4.5 Master Method

- used for solving recurrences like:

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$

where $a \geq 1$ and $b > 1$

and $f(n)$ is an asymptotically positive function.

The Master theorem says :

1. If $f(n) = O(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$

then $T(n) = \Theta(n^{\log_b a})$

2. If $f(n) = \Theta(n^{\log_b a})$

then $T(n) = \Theta(n^{\log_b a} \log n)$

3. If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$

~~then~~ and $a f(\frac{n}{b}) \leq c f(n)$ for $c < 1$

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~~then~~ and $a f(\frac{n}{b}) \leq c f(n)$ for $c < 1$

then $T(n) = \Theta(f(n))$

Solve : $T(n) = 4\,T\left(\frac{n}{3}\right) + n$

$a = 4$

$b = 3$

$f(n) = n$

compare $f(n)$ with $n^{\log_b a}$

$$n < n^{\log_3 4}$$

$$\therefore f(n) = O(n^{\log_b a - \varepsilon})$$

$\therefore$ case 1.

$$T(n) = \Theta(n^{\log_3 4})$$

Solve : $T(n) = 25\,T\left(\frac{n}{5}\right) + n^2$

$a = 25$
$b = 5$
$f(n) = n^2$

compare n^2 with $n^{\log_5 25}$
$= n^2$

$\therefore f(n) = \Theta(n^{\log_5 a})$

$\therefore$ Case 2.

$\therefore T(n) = \Theta(n^{\log_5 25} \cdot \log n)$

$= \Theta(n^2 \log n)$

<u>Solve</u> : $T(n) = 15\,T\left(\frac{n}{4}\right) + n^2$

$a = 15$	Compare n^2 with $n^{\log_4 15}$
$b = 4$	$n^2 > n^{\log_4 15}$.
$f(n) = n^2$	

$\therefore\ f(n) = \Omega\left(n^{\log_b a}\right)$

$\therefore$ Case 3.

<u>Check</u> : $a f\left(\frac{n}{b}\right) \leq c f(n)$ for some $c < 1$.

<u>Check</u>: $a f\left(\frac{n}{b}\right) \leq c f(n)$ for some $c < 1$.

$$15\left(\frac{n}{4}\right)^2 \leq c n^2$$

$$\iff \frac{15}{16} n^2 \leq c n^2$$

$$\iff \frac{15}{16} \leq c.$$ choose $c = \frac{15}{16} < 1$.

$\therefore \quad T(n) = \Theta(n^2)$