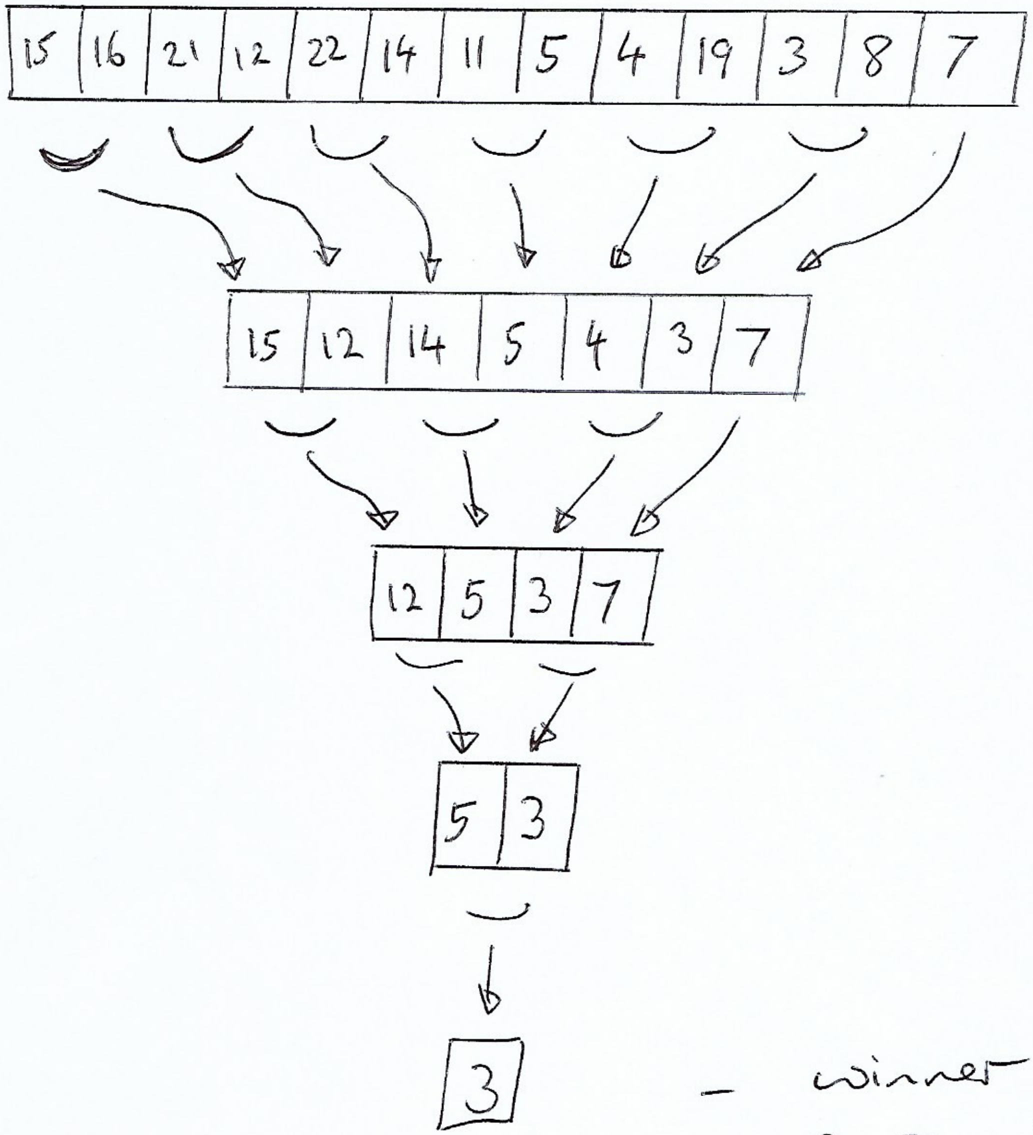
9.1-1

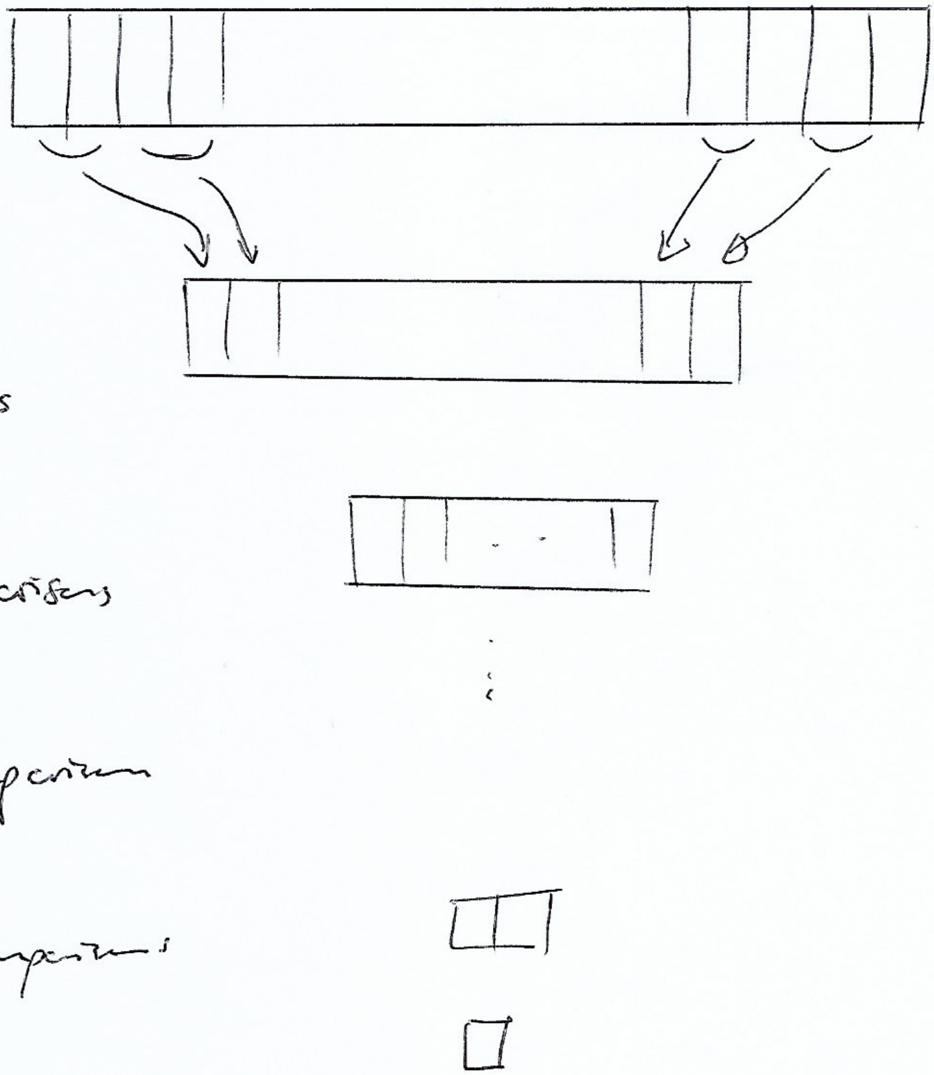


– winner!
3 is minimum

Search through list of all elements "knocked out" by 3:

5, 7, 4, 8

of these, 4 is smallest, so 4 is second smallest.



$\lfloor \frac{n}{2} \rfloor$ comparisons

