SNS Research Project

Fourier Analysis in Cardiovascular Signal Processing

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Milestone 3

In this milestone, we build on the foundation laid in our first report by explaining the implementation of our approach using mathematical expressions and simulations. We explore Fourier analysis through two methods: the **Direct Fourier Series approach** and the **Transform-based approach**.

The direct method focuses on noise reduction, data compression, and cardiac disorder detection using straightforward Fourier series analysis. In contrast, the transform-based method applies the **Discrete Fourier Transform (DFT)**, with simulations and examples to demonstrate its effectiveness in processing cardiovascular signals.

Direct Fourier Series Approach:

The ECG signal, being quasi-periodic, is well-suited for analysis using the Direct Fourier Series (DFS) method. This approach, as we have studied, represents the ECG as a sum of sine and cosine functions, making it easier to analyze, compress and reduce the noise of the signal without complex transformations.

The general Fourier Series formula is:

$$x(t) = a_0 + \sum_{k=1}^{\infty}\left[a_k \cos(kw_0 t) + b_k \sin(kw_0 t)\right]$$

From this formula, we know:

- $w_0 = \frac{2\pi}{T}$ is the fundamental angular frequency
- $a_0 = \frac{1}{T}\int_0^T f(t)\,dt$ which is our DC component
- $a_k = \frac{2}{T}\int_0^T f(t)\cdot\cos\left(\frac{2\pi kt}{T}\right)dt$ which is our cosine coefficient
- $b_k = \frac{2}{T}\int_0^T f(t)\cdot\sin\left(\frac{2\pi kt}{T}\right)dt$ which is our sine coefficient and T is the period of the signal.

These coefficients capture key features of the ECG's shape. By using only the first few harmonics, we can approximate the signal with good accuracy. This makes the direct method useful for applications like **noise reduction**, **data compression**, and **cardiac abnormality detection**, especially when simple and interpretable techniques are preferred.

Noise Reduction:

Noise reduction in ECG processing is crucial for improving signal clarity and accuracy. It helps remove baseline drift, powerline interference, and muscle noise, making ECG waveforms more reliable for analysis.

To demonstrate noise reduction using the direct Fourier approach, we simulated a clean ECG-like signal composed of two sine waves and then added random noise to mimic real-world disturbances. Using the Fourier Series, we computed the signal's frequency components (**Fourier coefficients**) and reconstructed it using a limited number of terms. By doing this, we filtered out high-frequency noise, effectively smoothing the signal. The result shows that even a simple Fourier-based reconstruction can reduce noise and recover the underlying ECG waveform.

An example equation that we have used to achieve our simulation is:

$$ECG_{clean}(t) = 1.5 \cdot \sin(2\pi \cdot 5t) + 0.5 \cdot \sin(2\pi \cdot 15t)$$

