MT2123 - Advanced Linear Algebra

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Contents

Chapter 1

Fields and Vector Spaces

1.1 Groups

Definition A group $\langle G, * \rangle$ is a set G with a binary operation $*$ such that the following axioms are satisfied:

1. Closure: For all $a, b \in G$, $a * b \in G$.
2. Associativity: For all $a, b, c \in G$, $a * (b * c) = (a * b) * c$.
3. Identitiy Element: There exists an element $I \in G$ such that for all $I \in G$, $a * I = I * a = a$. Here, I is called as the identity element of $*$ in G.
4. Inverse: corresponding to every element $a \in G$, there exists an element $a' \in G$ such that $a * a' = a' * a = I$. Here, a' is called as the inverse of a in G.

1.2 Rings

Definition A ring $\langle R, +, \cdot \rangle$ is a set R with two binary operations $+$ and $\cdot$, which we call addition and multiplication, such that the following axioms are satisfied:

1. $\langle R, + \rangle$ is an abelian/commutative group.
2. Multiplication is associative: For all $a, b, c \in R$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
3. Distributive Property: For all $a, b, c \in R$, the Left Distributive Law, $a \cdot (b + c) = a \cdot b + a \cdot c$ and Right Distributive Law, $(a + b) \cdot c = a \cdot c + b \cdot c$.

1.3 Fields

Definition

Chapter 2

Linear Transformations

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