Applied Nonlinear Control

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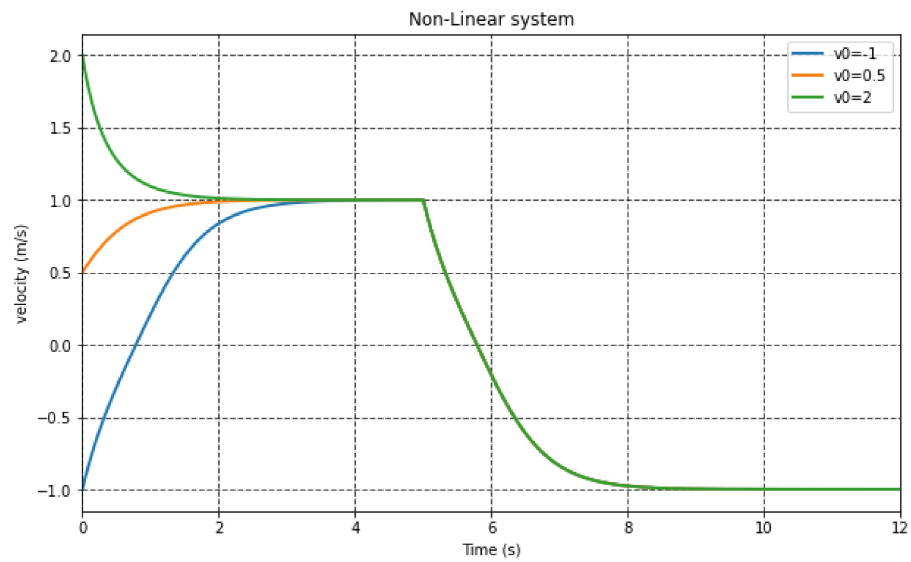
B19-RO-01

HW1_Report

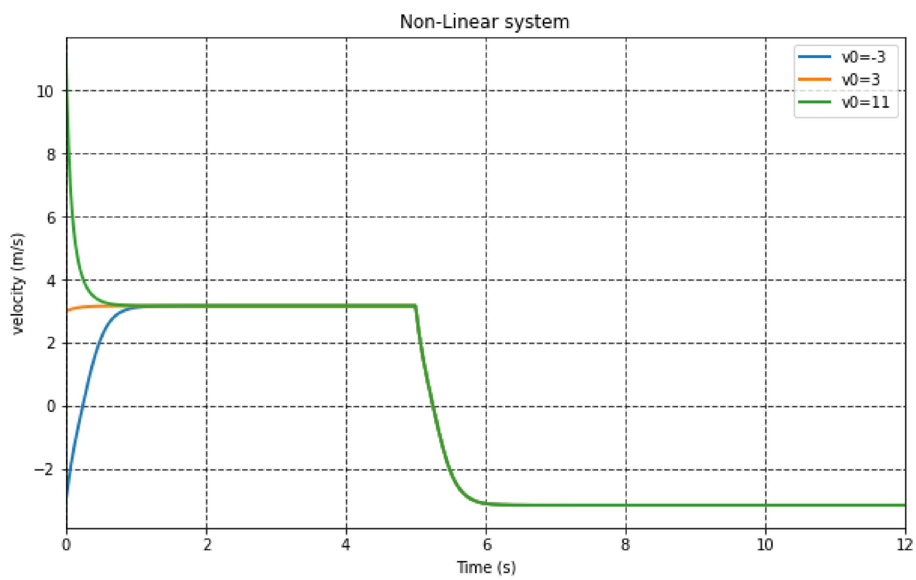
Problem 1

Response of **Non-Linear** System $\dot{v} = -|v|v + u$:

- In case of $u = \begin{cases} 1 & , t < 5 \\ -1 & , t > 5 \end{cases}$

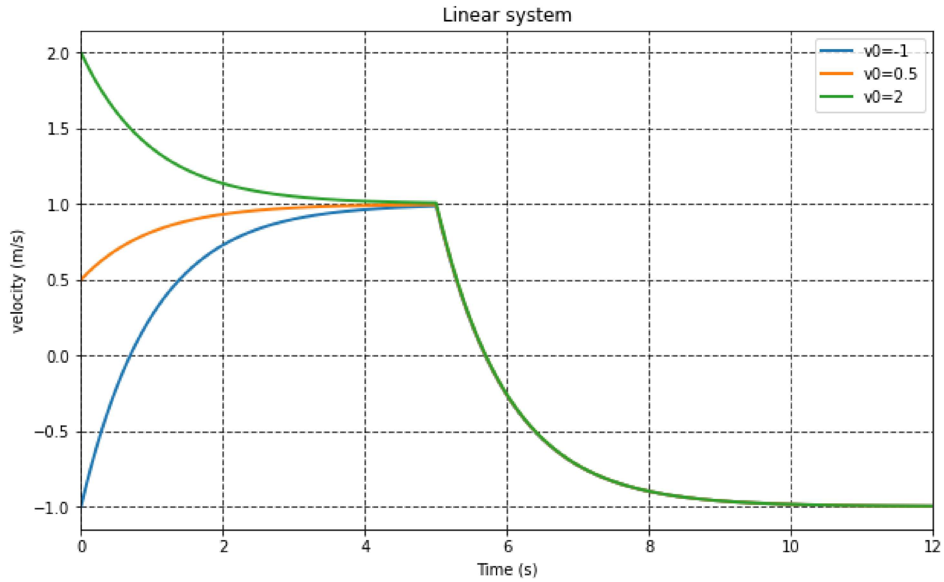


- In case of $u = \begin{cases} 10 & , t < 5 \\ -10 & , t > 5 \end{cases}$

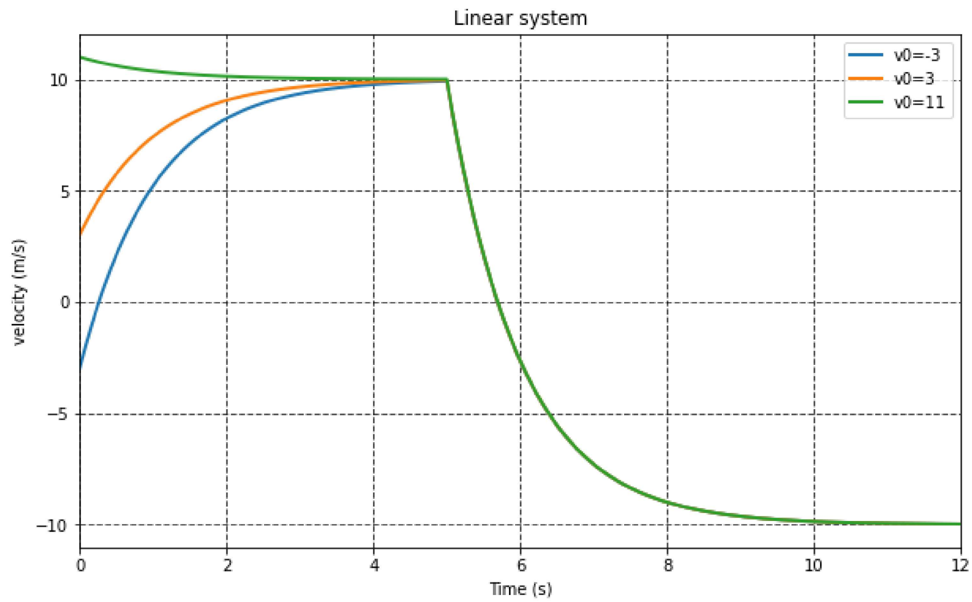


Response of **Linear** System $\dot{v} = -v + u$:

- In case of $u = \begin{cases} 1 & , t < 5 \\ -1 & , t > 5 \end{cases}$



- In case of $u = \begin{cases} 10 & , t < 5 \\ -10 & , t > 5 \end{cases}$



We can notice that:

