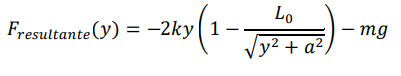
** UNIVERSIDAD DE BUENOS AIRES  
 FACULTAD DE INGENIERÍA  
 <75.12> ANÁLISIS NUMÉRICO**

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| **CORRECCIONES** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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***Introducción:***

El objetivo del trabajo práctico fue utilizar métodos numéricos para hallar los puntos de equilibrio de un sistema mecánico compuesto por una bola y dos resortes. Los métodos numéricos utilizados fueron Regula-Falsi, Punto Fijo y Newton-Raphson. La función a la cuál se le desean calcular las raíces es:



En primer lugar se consideró que la bola se encontraba sobre una mesa sin rozamiento de manera tal que las únicas fuerzas actuantes fueran solo las elásticas. En este caso se pueden hallar los puntos de equilibrio de manera analítica. La finalidad de utilizar métodos numéricos para hallar resultados que podían hallarse de manera analítica fue poder confirmar que los valores hallados por métodos numéricos eran iguales a los analíticos, mostrando que ambos métodos son válidos.

En segundo lugar se buscaron los puntos de equilibrio considerando que la bola estaba colgando, agregando así el peso a la ecuación. En este caso los puntos de equilibrio no se pueden hallar de forma analítica, solo se pueden aproximar de manera numérica. En el trabajo práctico estas aproximaciones se realizaron considerando 15 dígitos significativos.

***Desarrollo:***

El trabajo práctico consistió de 5 puntos (para las cuentas se consideró el Número de Padrón 99411):

En el primer punto se utilizaron los métodos numéricos para hallar la raíz positiva ya que es sencillo observar que las otras dos raíces son el cero y el valor negativo de la raíz positiva. Los datos obtenidos al implementar los métodos fueron los siguientes:

1. Regula-Falsi:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k | ak | bk | F(ak) | F(bk) | rk+1 | Δ rk+1 | Δ r/r | λ | p |
| 0 | 0.1 | 2 | 2.004029653 | -4.008249834 | 0.733313263 |  |  |  |  |
| 1 | 0.733313263 | 2 | 9.12978504 | -4.008249834 | 1.613549747 | 0.880236484 | 0.545527949 |  |  |
| 2 | 1.613549747 | 2 | 1.932894202 | -4.008249834 | 1.739277628 | 0.125727882 | 0.072287414 |  |  |
| 3 | 1.739277628 | 2 | 0.099490609 | -4.008249834 | 1.745592396 | 0.006314768 | 0.00361755 | 0.15296277 | 1.537060752 |
| 4 | 1.745592396 | 2 | 0.004533681 | -4.008249834 | 1.745879828 | 0.000287432 | 0.000164635 | 0.053772508 | 1.032906433 |
| 5 | 1.745879828 | 2 | 0.000205396 | -4.008249834 | 1.74589285 | 1.30E-05 | 7.46E-06 | 0.045872518 | 1.001534194 |
| 6 | 1.74589285 | 2 | 9.30E-06 | -4.008249834 | 1.74589344 | 5.90E-07 | 3.38E-07 | 0.045327912 | 1.000069583 |
| 7 | 1.74589344 | 2 | 4.21E-07 | -4.008249834 | 1.745893466 | 2.67E-08 | 1.53E-08 | 0.045294049 | 1.000003147 |
| 8 | 1.745893466 | 2 | 1.91E-08 | -4.008249834 | 1.745893467 | 1.21E-09 | 6.93E-10 | 0.045292177 | 1.000000265 |
| 9 | 1.745893467 | 2 | 8.64E-10 | -4.008249834 | 1.745893468 | 5.48E-11 | 3.14E-11 | 0.045290557 | 0.999998214 |
| 10 | 1.745893468 | 2 | 3.91E-11 | -4.008249834 | 1.745893468 | 2.48E-12 | 1.42E-12 | 0.045295523 | 1.000003553 |
| 11 | 1.745893468 | 2 | 1.78E-12 | -4.008249834 | 1.745893468 | 1.13E-13 | 6.45E-14 | 0.044766173 | 0.999506022 |
| 12 | 1.745893468 | 2 | 7.75E-14 | -4.008249834 | 1.745893468 | 4.88E-15 | 2.80E-15 | 0.066549284 | 1.014343586 |
| 13 | 1.745893468 | 2 | 7.75E-15 | -4.008249834 | 1.745893468 | 4.44E-16 | 2.54E-16 |  |  |

En 13 iteraciones se llega a la raíz positiva, r = 1.745893468. El orden de convergencia es aproximadamente 1 como era esperable en este método y la constante asintótica del error es muy baja, menor a 0.1.

b, c) Los métodos de Punto fijo y Newton-Raphson divergen para la función dada. Tras muchas pruebas con muchas semillas cercanas a las raíces para intentar asegurar que pertenezcan al intervalo de convergencia y no lograr llegar a la raíz con ninguna, se considera que dichos métodos divergen. (Ver datos adjuntos)

d) Debido a que dos de los tres métodos divergen, el gráfico consta del comportamiento del error relativo de Regula-Falsi únicamente:

El punto 2 consistía en hallar los 3 puntos de equilibrio del sistema considerando que la bola ejercía una fuerza con su peso en el sistema.

