MT3244 - Calculus on Manifolds

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Contents

Chapter 1

Parameterized Curves and Surfaces (in $\mathbb{R}^n$)

1.1 Paramterized Curves

Definition A paramterized curve is a continiuous function

$$f : I \to \mathbb{R}^n$$

where I is an open set in $\mathbb{R}$.

1.1.1 Examples

1. Straight line: $f : \mathbb{R} \to \mathbb{R}^2$ defined as $f(\alpha) = (\alpha, m\alpha + c)$ where m and c are constants.
2. Parabola: $f : \mathbb{R} \to \mathbb{R}^2$ defined as $f(\alpha) = (\alpha, m\alpha^2)$ where m is a constant.
3. Circle: $f : \mathbb{R} \to \mathbb{R}^2$ defined as $f(\alpha) = (\cos\alpha, \sin\alpha)$.
   - Function is bounded.
   - There is no polynomial paramterization of this curve.
   - Rational Parameterizations exists:

$$f(t) = \left(\frac{t^2 - 1}{t + 1}, \frac{2t}{t + 1}\right)$$

4. Not involving Modulus: $f : \mathbb{R} \to \mathbb{R}^2$ defined as $f(\alpha) = (\alpha^3, \alpha^2)$.

1.1.2 Differentiation Curves:

A function f which is differentiable for a $t \in I$ is called a Parameterized Differentiable Curves [for all possible paramterizations]. Let $f(t) = (f_1(t), f_2(t), \ldots, f_n(t))$, then its derivative is defined as: $f'(t) = (f_1'(t), f_2'(t), \ldots, f_n'(t))$. This is used to define the direction of the tangent at a point $f(t)$ as $f'(t)$.

Regular Curve: A differentiable curve such that the tangent vector is non-zero for all $t \in I$

1.2 Paramterized Surfaces