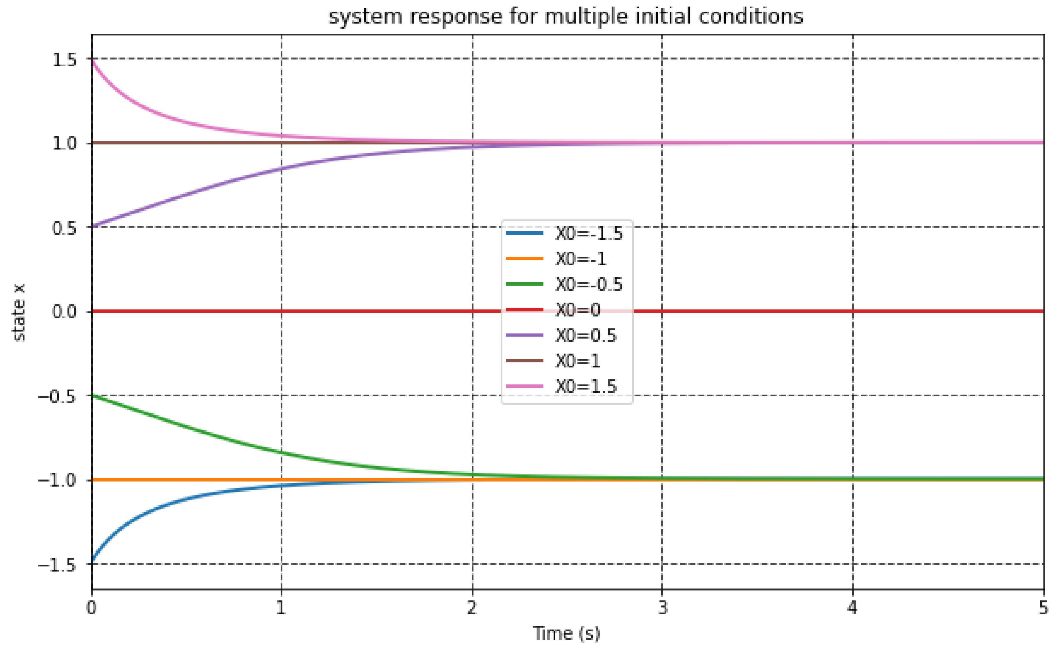
- in case of **unit step input,** both systems converges to ± 1. Because ± 1 are equilibrium points for both systems in this case. But the settling time of the linear system is longer than that of the non-linear system.
- In case of **10 times the step input** the linear system converges to ± 10 because they're also equilibrium points for the linear system. But the non-linear system converges to ± 3.16227 which comes from the equation of the system $v = \pm\sqrt{10}$, which are also equilibrium points for the non-linear system.

$$at\ \dot{v} = 0,\ -|v|v + u = 0$$
$$in\ case\ of\ u = 10$$
$$-|v|v + 10 = 0$$
$$|v|v = 10$$
$$v = \pm\sqrt{10}$$

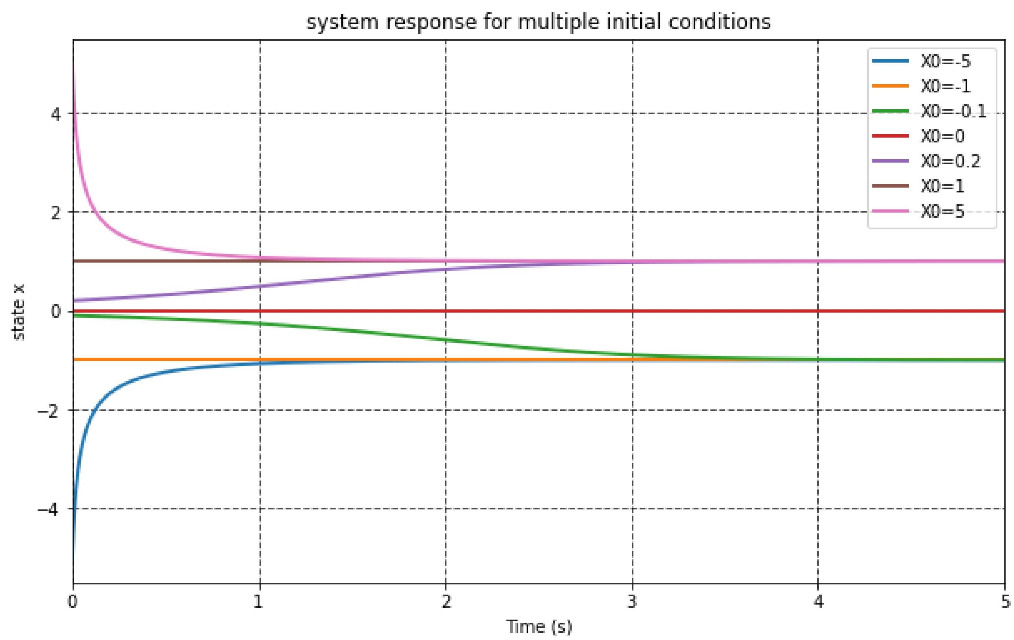
- both systems are stable.

