DEVELOPMENT ECONOMICS. HOMEWORK 2

QUESTION 1.1.

As a note regarding the methodology that allows to estimate the welfare gains for each of the cases of interest, for individual welfare gains we understand the proportional increase in each period´s consumption that would allow the consumer to remain indifferent. That is, if comparing when no seasonal risk, we should consider welfare gains g such that

$$\sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u((1+g)z_i e^{g(m)}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i}) = \sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u(z_i e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i})$$

And if comparing with no non-seasonal risk, then welfare gains g would be such that

$$\sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u((1+g)z_i e^{g(m)}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i}) = \sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u(z_i e^{g(m)})$$

1.1.a) Compute the welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality in Table 1.

The mean welfare gains are contained in Figure 1. For the case of η=1, individuals with log utility present risk aversion. This risk aversion results in higher utility from the certain outcome: the expected return presents a higher value for the consumer than the expectation of the utilities of all possible outcomes. Consequently, as consumption smooths higher levels of utility are reached, which is precisely what figure 1 shows, this is, in those cases in which seasonality presents a higher variability its removal will lead to higher efficiency gains.

Figure 1. Mean welfare gains of removing seasonal component by degree of seasonality

Low	Medium	High
0.00420	0.00862	0.01713

1.1.b) Compute the welfare gains of removing the non-seasonal consumption risk

The mean welfare gains and standard deviations are contained in figure 2. In contrast to the previous case, the nonseasonal component is individual specific and thus a distribution will emerge for each degree of seasonality. From the combined analysis of both moments we appreciate that the distributions of gains are almost, if not totally, identical across all degrees of seasonality, which makes sense if we consider that this component of risk is independent of the seasonal risk component.

In any case, as the non-seasonal consumption risk is removed, because of risk aversion, consumer´s utility will increase (unless persistence in positive shocks), leading to welfare gains.

Figure 2. Mean welfare gains of removing nonseasonal component by degree of seasonality

	Low	Medium	High
Mean	0.10956	0.10956	0.10956
Standard deviation	0.081644	0.081644	0.081644

1.1.c) Compare and discuss your results in (a) and (b)

While seasonal component is identical across individuals, nonseasonal component is induvial specific. This results in a rich dispersion of welfare gains when the nonseasonal component is removed which cannot be observed in the welfare gains from removing the seasonal component. Additionally, from the comparison of table 2 and table 3, it is clear that the variability in consumption produced by the nonseasonal component is considerably larger than that derived from seasons, resulting in higher welfare gains in the former´s case.

1.1.d) Redo for η = 2, 4

The results of removing the components of risk when η = 2, 4 are summarized in figures 3 and 4. The table clearly shows that the more risk averse individuals are (the bigger the η), the more pronounced the welfare gains are. However, for a given value of η, welfare gains from removing the nonseasonal component tend to be bigger than welfare gains from removing the seasonal component, which is a sign that the dispersion introduced in the model by this last component is stronger, as we already pointed out.

Figure 2. Mean welfare gains of removing nonseasonal component by degree of seasonality for η = 2, 4

	Low seasonality	Medium seasonality	High seasonality
η=2	0.0065698	0.018464	0.060107
η=4	0.011824	0.042584	0.18667

