Modeling Pan and Tilt of Laser Mount

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1   Description of Laser Mount Measurements
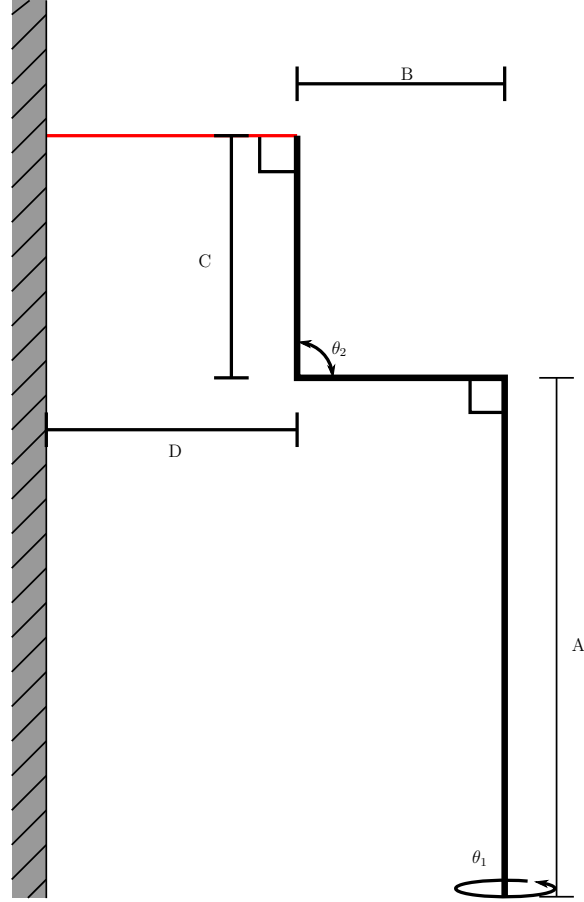

Figure 1: **A** is the height of the panning arm and θ_1 is the rotation of the panning arm. **B** is the offset distance of the tilting arm. It is perpendicular to the panning arm. **C** is the length of the tilting arm. **D** is the distance of the base of the tilting arm to the target. This distance changes with panning angle, but stays constant with tilt.

2 Finding Tilt Angle
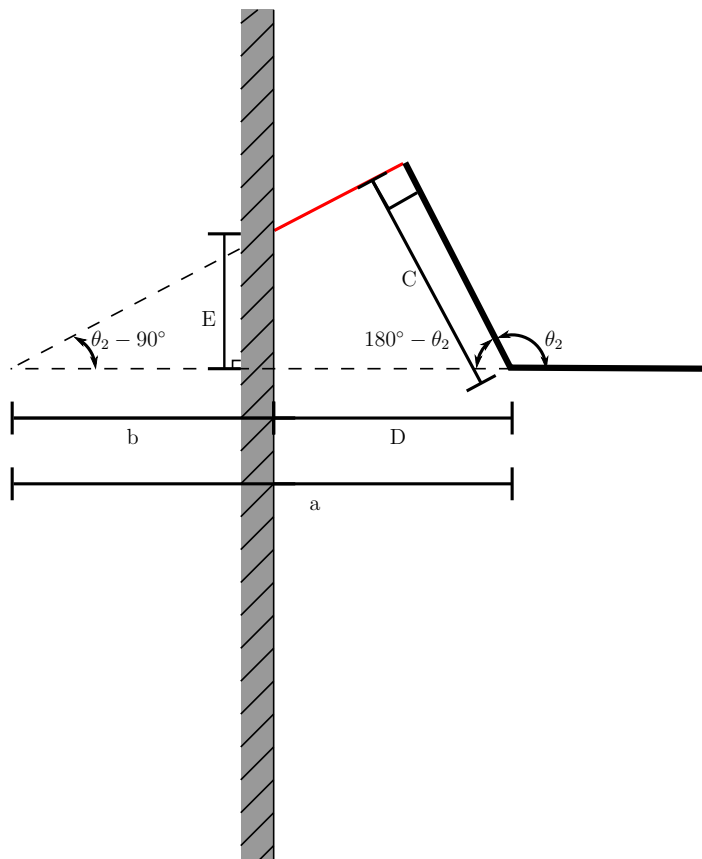
2.a Laser Hit above A and Below Horizontal

Figure 2: **D** is target's distance from the base of the tilting arm. **E** is the height above the base of the tilting arm that the laser hits its target. **a** is the hypotenuse of the extended triangle from the extrapolated intersection of the laser with the level of the base of the tilting arm. **b** is the hypotenuse of the extended triangle minus the distance of the base of the tilting arm to the target.

