

DISCRETE OPTIMIZATION: COMS4050 November 22, 2021

TOTAL MARK: [50 marks]

Time: 4 hours

QUESTION 1 [18 marks]

- (a) Consider a flow network $G(V, E)$ with source s and sink t . If the net flow from node i to node j , i.e., along the edge $e = (i, j)$, is given by $f(i, j) = p(i, j) - p(j, i)$ where $p(i, j)$ is the positive flow along (i, j) and $p(j, i)$ is the positive flow along (j, i) , then prove that $f(i, j) = -f(j, i)$. Why do you think $\sum_j f(i, j) = 0$ always hold for any node i ($i \neq s, i \neq t$)? [2 + 4 Marks]
- (b) State two important features of the minimum spanning tree graph. [4 Marks]
- (c) Describe the difference between the linear and the quadratic assignment problem (do not write mathematical formula or expressions). [3 Marks]
- (d) Give an example of a binary integer programming problem and define its relaxed version. [3 + 2 Marks]

QUESTION 2 [20 marks]

- (a) The quadratic knapsack problem (QKP) is defined as follows: Given a number of items n , positive integer weights w_i (say cost of investing in a share of the company i) for each item i , a knapsack capacity c (total money available to invest, or the maximal weight that can be contained by the knapsack) and the profit matrix $P = (p_{ij})$, $p_{ij} \geq 0$, $P \in R^{n \times n}$. Selections of items are made pairwise since profit of any two items (shares) are linked, i.e. profit hailed from items (i, j) is $p_{ij} + p_{ji}$; p_{ii} is the profit hailed from the selection of the same item (i, i) . Define the optimization variables and write down the mathematical model (including the constraint) of the problem to maximize the profit. [2 + 5 + 3 Marks]
- (b) In the context of Branch and Bound algorithm for minimization, give two cases when a subproblem (or node in the tree) is deleted (not considered for further investigation). [2 + 2 Marks]

DISCRETE OPTIMIZATION: COMS4050

- (c) Define an α -approximation algorithm for a minimization problem. What is the value of α for Christofides Algorithm for STSP (symmetric travelling salesman problem)?
[5 + 1 Marks]

QUESTION 3 [12 marks]

- (a) Suppose GA is used to solve a binary optimization problem where crossover is performed pairwise. Suppose that two parents solutions $x_1 = 10101$ and $x_2 = 01110$ are chosen for reproduction. Obtain two new solutions (y_1 and y_2) using a double point crossover with the integers $n_1=3$ and $n_2=4$.
[5 Marks]
- (b) What is the main objective of the Metropolis Algorithm in the context of the simulated annealing (SA) algorithm? Suppose you are minimizing a traveling salesman problem (TSP) involving 6 cities using SA. If your current solution is $x=(253416)$ with $E_x = 50$ and your trial solution $y=(542316)$ with $E_y = 150$, define the Metropolis acceptance probability in SA with the temperature parameter $T=100$, and use this to calculate the probability of accepting y . Decide if you would accept the solution y if you were given a uniform random number $r \in (0, 1)$, $r = 0.25$ (you may use calculator for this calculation).
[2 + 3 + 2 Marks]

–END–