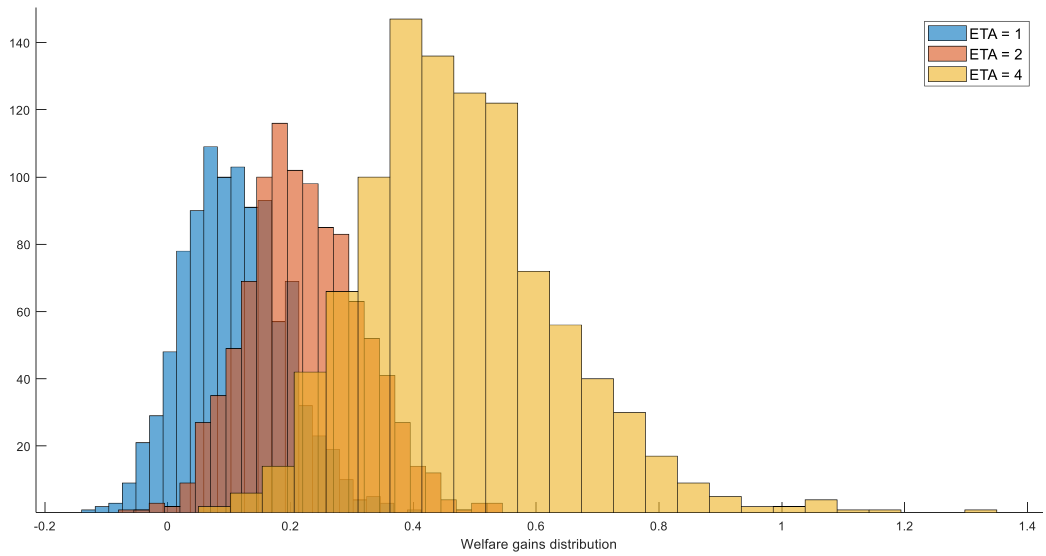
Moreover, when comparing across η, the impact of reducing the nonseasonal risk component, we have that not only mean welfare gains increase, but also its dispersion. The second and third moments (see histogram) increase as η grows. Once again, this is a result of the increasing risk aversion, the same patterns of consumption across individuals will result in increasingly different and smaller utility levels, consequently welfare gains will be larger on average but also more disperse.

Figure 3. Mean welfare gains of removing nonseasonal component by degree of seasonality for η = 2, 4

		Low seasonality	Medium seasonality	High seasonality
$\eta = 2$	Mean	0.22352	0.22352	0.22352
	Standard dev.	0.093859	0.093859	0.093859
$\eta = 4$	Mean	0.47825	0.47825	0.47825
	Standard dev.	0.16173	0.16173	0.16173

In conclusion, the distribution of welfare gains from removing the nonseasonal component has a similar shape for all η. However, as risk aversion increases it becomes more dispersed and skewed, in other words, risk aversion amplifies the first, second and third moments of the distribution.

Figure 4. Histogram of welfare gains of removing nonseasonal component for η = 1,2, 4



QUESTION 1.2.

As previously, for individual welfare gains we understand the proportional increase in each period´s consumption that would allow the consumer to remain indifferent. That is, if comparing when no seasonal risk, we should consider welfare gains g such that

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+g) z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i})$$

Lastly, if we remove the nonseasonal risk component, welfare gains g are those for which

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+g) z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i})$$

On the other hand, we need to highlight that, in total, there are potentially 9 feasible combinations of seasonal risk, 3 come from the deterministic part and 3 from the stochastic part. All these leads to focusing the analysis in the mean welfare gains and not in other moments of the distribution of gains (see figure 5).

However, if we consider that the seasonality non-stochastic and stochastic risk components are linked, in figure 6 it is showed that the bigger the seasonality component the higher the gains from its removal and that this effect intensifies even more when eta increases. This is a clear result of the nature of the utility function, as risk aversion increases, because of the addition of an stochastic seasonal component, the removal of the whole seasonal risk component leads to a higher utility and therefore, to potentially bigger welfare gains.

For the rest of the moments, we disclose a table (figure 7) containing a summary of the standard deviations of the welfare gains of removing the seasonal component that follows the commented pattern.

