



Structured  
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# Structured Decision Problems

## Actions and Decisions

Professor Ajoodha

Lecture 10

School of Computer Science and Applied Mathematics  
The University of the Witwatersrand, Johannesburg



ExplainableAI Lab

— MODELLING. DECISION MAKING. CAUSALITY —



# Problem Statement

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We now move from the problem of reasoning under uncertainty ...

... to the task of deciding how to act in the world.

## Problem

*In decision-making, an agent chooses from a set of actions that can lead to various outcomes preferred to different extents.*

**Situation 1:** Outcome of action **is** fully determined:

- Agent must determine which possible outcomes is most preferred

**Situation 2:** Outcome of action **is not** fully determined:

- We need to determine preferences to complex scenarios involving probability distributions over possible outcomes.



# Situation 1: Fully determined

- Imagine you are at a restaurant deciding between two dishes based on dietary restrictions or personal preferences:

Dish A	Dish B
Grilled chicken salad with vinaigrette dressing	Creamy chicken alfredo pasta
450 calories, 20g protein, 10g fat, 5g carbs	700 calories, 25g protein, 35g fat, 50g carbs.
Contains no dairy or gluten	Contains dairy and gluten



# Utility Functions and Preferences

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- Represent how much an individual values different outcomes in a lottery.
- For example:

Option 1			Option 2		
$P(x)$	0.2	0.8	$P(x)$	0.25	0.75
Reward	$R4 \text{ mill}$	$R0$	Reward	$R3 \text{ mill}$	$R0$
$(0.2 \times 4) + (0.8 \times 0)$			$(0.25 \times 3) + (0.75 \times 0)$		

- People **do not always** choose lotteries with higher expected payoffs.

Option 1			Option 2		
$P(x)$	0.8	0.2	$P(x)$	1	0
Reward	$R4 \text{ mill}$	$R0$	Reward	$R3 \text{ mill}$	$R0$
$(0.8 \times 4) + (0.2 \times 0) = 3.2$			$(1 \times 3) + (0 \times 0) = 3$		



# Insurance Premium

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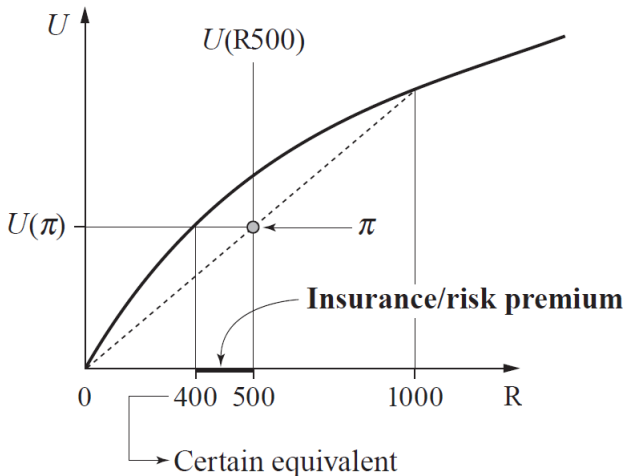
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Credit: Adapted from Koller Textbook [2014], pp 1067



# Utility Curves

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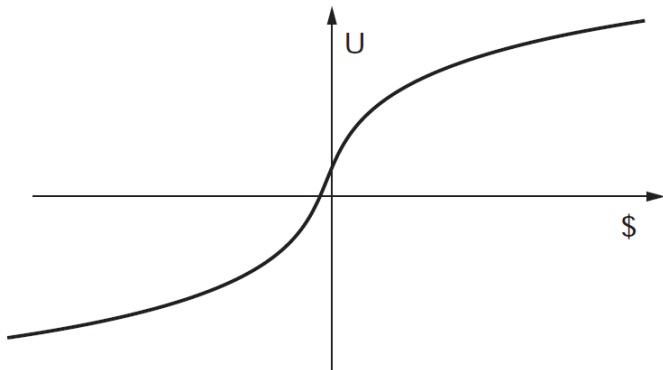
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Credit: *Koller Textbook [2014], pp 1066*



# Example

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- Suppose you are a rational decision-maker with a utility function defined over two goods: puppies (P) and apples (A). Your utility function is given by

$$U(P, A) = \sqrt{P} \times \sqrt{A}$$

- Q.1) You currently have 10 puppies and 20 apples. What is your utility?
- Q.2) Should you trade 3 puppies for 6 apples?
- Q.3) Does the utility function exhibit diminishing marginal utility for puppies and apples?