Problem 2

Response of **Non-Linear** System $\dot{x} = x - x^3$:



we can try to change the initial conditions:



We can notice that this non-linear system has 3 equilibrium points:

$$at\ \dot{x} = 0,\ x\left(1 - x^2\right) = 0$$

$$x\ = 0,\ x = 1,\ x = -1$$

- So, For the initial conditions $x = 0, 1, -1$ the system behavior stays the same because they're also the equilibrium points.
- For any initial condition $x\ in\ range\ (0, 1)$ the system converges to 1.
- For any initial condition $x\ in\ range\ (0, -1)$ the system converges to -1.
- For any initial condition x is bigger than 1, the system converges to 1.
- For any initial condition less than -1 the system converges to -1.

we can conclude that the system is stable at its equilibrium points.

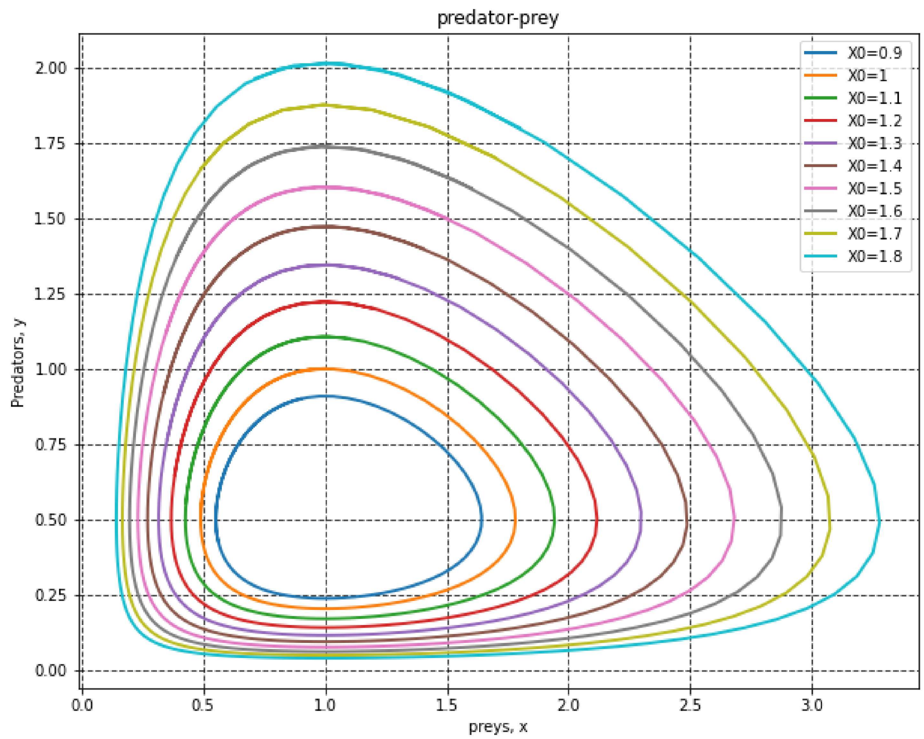
Problem 3

Response of **Lotka-Volterra (predator-prey)** equations.

$$\begin{cases} \dot{x} = \alpha x - \beta x y \\ \dot{y} = \delta x y - \gamma y \end{cases}$$

$$for\ \alpha = \frac{2}{3},\ \beta = \frac{4}{3},\ \gamma = \delta = 1$$

$$x_0 = y_0 = [0.9, 1, 8]$$



The resulting trajectory doesn't represent a limit cycle because non of the responses spiral into one circle. The system oscillates for each different initial condition.

