Kinematics Calculations for Hexapod Robot

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1 Servo Angles for each Leg

Looking from above, the first angle of each leg that makes contact with the floor at $P(x, y, z)$ is found as $\theta_C = \tan^{-1}\left(\frac{L_y}{L_x}\right) = \tan^{-1}\left(\frac{P_y}{P_x}\right)$.

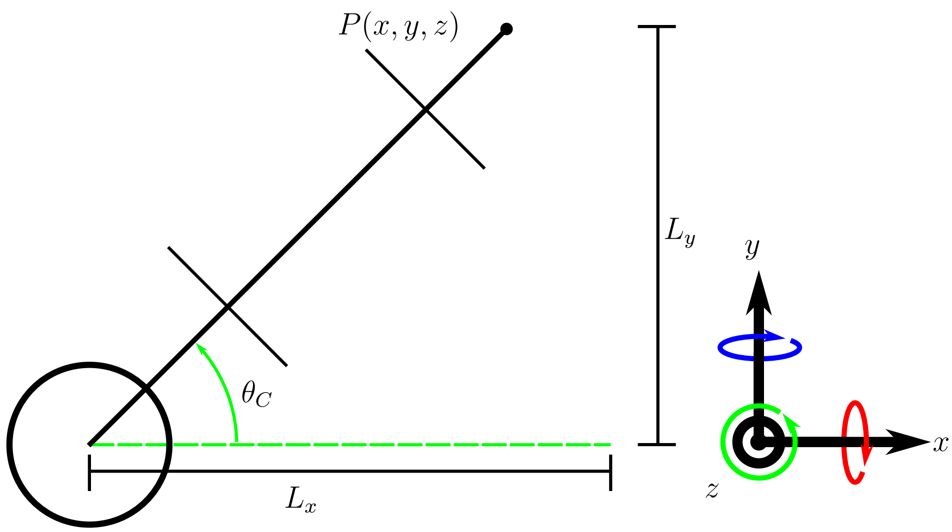

Figure 1: The top view of the leg. θ_C is the Coax angle of the servo attached to the body of the Hexapod. L_x is the length of the leg from the body in the x direction and L_y is the length of the leg in the y direction.

Looking from the side of the leg, we can define several parameters.

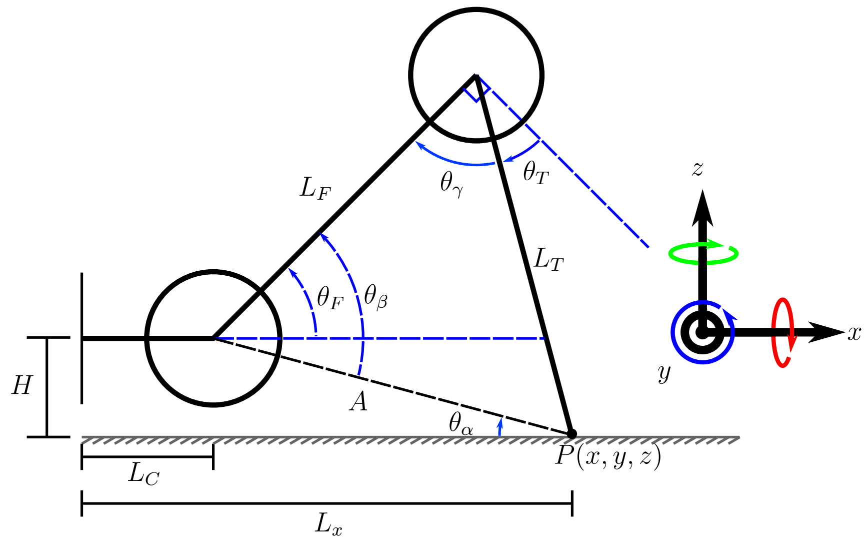

Figure 2: The side view of the leg. θ_F is the Femur angle and θ_T is the tibia angle. L_C is the offset of the femur servo from the body, or the coax length. L_F is the length of the Femur and L_T is the length of the Tibia. L_x is the x distance of the leg's point of contact with the ground. H is the height of the body off of the ground. The rest of the parameters are needed in interim steps to find θ_F and θ_T.