## Q.1. Example

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Q.1) You currently have 10 puppies and 20 apples. What is your utility?

$$\begin{aligned}U(P, A) &= \sqrt{P} \times \sqrt{A} \\U(10, 20) &= \sqrt{10} \times \sqrt{20} \\&\approx 14.14\end{aligned}$$



## Q.2. Example

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Q.2) Should you trade 3 puppies for 6 apples?

$$U(P, A) = \sqrt{P} \times \sqrt{A}$$

$$U^1(10, 20) = \sqrt{10} \times \sqrt{20} \\ \approx 14.14$$

$$U^2(7, 26) = \sqrt{7} \times \sqrt{26} \\ \approx 13.49$$



## Q.3. Example

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Q.3) Does the utility function exhibit diminishing marginal utility for puppies and apples?

$$\frac{\partial U}{\partial P} = \frac{1}{2\sqrt{P}} \times \sqrt{A}$$

$$\frac{\partial U}{\partial A} = \sqrt{P} \times \frac{1}{2\sqrt{A}}$$

- Diminishing marginal utility happens when the utility gained from each extra unit of a good diminishes as the quantity of that good increases.
- The partial derivative  $\frac{\partial U}{\partial P}$  represents the marginal utility of puppies.
- As more puppies are added, their marginal utility decreases due to the exponent of -0.5 on P.
- Diminishing marginal utility for puppies.



# Picking Utilities

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- Picking utilities is therefore a difficult task, here are some tips to help you:
- **Determine a scale** for the utility values. Use a scale (0-10) to be consistent when assigning utilities across values.
- Consider the **shape of your utility function**: Do you have diminishing marginal utility, that is are you risk averse (concave); risk seeking (convex), risk neutral (linear)?
- Take into account **abstract attributes** with each outcome which may affect your satisfaction or dissatisfaction e.g. personal preferences, goals and aspirations, risk aversion, time, social norms and values.



# Situation 2: Modelling Uncertainty in Decision Making

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What do we need to consider when making a decision  $\mathcal{D}$ ?

- 1 A set of **actions**  $Val(A) = \{a^1, \dots, a^k\}$
- 2 A set of **state** of the world  $Val(\mathbf{X}) = \{\mathbf{x}^1, \dots, \mathbf{x}^k\}$
- 3 A **distribution** of these states given the actions:  $P(\mathbf{X} | A)$
- 4 A **utility** (agent preference) for each action and state the world:  $U(\mathbf{X}, A)$

E.g. Your vacuum robot cleans a living room with fixed furniture and occasional obstacles like toys.

Using sensor data, it decides its movement based on probable outcomes when encountering obstacles.

**Go Left**

40% Open route (5)

30% Another obstacle (0)

30% Get stuck (-5)

**Go Right**

50% Open route

20% Another obstacle

30% Get stuck (-5)

**Go Backward**

10% Open route

20% Another obstacle

70% Retrace route (-1)



# Expected Utility

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- With this context we can calculate the **expected utility** of a decision  $\mathcal{D}$ :

$$EU[\mathcal{D}[a]] = \sum_{\mathbf{x}} P(\mathbf{x} \mid a) U(\mathbf{x}, a)$$

- For  $P(\mathbf{x} \mid a)$ , not all states are affected by the action.
- Often what we want to do is choose the **best action** (maximum expected utility):

$$\begin{aligned} a^* &= \operatorname{argmax}_a EU[\mathcal{D}[a]] \\ &= \operatorname{argmax}_a \sum_{\mathbf{x}} P(\mathbf{x} \mid a) U(\mathbf{x}, a) \end{aligned}$$



# Influence Diagrams (Decision Network)

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The major theme of this module is that:

- **The world is structured and that we can obtain representational and computational efficiency by exploiting this structure.**

Influence diagram ( $\mathcal{I}_F$ ) is a directed acyclic graph which encodes the decision scenario with a set of variables:



**Chance Variables:** Random variables. Values are selected by nature using some probabilistic model. Associated with a CPD.



