4.4 Recursion-Tree Method

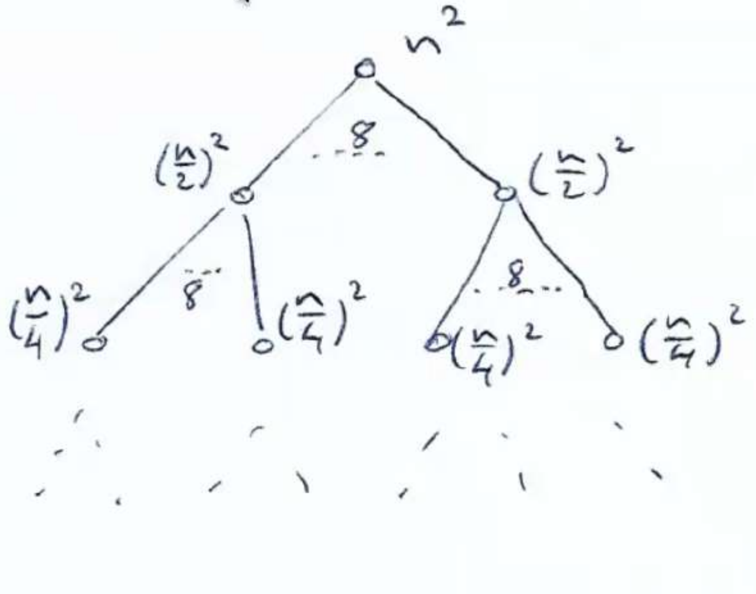
Example: Solve $T(n) = \begin{cases} c & \text{if } n = 1 \\ 8\,T(\frac{n}{2}) + n^2 & \text{if } n > 1 \end{cases}$

