

Chapter 9 Medians and Order-Statistics

Given a list of n elements,
the i^{th} order statistic is the
 i^{th} smallest element in the list.

So the 1^{st} order statistic is the minimum

The n^{th} order statistic is the maximum

The median is the $\lfloor \frac{(n+1)}{2} \rfloor^{\text{th}}$ order statistic

Consider the following problem:

Given a list A of n numbers
and an integer i such that $1 \leq i \leq n$
find the i^{th} order statistic of A .

e.g. $A =$

17	21	3	19	51	11	26	42
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 $i = 5$

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17	21	3	19	51	11	26	42
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$i = 5$

the i^{th} order statistic is 21
 5^{th}

Note : there are 4 smaller elements than 21
in A .

Observe that if we sort A as follows:

1	2	3	4	5	6	7	8
3	11	17	19	21	26	42	51



then it is easy to find the 5th order statistic.
In fact we can easily find any i^{th} order statistic -

But sorting is $O(n \log n)$,
so we seek algorithms that
are $O(n)$.

9.1 . Finding Min and Max .

Minimum (A)

min = A[1]

for $i = 2$ to n

if min > A[i]

min = A[i]

return min .

Note : ↑ requires $n-1$ comparisons

Finding Max is similar and also requires $n-1$ comparisons.

However, we can simultaneously find Min and Max with $3 \lceil \frac{n}{2} \rceil$ comparisons.

- Please read through this on page 214 -