**Decision Variables:** Under the control of the agent. Value reflects agents choice.



**Utility Variables:** Numerically valued variables encoding the agent's utility. Utility nodes have no children. Associated with a deterministic function.



# Attending Class Example

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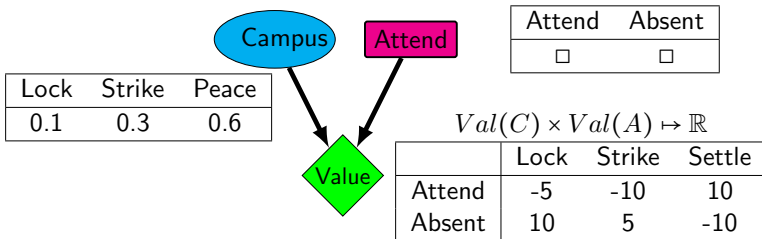
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- What if you need to decide whether to go to campus to attend teaching activities?
- The campus can be in one of three possible states:
  - Lock:** This is the lockdown/shutdown scenario
  - Strike:** Here there may be classroom disruptions
  - Settle:** There is no conflict on campus
- Ignoring political involvement, depending on the situation on campus your utility of learning is affected (utility  $U$ ) depending on your decision to attend class.
- For example, if you attend class and there are disruptions, then you may take a long time to get on and off campus which also means you didn't learn much.
- Lets explore these scenarios.



# Influence Diagram for Attending University



$$\begin{aligned}
 EU[\mathcal{D}[\text{Attend}]] &= \sum_{\mathbf{x}} P(\mathbf{x} \mid \text{Attend}) U(\mathbf{x}, \text{Attend}) \\
 &= (0.1 \times -5) + (0.3 \times -10) + (0.6 \times 10) = 2.5
 \end{aligned}$$

$$\begin{aligned}
 EU[\mathcal{D}[\text{Absent}]] &= \sum_{\mathbf{x}} P(\mathbf{x} \mid \text{Absent}) U(\mathbf{x}, \text{Absent}) \\
 &= (0.1 \times 10) + (0.3 \times 5) + (0.6 \times -10) = -3.5
 \end{aligned}$$



# Influence diagram for the Student Network

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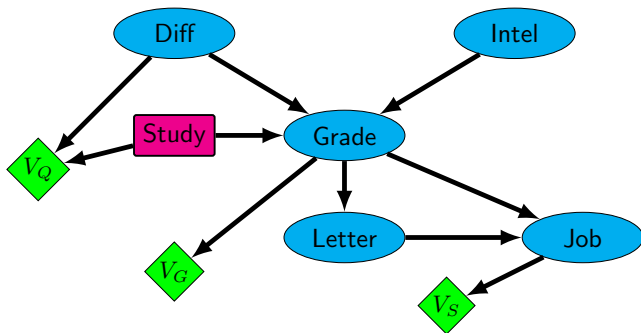
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- Overall Utility =  $V_Q + V_G + V_S$
- Decomposed Utility Function helps with reducing complexity



# Entrepreneur Example

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- Consider another example: An entrepreneur wants to startup a company that sells widgets.
- She doesn't know how large the market demand for widgets are:
  - $m^0$  Nonexistent demand: 0.5
  - $m^1$  Low demand: 0.3
  - $m^2$  High demand: 0.2
- There are variables associated outcomes that we need to consider w.r.t entrepreneur's earnings (utility  $U$ ).
- If she does not start the company, she neither earns nor loses anything.
  - $o_0$ : No loss no gain:  $U(o_0) = 0$ .
- If she decides to start the company:
  - $o_1$ : if  $m^0$  = Lose money:  $U(o_1) = -7$ .
  - $o_2$ : if  $m^1$  = Sells company makes some profit:  $U(o_2) = 5$ .
  - $o_3$ : if  $m^2$  = Goes public and makes a fortune:  $U(o_3) = 20$ .