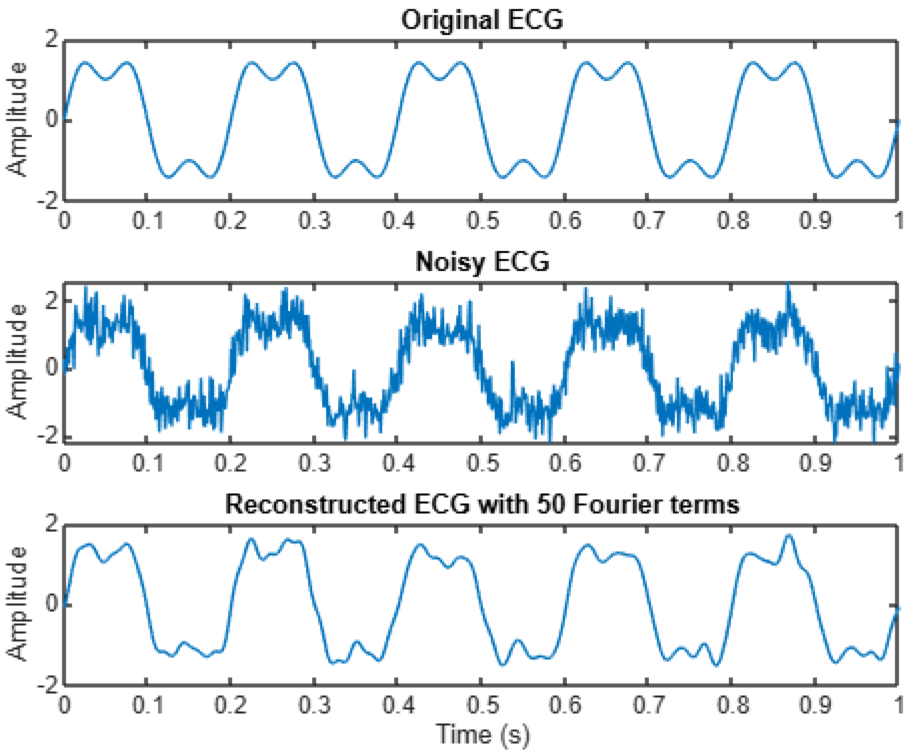


Figure 1: Noise Reduction of an ECG signal

Data Compression:

Data compression plays a key role in handling the large volumes of data from continuous ECG monitoring. Reducing redundancy in the signal using direct or transform-based techniques enables efficient storage and transmission while maintaining the essential features needed for accurate diagnosis.

In the simulation, we generated a clean ECG-like signal and used the Fourier Series to compress it by calculating the coefficients with a Riemann sum. The signal was then reconstructed using a limited number of terms, and the comparison between the original and compressed signals demonstrates how the Fourier Series can effectively reduce data while retaining key features of the ECG waveform. We are using the same equation for the simulation here.

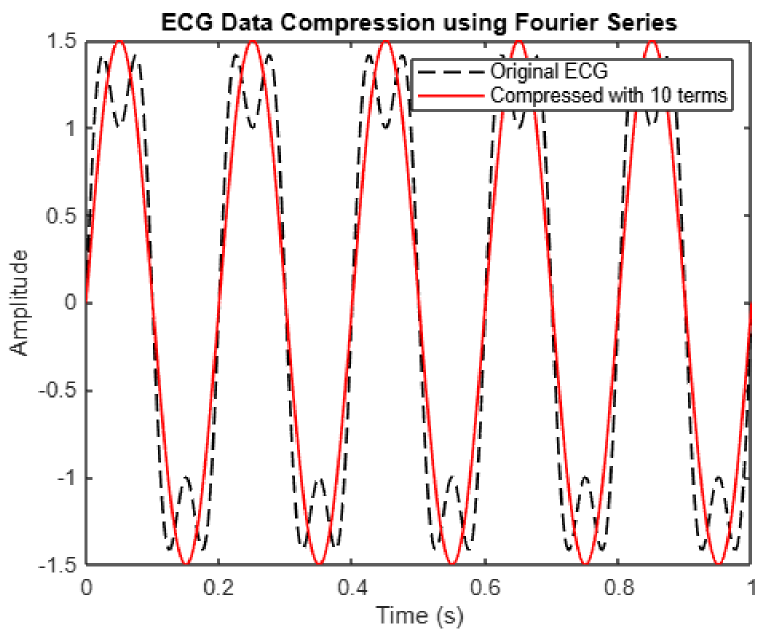


Figure 2: Data Compression of an ECG signal

Detection of Cardiac Disorders:

Cardiac disorder detection uses advanced algorithms to analyse QRS complexes and wave morphology, helping to identify arrhythmias and abnormalities. Techniques like high-resolution ECG and T-wave alternans analysis offer insights into heart rhythm and beat patterns, aiding in the detection of life-threatening arrhythmias.

The simulation generates a clean ECG signal and simulates **ventricular fibrillation** by adding random noise, mimicking the irregular electrical activity seen in VF. The clean and disorder-affected signals are then plotted to show how arrhythmias can disrupt normal heart rhythms.