Figure 6. Summary table of mean welfare gains for η = 1,2, 4

		Low/Low	Med/Med	High/High
η=1	Removing seasonal component	0.055941	0.11527	0.24167
	Removing nonseasonal component	0.10956	0.10956	0.10956
η=2	Removing seasonal component	0.1112	0.23632	0.53436
	Removing nonseasonal component	0.22394	0.22425	0.22272
η=4	Removing seasonal component	0.22761	0.50818	1.2536
	Removing nonseasonal component	0.47757	0.47683	0.45654

Figure 7. Summary table of standard deviations welfare gains (removing seasonal comp.) for η = 1,2, 4

		Low/Low	Med/Med	High/High
η=1	Removing seasonal component	0.0148	0.0225	0.0358
η=2	Removing seasonal component	0.0176	0.0283	0.0522
η=4	Removing seasonal component	0.0438	0.1137	0.2879

Figure 5. Mean welfare gains of removing nonseasonal and seasonal component for η = 1,2, 4

		Low/Low	Low/Med	Low/High	Med/Low	Med/Med	Med/High	High/Low	High/Med	High/High
η=1	Removing seasonal component	0.055941	0.1104	0.22591	0.060567	0.11527	0.23128	0.069521	0.12468	0.24167
	Removing nonseasonal component	0.10956	0.10956	0.10956	0.10956	0.10956	0.10956	0.10956	0.10956	0.10956
η=2	Removing seasonal component	0.1112	0.22781	0.50201	0.12153	0.23632	0.50539	0.16103	0.27325	0.53436
	Removing nonseasonal component	0.22394	0.22422	0.22279	0.22391	0.22425	0.22277	0.22384	0.22432	0.22272
η=4	Removing seasonal component	0.22761	0.50955	0.24363	0.24363	0.50818	1.2708	0.35925	0.58795	1.2536
	Removing nonseasonal component	0.47757	0.47596	0.45093	0.47755	0.47683	0.45276	0.47714	0.47821	0.45654

Note: for each column A/B, A is the non-stochastic seasonal component of the risk and B is the stochastic seasonal component of risk

QUESTION 2

As it is explained in the code, some critical assumptions have been made to construct the seasonal components of labor supply. In relation to the covariance matrix, the highest possible covariances have been chosen so as to guarantee the invertibility of the matrix.

2.a) Assume a deterministic seasonal component and a stochastic seasonal component for labor supply both of which are highly positively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure

Figure 8 contains the mean welfare gains of removing the seasonal effect distinguishing by the effect of consumption and leisure. Firs of all, note that the labor effect clearly dominates: considering that, on the one hand, the units in which it is measured (monthly hours) are much bigger than those used for consumption and, on the other hand, the dispersion levels of both variables are similar because of the way in which they have been constructed, in absolute terms dispersion is bigger in the number of hours worked and, consequently, welfare gains from labor smoothing are bigger than those from consumption smoothing.

Moreover, we can also see, because of the positive correlation of the consumption counterparts, that both effects move in the same direction, that is, the bigger the dispersion in consumption leads to a bigger dispersion in labor, which results in an even higher mean welfare gain from removing the season effect.

Figure 8. Mean welfare gains of removing seasonal component by effect. Positive correlation

	Low	Medium	High
Total effect	0.26466	0.91103	4.1037
Consumption effect	0.056158	0.11424	0.24476
Labor effect	0.19753	0.72586	3.3957

(b) Assume a deterministic seasonal component and a stochastic seasonal component for labor supply both of which are highly negatively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

Figure 9 shows a similar pattern to figure 8: because of the labor effect dominates, even though negatively correlated, as the seasonal effect intensifies (from low to high), the total effect increases. However, it is worth noting that, mean welfare gains from consumption reduce, precisely as a result of the negative correlation of the seasonal components of consumption and labor. In other words, because the seasonal components take opposing directions, they somehow smooth the aggregate utility, leading to less potential welfare gains.

Figure 9. Mean welfare gains of removing seasonal component by effect. Negative correlation

	Low	Medium	High
Total effect	0.25484	0.8127	3.6114
Consumption effect	0.04719	0.095236	0.20113
Labor effect	0.19827	0.65935	3.0393

(c) How do your answers to (a) and (b) change if the nonseasonal stochastic component of consumption and leisure are correlated?

Intuitively, a positive intense correlation would amplify the commented effects while a negative correlation could potentially soften these problems, leading to a potential reduction in the potential welfare gains.