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Exercise 7:

(1) $A \in \mathbb{R}^{n \times m}$, $A_{ij} \geq 0$ 19×19 face image ($M = 361$)

$A \approx WH$

$W \in \mathbb{R}^{n \times R}$

$H \in \mathbb{R}_+^{R \times M}$, $R \rightarrow$ chosen rank

$$L(W, H) = \|A - WH\|_F^2$$

$$\Rightarrow L(W, H) = \sum_{ij} (A_{ij} - (WH)_{ij})^2$$

Gradient w.r.t. W

$E = WH - A$, $L = \|E\|_F^2$

$\nabla_W L = 2(WH - A)H^T$

Gradient w.r.t H

$\nabla_H L = 2W^T(WH - A)$

$\alpha > 0$

$W^{(t+1)} = [W^{(t)} - \alpha \nabla_W L]_+$

$H^{(t+1)} = [H^{(t)} - \alpha \nabla_H L]_+$

$[X]_+ = \max(X, 0)$. $\rightarrow$ ensure non-negativity