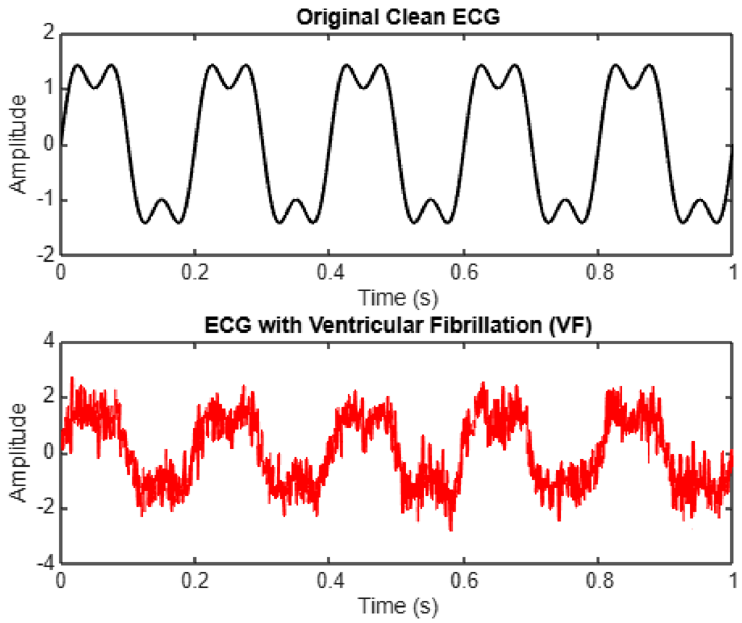


Figure 3: Ventricular Fibrillation depicted by adding random noise to a clean ECG signal

Discrete Fourier Transform Approach:

In this method, we apply DFT (Discrete Fourier Transform) to convert ECG into frequency domain.This makes sure that only the few dominant frequencies (e.g., top 10) are retained. The signal is then reconstructed using Inverse DFT (IDFT).

General Formula for DFT:

$$X[k]=\sum_{n=0}^{N-1} x[n]\cdot e^{-j\frac{2\pi}{N}kn}$$

Inverse DFT:

$$x[n]=\frac{1}{N}\sum_{k=0}^{N-1} X[k]\cdot e^{j\frac{2\pi}{N}kn}$$

This gives us the reconstructed signal after compressing it and storing only the dominant frequencies

RMS Error:

$$RMS=\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}(x[n]-\tilde{x}[n])^2}$$

This shows the average reconstruction error.

PRD (Percent Root Mean Square Difference):

$$PRD = 100 \times \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (x[n] - \tilde{x}[n])^2 \Big/ \sum_{n=0}^{N-1} x[n]^2}$$

This shows relative energy loss, useful for comparing compression quality.

Example Calculation:

Original signal:

x[n]=[3,4,5,6]

DFT yields:

X=[18,−2+2j,−2,−2−2j]

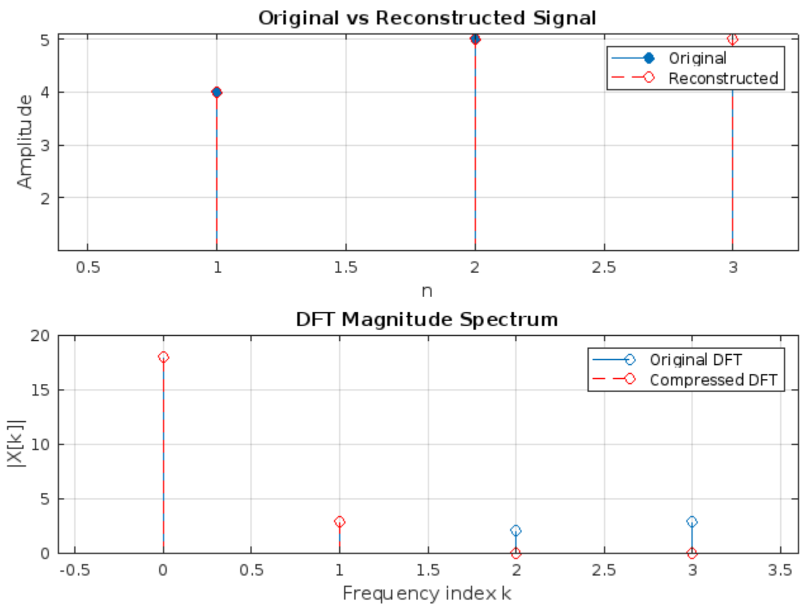
Keeping X[0],X[1]; truncating the rest gives:

$\tilde{x}[n] = [4, 4, 5, 5]$

RMS Error ≈ 0.7071

PRD ≈ 15.25%

MATLAB Simulation

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RMS Error: 0.7071
PRD: 15.25%
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