$\frac{n}{4}$ comparisons

$\frac{n}{2^k}$ comparisons

$1 = \frac{n}{2^{\log n}}$ comparisons

$$\# \text{ comparisons} = \sum_{k=1}^{\log n} \frac{n}{2^k} = n-1$$

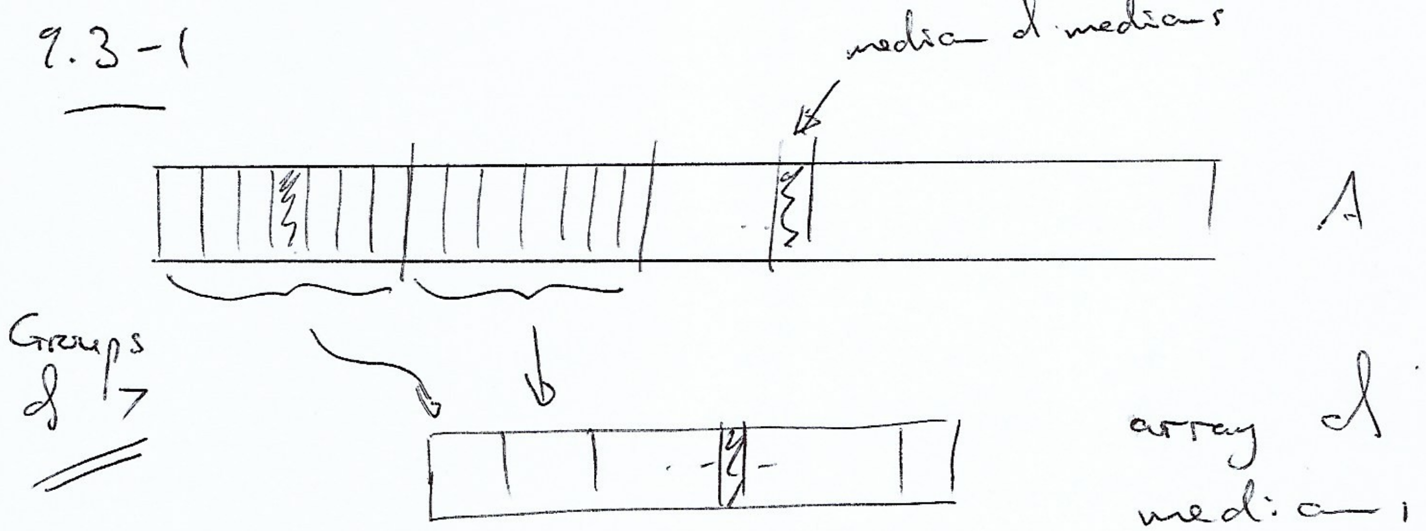
Search through path of the "winner":

