33.3 Finding the Convex Hull.

Given a set of points in $\mathbb{R}^2$:

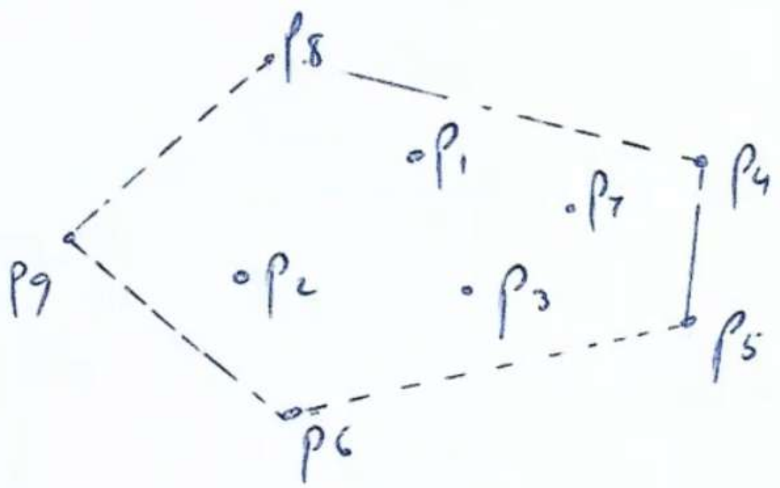
$Q = \{p_1, p_2, \ldots p_n\}$

the convex hull of Q or $CH(Q)$

is the smallest convex polygon

containing all points in Q.

e.g.



The convex hull is $\{p_4, p_5, p_6, p_8, p_9\}$.

GRAHAM-SCAN (Q).

Given a set Q of points in $\mathbb{R}^2$.

- Let p_0 be the point in Q with the minimum y-co-ord.
- Let $\langle p_1, p_2, \dots, p_m \rangle$ be the remaining points in Q, sorted by polar angle in counterclockwise order around p_0.
- Let S be an empty stack.

Push(p_0, S)

- Let $\underline{S}$ be an empty stack.

  Push (p_0, S)

  Push (p_1, S)

  Push (p_2, S)

for $i = 3$ to m

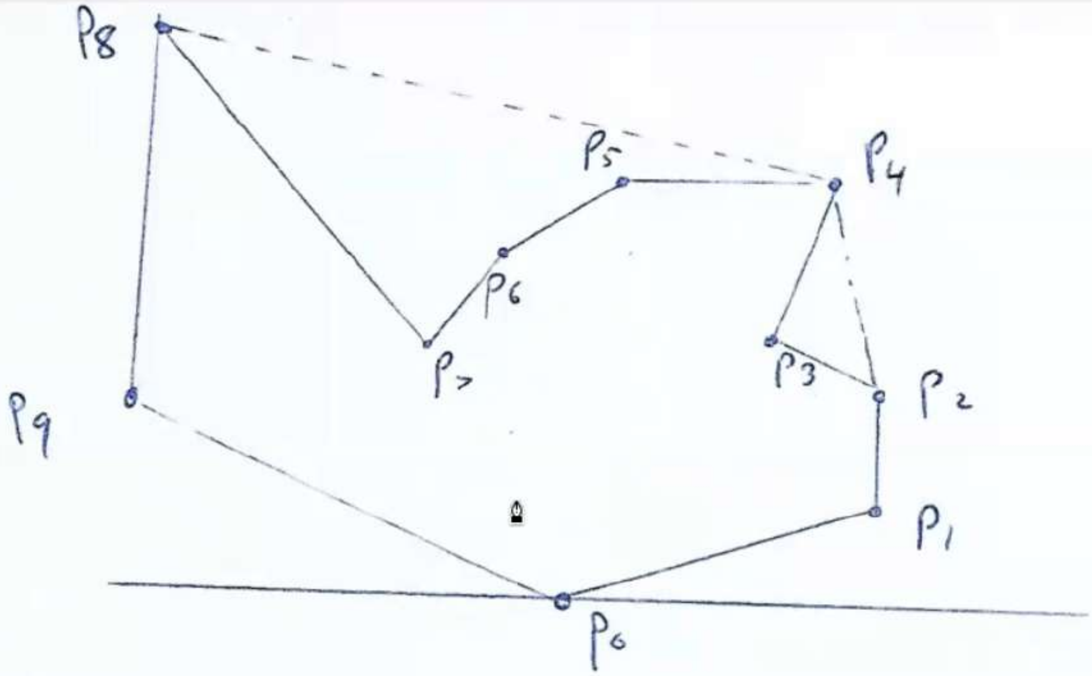
while angle formed by Next-to-top(S),

top(S) and p_i makes nonleft turn,

Pop (S)

