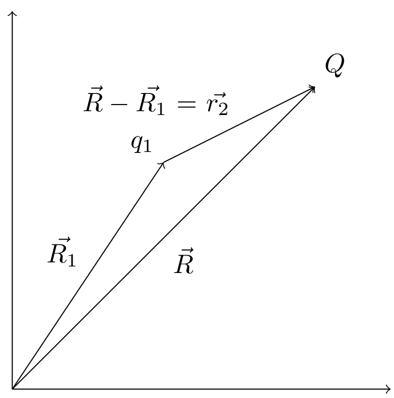
Introduction to Electromagnetism

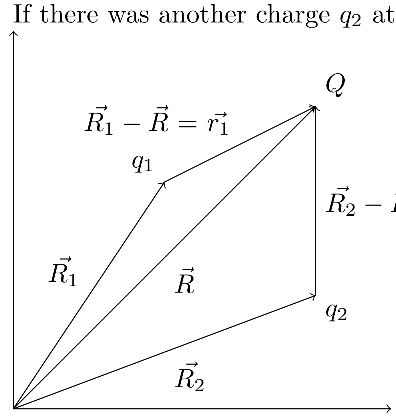
1 Coulomb's Law



the force on Q due to q_1 is given by:

$$\vec{F_1} = \frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{r_1^2}\hat{r_1}$$

If there was another charge q_2 at $\vec{r_2}$:



The force on Q due to q_2 is given by:

$$\vec{F_2} = \frac{1}{4\pi\epsilon_0}\frac{q_2 Q}{r_2^2}\hat{r_2}$$

And the net charge on Q is given by:

$$\vec{F} = \vec{F_1} + \vec{F_2}$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{r_1^2}\hat{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2 Q}{r_2^2}\hat{r_2}$$

If there were n charges, the net force on Q would be:

$$\vec{F} = \sum_{i=1}^{n}\frac{1}{4\pi\epsilon_0}\frac{q_i Q}{r_i^2}\hat{r_i}$$

$$\vec{F} = Q\sum_{i=1}^{n} \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i^2}\hat{r}_i$$

where $\sum_{i=1}^{n} \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i^2}\hat{r}_i$ is the electric field $\vec{E}$ at $\vec{R}$ due to the n charges.

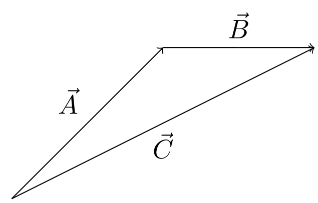
Field: A value, vector or tensor, that is defined for every point in space and time.

2 Vector Calculus

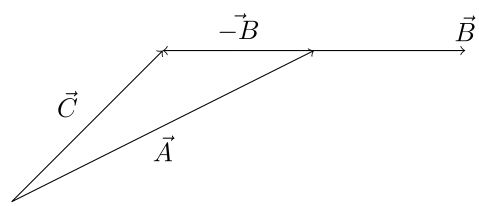
Vectors A vector is a quantity that is said to transform like a displacement.

Operations on Vectors

- **Addition**: $\vec{A} + \vec{B} = \vec{C}$



- **Subtraction:** $\vec{A} - \vec{B} = \vec{C}$



- **Multiplication by a scalar:** $c\vec{A} = \vec{C}$
- **Dot Product:** $\vec{A}.\vec{B} = AB\cos(\theta)$. Dot product is a scalar. It is commutative and distributive.
- **Cross Product:** $\vec{A} \times \vec{B} = AB\sin(\theta)\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

Cross product is a vector. It is anti-commutative and distributive.

Component form of a Vector:

A vector $\vec{A}$ can be written as:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x, y, z axes respectively.

Given that $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, we can perform the following operations:

- **Addition (and Subtraction):** $\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$
- **Multiplication by a scalar:** $c\vec{A} = cA_x\hat{i} + cA_y\hat{j} + cA_z\hat{k}$, where c is a scalar.
- **Dot Product:** $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$
- **Modulus:** $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
- **Cross Product:** $\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$ This can also be written in a determinant form:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Triple Product:

Scalar Triple Product: $\vec{A}.(\vec{B}\times\vec{C}) = \vec{B}.(\vec{C}\times\vec{A}) = \vec{C}.(\vec{A}\times\vec{B})$ Geometrically the scalar triple product is the volume of the parallelopiped formed by the three vectors.

Vector Triple Product: $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$