There are $\lceil \log n \rceil$ elements in the path, so we need $\lceil \log n \rceil - 1$ comparisons.

Thus, total comparisons $= n - 1 + \lceil \log n \rceil - 1$

$= n + \lceil \log n \rceil - 2$

9.3-1



$$T(n) = T(\lceil \tfrac{n}{7} \rceil) + T(\text{Recursive call to left or right}) + \Theta(n)$$

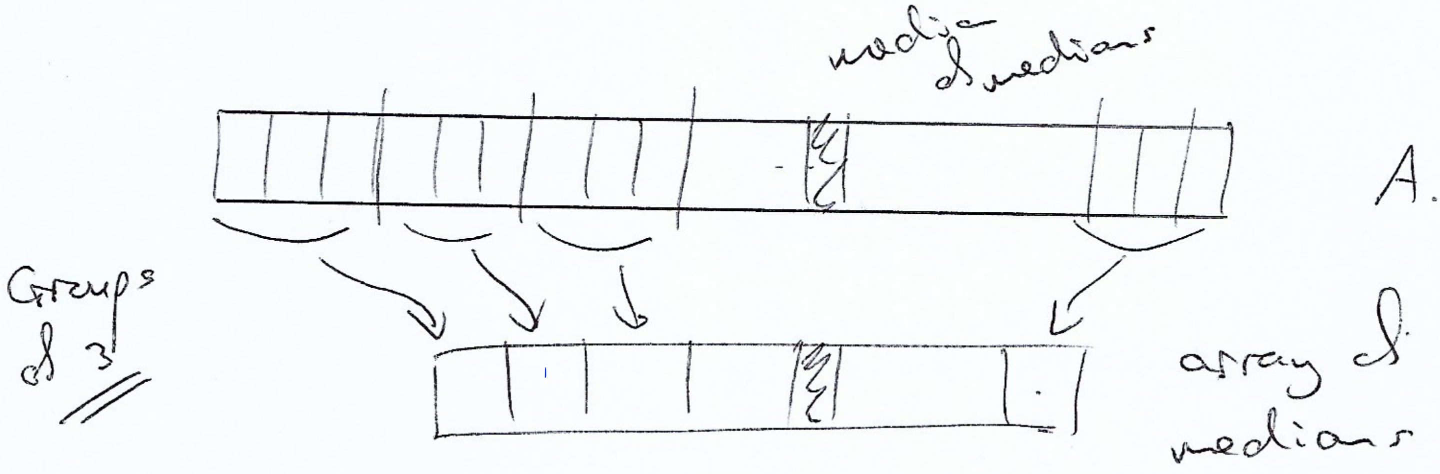
median of medians $\geq 4\left(\lceil \tfrac{1}{2} \lceil \tfrac{n}{7} \rceil \rceil - 2\right)$

$\geq \frac{2}{7}n - 8$

$\therefore$ worst case recursive call is to an array of size $n - (\frac{2n}{7} - 8) = \frac{5n}{7} + 8$.

Thus, $T(n) = T(\lceil \tfrac{n}{7} \rceil) + T(\frac{5n}{7} + 8) + \Theta(n)$

Can show $T(n) = \Theta(n)$



median of medians $\geq 2\left(\left\lceil \frac{1}{2}\left\lceil \frac{n}{3} \right\rceil \right\rceil - 2\right)$

$\geq \frac{n}{3} - 4$

$\therefore$ worst case recursive call is to array of size: $n - \left(\frac{n}{3} - 4\right) = \frac{2n}{3} + 4$

Thus, $T(n) = T\left(\left\lceil \frac{n}{3} \right\rceil\right) + T\left(\frac{2n}{3} + 4\right) + \Theta(n)$

In this case, $T(n) = \Theta(n \log n)$

Note: For 7: $\frac{1}{7} + \frac{5}{7} = \frac{6}{7} < 1$

For 3: $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$

For 5: $\frac{1}{5} + \frac{7}{10} = \frac{9}{10} < 1$.