# Simple Entrepreneur Example

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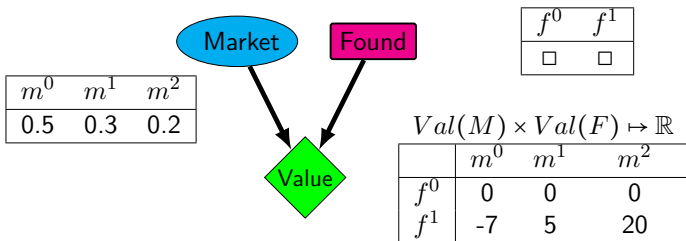
## Utility Functions

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$$\begin{aligned}
 EU[\mathcal{D}[f^0]] &= \sum_{\mathbf{x}} P(\mathbf{x} \mid f^0) U(\mathbf{x}, f^0) \\
 &= (0.5 \times 0) + (0.3 \times 0) + (0.2 \times 0) = 0 \\
 EU[\mathcal{D}[f^1]] &= \sum_{\mathbf{x}} P(\mathbf{x} \mid f^1) U(\mathbf{x}, f^1) \\
 &= (0.5 \times -7) + (0.3 \times 5) + (0.2 \times 20) = 2
 \end{aligned}$$



# Influence Diagram for the Entrepreneur Example

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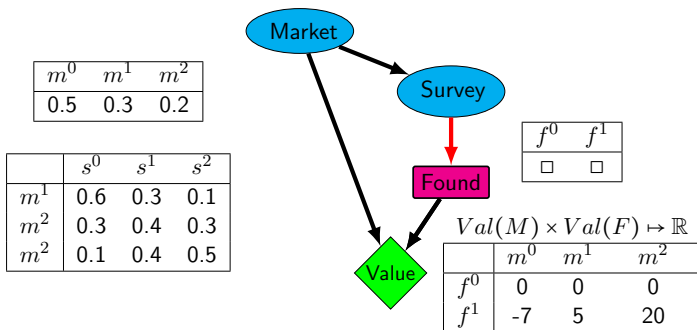
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Considering all of these parameters we get the following influence diagram:



This framework provides a factored representation **of both** the action and the outcome space.



# Decision Rules and Complete Strategies

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- Influence diagrams have implicit causal semantics.
- The agent can select a decision variable's value with the parent node's values as known, enabling intervention.
- Edges entering the decision node are therefore referred to as **information edges**.
- This decision will affect the values of variables downstream.
- When there is a complex relationship between information edges and the decision, a **decision rule** ( $\delta_D$ ) can be created.
- A complete mapping,  $\sigma$ , of decision rules to every possible assignment is called a **complete strategy**.
- Complete strategy makes decision node a chance node



# Maximising Expected Utility

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- With decision rules in mind we can have **expected utility with information**:

$$EU[\mathcal{D}[\delta_A]] = \sum_{\mathbf{x}, a} P_{\delta_A}(\mathbf{x}, a) U(\mathbf{x}, a)$$

- A Maximal Expected Utility (MEU) (non-unique) strategy,  $\sigma^*$ , is one that maximises  $EU[\mathcal{I}[\sigma]]$ .
- We need to find:

$$\operatorname{argmax}_{\delta_{D_1}, \dots, \delta_{D_k}} EU[\mathcal{I}[\delta_{D_1}, \dots, \delta_{D_k}]]$$

- This is not trivial since each decision rule is a complex function.



# Optimising Decision Rules for MEU

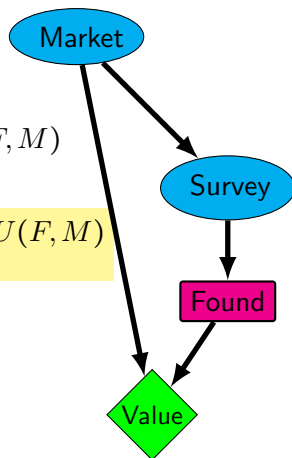
Optimise:

$$EU[\mathcal{D}[\delta_A]] = \sum_{\mathbf{x}, a} P_{\delta_A}(\mathbf{x}, a) U(\mathbf{x}, a)$$

$$\sum_{M, S, F} P(M) P(S | M) \delta_F(F | S) U(F, M)$$

$$= \sum_{S, F} \delta_F(F | S) \sum_M P(M) P(S | M) U(F, M)$$

$$= \sum_{S, F} \delta_F(F | S) \mu(F, S)$$



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# Optimising Decision Rules for MEU

