Digital Signal Processing

Nabila Adawy

n.roshdy@innopolis.university

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HA2_Report

Task 1: Moving Average Filter

For the Moving Average Filter with $M = 3$:

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

We can compute the system function $H(z)$ from its following Transfer Function as follows:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{N}\sum_{i=0}^{M-1} = z^{-i}$$

$$H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2} \tag{1}$$

So the system function $H(z)$ is:

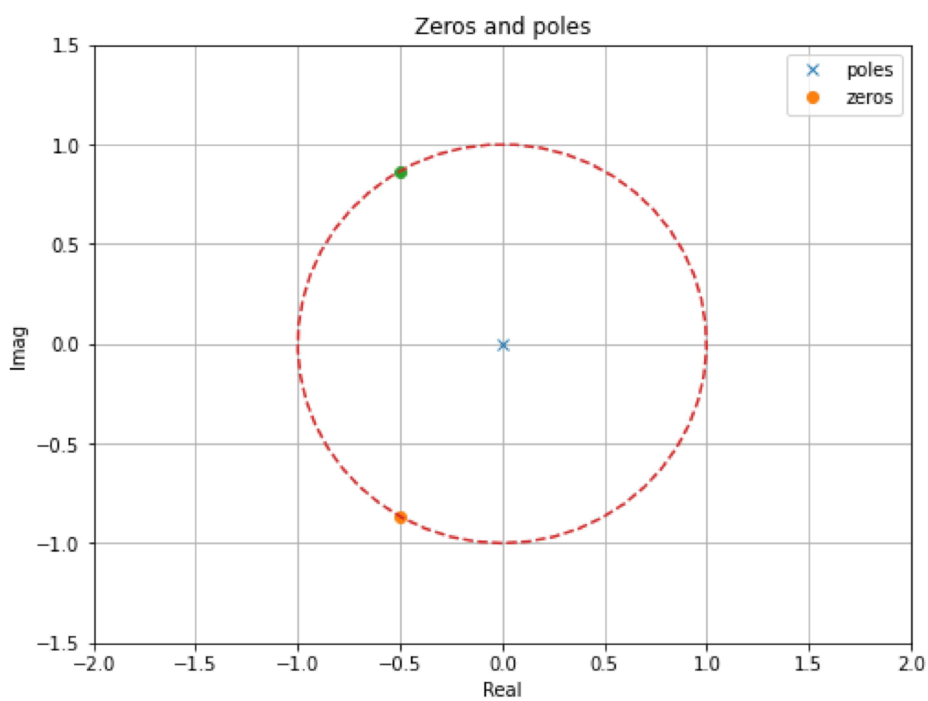
$$H(z) = \frac{1}{3}\frac{z^2+z+1}{z^2}$$

It has 2 zeros located at:

$$z_1 = -0.5 + j\frac{\sqrt{3}}{2}, \ z_2 = -0.5 - j\frac{\sqrt{3}}{2}$$

and 2 poles located at the origin.

Plotting the zeros and poles of the function $H(z)$ in the complex plane, we obtain:



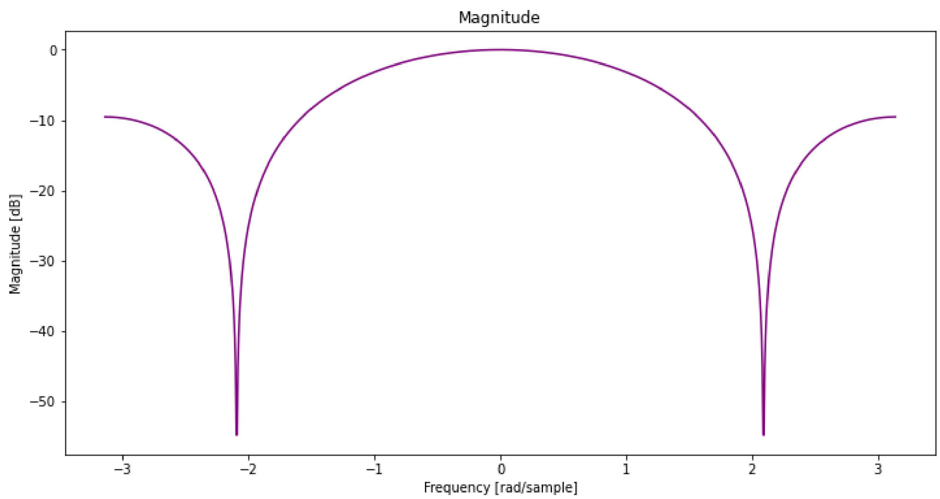
To Compute the Magnitude $|H(z)|$ we can substitute $z = e^{i\omega}$ in equation (1)

