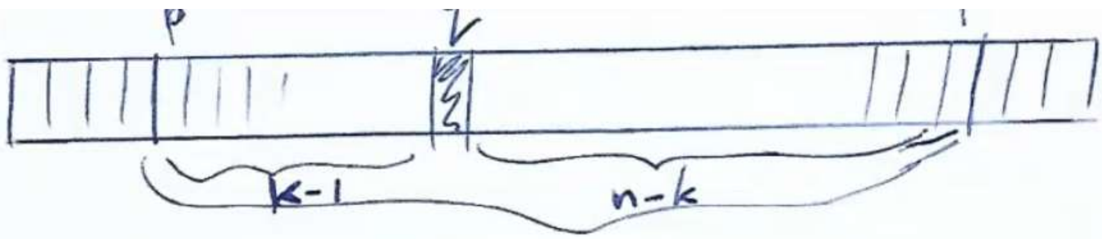

Let X_k be the indicator random variable:

$X_k = I\{\text{the subarray } A[p..q] \text{ has } k \text{ elements}\}$.

$E[X_k] = \frac{1}{n}$ (where n is length of $A[p..,r]$.)

$T(n) \leq O(n) + X_1 \cdot T(n-1) + X_2 \cdot T(n-2) + X_3 \cdot T(n-3) + \ldots$

$+ \ldots + X_k T(\max(k-1, n-k)) + \ldots + X_n T(n-1)$

$$E[T(n)] \leq E[O(n)] + E\left[\sum_{k=1}^{n} X_k \cdot T(\max(k-1, n-k))\right]$$

$$= O(n) + \sum_{k=1}^{n} E[X_k] \cdot E[T(\max(k-1, n-k))]$$

$$= O(n) + \sum_{k=1}^{n} \frac{1}{n} E[T(\max(k-1, n-k))]$$

$$= O(n) + \frac{2}{n} \sum_{k=\lfloor \frac{n}{2} \rfloor}^{n-1} E[T(k)]$$

<u>Note</u>: $\max(k-1, n-k) = \begin{cases} n-k & \text{if } k \leq \lceil \frac{n}{2} \rceil \\ k-1 & \text{if } k > \lceil \frac{n}{2} \rceil \end{cases}$

$$E[T(n)] \le \frac{2}{n} \sum_{k=\lfloor \frac{n}{2} \rfloor}^{n-1} E[T(k)] + \underline{an}$$

Use substitution method to show

$$E[T(n)] \le cn$$

Assume $E[T(m)] \le cm$ for all $m < n$

Then $E[T(n)] \le \frac{2}{n} \sum_{k=\lfloor \frac{n}{2} \rfloor}^{n-1} ck + an$

$\vdots$

$$\leq cn - \underline{\left(\frac{cn}{4} - \frac{c}{2} - an \right)}$$

$$\leq \underline{cn} \quad \text{iff} \quad \underline{\frac{cn}{4} - \frac{c}{2} - an \geq 0}$$

$$\text{iff} \quad n \geq \frac{2c}{c-4a}$$

Choose $c > 4a$ and $n_0 = \dfrac{2c}{c-4a}$

e.g $c = 5a$ and $n_0 = 10$.

Then $E[T(n)] \le cn$

$\therefore \quad E[T(n)] = O(n)$