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$$\sum_{S,F} \delta_F(F | S) \mu(F, S)$$

- To maximize the expression we need to pick:

- $f^0$  if  $s^0$
- $f^1$  if  $s^1$  or  $s^2$

- What is the overall EU for this complete strategy:

$$\text{Overall Utility} = 0 + 1.15 + 2.1 = 3.25$$

- Recall that without the survey it was 2.
- With the survey we gain **1.25** utility.

$m^0$	$m^1$	$m^2$
0.5	0.3	0.2

	$s^0$	$s^1$	$s^2$
$m^1$	0.6	0.3	0.1
$m^2$	0.3	0.4	0.3
$m^2$	0.1	0.4	0.5

	$m^0$	$m^1$	$m^2$
$f^0$	0	0	0
$f^1$	-7	5	20

$$\mu(F, S) \mapsto \mathbb{R}$$

	$f^0$	$f^1$
$s^0$	0	-1.25
$s^1$	0	1.15
$s^2$	0	2.1



# Generalised Variable Elimination

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Complete Joint Distribution

$$EU[\mathcal{D}[\delta_A]] = \sum_{\mathbf{x}, a} \overbrace{P_{\delta_A}(\mathbf{x}, a)} \quad U(\mathbf{x}, a)$$

Information Edges

$$= \sum_{X_1, \dots, X_n, A} \left( \left( \prod_i P(X_i \mid Pa_{X_i}) \right) U(Pa_U) \delta_A(A \mid \overbrace{Pa_A}) \right)$$

$$= \sum_{Pa_{A,A}} \delta_A(A \mid Pa_A) \sum_{\{X_1, \dots, X_n\} - Pa_A} \left( \left( \prod_i P(X_i \mid Pa_{X_i}) \right) U(Pa_U) \right)$$

$$= \sum_{Pa_{A,A}} \delta_A(A \mid Pa_A) \mu(A, Pa_A)$$

$$\delta_A^*(a \mid \mathbf{z}) = \begin{cases} 1 & a = \operatorname{argmax}_A \mu(\mathbf{A}, z) \\ 0 & \text{otherwise} \end{cases}$$



# Complete MEU Algorithm

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To compute the MEU with the optimised decision at a decision node  $A$

- 1 Treat  $A$  as a random variable with an arbitrary CPD
- 2 Introduce utility factor with scope  $Pa_U$
- 3 Eliminate all variables except  $A$  and its parents to produce  $\mu(A, \mathbf{Z})$
- 4 For each parent-assignment to the decision pick the assignment where:

$$\delta_A^*(a \mid \mathbf{z}) = \begin{cases} 1 & a = \operatorname{argmax}_A \mu(\mathbf{A}, z) \\ 0 & \text{otherwise} \end{cases}$$

Approach can be extended for:

- Multiple utility components
- Multiple decisions



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- A different kind of problem is identifying the variables that need to be observed to generate an accurate prediction.
- For example, in a diagnostic setting we need to choose which tests to perform to make a diagnosis.
- Other more broad applications include:
  - ① **Assessing** the worth of collecting additional information before making a decision.
  - ② **Evaluating** the potential benefit of experimentation in a particular field.
  - ③ **Identifying** where the acquisition of additional data can improve decision-making.
  - ④ **Optimizing** decision strategies by accounting for potential value of additional information.



