Digital Signal Processing EE 351M
Prof. Haris Vikalo HW #8
March 29, 2018 Due: 04/05/18

Homework Set #8

1. (4 points) Let $x(t) = 0.4 + 0.5cos(2\pi f_1 t) + 2cos(2\pi f_2 t) + sin(2\pi f_2 t) + 1.5cos(2\pi f_3 t)$, where $f_1 = 3kHz$, $f_2 = 5kHz$, and $f_3 = 8kHz$. If $x[n]$ is obtained by sampling $x(t)$ with a sampling frequency $f_s = 16kHz$, what is the minimum number of data points needed for a DFT in order to resolve all the frequencies without spectrum leakage?

2. (5 points) In sound systems, if the music is turned up very loud, the sound begins to distort. Typically, the reason for the distortion is that the magnitude of the amplified output signal exceeds the DC power supply level of the amplifier (and thus the device no longer operates in its linear range). As a result, the signal is *clipped* at some lower and upper limits. To illustrate this phenomenon, consider a cosine input signal which is sampled to give

$$x[n] = 10\cos(\pi n/16).$$

Suppose this signal is passed through a nonlinearity with a clipping interval $[-8, 8]$, resulting in a signal

$$y[n] = clip[x[n], -8, 8] \triangleq \begin{cases} -8, & -\infty < x[n] < -8 \\ x[n], & -8 \leq x[n] \leq 8 \\ 8, & 8 < x[n] < \infty \end{cases}$$

Plot the magnitudes of the 32-point DFTs of $x[n]$ and $y[n]$ (for $0 \leq n \leq 31$) on two separate graphs. Please comment on the result.

3. (5 points) Let $x[n] = cos(\frac{17\pi n}{64}) + 2sin(\frac{23\pi n}{32})$ for $0 \leq n \leq 255$, and 0 otherwise. The signal $x[n]$ is passed through an LTI system with transfer function $H(z) = (1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})$. Denote the signal at the output of the system by $y[n]$. We want to recover $x[n]$ by passing $y[n]$ through an inverse system.

   (a) Find (analytically) an impulse response $g[n]$ of the inverse system, $G(z) = \frac{1}{H(z)}$. (Note: the inverse system is an IIR system.)

   (b) Apply a rectangular window of length $M = 8$,

$$w[n] = \begin{cases} 1 & 0 \leq n \leq 7, \\ 0 & \text{otherwise,} \end{cases}$$

   to obtain an FIR filter $r[n] = g[n] \cdot w[n]$. Please plot the frequency response of the FIR filter. Compare it with the frequency response of the IIR inverse system Is $M = 8$ an appropriate choice?

   (c) Compute $\hat{x}[n] = r[n] * y[n]$. Plot $x[n]$, $y[n]$, and $\hat{x}[n] - x[n]$.

4. (4 points) A continuous-time filter has the following frequency response:

$$H_c(j\Omega) = \begin{cases} |\Omega|, & |\Omega| < 10\pi \\ 0, & |\Omega| > 10\pi \end{cases}$$

If a discrete-time filter is designed by applying the impulse invariance method to $H_c(j\Omega)$, find the frequency response of the discrete-time filter, $H(e^{j\omega})$. [Assume $T_d \geq 0.1$.]