First, we need to find the length of A.

$$A = \sqrt{H^2 + (L_x - L_C)^2} = \sqrt{P_z^2 + (P_x - L_C)^2}$$

Using A we can use the law of cosines to find θ_F.

$$\theta_F = \theta_\beta - \theta_\alpha$$

$$\theta_\alpha = \tan^{-1}\left(\frac{H}{L_x - L_C}\right)$$

$$L_T^2 = L_F^2 + A^2 - 2L_F A\cos(\theta_\beta)$$

$$\therefore \theta_F = \cos^{-1}\left(\frac{L_T^2 - L_F^2 - A^2}{-2L_F A}\right) - \tan^{-1}\left(\frac{H}{P_x - L_C}\right)$$

We can also use the law of cosines to find the tibia angle as well.

$$
\begin{aligned}
-\theta_T &= 90^\circ - \theta_\gamma \\
A^2 &= L_F^2 + L_T^2 - 2L_F L_T \cos(\theta_\gamma) \\
\therefore \theta_T &= \cos^{-1}\left(\frac{A^2 - L_F^2 - L_T^2}{-2L_F L_T}\right) - 90^\circ
\end{aligned}
$$

In summary:

$$
\boxed{\theta_C = \tan^{-1}\left(\frac{P_y}{P_x}\right)}
$$

$$
\boxed{\theta_F = \cos^{-1}\left(\frac{L_T^2 - L_F^2 - P_z^2 - (P_x - L_C)^2}{-2L_F\left(\sqrt{P_z^2 + (P_x - L_C)^2}\right)}\right) - \tan^{-1}\left(\frac{-P_z}{P_x - L_C}\right)}
$$

$$
\boxed{\theta_T = \cos^{-1}\left(\frac{P_z^2 + (P_x - L_C)^2 - L_F^2 - L_T^2}{-2L_F L_T}\right) - 90^\circ}
$$

2 Converting Leg Origin to Body Origin

In order to find how all of the legs will move together, the origins of each leg need to be shifted from their body attachment points to the body center. Shown in the image below is a top down view of the hexapod body with the leg attachment points labeled as x_n, y_n, z_n with $n = 0, 1, 2, 3, 4, 5$ to correspond to the leg numbers in the code. in the default position, the offset of the legs from the body are a known number, but each offset will change as the body rotates in place. This will be explored later.

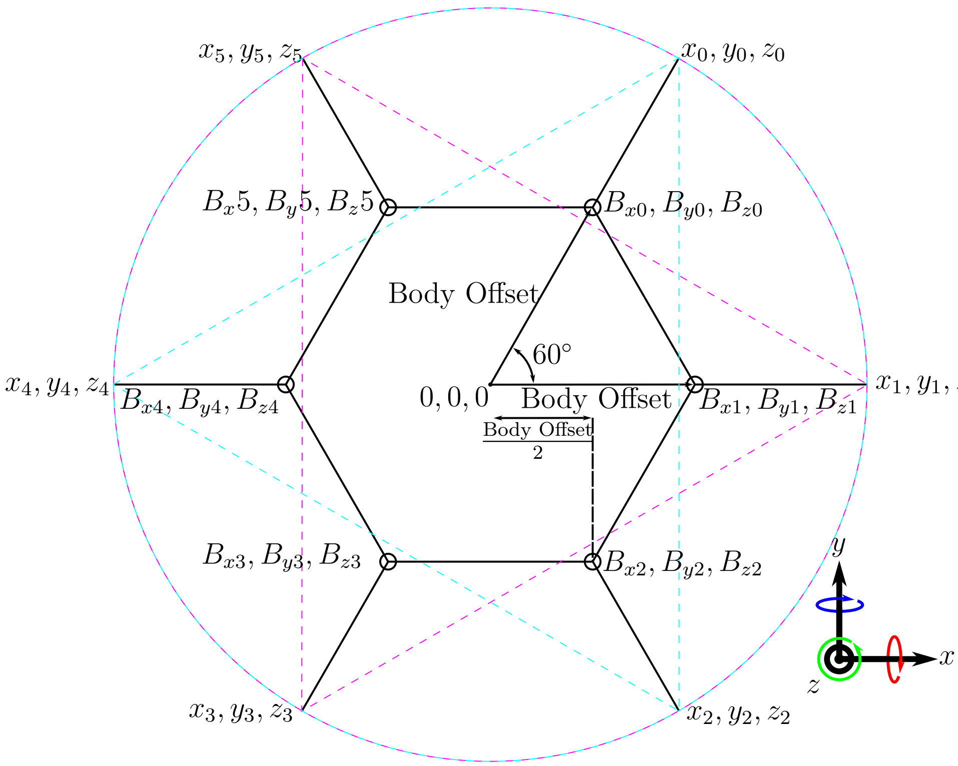

Figure 3: The top view of the whole body with each leg attachment point labeled with x, y, and x values.

The position of each leg endpoint can be found using the body offset plus the leg's position from the body attachment.

$x_n = B_{xn} + P_{xn}$ where: $n = 0, 1, 2, 3, 4, 5$
$y_n = B_{yn} + P_{yn}$ where: $n = 0, 1, 2, 3, 4, 5$
$z_n = B_{zn} + P_{zn}$ where: $n = 0, 1, 2, 3, 4, 5$

2.1 Body Offset Changes Due to Rotation

Each body offset will change based on the rotation of the body. This could be solved using trigonometry like for the legs, but a simpler, global solution is to use rotation matrices and find where each leg attachment point is moved to in space. A change in pitch is a rotation around the x axis, a change in roll is a rotation around the y axis, and a change in yaw is a rotation around the z axis.

