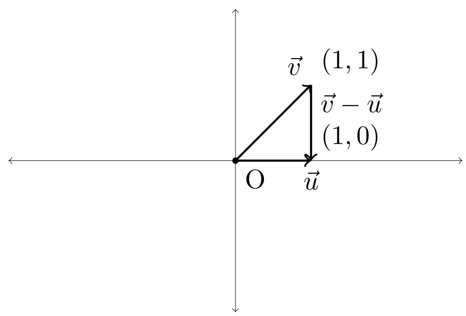
Calculus II

1 Vector Calculus

$\mathbb{R}$ represents the set of real numbers.
$\mathbb{R}^2$ represents a 2 dimensional real plane.

$$\mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}$$



Normally elements of $\mathbb{R}$ are known as scalers.
We can

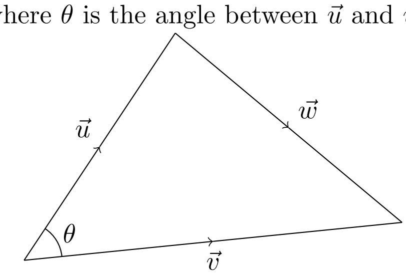
- add (or subtract) two vectors
- if $c \in \mathbb{R}$ and $v \in \mathbb{R}^2$, $c\vec{v}$
- $u = (u_1, u_2)$ and $v = (v_1, v_2)$

$$u.v = u_1.v_1 + u_2.v_2$$

Theorem

$$u.v = |u||v| \cos(\theta)$$

where θ is the angle between $\vec{u}$ and $\vec{v}$ and $|u|$ is the length of vector $\vec{u}$



$$w^2 = u^2 + v^2 - 2|u||v| \cos(\theta) \ldots (1)$$

$$\vec{w} = \vec{v} - \vec{u}$$

$$w = (v_1 - u_1, v_2 - u_2)$$

$$w^2 = (v_1 - u_1)^2 + (v_2 - u_2)^2$$

$$w^2 = v_1^2 - 2v_1u_1 + u_1^2 + v_2^2 - 2v_2u_2 + u_2^2$$
$$w^2 = v^2 + u^2 - 2v_1u_1 - 2v_2u_2 \ldots (2)$$

now, as (1) = (2)

$$u^2 + v^2 - 2|u||v|\cos(\theta) = v^2 + u^2 - 2v_1u_1 - 2v_2u_2$$
$$|u||v|\cos(\theta) = v_1u_1 + v_2u_2$$
$$|u||v|\cos(\theta) = \vec{u}.\vec{v}$$

Hence proved.

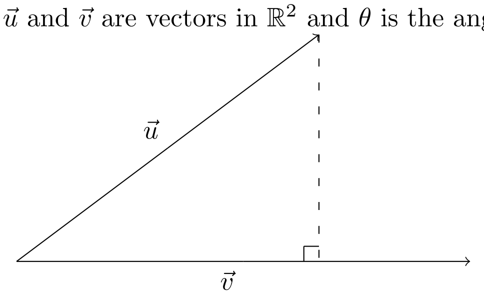
Unit Vectors

If $\vec{v} \in \mathbb{R}^2$ is a vector. then,

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

projections

If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^2$ and θ is the angle between them:



let $\vec{w}$ be the projection of $\vec{u}$ on $\vec{v}$

$$\vec{w} = |u|\cos(\theta)\hat{v}$$
$$\vec{w} = \frac{|u|\cos(\theta)|v|\vec{v}}{|v|^2}$$
$$\vec{w} = \frac{\vec{u}.\vec{v}}{|\vec{v}|^2}\vec{v}$$