In order to find the height of the height of the target hit location, $\mathbf{E}$, we need to find $\mathbf{b}$.

$$b = a - D$$

$$a = \frac{C}{\sin(\theta_2 - 90°)} = \frac{C}{-\cos(\theta_2)}$$

$$\therefore b = -\frac{C}{\cos(\theta_2)} + D$$

$$-\frac{\mathbf{E}}{\frac{C}{\cos(\theta_2)} + D} = \tan(\theta_2 - 90°) = -\cot(\theta_2)$$

$$\therefore \mathbf{E} = \left[D + \frac{C}{\cos(\theta_2)}\right]\cot(\theta_2) = D\cot(\theta_2) + \frac{C}{\sin(\theta_2)}$$

Now assuming we want to input in a target height to hit, we need to extract θ_2 from the equation.

$$\text{If } x = \tan(\frac{\theta_2}{2}), \ \sin(\theta_2) = \frac{2x}{1+x^2}, \ \cos(\theta_2) = \frac{1-x^2}{1+x^2}, \ \tan(\theta_2) = \frac{2x}{1-x^2}$$

$$\mathbf{E} = D\frac{1-x^2}{2x} + C\frac{1+x^2}{2x}$$

$$2\mathbf{E}x = D(1-x^2) + C(1+x^2)$$

$$0 = (C - D)\,x^2 - 2\mathbf{E}x + C + D$$

$$x = \frac{2\mathbf{E} \pm \sqrt{4\mathbf{E}^2 - 4\,(C - D)}}{2\,(C - D)} = \tan(\frac{\theta_2}{2})$$

$$\boxed{\therefore \ \theta_2 = 2\tan^{-1}\left[\frac{2\mathbf{E} \pm \sqrt{4\mathbf{E}^2 - 4\,(C - D)}}{2\,(C - D)}\right]}$$

The actual value of θ_2 is the positive angle that results from the inverse tangent calculation.

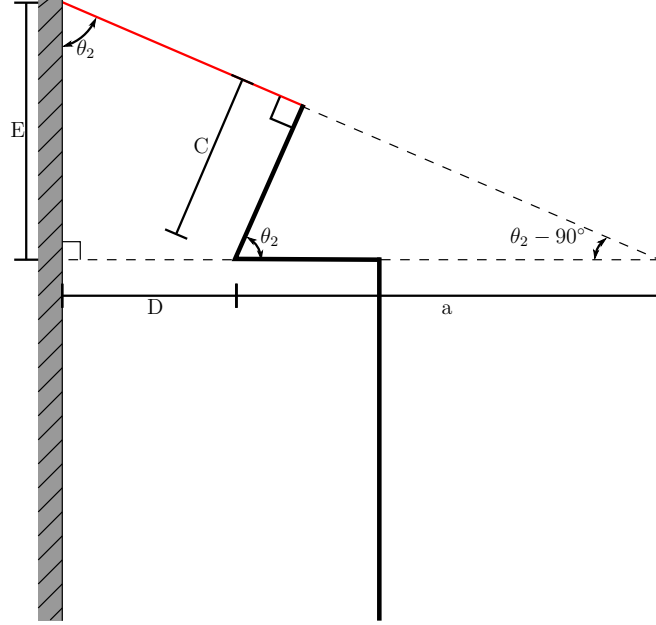## 2.b Laser Hit above A and Above Horizontal

Figure 3: **D** is target's distance from the base of the tilting arm. **E** is the height above the base of the tilting arm that the laser hits its target. **a** is the hypotenuse of the smaller extended triangle from the extrapolated intersection of the laser with the level of the base of the tilting arm.

In order to find θ_2, we first need to find **E**.

$$a = \frac{C}{\cos(\theta_2)}$$

$$\therefore \mathbf{E} = \frac{D + a}{\tan(\theta_2)} = \frac{D + \frac{C}{\cos(\theta_2)}}{\tan(\theta_2}$$

$$\mathbf{E} = D \cot(\theta_2) + \frac{C}{\sin(\theta_2)}$$

This is the same result as the previous calculation. Therefore,

$$\theta_2 = 2\tan^{-1}\left[\frac{2\mathbf{E} \pm \sqrt{4\mathbf{E}^2 - 4(C - D)}}{2(C - D)}\right]$$
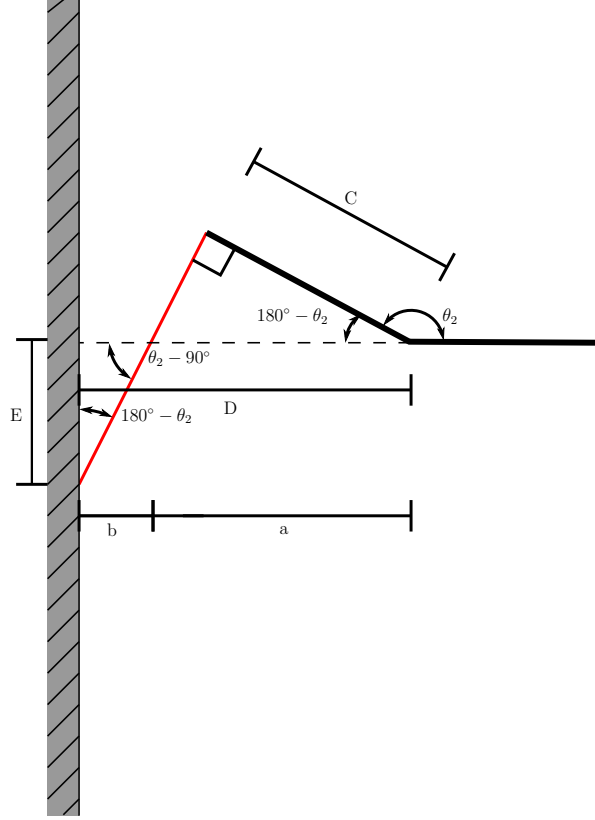
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2.c   Laser Hit below A