1. Regula-Falsi:

En este caso son 3 las tablas con datos ya que son 3 las raíces.

Raíz 1:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k | ak | bk | F(ak) | F(bk) | rk+1 | Δ rk+1 | Δ r/r | λ | p |
| 0 | 0.1 | 1 | -0.956628347 | 5.493318875 | 0.233484117 |  |  |  |  |
| 1 | 0.1 | 0.233484117 | -0.956628347 | 1.518982842 | 0.151581077 | -0.081903041 | 0.54032497 |  |  |
| 2 | 0.1 | 0.151581077 | -0.956628347 | 0.038452928 | 0.149587829 | -0.001993248 | 0.013324932 |  |  |
| 3 | 0.1 | 0.149587829 | -0.956628347 | 0.000762461 | 0.149548338 | -3.95E-05 | 0.000264072 | 0.027951767 | 1.055349257 |
| 4 | 0.1 | 0.149548338 | -0.956628347 | 1.50E-05 | 0.149547559 | -7.78E-07 | 5.20E-06 | 0.019983265 | 1.001379206 |
| 5 | 0.1 | 0.149547559 | -0.956628347 | 2.96E-07 | 0.149547544 | -1.53E-08 | 1.03E-07 | 0.019711221 | 1.000027346 |
| 6 | 0.1 | 0.149547544 | -0.956628347 | 5.83E-09 | 0.149547544 | -3.02E-10 | 2.02E-09 | 0.019703787 | 1.000000528 |
| 7 | 0.1 | 0.149547544 | -0.956628347 | 1.15E-10 | 0.149547544 | -5.95E-12 | 3.98E-11 | 0.019702555 | 0.999997051 |
| 8 | 0.1 | 0.149547544 | -0.956628347 | 2.26E-12 | 0.149547544 | -1.17E-13 | 7.84E-13 | 0.019732398 | 1.0000661 |
| 9 | 0.1 | 0.149547544 | -0.956628347 | 4.44E-14 | 0.149547544 | -2.30E-15 | 1.54E-14 | 0.020056122 | 1.00069567 |
| 10 | 0.1 | 0.149547544 | -0.956628347 | 1.33E-15 | 0.149547544 | -8.33E-17 | 5.57E-16 |  |  |

Se necesitaron 10 iteraciones para llegar a la aproximación deseada de la raíz.

Raíz 2:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k | ak | bk | F(ak) | F(bk) | rk+1 | Δ rk+1 | Δ r/r | λ | p |
| 0 | 1 | 2 | 5.493318875 | -6.968907834 | 1.44079754 |  |  |  |  |
| 1 | 1.44079754 | 2 | 1.281271074 | -6.968907834 | 1.527642922 | 0.086845382 | 0.056849268 |  |  |
| 2 | 1.527642922 | 2 | 0.154461151 | -6.968907834 | 1.537885381 | 0.010242459 | 0.006660093 |  |  |
| 3 | 1.537885381 | 2 | 0.01685117 | -6.968907834 | 1.539000102 | 0.001114721 | 0.000724315 | 0.129285299 | 1.037589162 |
| 4 | 1.539000102 | 2 | 0.001817761 | -6.968907834 | 1.539120317 | 0.000120215 | 7.81E-05 | 0.110907 | 1.004119957 |
| 5 | 1.539120317 | 2 | 0.000195845 | -6.968907834 | 1.539133269 | 1.30E-05 | 8.41E-06 | 0.108169925 | 1.000444698 |
| 6 | 1.539133269 | 2 | 2.11E-05 | -6.968907834 | 1.539134664 | 1.40E-06 | 9.06E-07 | 0.107783212 | 1.000047915 |
| 7 | 1.539134664 | 2 | 2.27E-06 | -6.968907834 | 1.539134814 | 1.50E-07 | 9.77E-08 | 0.107731363 | 1.000005161 |
| 8 | 1.539134814 | 2 | 2.45E-07 | -6.968907834 | 1.539134831 | 1.62E-08 | 1.05E-08 | 0.107724687 | 1.000000565 |
| 9 | 1.539134831 | 2 | 2.64E-08 | -6.968907834 | 1.539134832 | 1.74E-09 | 1.13E-09 | 0.107723896 | 1.000000097 |
| 10 | 1.539134832 | 2 | 2.84E-09 | -6.968907834 | 1.539134832 | 1.88E-10 | 1.22E-10 | 0.107720996 | 0.999998596 |
| 11 | 1.539134832 | 2 | 3.06E-10 | -6.968907834 | 1.539134833 | 2.02E-11 | 1.31E-11 | 0.107741089 | 1.000007845 |
| 12 | 1.539134833 | 2 | 3.30E-11 | -6.968907834 | 1.539134833 | 2.18E-12 | 1.42E-12 | 0.107806816 | 1.000035077 |
| 13 | 1.539134833 | 2 | 3.55E-12 | -6.968907834 | 1.539134833 | 2.35E-13 | 1.53E-13 | 0.107190115 | 0.999802093 |
| 14 | 1.539134833 | 2 | 3.87E-13 | -6.968907834 | 1.539134833 | 2.55E-14 | 1.66E-14 | 0.097111411 | 0.996124664 |
| 15 | 1.539134833 | 2 | 3.95E-14 | -6.968907834 | 1.539134833 | 2.66E-15 | 1.73E-15 |  |  |

