School of Computer Science & Applied Mathematics

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

DISCRETE OPTIMIZATION & OPTIMIZATION IN BIG DATA: COMS4050

TEST-I **Time:14H00-16H00** **Date: 13 October, 2022**

Answer all questions **Total Marks: 30**

QUESTION 1 [15 marks]

(a) Consider the max-flow problem of a directed network with a source node s and the sink node t. Let x_{ij} (respectively, x_{ji}) denote the amount of positive flow from node i to node j (respectively, from node j to node i). Write the node balanced equations of a nodes i and j where $i, j \neq s$ and $i, j \neq t$. [4 Marks]

(b) Suppose you are solving a network flow problem using the Ford-Fulkerson algorithm and at a particular iteration, the remaining capacity of the edge (i, j) is r_{ij} (respectively, r_{ji} in (j, i)) and you send a feasible flow $f(i, j)$ along (i, j) then show that $r_{ji} = r_{ji} + f(i, j)$ holds for the residual network. [3 Marks]

(c) Given a set of items $S = \{1, 2, \cdots, n\}$ and m proper subsets S_j of S, $j = 1, 2, \cdots, m$. You wants to covers all items in S by choosing a number of S_j with minimum cost; the cost of choosing S_j is c_j. Write down the mathematical model of the weighted set covering problem by defining decision variables for optimization. Describe the physical meaning of the constraint sets in the mathematical model. [3+2 Marks]

(d) Construct a directed graph network with s and t as the source and terminal node, respectively, and define an $(s - t)$ cut-(A, B) together with the corresponding capacity of the cut, CAP(A, B). [3 Marks]

QUESTION 2 ON PAGE 2

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QUESTION 2 **[15 marks]**

(a) State one of the most important properties of the MST subgraph of a graph. Find the minimum spanning tree (MST) sub-graph by inspection of the undirected graph below in Fig. 1, where the distance or cost of various edges are $c_{12} = c_{13} = c_{24} = c_{34} = 1$, $c_{14} = 2$ and $c_{23} = 3$ [$1 + 3$ Marks]

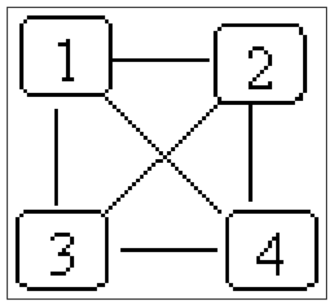

Figure 1: Undirected Graph

(b) Explain why there are $(n-1)!$ possible distinct routes (solutions) in a Traveling Salesman Problem (TSP) with n cities. How many solutions are there in the quadratic assignment problem with n facilities and n locations? Why? [$2 + 2$ Marks]

(c) Define an α-approximation algorithm of a minimization problem in combinatorial optimization. [3 Marks]

(d) Consider the 7 city symmetric TSP with a feasible tour $x = (1, 3, 2, 6, 7, 4, 5)$ where the 1st city is the starting city. Answer the following questions:

(i) Construct two 2-Opt neighbors of x. [2 Marks]

(ii) Construct an example of a sub-tour that may occur in the solution process. [2 Marks]