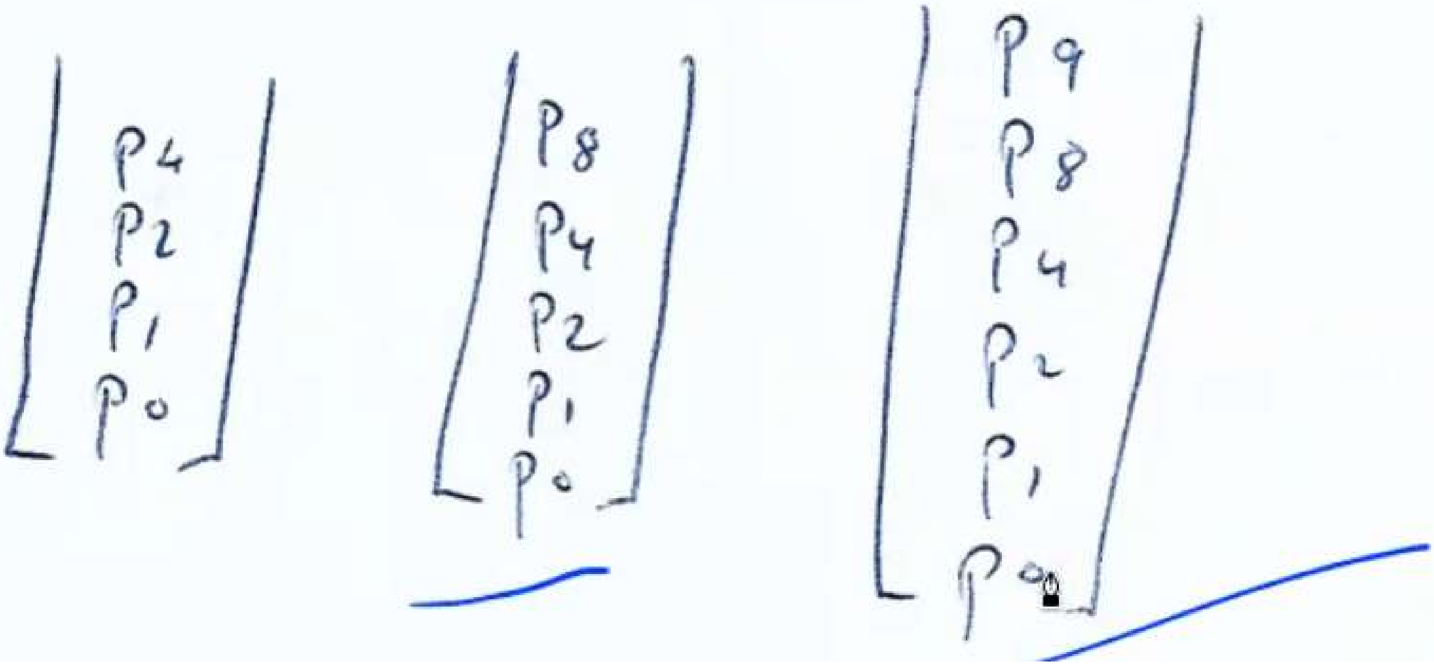
Push(p_i, S)

return S.



Stack S

$$\begin{bmatrix} p_2 \\ p_1 \\ p_0 \end{bmatrix} \quad \begin{bmatrix} p_3 \\ p_2 \\ p_1 \\ p_0 \end{bmatrix} \quad \begin{bmatrix} p_2 \\ p_1 \\ p_0 \end{bmatrix} \quad \begin{vmatrix} p_4 \\ p_2 \\ p_1 \end{vmatrix}$$


P4
P2
P1
P0
P8
P4
P2
P1
P0
P9
P8
P4
P2
P1
P0

Note that the final stack contains the points of the convex hull in counterclockwise order.