# Value of Information

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- Influence diagrams aid structured decision problems and efficient decision-making.
- Using a straightforward graph-based criterion, we can calculate the value of information via this representation.
- The value of perfect information,  $VPI(A \mid X)$ , is the value of observing  $X$  before choosing an action at  $A$ .
- We can learn this by comparing:
  - ①  $\mathcal{D}$  = original influence diagram.
  - ②  $\mathcal{D}_{X \rightarrow A}$  = influence diagram with edge  $X \rightarrow A$ .

$$VPI(A \mid X) := MEU(\mathcal{D}_{X \rightarrow A}) - MEU(\mathcal{D})$$



# Finding MEU Decision Rules

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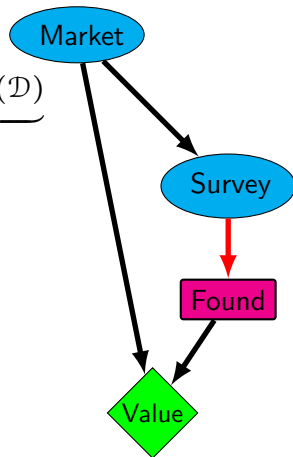
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$$\underbrace{\text{VPI}(A \mid X)}_{1.25} := \underbrace{\text{MEU}(\mathcal{D}_{X \rightarrow A})}_{3.25} - \underbrace{\text{MEU}(\mathcal{D})}_2$$

- The agent should be willing to pay anything up to 1.25 utility points to conduct the survey.
- 1  $\text{VPI}(A \mid X) \geq 0$ .
  - 2  $\text{VPI}(A \mid X) = 0 \iff$  the optimal decision rule for  $\mathcal{D}$  is still optimal for  $\text{MEU}(\mathcal{D}_{X \rightarrow A})$





# When is Information Useful?

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$$\text{VPI}(A \mid X) := \text{MEU}(\mathcal{D}_{X \rightarrow A}) - \text{MEU}(\mathcal{D})$$

- 1 Information is useful when it **changes my decision** towards the optimal strategy.
- 2 If  $\text{VPI}(A \mid X)$  is 0, then I can learn nothing from the edge.



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- ① During 2021, many people were being vaccinated against COVID-19.
- ② Suppose that you are deciding to take either Johnson & Johnson or Pfizer.
- ③ Either has a credibility profile [ $r^1$  (bad) to  $r^3$  (good)], which is inferred by multiple observations not mentioned in this case-study: e.g. RCTs, systematic reviews, samples tested, side-effects.
- ④ Either vaccinations can be endorsed by the South African Health Products Regulatory Authority (SAHPRA) or not depending on their credibility.
- ⑤ You need to decide whether to take which vaccine.
- ⑥ **DISCLAIMER: All the probabilities here are made-up**



# Example 1: Johnson& Johnson is Reliable

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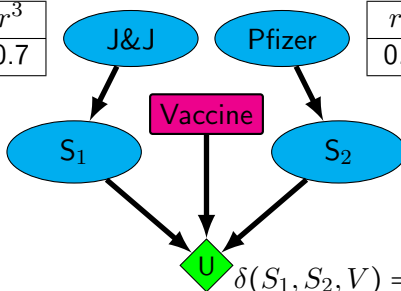
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$r^1$	$r^2$	$r^3$
0.1	0.2	0.7

$j$	$p$
<input type="checkbox"/>	<input type="checkbox"/>



$r^1$	$r^2$	$r^3$
0.4	0.5	0.1

	$a^0$	$a^1$
$r^1$	0.9	0.1
$r^2$	0.6	0.4
$r^3$	0.1	0.9

$$\delta(S_1, S_2, V) = \begin{cases} 1 & \text{if approved} \\ 0 & \text{otherwise} \end{cases}$$

Two Strategies without information:

$$EU(\mathcal{D}[j]) = \sum_V \delta_V(j) \mu(j) = 0.72$$

$$EU(\mathcal{D}[p]) = \sum_V \delta_V(p) \mu(p) = 0.33$$



# Neither Vaccine is Reliable

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- 1 So it looks like the Johnson & Johnson vaccine will maximise our utility.

$$EU(\mathcal{D}[j]) = 0.72 > EU(\mathcal{D}[p]) = 0.33$$

- 2 Now suppose that both companies are unreliable.
- 3 And further suppose the agent makes an observation about the reliability of the Pfizer vaccine.
- 4 E.g. maybe your doctor who works for Pfizer tells you some information about the reliability of the vaccine.

