MT3164 - Fields and Galois Theory

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Contents

Chapter 1

Basic Ring and Field Theory

1.1 Rings and Fields

A Ring, R, is a non-empty set with two operations $+$ and . with the following properties:

- $(R, +)$ is an abelian group.
- . is a binary operation on R
- . is associative.
- . distributes over $+$.

Multiplicative Identity: Some Rings have a Multiplicative Identity, $1 \neq 0$ such that, $\forall x \in R$:

$$x.1 = x = 1.x$$

Commutative Ring: Where the multiplication operation is Commutative.
A field is a commutative ring with identity, where every non-zero element has an inverse.