Exercise Sheet 2
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NOTE: For this exercise sheet, you must submit a jupyter notebook. You can use pen and paper to solve the first exercise but provide it (e.g. as screenshot png) in the notebook in a markdown cell At the beginning of your submission, list the **names** and **Uni ID (Unikennung; not Matrikelnummer)** of each team member.

Exercise 1: Conjugate Priors for Gaussians (2 Points)

Let us assume that our data distribution is Gaussian, i.e.

$$p(x|\theta) \sim \mathcal{N}(x|\mu, \sigma^2) \tag{0.1}$$

where our parameters θ are $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}_+$. Also assume that our prior distribution on the data distribution mean is given by

$$p(\mu|\mu_0, \sigma_0^2) \sim \mathcal{N}(\mu|\mu_0, \sigma_0^2) \tag{0.2}$$

Let us assume that our observed data is $\mathcal{D} = (x^1, \ldots, x^n)$. Show that the posterior is given by

$$p(\mu|\mathcal{D}, \mu_0, \sigma_0^2) \sim \mathcal{N}\left(\frac{1}{n}\left(\frac{\sigma_0^2}{\frac{\sigma^2}{n} + \sigma_0^2}\bar{x} + \frac{\sigma^2}{\frac{\sigma^2}{n} + \sigma_0^2}\mu_0\right), \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right)^{-1}\right) \tag{0.3}$$

where $\bar{x} = \sum_{i=1}^{n} x^i$.

Exercise 2: Parameter Estimation for Multinomial Distribution (3 Points)

(Solve this exercise in the Exercise2.ipynb)
Consider a dataset containing observations from $K = 200$ distinct categories. Given this data, your task is to estimate the parameter vector $\boldsymbol{\theta}$ of the underlying multinomial distribution using two different approaches:

a) Maximum Likelihood Estimation (MLE)

b) Maximum A Posteriori (MAP) estimation with a symmetric Dirichlet(α) prior.