$T(n)$

$T(\frac{n}{2})$

$T(\frac{n}{4})$

$\vdots$



n^2

$8\,(\frac{n}{2})^2$

$64\,(\frac{n}{4})^2$

$\vdots$

$8^i\,(\frac{n}{2^i})^2$

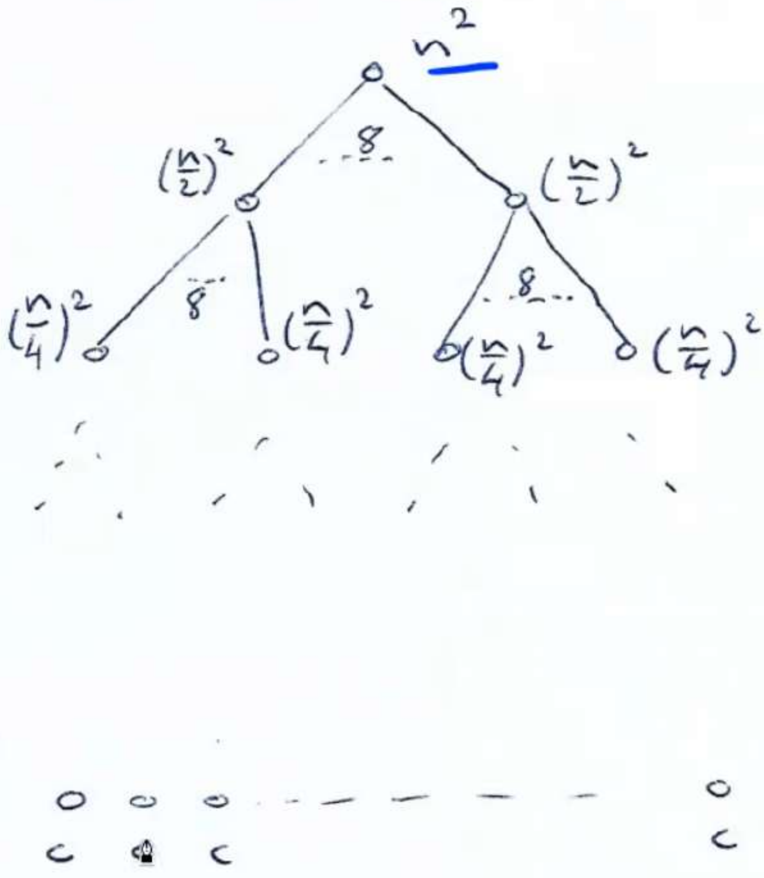
$T(n)$

$T\left(\frac{n}{2}\right)$

$T\left(\frac{n}{4}\right)$

$\vdots$

$T(1)$



n^2

$8\left(\frac{n}{2}\right)^2$

$64\left(\frac{n}{4}\right)^2$

$\vdots$

$8^i\left(\frac{n}{2^i}\right)^2$

$\vdots$

$c\cdot(\#\text{leaves})$

For height: Solve $\frac{n}{2^k} \leq 1$

$$\iff n \leq 2^k$$

$$\iff \log n \leq k \qquad \therefore \text{height} = \log n$$

$$\#\text{leaves} = (\text{branching factor})^{\text{height}}$$

$$= 8^{\log n}$$

$$T(n) = \sum_{i=0}^{\log n - 1} 8^i \left(\frac{n}{2^i}\right)^2 + c.n^3$$

$$= n^2 \sum_{i=0}^{\log n - 1} \frac{8^i}{2^{2i}} + cn^3$$

$$= n^2 \sum_{i=0}^{\log n - 1} 2^i + cn^3$$

$$= n^2\left(\frac{2^{\log n} - 1}{2 - 1}\right) + cn^3$$

$$= n^2 2^{\log n} - n^2 + cn^3$$

$$= n^3 - n^2 + cn^3$$

$$= \Theta(n^3)$$

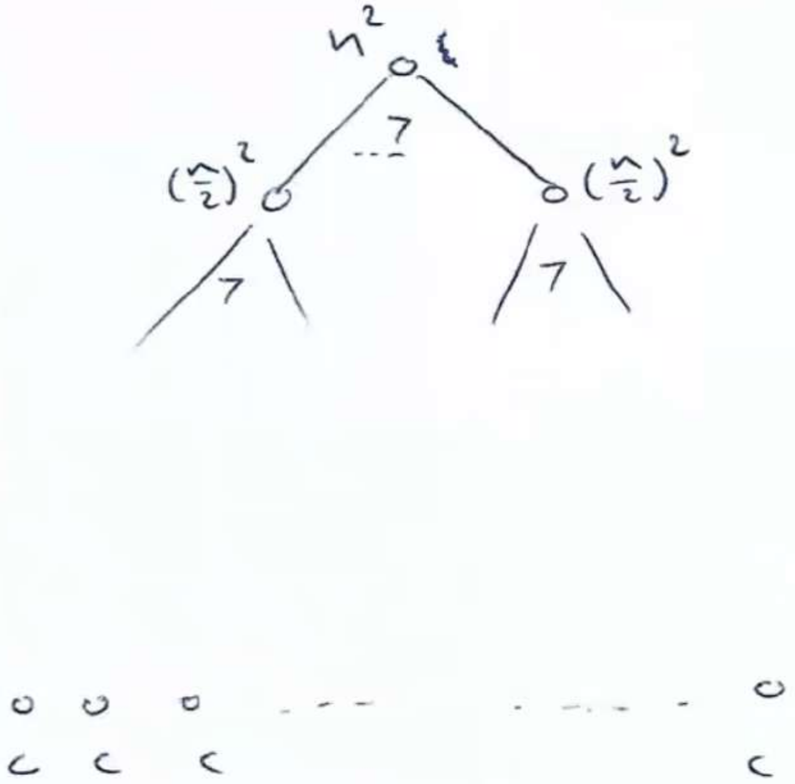
Example: Solve $T(n) = \begin{cases} c & \text{if } n = 1 \\ 7\,T(\frac{n}{2}) + n^2 & \text{if } n > 1 \end{cases}$

$T(n)$

$T(\frac{n}{2})$

$\vdots$

$T(1)$



n^2

$7(\frac{n}{2})^2$

$\vdots$

$7^i(\frac{n}{2^i})^2$

$\vdots$

$c(\#\text{leaves})$

height = $\log n$ $\quad$ # leaves = $7^{\log n}$

$$= n^{\log 7}$$

$$T(n) = \sum_{i=0}^{\log n - 1} 7^i \left(\frac{n}{2^i}\right)^2 + c \cdot n^{\log 7}$$

$$= n^2 \sum_{i=0}^{\log n - 1} \left(\frac{7}{4}\right)^i + c \cdot n^{\log 7}$$

$$= n^2 \left(\left(\frac{7}{4}\right)^{\log n} - 1\right) + cn^{\log 7}$$

$$= n^{\log 7} - n^2 + cn^{\log 7}$$

$$= \Theta(n^{\log 7}) \approx \Theta(n^{2.81})$$