MT3434 - Topics in Number Theory

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Contents

Chapter 1

Basics

1.1 Arithmetic Functions

An Arithmetical function is any function defined as $f : \mathbb{N} \to \mathbb{C}$.

Additive Arithmetic Funtion: An Arithmetic function is Additive if for all relatively primes $m, n \in \mathbb{N}$:

$$f(m \cdot n) = f(m) + f(n)$$

If the above function holds for all $m, n \in \mathbb{N}$ then f is completely additive.

Multiplicative Arithmetic Funtion: An Arithmetic function is Multiplicative if for all relatively primes $m, n \in \mathbb{N}$:

$$f(m \cdot n) = f(m) \cdot f(n)$$

If the above function holds for all $m, n \in \mathbb{N}$ then f is completely multiplicative.

1.1.1 Examples:

1. $\omega(x) =$ No. of distinct Prime divisors of x.
   $\Rightarrow$ Additive, but not completely.
2. $\Omega(x) =$ No. of Prime divisors of x, counted with multiplicity.
   $\Rightarrow$ Completely Additive.