School of Computer Science & Applied Mathematics

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

DISCRETE OPTIMIZATION & OPTIMIZATION IN BIG DATA: COMS4050

TEST-I **Time:12H30-13H30** **Date: 18 April, 2017**

Answer all questions **Total Marks: 35**

QUESTION 1 [14 marks]

(a) Describe the difference between the Hamiltonian and the Eulerian cycle in graph theory. Use an example graph for each concept. [4 Marks]

(b) Consider the max-flow problem of a directed network with a source node s and the sink node t. Let x_{ij} denote the amount of flow between the node i and node j. Write the node balanced equation of a node i where $i \neq s$ and $i \neq t$. [2 Marks]

(c) (i) Define the following concepts in the Ford Fulkerson algorithm for max-flow problem: (i) The Augmenting Path; (ii) the Residual Network corresponding to a feasible flow of amount x. (iii) Use these concepts to find the maximum flow of the network in Fig. 1:

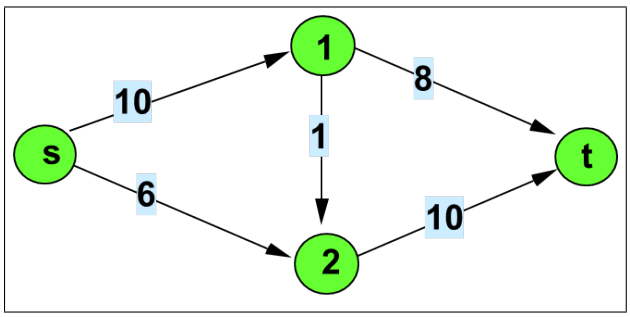

Figure 1: Flow network with capacities

(iv) Find the minimum s-t cut (A, B) of the max-flow solution for the above network. [$1\frac{1}{2} + 1\frac{1}{2} + 4 + 1$ Marks]

QUESTION 2 ON PAGE 2

DISCRETE OPTIMIZATION & OPTIMIZATION IN BIG DATA: COMS4050

QUESTION 2 [12 marks]

(a) Find the minimum spanning tree (MST) of the undirected graph below in Fig. 2, where the distance or cost of various edges are $c_{12} = c_{13} = c_{24} = c_{34} = 1$, $c_{14} = 2$ and $c_{23} = 3$ [3 Marks]

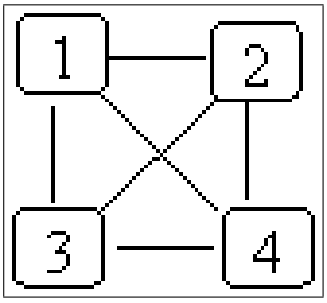

Figure 2: Undirected network

(b) Define mathematically the sub-tour elimination constraints for asymmetric traveling salesman problem (TSP). [3 Marks]

(c) Define an α-approximation algorithm of a minimization problem. [3 Marks]

(d) Consider the 7 city symmetric TSP with a feasible tour $x = (1, 3, 2, 6, 7, 4, 5)$ where the 1st city is the starting city. Construct a single 2-Opt neighbor of x. [3 Marks]

QUESTION 3 [9 marks]

(a) Write down the mathematical model of the set covering problem. Describe the physical meaning of the constraint sets in the mathematical model. [$3\frac{1}{2}$ Marks]

(b) A constraint of the mathematical model of the facility location problem is given by $\sum_{i=1}^{m} x_{ij} = 1, \forall j$, where i represents the warehouses and j is a customer; x_{ij} is the variable representing whether customer j is served by the warehouse i. Described the meaning of this constraint. [2 Marks]

(c) Write down the mathematical model of the linear knapsack problem. Describe the physical meaning of the main constraint in the mathematical model. [$3\frac{1}{2}$ Marks]