School of Computer Science & Applied Mathematics

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

DISCRETE OPTIMIZATION & OPTIMIZATION IN BIG DATA: COMS4050

TEST-I **Time:11H00-12H00** **Date: 21 September, 2016**

Answer all questions **Total Marks: 35**

QUESTION 1 [14 marks]

(a) Describe the difference between the Hamiltonian and the Eulerian cycle in graph theory. Use an example graph for each concept. [4 Marks]

(b) Consider the max-flow problem of a directed network with a source node s and the sink node t. Let x_{ij} denote the amount of flow between the node i and node j. Write the node balanced equation of a node i where $i \neq s$ and $i \neq t$. [2 Marks]

(c) Define the following concepts in the Ford Fulkerson algorithm for max-flow problem: (a) An Augmenting Path; (b) the Residual Network corresponding to a feasible flow of amount x. Be brief in your answer. [4 Marks]

(d) The manager of Hotel in Johannesburg have four workers: Agness, Rose, Joyce and Zainab. The manager needs to have one of them clean the bathroom of the hotel, another sweeps the floor of the hotel, the third to be a receptionist and the last person to cook food for the guests and management, but they each demand different pay for their different tasks. The table below represents the costs c_{ij} of the workers i doing the jobs (tasks) j, where ZAR is the South African Rand. Formulate the linear assignment problem. [4 Marks]

Table 1: Data showing the cost of workers doing different jobs

	Jobs			
Workers	Cleaning	Sweeping	Receptionist	Cooking
Agness	ZAR9000	ZAR7500	ZAR7500	ZAR800
Rose	ZAR3500	ZAR8500	ZAR5500	ZAR6500
Joyce	ZAR12500	ZAR9500	ZAR9000	ZAR10500
Zainab	ZAR4500	ZAR11000	ZAR9500	ZAR11000

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QUESTION 2 [12 marks]

(a) Differentiate between a Linear Programming (LP) Problem and an integer Linear Programming (ILP) Problem. Which of the problems in your view is most difficult to solve? Why? [3 Marks]

(b) Explain why there are $(n-1)!$ possible distinct routes in a Traveling Salesman Problem (TSP) with n cities. Described what do you mean by the sub-tour elimination constraints in the mathematical model of the TSP? [2 + 2 Marks]

(c) Define an α-approximation algorithm of a minimization problem. What is the value of α in the Christofides algorithm for TSP? [2 + 1 Marks]

(d) Consider the 7 city TSP with a feasible tour $x = (1, 3, 2, 6, 7, 4, 5)$ where the 1st city is the starting city. Construct a 2-Opt neighbor of x. [2 Marks]

QUESTION 3 [9 marks]

(a) Define the set covering problem and write down its mathematical model. Describe the physical meaning of the constraint sets in the mathematical model. [4 Marks]

(b) A constraint of the mathematical model of the facility location problem is given by $\sum_{i=1}^{m} x_{ij} = 1, \forall j$, where i represents the warehouses and j is a customer; x_{ij} is the variable representing whether customer j is served by the warehouse i. Described the meaning of this constraint. [2 Marks]

(c) Write down the mathematical model of the weighted vertex cover problem where $c_i > 0$ is the weight of each vertex i. [3 Marks]