MT2123 - Advanced Linear Algebra

Nachiketa Kulkarni

Contents

Chapter 1

Fields and Vector Spaces

1.1 Groups

Definition A group $\langle G, * \rangle$ is a set G with a binary operation $*$ such that the following axioms are satisfied:

1. Closure: For all $a, b \in G$, $a * b \in G$.
2. Associativity: For all $a, b, c \in G$, $a * (b * c) = (a * b) * c$.
3. Identity Element: There exists an element $I \in G$ such that for all $I \in G$, $a * I = I * a = a$. Here, I is called as the identity element of $*$ in G.
4. Inverse: corresponding to every element $a \in G$, there exists an element $a' \in G$ such that $a*a' = a'*a = I$. Here, a' is called as the inverse of a in G.

1.2 Rings

Definition A ring $\langle R, +, \cdot \rangle$ is a set R with two binary operations $+$ and $\cdot$, which we call addition and multiplication, such that the following axioms are satisfied:

1. $\langle R, + \rangle$ is an abelian/commutative group.
2. Multiplication is associative: For all $a, b, c \in R$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
3. Distributive Property: For all $a, b, c \in R$, the Left Distributive Law, $a \cdot (b + c) = a \cdot b + a \cdot c$ and Right Distributive Law, $(a + b) \cdot c = a \cdot c + b \cdot c$.

1.3 Fields

Definition A field $\langle F, +, \cdot \rangle$ is a set F with two binary operations $+$ and $\cdot$, which we call addition and multiplication, such that the following axioms are satisfied:

1. Closure: For all $a, b \in F$, $a + b \in F$ and $a \cdot b \in F$.
2. Associativity: For all $a, b, c \in F$, $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
3. Commutativity: For all $a, b \in F$, $a + b = b + a$ and $a \cdot b = b \cdot a$.
4. Identity Elements: There exist two elements $I, O \in F$ such that for all $a \in F$, $I \cdot a = a$ and $O + a = a$. Here, I is called as the multiplicative identity and O is called as the additive identity.
5. Additive Inverse: For all $a \in F$, there exists an element $-a \in F$ such that $a + (-a) = O$. Here, $-a$ is called as the additive inverse of a.

6. Multiplicative Inverse: For all $a \neq O \in F$, there exists an element $a^{-1} \in F$ such that $a \cdot a^{-1} = I$. Here, a^{-1} is called as the multiplicative inverse of a.

7. Distributivity: For all $a, b, c \in F$, the Left Distributive Law, $a \cdot (b + c) = a \cdot b + a \cdot c$ and Right Distributive Law, $(a + b) \cdot c = a \cdot c + b \cdot c$.

Chapter 2

Linear Transformations

long wall of text incoming