Kinematics Calculations for Hexapod Robot

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1 Servo Angles for each Leg

Looking from above, the first angle of each leg that makes contact with the floor at $P(x, y, z)$ is found as $\theta_C = \tan^{-1}\left(\frac{L_y}{L_x}\right) = \tan^{-1}\left(\frac{P_y}{P_x}\right)$.

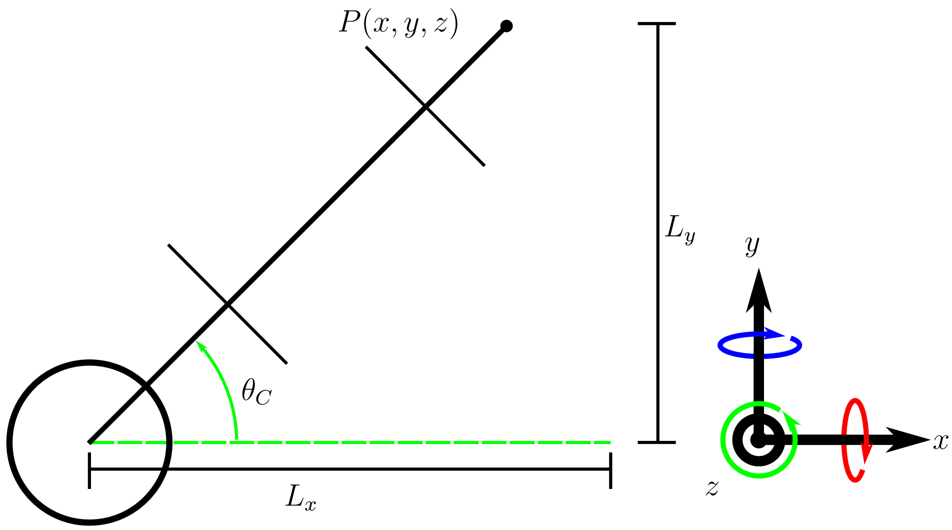

Figure 1: The top view of the leg. θ_C is the Coax angle of the servo attached to the body of the Hexapod. L_x is the length of the leg from the body in the x direction and L_y is the length of the leg in the y direction.

Looking from the side of the leg, we can define several parameters.

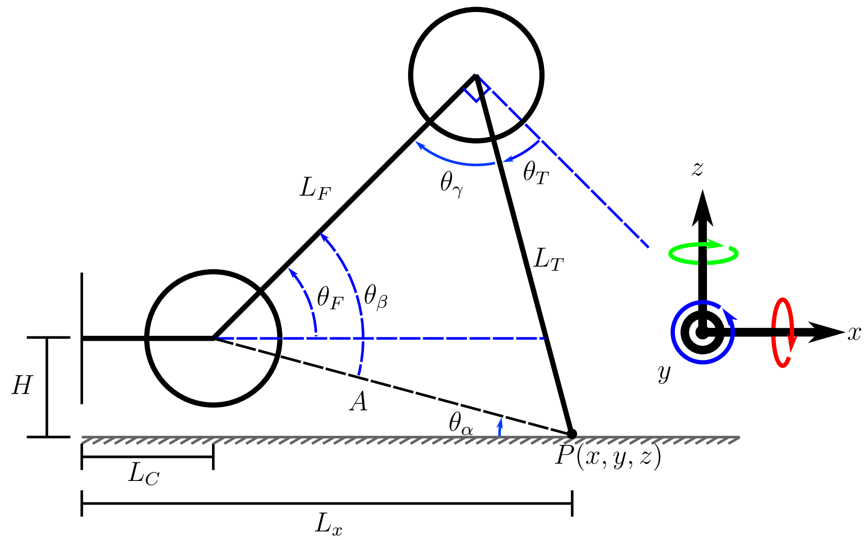

Figure 2: The side view of the leg. θ_F is the Femur angle and θ_T is the tibia angle. L_C is the offset of the femur servo from the body, or the coax length. L_F is the length of the Femur and L_T is the length of the Tibia. L_x is the x distance of the leg's point of contact with the ground. H is the height of the body off of the ground. The rest of the parameters are needed in interim steps to find θ_F and θ_T.

First, we need to find the length of A.

$$A = \sqrt{H^2 + (L_x - L_C)^2} = \sqrt{P_z^2 + (P_x - L_C)^2}$$

Using A we can use the law of cosines to find θ_F.

$$\theta_F = \theta_\beta - \theta_\alpha$$

$$\theta_\alpha = \tan_1\left(\frac{H}{L_x - L_C}\right)$$

$$L_T^2 = L_F^2 + A^2 - 2L_F A\cos(\theta_\beta)$$

$$\therefore \theta_F = \cos^{-1}\left(\frac{L_T^2 - L_F^2 - A^2}{-2L_F A}\right) - \tan^{-1}\left(\frac{H}{P_x - L_C}\right)$$

We can also use the law of cosines to find the tibia angle as well.

$$
\begin{aligned}
-\theta_T &= 90^\circ - \theta_\gamma \\
A^2 &= L_F^2 + L_T^2 - 2L_F L_T \cos(\theta_\gamma) \\
\therefore \theta_T &= \cos^{-1}\left(\frac{A^2 - L_F^2 - L_T^2}{-2L_F L_T}\right) - 90^\circ
\end{aligned}
$$

In summary:

$$\boxed{\theta_C = \tan^{-1}\left(\frac{P_y}{P_x}\right)}$$

$$\boxed{\theta_F = \cos^{-1}\left(\frac{L_T^2 - L_F^2 - P_z^2 - (P_x - L_C)^2}{-2L_F\left(\sqrt{P_z^2 + (P_x - L_C)^2}\right)}\right) - \tan^{-1}\left(\frac{-P_z}{P_x - L_C}\right)}$$

$$\boxed{\theta_T = \cos^{-1}\left(\frac{P_z^2 + (P_x - L_C)^2 - L_F^2 - L_T^2}{-2L_F L_T}\right)} - 90^\circ$$