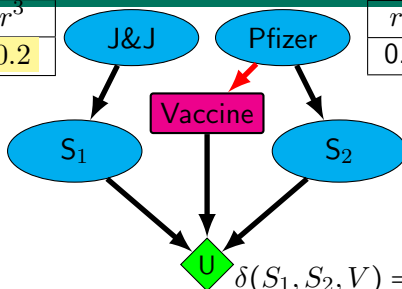


## Example 2: Neither Vaccine is Reliable

$r^1$	$r^2$	$r^3$
0.5	0.3	0.2

$r^1$	$r^2$	$r^3$
0.4	0.5	0.1

$j$	$p$
<input type="checkbox"/>	<input type="checkbox"/>



	$a^0$	$a^1$
$r^1$	0.9	0.1
$r^2$	0.6	0.4
$r^2$	0.1	0.9

$$\delta(S_1, S_2, V) = \begin{cases} 1 & \text{if approved} \\ 0 & \text{otherwise} \end{cases}$$

We get more information **changing our mind**:

$$EU(\mathcal{D}[j]) = 0.35 \quad \bigg| \quad MEU(\mathcal{D}_{P \rightarrow V}) = 0.43$$

$$EU(\mathcal{D}[p]) = 0.33$$

$$\delta^*(V \mid P) = \begin{cases} P(p) = 1 & \text{if } P=r^2, r^3 \\ P(j) = 1 & \text{otherwise} \end{cases} \quad EU[p] = \begin{cases} P = r^1 & \text{Prefer } j \\ P = r^2 & \text{Prefer } p \\ P = r^3 & \text{Prefer } p \end{cases}$$



## Example 3: Everyone gets Funded

Structured  
Decision  
Problems

Professor  
Ajoodha

Problem  
Statement

Utility  
Functions

Decision  
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Influence  
Diagrams

Strategy

Value of  
Perfect  
Information

- ① Now suppose that, for whatever reason, SAHPRA standard drop tremendously.
- ② In this case SAHPRA approves just about any vaccine which is offered by private companies.
- ③ Maybe SAHPRA takes bribes from pharmaceutical companies.



# Example 3: Everyone gets Funded

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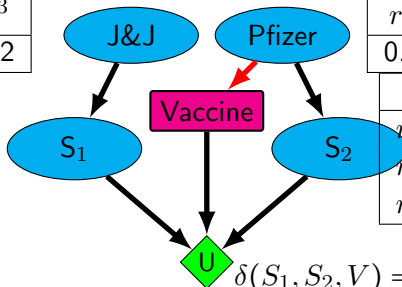
Influence  
Diagrams

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Information

$r^1$	$r^2$	$r^3$
0.5	0.3	0.2

$j$	$p$
<input type="checkbox"/>	<input type="checkbox"/>



$r^1$	$r^2$	$r^3$
0.4	0.5	0.1

	$a^0$	$a^1$
$r^1$	0.3	0.7
$r^2$	0.2	0.8
$r^2$	0.01	0.99

$$\delta(S_1, S_2, V) = \begin{cases} 1 & \text{if approved} \\ 0 & \text{otherwise} \end{cases}$$

EU is the **same** for both:

$$EU(\mathcal{D}[j]) = 0.788 \quad \bigg| \quad MEU(\mathcal{D}_{P \rightarrow V}) = 0.8142$$

$$EU(\mathcal{D}[p]) = 0.779$$

$$\delta^*(V \mid P) = \begin{cases} P(p) = 1 & \text{If } P=r^2, r^3 \\ P(j) = 1 & \text{otherwise} \end{cases} \quad EU[p] = \begin{cases} P = r^1 & \text{Prefer } j \\ P = r^2 & \text{Prefer } p \\ P = r^3 & \text{Prefer } p \end{cases}$$



# Summary

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- Influence Diagrams provide **graphical representation** of decision problems
- Nodes represent decisions, uncertainties, and outcomes
- Arrows represent (as usual) **cause-and-effect** relationships or correlations
- Provides a structured approach to decision-making
- Helps identify critical variables and potential outcomes
- Can be used to calculate the optimal **decision strategies** over decisions and **value of information**