1)*A);
beq=zeros(sum(n_is1),1);

row_index=0;
for t=1:T
    for i=1:length(is1{t})
        row_index=row_index+1;
        for a=1:A %As there is summation a_t in R_{I_t}
            I=is1{t}(i,:); %for selecting each row from the information set of player 1
            [col_index_RIt] = RIt_col_index_P1(t,I,A,B,k,a,n_is1); %finding the column index for that particular information set row
            Aeq(row_index,col_index_RIt)=1;
        end
        kt=I(end);
        if t==1
            beq(row_index,1)=p(kt); %As at t=1, P_{a_0,b_0}(k_0,k_1)=1 and R_{I_0}(a_0)=Pr(k1)
        else
            Ipre=is1{t}(i,1:(length(is1{t}(i,:))-3));
            apre=is1{t}(i,(length(is1{t}(i,:))-2));
            bpre=is1{t}(i,(length(is1{t}(i,:))-1));
            [col_index_RItprev] = RIt_col_index_P1(t-1,Ipre,A,B,k,aprev,n_is1);
            ktpre=I(end-3);
            Aeq(row_index,col_index_RItprev)=-P{aprev,bprev}(ktpre,kt);
        end
    end
end
end
```

## The inequality constraint

```
%Creating the equation as matrix multiplication [Ain]*[variables]=[bin]. In
%this equation the variables are R_{I_t}, (U_{J_t}, U_{J_{t+1}})
[kset,n_kset] = Kset(T,k); %to get player 1's state information only

Ain=zeros(sum(n_is2)*B,(sum(n_is1)*A+sum(n_is2)));
bin=zeros(sum(n_is2)*B,1);

row_index=0;
for t=1:T
    for j=1: length (is2{t}) %for different J_t and different b_t there will be different row
        for b=1:B
            row_index=row_index+1;
            for kn=1:length(kset{t})
```

```

        for a=1:A
            [I] = construct_Is(j,kn,t,kset,is2);
            [col_index] = RIt_col_index_P1(t,I,A,B,k,a,n_is1);
            Ain(row_index,col_index)=lm^(t-1)*G{I(3*t-2),(is2{t}(j,(3*t-2)))}(a,b)
; %R_{I_t} coefficient
        end
    end
    %To find the term for U_{J_{t+1}}
    if t<T %As for U_{J_{N+1}}=0
        for a1=1:A
            for lplus=1:l
                Jplus=[is2{t}(j,:) a1 b lplus];
                [col_index_Jplus] = J_col_index(n_is1,A,B,l,Jplus,n_is2,t+1);
                Ain(row_index,col_index_Jplus)=Q{a1,b}(is2{t}(j,end),lplus);%The
coefficient for U_{J_{t+1}}
            end
        end
    end
    %To find the term for U_{J_t}
    Jpre=is2{t}(j,:);
    [col_index_Jpre] = J_col_index(n_is1,A,B,l,Jpre,n_is2,t);
    Ain(row_index,col_index_Jpre)=-1;
end
end
end
end

```

## Construct the objective function: $\max_{\{R\}} \max_{\{U\}} \sum_{\{I\}} q(I) * U_{\{\mathcal{J}\}_1}$

```

f=[zeros(1,sum(n_is1)*A) q zeros(1,sum(n_is2(2:end)))]; %zeros(1,sum(n_is1)*A) is for R_{I_t} as there is no R_{I_t} in the objective function. zeros(1,sum(n_is2(2:end))) is for U_{J_{2:T}} as in objective function there is only U_{\mathcal{J}_1}

