MT2213 - Group Theory

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Contents

Chapter 1

Definitions

1.1 Group

1.1.1 Examples:

1. $(\mathbb{Z}, +)$ is a group:
   (a) Associativity: Addition is associative.
   (b) Identity: 0 is the identity. Let $x \in Z$
2. $(\mathbb{Q}^+, \times)$ is a group:
   (a) Associativity: Multiplication is associative.
   (b) Identity: 1 is the identity: Let $x \in \mathbb{Q}^+$. Now, $1 \times x = x \times 1 = x$. Hence, it is an identity.
   (c) Inverse: Let $x \in \mathbb{Q}^+$, Now, $x \times \frac{1}{x} = \frac{1}{x} \times x = 1$, where 1 is the multiplicative inverse.

Check if:

1. $(\mathbb{R}, \times)$ is a group or not.
   $0 \in \mathbb{R}$, 0 does not have an inverse. Hence, it is not a group.
2. $(\mathbb{C}, \times)$ is a group or not.
   $0 \in \mathbb{C}$, 0 does not have an inverse. Hence, it is not a group.
3. $(\mathbb{R}/\{0\}, \times)$ is a group or not.
   Yes its a group:
   (a) Associativity: Multiplication is associative.
   (b) Identity: 1 is an identity: Let $x \in \mathbb{R}/\{0\}$. Now, $1 \times x = x \times 1 = x$. Hence, it is an identity.
   (c) Inverse: Let $x \in \mathbb{R}/\{0\}$, Now, $x \times \frac{1}{x} = \frac{1}{x} \times x = 1$, where 1 is the multiplicative inverse.
4. $(\mathbb{C}/\{0\}, \times)$ is a group or not. Yes it is a group:
   (a) Associativity: Multiplication is associative.
   (b) Identity: 1 is an identity: Let $x \in \mathbb{C}/\{0\}$. Now, $1 \times x = x \times 1 = x$. Hence, it is an identity.
   (c) Inverse: Let $x \in \mathbb{C}/\{0\}$, Now, $x \times \frac{1}{x} = \frac{1}{x} \times x = 1$, where 1 is the multiplicative inverse.