Problem 4

For a double pendulum system (I used the dynamics equations from this [link]()):

$$\theta_1'' = \frac{-g\,(2\,m_1 + m_2)\sin\theta_1 - m_2\,g\sin(\theta_1 - 2\,\theta_2) - 2\sin(\theta_1 - \theta_2)\,m_2\,(\theta_2'^2\,L_2 + \theta_1'^2\,L_1\cos(\theta_1 - \theta_2))}{L_1\,(2\,m_1 + m_2 - m_2\cos(2\,\theta_1 - 2\,\theta_2))}$$

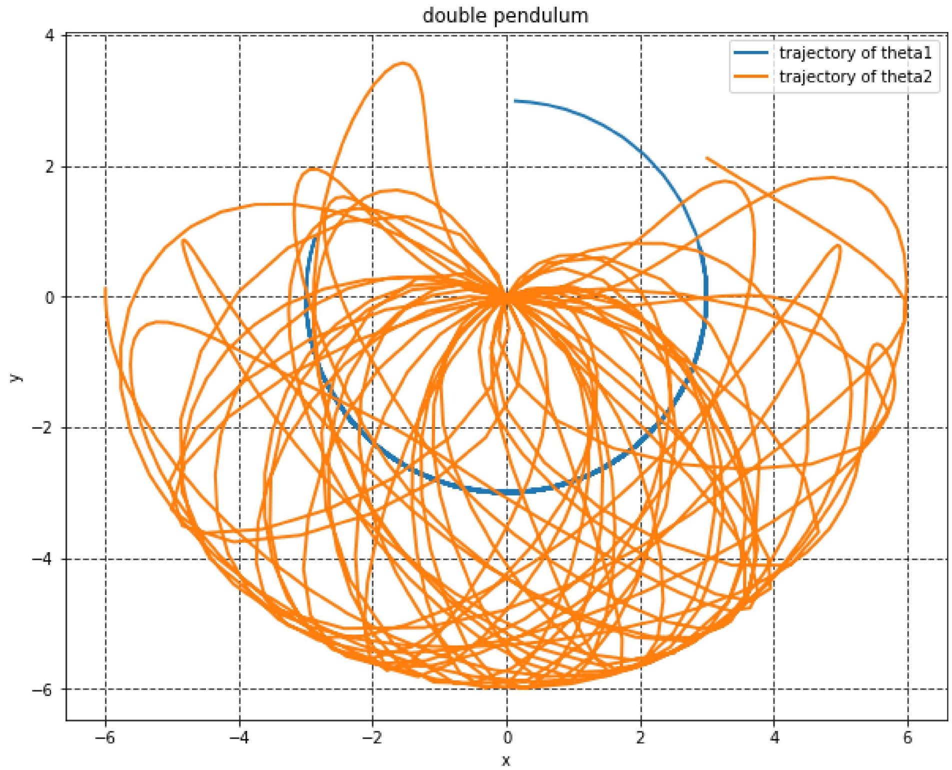
$$\theta_2'' = \frac{2\sin(\theta_1 - \theta_2)\,(\theta_1'^2\,L_1\,(m_1 + m_2) + g(m_1 + m_2)\cos\theta_1 + \theta_2'^2\,L_2\,m_2\cos(\theta_1 - \theta_2))}{L_2\,(2\,m_1 + m_2 - m_2\cos(2\,\theta_1 - 2\,\theta_2))}$$

These are the equations of motion for the double pendulum.

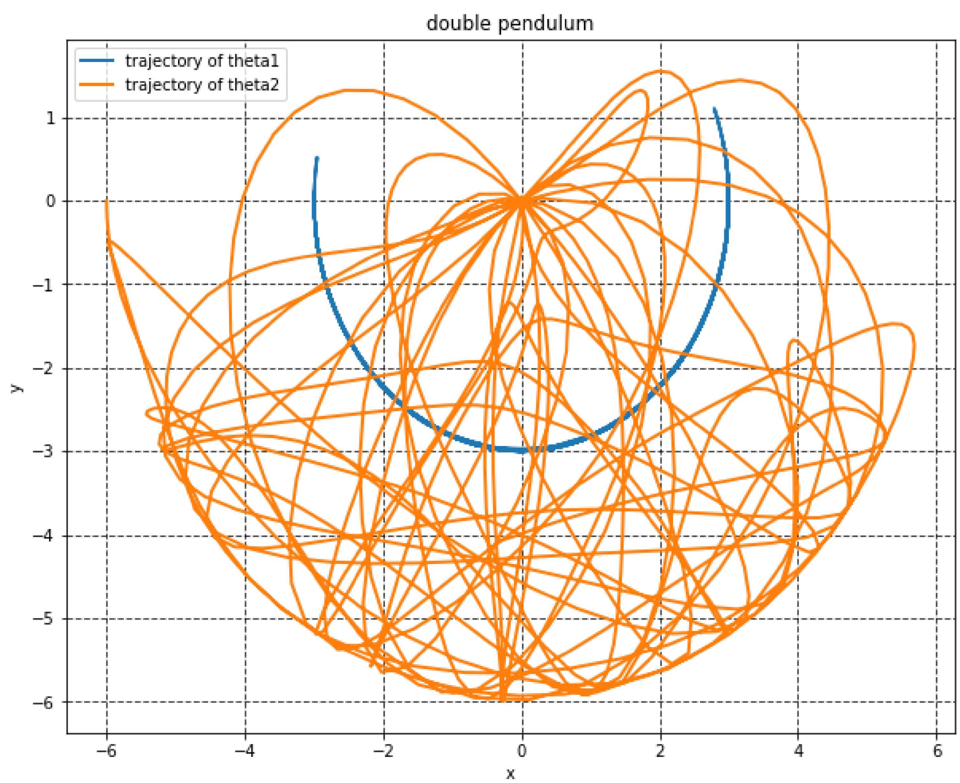
with parameters:

$$g = 9.81,\ \ m1 = 2,\ m2 = 1,\ L1 = 3,\ L2 = 2,\ \ t = [0, 20]$$

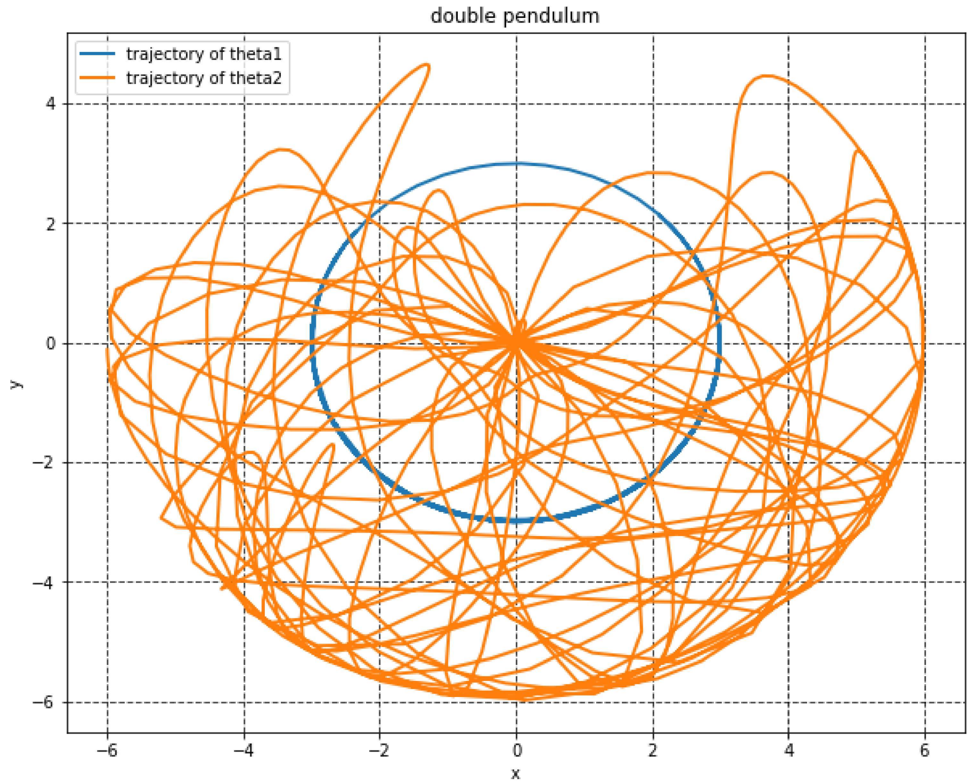
Response for for initial condition $\theta_1 = \theta_2 = \left(\frac{\pi}{2} + 0.02\right)$:



Response for for initial condition $\theta_1 = \theta_2 = \left(\frac{\pi}{2}\right)$:



Response for for initial condition $\theta_1 = \theta_2 = \left(\frac{\pi}{2} - 0.02\right)$:



we can see that the double pendulum shows a chaotic behavior nearby slightly different initial conditions which results in different trajectories.
