9.3 Select

Given array A and integer i such that $1 \le i \le n$

find i^{th} order statistic of A.

Select differs from Randomised-Select in the way the ~~key~~ 'pivot' element is chosen.

A	29	17	3	21	9	85	42	13	101	6	37	38	15	14	88	7	2	8

A	3	9	17	21	29	6	13	42	85	101	14	15	37	38	88	2	7	8

Array of Medians

17	42	37	7

obtain Median of Medians by a
recursive call to Select on Array of Medians

with

$i = \left\lfloor \frac{n+1}{2} \right\rfloor$

In this example we get:

Median of Medians = 17

Using 17 as pivot, partition A

8	3	9		$\overset{\Downarrow k}{17}$		19	101	38

if $i < k$ recurse on left

if $i > k$ recurse on right

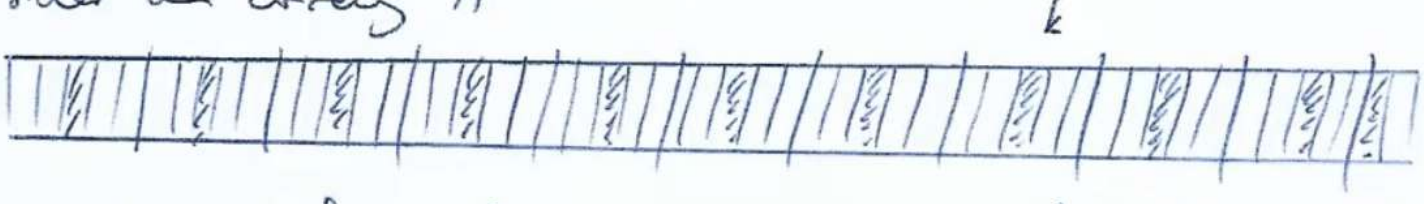
if $i = k$ return 17

Recurrence for Select:

$$T(n) = \Theta(n) + T(\lceil \tfrac{n}{5} \rceil) + \Theta(n) + T(\max(k-1, n-k))$$

Claim: $\max(k-1, n-k)$ is $\leq \frac{7n}{10} + 6$

consider an array A:



Let Median of Medians occur at k.

pivot

1. Group A into groups of size 5
with last group possibly having less than 5 elements.

$\therefore$ Number of groups is $\lceil \frac{n}{5} \rceil$.

2. Sort each group of 5 using InsertionSort
Then the middle element of each group of 5 is the median of that group.

3. Find Median of Medians using a recursive call to Select - this is the pivot.

4. Partition A around the pivot (at k).
5. Either return pivot or recurse left or recurse right depending on whether $i = k$, $i < k$ or $i > k$.

Let median of medians occur at k.

pivot
$\therefore A[k] \geq$ half of the medians

& each median $\geq$ 3 elements of its group.

except perhaps k's group & last group

So $A[k] \geq 3\left(\left\lceil \frac{1}{2}\left\lceil \frac{n}{5} \right\rceil \right\rceil - 2\right) \geq \frac{3n}{10} - 6$

So after the partitioning A around pivot there are at least $\frac{3n}{10} - 6$ elements left of the pivot.

In the worst case there are

$$n - \left(\frac{3n}{10} - 6\right) = \frac{7n}{10} + 6$$

elements to the right. Thus the Claim holds

$$T(n) = \Theta(n) + T(\lceil \tfrac{n}{5} \rceil) + \Theta(n) + T(\tfrac{7n}{10} + 6)$$

Use substitution method to show that

$T(n) \leq cn$

$$T(n) = T(\lceil \tfrac{n}{5} \rceil) + T(\tfrac{7n}{10} + 6) + an$$

Assume $T(m) \leq cm$ for all $m < n$.

Then $T(n) \leq c\lceil \tfrac{n}{5} \rceil + c(\tfrac{7n}{10} + 6) + an$

$\leq c(\tfrac{n}{5} + 1) + c(\tfrac{7n}{10} + 6) + an$

$$= \frac{9}{10} cn + 7c + an$$

$$= cn + \left(-\frac{1}{10} cn + 7c + an \right)$$

$$\leq cn \quad \text{iff} \quad -\frac{1}{10} cn + 7c + an \leq 0$$

$$\iff \quad an \leq c\left(\frac{n}{10} - 7 \right)$$

$$\iff \quad c \geq \frac{a}{\frac{1}{10} - \frac{7}{n}} \qquad (n > 70)$$

choose $n_0 = 140$, $c = 20a$

Thus, $T(n) \leq cn$ so $T(n) = O(n)$

$$\therefore \quad T(n) = \Theta(n)$$