%Rearrange every coefficient to use in linprog
Ain1=-Ain; %As linprog use <=
bin1=-bin; %As linprog use <=
f1=-[zeros(1,size(Ain1,2)-size(f,2)) f]; %as linprog works for only minimize obj function thats why -ve. To ensure that matrix of objective function and Ain,Aeq are of same size we add zeroes
Aeq1=padarray(Aeq,[0, (size(Ain1,2)-size(Aeq,2)),0,'post']); % to use linprog Ain and Aeq must have same column number as column number indicates variables. Both equation must have same variables
lb=[zeros(sum(n_is1)*A,1);-Inf((size(Ain1,1)-sum(n_is1)*A),1)];
ub=+Inf;

options = optimoptions('linprog','Display','none');
[x,v1]=linprog(f1,Ain1,bin1,Aeq1,beq,lb,ub,options); %x=[R_{I_t} U_{J_t}] and v1 is the game value

```

## Game value

v1

v1 =

-103.6310

## Finding the optimal strategy from the realization plan we got from previous linprog result

```
%sigma=R_{I_t}/(P_{a_{t-1},b_{t-1}}(k_{t-1},k_t)*RI_{t-1})

sigma=zeros(1,sum(n_is1));
sigma_col=0;
for t=1:T
    for i=1:length(is1{t})
        I=is1{t}(i,:); %for selecting each row from the information set of player 1
        sigma_col=sigma_col+1;
        for a=1:A %As there is summation a_t in R_{I_t}
            [row_index_RIt] = RIt_col_index_P1(t,I,A,B,k,a,n_is1); %finding the column index for that particular information set row. The number of column index in RIt is the number of row index in optimal realization plan x
            RIt=x(row_index_RIt,1);
            kt=I(end);
            if t==1
                RItprev=p(kt);
                pvalue=1;
            else
                Ipre=is1{t}(i,1:(length(is1{t}(i,:))-3));
                apre=is1{t}(i,(length(is1{t}(i,:))-2));
                bpre=is1{t}(i,(length(is1{t}(i,:))-1));
                [row_index_RItprev] = RIt_col_index_P1(t-1,Ipre,A,B,k,aprev,n_is1);
                ktpre=I(end-3);
                RItprev=x(row_index_RItprev,1);
                pvalue=P{aprev,bprev}(ktpre,kt);
            end
            sigma(a,sigma_col)=RIt/(RItprev*pvalue);
        end
    end
end
```

## sigma is player 1's optimal strategy

sigma %sigma(1st row) for a=1 and sigma(2nd row) for a=2. Each column indicates each information set at different stages. For example. 5th column is for T=2 (k\_1=1, a\_1=1, b\_1=1, k\_2=2)

sigma =

Columns 1 through 7

1.0000	0.0000	1.0000	0	0	0	0
0	1.0000	0	1.0000	1.0000	1.0000	1.0000

Columns 8 through 14

0	0	NaN	NaN	NaN	NaN	NaN
---	---	-----	-----	-----	-----	-----

1.0000	1.0000	NaN	NaN	NaN	NaN	NaN
--------	--------	-----	-----	-----	-----	-----

Columns 15 through 21

NaN	0	0	0	0	0	0
NaN	0	0	0	0	0	0

Columns 22 through 28

0	0.2774	0	0	0.4392	0	0
1.0000	0.7226	1.0000	1.0000	0.5608	1.0000	1.0000

Columns 29 through 35

0.4899	0	0	0.3135	0	NaN	NaN
0.5101	1.0000	1.0000	0.6865	1.0000	NaN	NaN

Columns 36 through 39

NaN	NaN	NaN	NaN
NaN	NaN	NaN	NaN

## to find optimal nu

```
%to find UJ1
for l_present=1:l
    t=1;
    info=l_present;
    [col_index_UJ1] = J_col_index(n_is1,A,B,l,info,n_is2,t);
    UJ1=x(col_index_UJ1,1);
    nu(l_present)=[UJ1];
end
```

## nu is the initial vector payoff over player 2's state

```
nu=-nu %as nu=-Z_{I_1}. nu[1] is for nu(l=1) and nu[2] is for nu(l=2)
```

```
nu =
```

```
-89.5061 -117.7559
```