Figure 4: **D** is target's distance from the base of the tilting arm. **E** is the height below the base of the tilting arm that the laser hits its target. **a** is the distance from the base of the tilting arm to where the laser crosses the horitontal axis with respect to the base of the tilting arm. It is also the hypotenuse of the triangle made by the tilting arm and the laser crossing the horizontal made by the base of the tilting arm. **b** is the distance from the crossing to the target wall.

Once again, we need to find $\mathbf{E}$ based on the tilt angle.

$$b = D - a$$

$$a = \frac{C}{\cos(180° - \theta_2)} = -\frac{C}{\cos(\theta_2)}$$

$$\mathbf{E} = \frac{b}{\tan(180° - \theta_2)} = \frac{D + \frac{C}{\cos(\theta_2)}}{\tan(180° - \theta_2)} = -\frac{\frac{C}{\cos(\theta_2)} + D}{\tan(\theta_2)}$$

$$\mathbf{E} = -\frac{C}{\sin(\theta_2)} - D\cot(\theta_2)$$

Solving for θ_2:

If $x = \tan(\frac{\theta_2}{2})$, $\sin(\theta_2) = \frac{2x}{1 + x^2}$, $\cos(\theta_2) = \frac{1 - x^2}{1 + x^2}$, $\tan(\theta_2) = \frac{2x}{1 - x^2}$

$$\mathbf{E} = -C\frac{1 + x^2}{2x} - D\frac{1 - x^2}{2x}$$

$$2\mathbf{E}x = -C(1 + x^2) - D(1 - x^2)$$

$$0 = (C - D)x^2 + 2\mathbf{E}x + C + D$$

If we define E as negative when below the horizontal made by the base of the tilting arm, we get the same result as previously where.

$$\theta_2 = 2\tan^{-1}\left[\frac{2\mathbf{E} \pm \sqrt{4\mathbf{E}^2 - 4(C - D)}}{2(C - D)}\right]$$
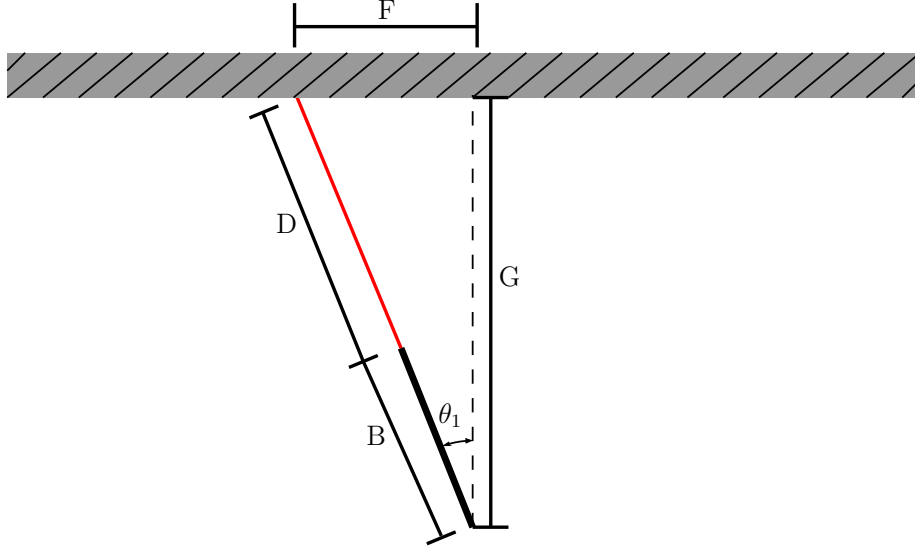
3   Finding Pan Angle

Figure 5: **B** is the length of the base of the tilting arm. **D** is target's distance from the base of the tilting arm. **F** horizontal distance of the target from center. **G** is the target's perpendicular distance from the panning arm.

Solving for θ_1:

$$\tan(\theta_1) = \frac{F}{G}$$

$$\boxed{\therefore \theta_1 = \tan^{-1}(\frac{F}{G})}$$

D is a different distance now that the arm has panned. To find the new distance of D needed in the tilt angle calculations, we do the following:

$$\cos(\theta_1) = \frac{G}{D + B}$$

$$D = \frac{G}{\cos(\theta_1)} - B$$