$$H(\omega) \ = \ \frac{1}{3}e^{-i\omega} \ (1 + 2\ cos(\omega))$$

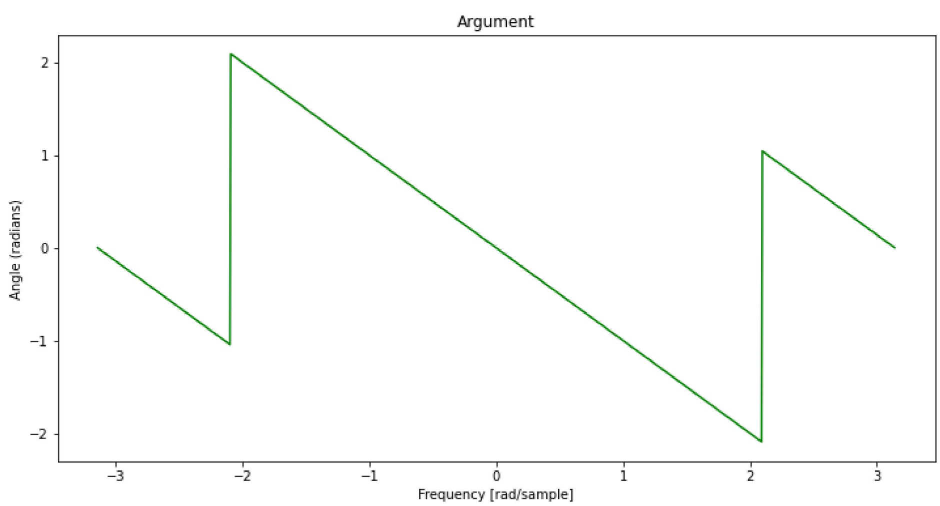
Or in the simplified equation mentioned in the lab (both gave the same results):

$$H(\omega) = \frac{1}{3}\ \frac{z^3 - 1}{z^2(z-1)}$$

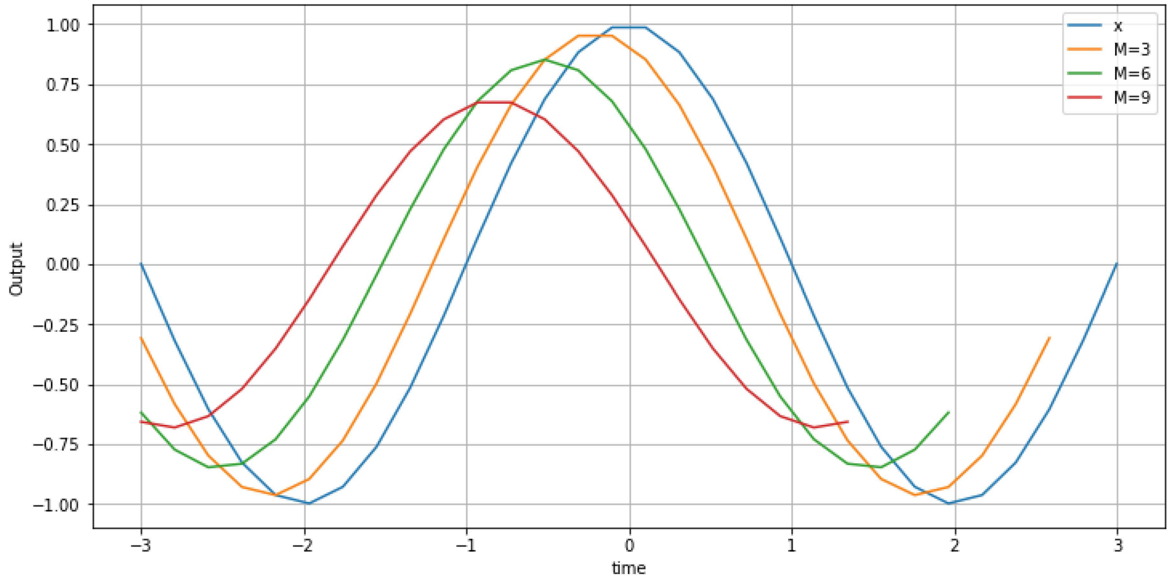
Then compute the absolute and logarithm for better visualization then plot the results in frequency domain, we obtain:



To Compute the Argument of $H(z)$ we can use the function $np.angle(H)$ which returns the angles (phase) of $H(\omega)$, then plot the results in frequency domain:



For Implementing the Moving Average Filter for the given signal $x = cos\left(\frac{1}{2}\pi t\right)$, I used $np.convolve()$ and divided it by M, Then tested my implementation by substituting $M = 3, 6, 9$ and plotting the results, we obtain:



Task 2: The Window Method

For Implementation of the 4 functions of window, I used the definition of each window as following:

The Bartlett window is defined as:

$$w(n) = \frac{2}{M-1}\left(\frac{M-1}{2} - \left| n - \frac{M-1}{2} \right|\right)$$

The Blackman window is defined as:

$$w(n) = 0.42 - 0.5\, cos(2\pi n/M) + 0.08\, cos(4\pi n/M)$$

The Hamming window is defined as:

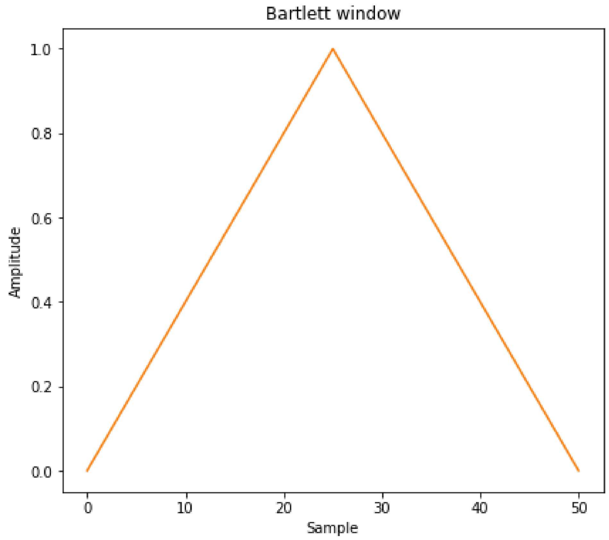
$$w(n) = 0.54 - 0.46\, cos\left(\frac{2\pi n}{M-1}\right),\ 0 \le n \le M-1$$

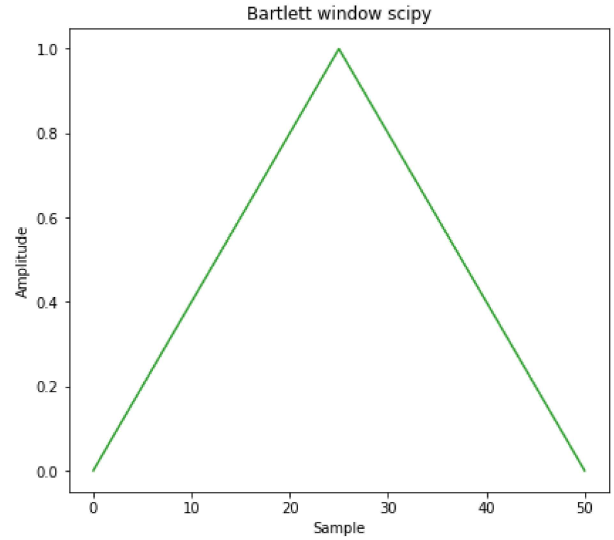
The Hann window is defined as:

$$w(n) = 0.5 - 0.5\, cos\left(\frac{2\pi n}{M-1}\right),\ 0 \le n \le M-1$$

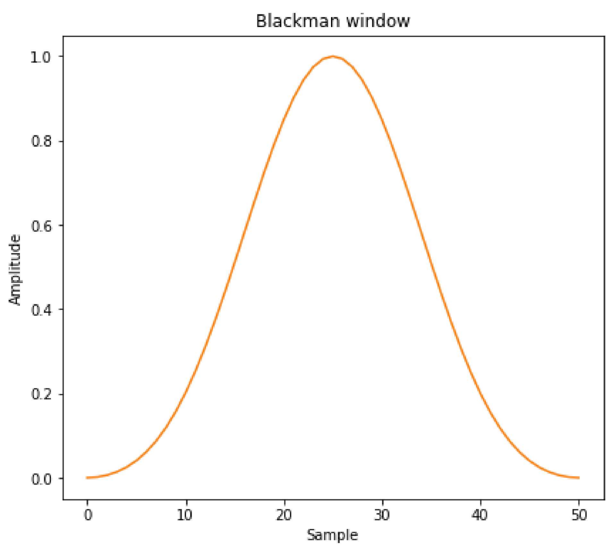
And comparing Each of the previous implementation with the ones in the scipy.signal.windows library, Then plotting, we obtain the following graphs:

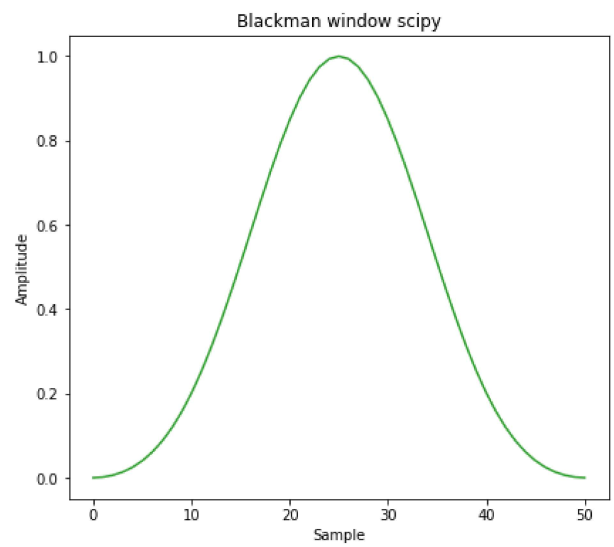
Bartlett window:



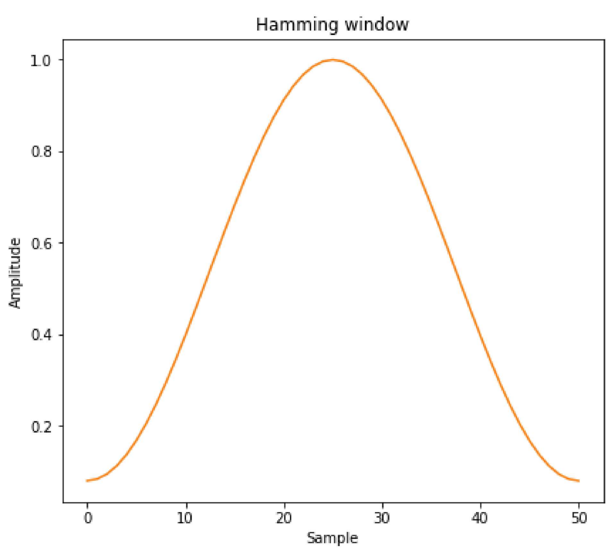


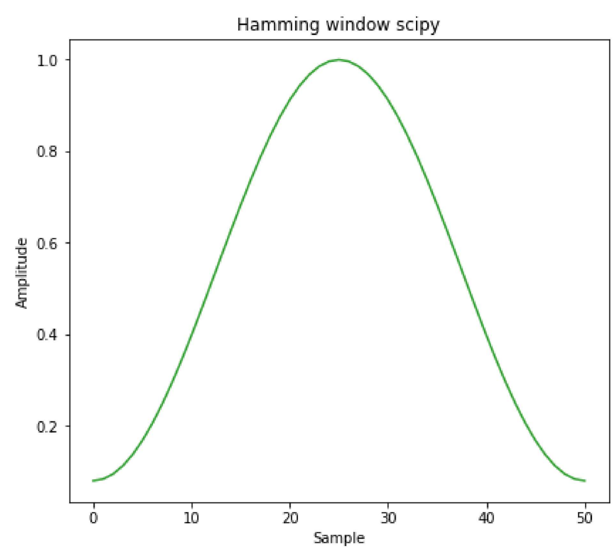
Blackman window:



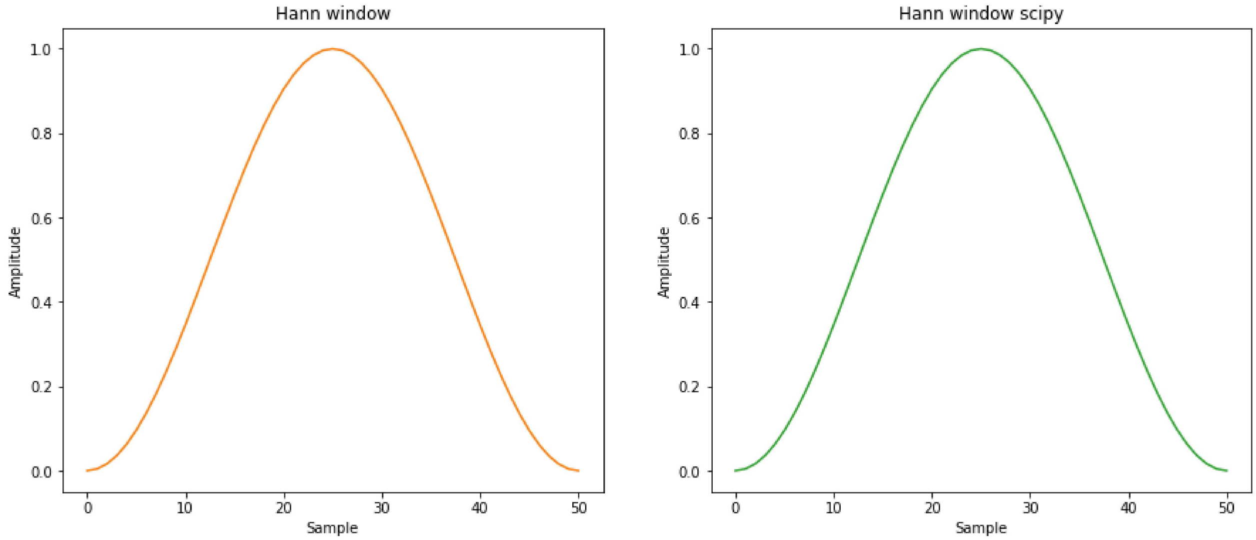


Hamming window:

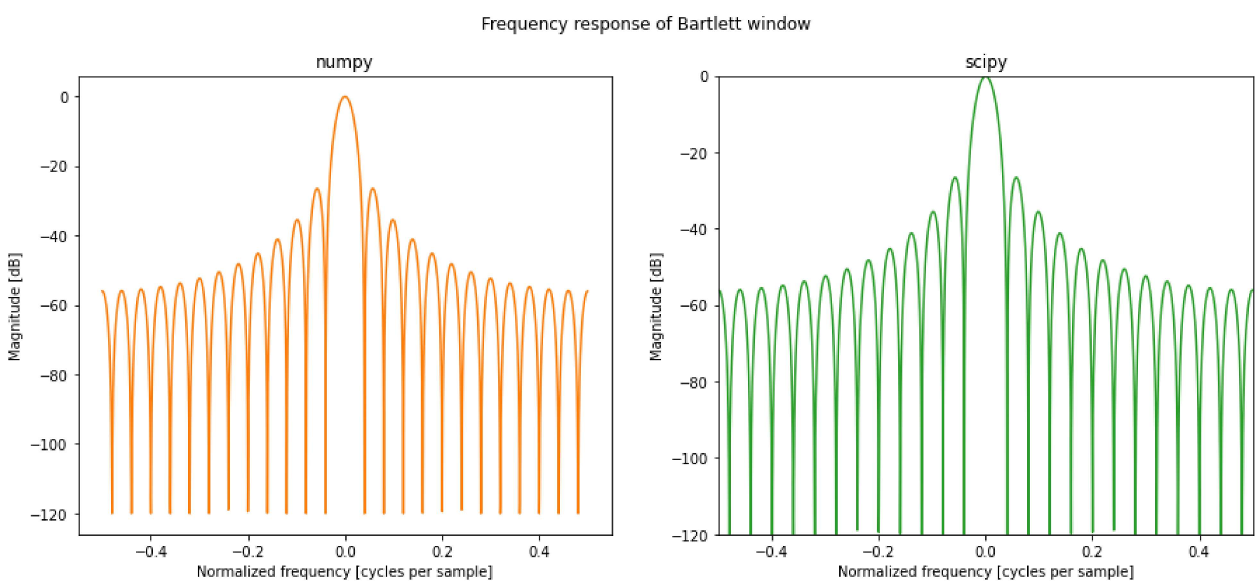


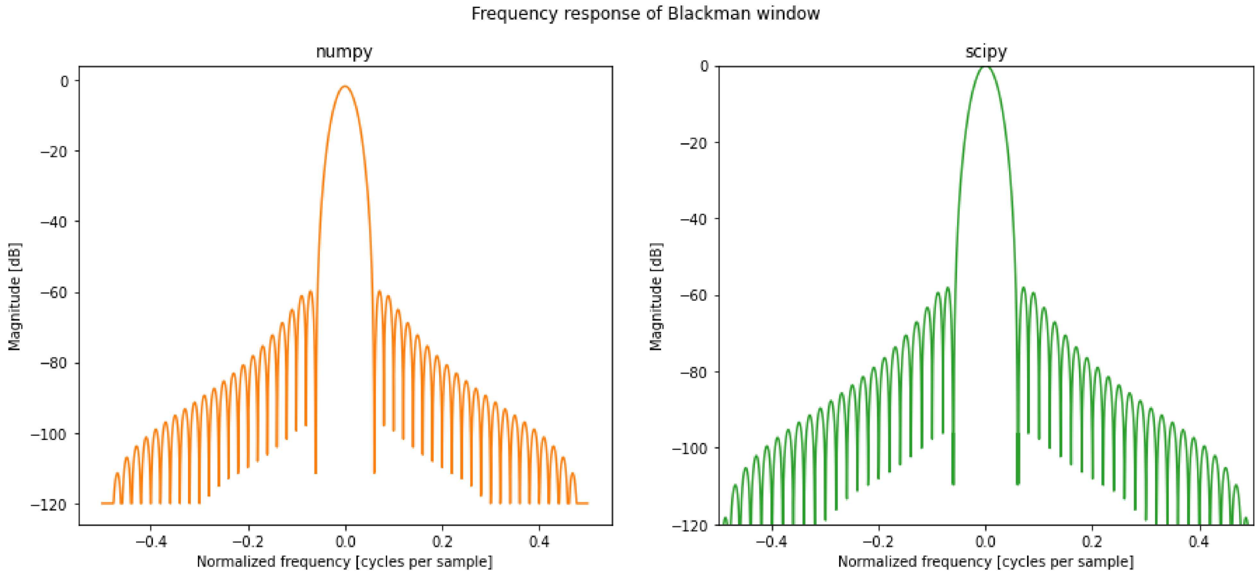


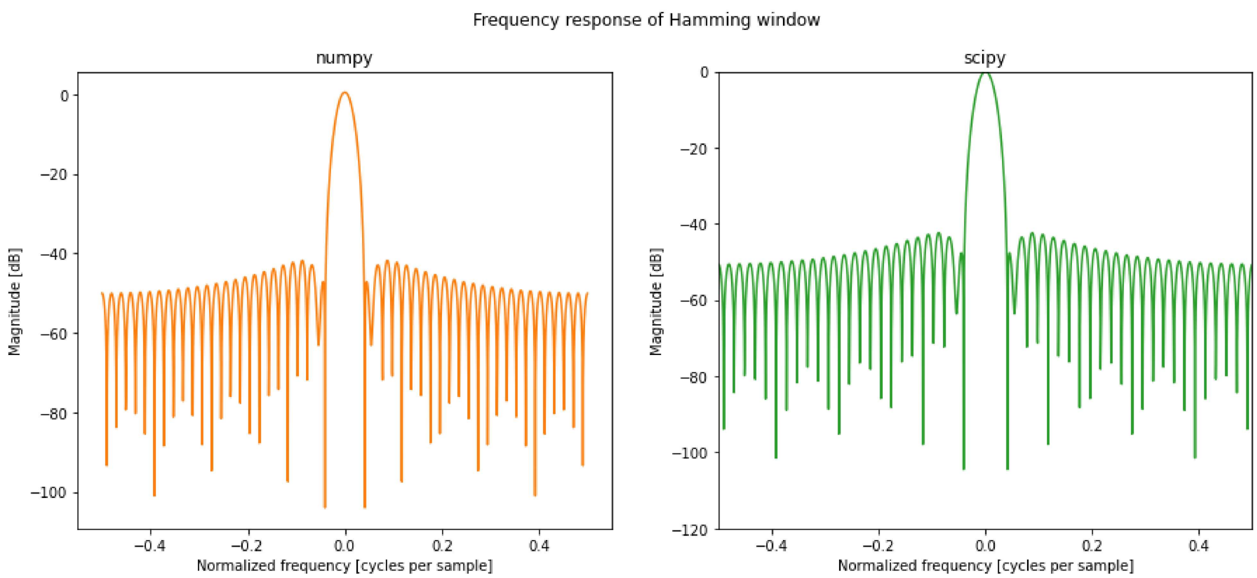
Hann window:

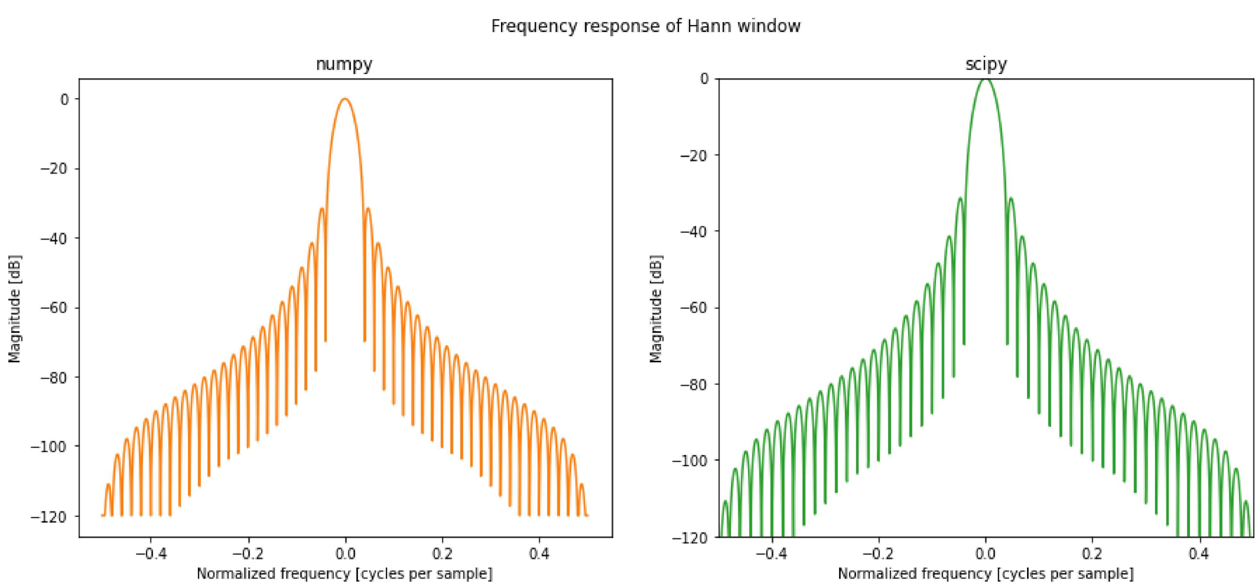


To compute the Frequency Response, I applied the Fourier Transform $fft()$ for all window functions output, then shifted the spectrum to the center and took the absolute value and logarithm for better visualization, and finally plotting the responses in the frequency domain, we obtain the following graphs:









We can notice that my implementation is almost identical to the one in signal.window library, I also used $np.bartlett()$, $np.blackman()$,$np.hamming()$, $np.hanning()$ to compare with my implementation and it also leads to the same results.