Running-time of GRAHAM-SCAN

On input set of size m.

The for loop runs from 3 to m.

In the while loop any number of pops can be made ...

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Using Amortized Analysis, we see that we can only Pop as many elements as we push on the stack.
Each element is Pushed once onto the stack so we do at most n Pops.

The amortized cost of Pop over the for loop of $m-2$ executions is thus $O(1)$

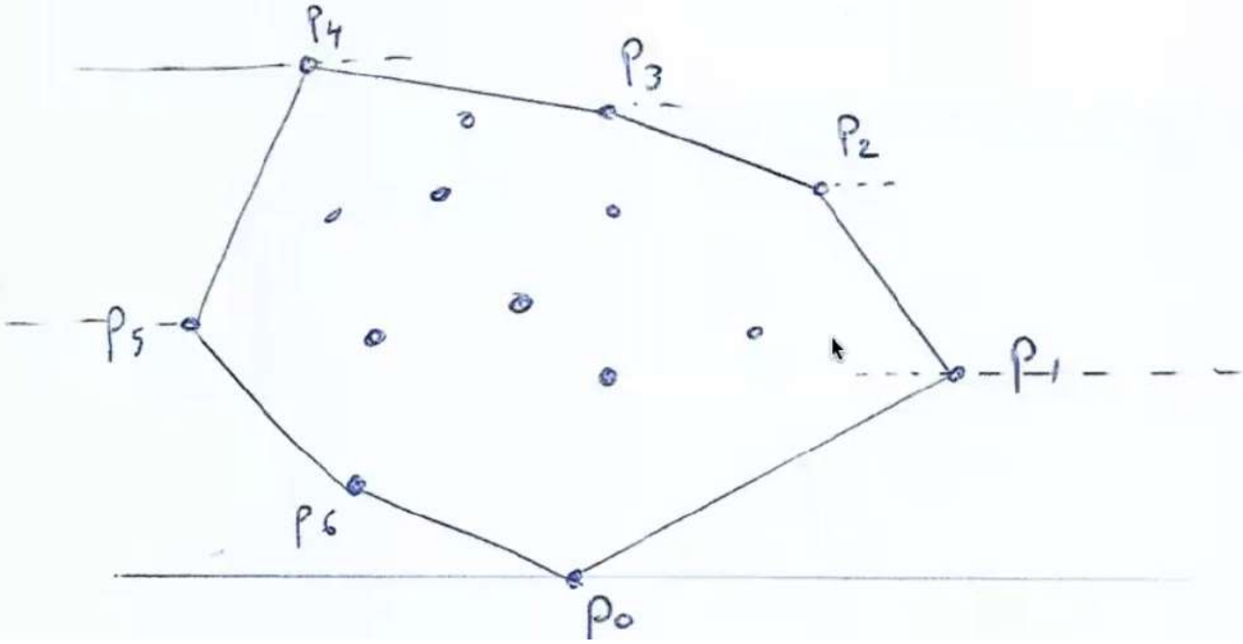
so the loop runs in $O(m)$

The initial sorting takes $O(m \log m)$

hence the run-time is $O(m \log m)$

Jarvis's March

Given a set of points in $\mathbb{R}^2$.



Let p_0 be the point with min. y-co-ord

Let p_1 be the point with min polar angle with respect to p_0 and horizontal.

Let p_2 be the point with min polar angle with respect to p_1 & horizontal

etc.

From p_4 (here top) we look for the point with minimum polar angle w.r.t. negative

x-axis

& continue until we get to p_0.

Running-time of Jarvis's March:

Recall that find min of a set of n points is $\mathcal{O}(n)$.

To find p_1 : $\mathcal{O}(n)$

To find p_2 : $\mathcal{O}(n)$

To find p_3 : $\mathcal{O}(n)$

$\vdots$

To find p_k : $\mathcal{O}(n)$

Run-time is $O(nh)$

where h is the number of points on the Convex Hull.

In worst case this is $O(n^2)$

Compared to GRAHAM-SCAN, JARVIS'S-MARCH is better if $h = o(\log n)$.