Neil Adrian B. Baltar

NO.:
DATE: 8/20/25

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Given:

$R_{school} = \begin{bmatrix} +5 \\ +3 \end{bmatrix}$ $R_{stay} = \begin{bmatrix} -5 \\ +1 \end{bmatrix}$ $P_{school} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$ $P_{stay} = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$

State wise average reward (r_π)

1. Sunny

$r_\pi = 0.5(5) + 0.5(-5) = 2.5 + (-2.5) = \boxed{0}$

2. Cloudy

$r_\pi = 0.5(3) + 0.5(1) = 1.5 + 0.5 = \boxed{2}$

3. Matrix

$$\boxed{r_\pi = \begin{bmatrix} 0 \\ 2 \end{bmatrix}}$$

Policy Transition Matrix

Row 1 (Sunny)

- $P_\pi(1,1) = 0.5 \times 0.8 + 0.5 \times 0.9 = 0.40 + 0.45 = 0.85$
- $P_\pi(1,2) = 0.5 \times 0.2 + 0.5 \times 0.1 = 0.10 + 0.05 = 0.15$

Row 2 (Cloudy)

- $P_\pi(2,1) = 0.5 \times 0.4 + 0.5 \times 0.3 = 0.20 + 0.15 = 0.35$
- $P_\pi(2,2) = 0.5 \times 0.6 + 0.5 \times 0.7 = 0.30 + 0.35 = 0.65$

4. Matrix

$$P_\pi = \begin{bmatrix} 0.85 & 0.15 \\ 0.35 & 0.65 \end{bmatrix}$$

Bellman Equation

Sunny:

$V_1 = 0 + 0.9(0.85V_1 + 0.15V_2)$

$V_1 = 0 + 0.765V_1 + 0.135V_2$

$V_1 - 0.765V_1 - 0.135V_2 = 0$

$0.235V_1 - 0.135V_2 = 0$

Cloudy:

$V_2 = 2 + 0.9(0.35V_1 + 0.65V_2)$

$V_2 = 2 + 0.315V_1 + 0.585V_2$

$V_2 - 0.315V_1 - 0.585V_2 = 2$

$-0.315V_1 + 0.415V_2 = 2$

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Solve for V_1:

$0.235V_1 - 0.135V_2 = 0$

$0.235V_1 = 0 + 0.135V_2$

$\frac{0.235V_1}{0.235} = \frac{0 + 0.135V_2}{0.235}$

5. $\boxed{V_1 = \frac{0 + 0.135V_2}{0.235}}$

Solve for V_2:

$-0.315V_1 + 0.415V_2 = 2$

$-0.315\left(\frac{0 + 0.135V_2}{0.235}\right) + 0.415V_2 = 2$

$(-0.315 \times 0) - (0.315 \times \frac{0.135V_2}{0.235}) + 0.415V_2 = 2$

$(-0.315 \times 0) - (0.315 \times 0.574468V_2) + 0.415V_2$

$-0.1810V_2 + 0.415V_2 = 2$

$0.234V_2 = 2$

6. $\boxed{V_2 = \frac{2}{0.234}}$

7. Find $V\pi$ (cloudy)

$V\pi(\text{cloudy}) = \frac{2}{0.234}$

$\boxed{V\pi(\text{cloudy}) = 8.547}$

8. Find $V\pi$ (sunny)

$V_1 = \frac{0 + 0.135V_2}{0.235}$

$V_1 = \frac{0 + 0.135(8.547)}{0.235} = \frac{1.1538}{0.235}$

$\boxed{V\pi(\text{sunny}) = 4.910}$

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Bellman Equation (Optimality)

Sunny (V_1) using school:

9. $V^*(Csunny) = 5 + 0.9(0.8V_1 + 0.2V_2)$

Cloudy (V_2) using school:

10. $V^*(cloudy) = 3 + 0.9(0.4V_1 + 0.6V_2)$

Find V_1 and V_2.

$V_1 = 5 + 0.72V_1 + 0.18V_2$

$V_1 - 0.72V_1 - 0.18V_2 = 5$

$0.28V_1 - 0.18V_2 = 5$

$0.28V_1 = 5 + 0.18V_2$

$V_1 = \frac{5 + 0.18V_2}{0.28}$

$V_2 = 3 + \cancel{0.9(0.4V_1 + 0.6V_2)}$ $0.36V_1 + 0.54V_2$

$V_2 - 0.36V_1 - 0.54V_2 = 3$

$-0.36V_1 + 0.46V_2 = 3$

Find $V^*(Csunny)$ and $V^*(cloudy)$

$V^*(Cloudy)$:

$-0.36\left(\frac{5 + 0.18V_2}{0.28}\right) + 0.46V_2 = 3$

$(-0.36 \times \frac{5}{0.28})(-0.36 \times \frac{0.18V_2}{0.28}) + 0.46V_2 = 3$

$(-0.36 \times 17.857)(-0.36 \times 0.642857V_2) + 0.46V_2 = 3$

$-6.42852 - 0.23143V_2 + 0.46V_2 = 3$

$0.22857V_2 = 3 + 6.42852$

$\frac{0.22857V_2}{0.22857} = \frac{9.42852}{0.22857}$

11. $V^*(cloudy) = 41.25$

$V^*(sunny)$:

$V_1 = \frac{5 + 0.18(41.25)}{0.28}$

$V_1 = \frac{12.425}{0.28}$

12. $V^*(sunny) = 44.375$

Solve for q^*:

13. $q(1, school) = 44.375$

14. $q(1, Home)$

15. $q(2, school) = 41.25$

16. $q(2, Home)$