

# Kinematics Calculations for Hexapod Robot

Nabeel Chowdhury

October 6, 2021

## 1 Servo Angles for each Leg

Looking from above, the first angle of each leg that makes contact with the floor at  $P(x, y, z)$  is found as  $\theta_C = \tan^{-1} \left( \frac{L_y}{L_x} \right) = \tan^{-1} \left( \frac{P_y}{P_x} \right)$ .

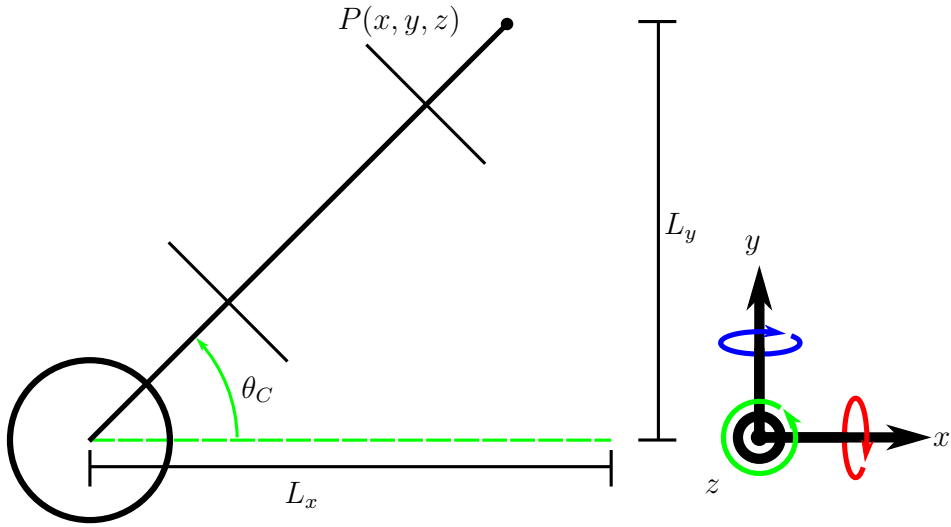


Figure 1: The top view of the leg.  $\theta_C$  is the Coax angle of the servo attached to the body of the Hexapod.  $L_x$  is the length of the leg from the body in the x direction and  $L_y$  is the length of the leg in the y direction.

Looking from the side of the leg, we can define several parameters.

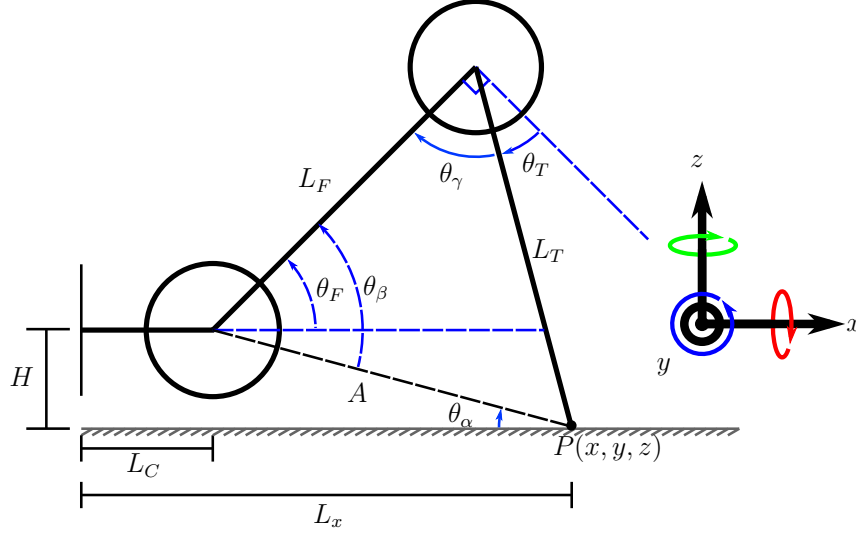


Figure 2: The side view of the leg.  $\theta_F$  is the Femur angle and  $\theta_T$  is the tibia angle.  $L_C$  is the offset of the femur servo from the body, or the coax length.  $L_F$  is the length of the Femur and  $L_T$  is the length of the Tibia.  $L_x$  is the x distance of the leg's point of contact with the ground.  $H$  is the height of the body off of the ground. The rest of the parameters are needed in interim steps to find  $\theta_F$  and  $\theta_T$ .

First, we need to find the length of  $A$ .

$$A = \sqrt{H^2 + (L_x - L_C)^2} = \sqrt{P_z^2 + (P_x - L_C)^2}$$

Using  $A$  we can use the law of cosines to find  $\theta_F$ .

$$\begin{aligned} \theta_F &= \theta_\beta - \theta_\alpha \\ \theta_\alpha &= \tan^{-1} \left( \frac{H}{L_x - L_C} \right) \\ L_T^2 &= L_F^2 + A^2 - 2L_F A \cos(\theta_\beta) \\ \therefore \theta_F &= \cos^{-1} \left( \frac{L_T^2 - L_F^2 - A^2}{-2L_F A} \right) - \tan^{-1} \left( \frac{H}{P_x - L_C} \right) \end{aligned}$$

We can also use the law of cosines to find the tibia angle as well.

$$\begin{aligned}
-\theta_T &= 90^\circ - \theta_\gamma \\
A^2 &= L_F^2 + L_T^2 - 2L_FL_T \cos(\theta_\gamma) \\
\therefore \theta_T &= \cos^{-1} \left( \frac{A^2 - L_F^2 - L_T^2}{-2L_FL_T} \right) - 90^\circ
\end{aligned}$$

In summary:

$$\begin{aligned}
&\boxed{\theta_C = \tan^{-1} \left( \frac{P_y}{P_x} \right)} \\
&\boxed{\theta_F = \cos^{-1} \left( \frac{L_T^2 - L_F^2 - P_z^2 - (P_x - L_C)^2}{-2L_F \left( \sqrt{P_z^2 + (P_x - L_C)^2} \right)} \right) - \tan^{-1} \left( \frac{-P_z}{P_x - L_C} \right)} \\
&\boxed{\theta_T = \cos^{-1} \left( \frac{P_z^2 + (P_x - L_C)^2 - L_F^2 - L_T^2}{-2L_FL_T} \right) - 90^\circ}
\end{aligned}$$