2.1.1 Rotation Matrices

The rotation matrices are below:

Rotation About the X Axis

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

Rotation About the Y Axis

$$R_y = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

Rotation About the Z Axis

$$R_z = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find the new endpoints, we do the following calculation:

$$\begin{bmatrix} B'_{xn} \\ B'_{yn} \\ B'_{zn} \end{bmatrix} = (R_z R_y R_x)^{-1} \begin{bmatrix} B_{xn} \\ B_{yn} \\ B_{zn} \end{bmatrix} \text{ where: } n = 0, 1, 2, 3, 4, 5$$

$$\therefore \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = (R_z R_y R_x)^{-1} \begin{bmatrix} B_{xn} \\ B_{yn} \\ B_{zn} \end{bmatrix} + \begin{bmatrix} P_{xn} \\ P_{yn} \\ P_{zn} \end{bmatrix} \text{ where: } n = 0, 1, 2, 3, 4, 5$$

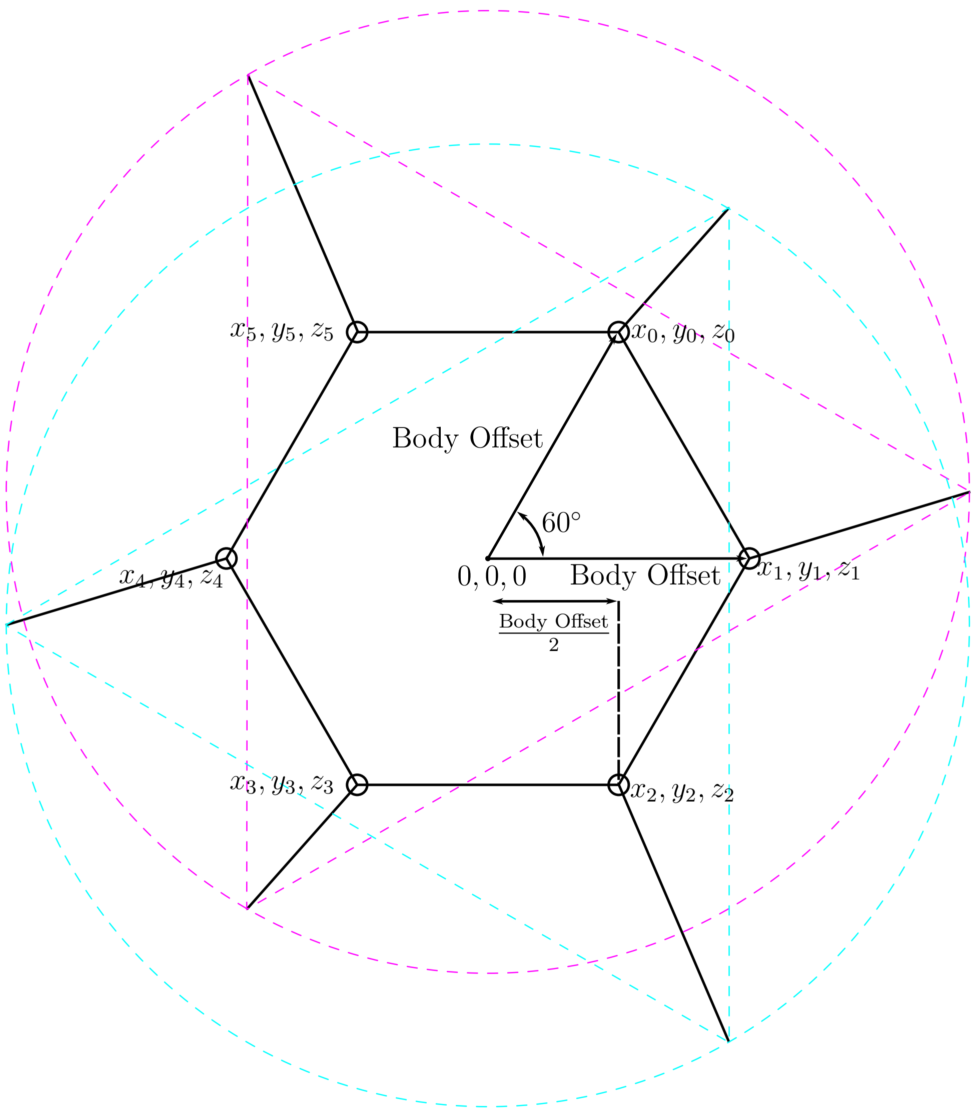

Figure 4: The top view of the whole body when the legs that make the magenta triangle take a forward step.