ANC HW 2: Phase Portraits and Linearization

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Problem 1 Consider one dimensional actuated rotational body:

$$\ddot{\varphi} = u$$

Assume that we apply the switching and rate feedback as follows:

$$u = -\text{sign}(k\varphi + \dot{\varphi})$$

Draw the phase portrait of the system and determine convergence of the trajectories starting from $\varphi_0 > 0$, $\varphi_0 < 0$ and $\varphi_0 = 0$

Problem 2 Consider the linear two dimensional system in form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}, \ \mathbf{x} \in \mathbb{R}^2$$

Study how the eigenvalues of $\mathbf{A}$ affect the phase portraits. Plot the different types of **phase portraits** (node, saddle, focus, center) with asociated **complex plane** representation of eigenvalues.

Hint: Read the chapter 2.4 of Slotine's textbook and repeat figure 2.9

Problem 3 Consider the following systems:

$$(a) \quad \begin{cases} \dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1) \\ \dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1) \end{cases}$$

$$(b) \quad \begin{cases} \dot{x}_1 = x_2 + x_1(x_1^2 + x_2^2 - 1) \\ \dot{x}_2 = -x_1 + x_2(x_1^2 + x_2^2 - 1) \end{cases}$$

$$(c) \quad \begin{cases} \dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1)^2 \\ \dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1)^2 \end{cases}$$

Draw the associated phase portraits, and describe the behavior of system trajectories.

Hint: use the figure 2.10 from the textbook.

Problem 4 Consider damped nonlinear pendulum with described by:

$$\ddot{x} = -\dot{x} - \sin x$$

Find the equlibrium points and deduce their stability using Lyapunov linearization method.

Problem 5 Implement software routine that will implement the Lyapunov linearization method including:

- Solving for equlibrium
- Symbolical linearization
- Checking for the stability of each equlibrium
- Drawing the phase portrait together with stable and unstable points.

Test developed routine on following system:

$$\begin{cases} \dot{x}_1 = x_1 - x_1^3 + 2x_1x_2 \\ \dot{x}_2 = -x_2 + \frac{1}{2}x_1x_2 \end{cases}$$