Se necesitaron 15 iteraciones para llegar a la aproximación deseada de la raíz.

Raíz 3:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k | ak | bk | F(ak) | F(bk) | rk+1 | Δ rk+1 | Δ r/r | λ | p |
| 0 | -1 | -3 | -11.41463487 | 18.86432609 | -1.753964767 |  |  |  |  |
| 1 | -1.753964767 | -3 | -2.83890122 | 18.86432609 | -1.916953013 | -0.162988246 | 0.085024643 |  |  |
| 2 | -1.916953013 | -3 | -0.298912188 | 18.86432609 | -1.933846606 | -0.016893593 | 0.008735746 |  |  |
| 3 | -1.933846606 | -3 | -0.027606048 | 18.86432609 | -1.935404534 | -0.001557929 | 0.000804963 | 0.113808101 | 1.051542503 |
| 4 | -1.935404534 | -3 | -0.002517028 | 18.86432609 | -1.935546562 | -0.000142028 | 7.34E-05 | 0.094055719 | 1.00482976 |
| 5 | -1.935546562 | -3 | -0.000229224 | 18.86432609 | -1.935559496 | -1.29E-05 | 6.68E-06 | 0.091424813 | 1.000441037 |
| 6 | -1.935559496 | -3 | -2.09E-05 | 18.86432609 | -1.935560674 | -1.18E-06 | 6.08E-07 | 0.091100696 | 1.000040171 |
| 7 | -1.935560674 | -3 | -1.90E-06 | 18.86432609 | -1.935560781 | -1.07E-07 | 5.54E-08 | 0.091063264 | 1.000003658 |
| 8 | -1.935560781 | -3 | -1.73E-07 | 18.86432609 | -1.935560791 | -9.77E-09 | 5.05E-09 | 0.091059119 | 1.000000324 |
| 9 | -1.935560791 | -3 | -1.58E-08 | 18.86432609 | -1.935560792 | -8.89E-10 | 4.59E-10 | 0.091058585 | 0.999999959 |
| 10 | -1.935560792 | -3 | -1.44E-09 | 18.86432609 | -1.935560792 | -8.10E-11 | 4.18E-11 | 0.091058798 | 1.000000086 |
| 11 | -1.935560792 | -3 | -1.31E-10 | 18.86432609 | -1.935560792 | -7.37E-12 | 3.81E-12 | 0.091086596 | 1.000014732 |
| 12 | -1.935560792 | -3 | -1.19E-11 | 18.86432609 | -1.935560792 | -6.72E-13 | 3.47E-13 | 0.09064362 | 0.999804931 |
| 13 | -1.935560792 | -3 | -1.08E-12 | 18.86432609 | -1.935560792 | -6.11E-14 | 3.15E-14 | 0.093143828 | 1.000866418 |
| 14 | -1.935560792 | -3 | -9.77E-14 | 18.86432609 | -1.935560792 | -5.55E-15 | 2.87E-15 | 0.090909091 | 1 |
| 15 | -1.935560792 | -3 | -1.20E-14 | 18.86432609 | -1.935560792 | -6.66E-16 | 3.44E-16 |  |  |

Se necesitaron 15 iteraciones para llegar a la aproximación deseada de la raíz.

Debido a que los métodos de Punto Fijo y Newton-Raphson divergen, no se pueden hallar las raíces en este caso tampoco, por lo tanto, no hay valores para comparar.

El tercer punto trata de hallar el intervalo máximo de convergencia de cada raíz según el método de Newton-Raphson. Debido a que el método diverge al intentar encontrar las raíces para esta función, puede que no exista ningún intervalo de convergencia.

En el cuarto punto existe el mismo problema que en los puntos anteriores ya que se trata de analizar el comportamiento del método Newton-Raphson y el mismo diverge.

***Conclusiones:***

Como conclusión podemos observar que el orden de convergencia del método de Regula-Falsi tiende a mantenerse en 1 a pesar de que cambien los valores de la función, y la convergencia incondicional del método resulta muy útil cuando los otros métodos considerados divergen.

Respecto de los métodos de Punto Fijo y Newton-Raphson se puede apreciar que dependerán de la función a evaluar ya que puede ser que no exista una semilla tal que el método converja, o hallarla es extremadamente difícil.