TEACHER'S GUIDE: The Modular Spectrum of π

Pentalogy of Interactive STEM Workshops

Based on the research by José Ignacio Peinador Sala

1 Introduction and Context

This series of five interactive notebooks (Google Colab) brings recent mathematical research into the High School (Bachillerato) and early University classroom.

The goal is to break the barrier between "textbook mathematics" and current research, allowing students to rediscover the number π and the structure of integers through a novel approach: **Modular Arithmetic**.

Unlike traditional teaching, where formulas are memorized, here the student uses programming (Python) to experiment, visualize, and verify theorems in real-time, connecting disparate areas such as Number Theory, Statistics, Quantum Physics, and Computing.

Research Reference

This educational material is based on the research:

> Peinador Sala, J. I. (2025). *The Modular Spectrum of π: From Prime Channel Structure to Elliptic Supercongruences* (Version 1). Zenodo. https://doi.org/10.5281/zenodo.17680024

Project Repository: GitHub Espectro Modular Pi

2 Course Structure

The course is divided into five sessions or modules, designed to progressively increase conceptual complexity, from basic arithmetic to theoretical physics.

1. **Arithmetic and Patterns:** Infinite sums and the 6k modular clock.
2. **Analysis and Connections:** Products, Statistics, and Complex Numbers.
3. **Geometry and Application:** Volumes, Monte Carlo, and the Impossible.
4. **Tools of Infinity:** Zeta Function, Engineering, and Algorithms.
5. **The Source Code:** Sequences, Modular Derivatives, and Matrices.

Recommended Level: Students aged 16–18 (Science/Technology High School) or undergraduate students in introductory subjects.

3 Session Details

3.1 Session 1: Unlocking the Secrets of π

Colab Link: [Insert Link Here]

- **Key Concept:** Convergence speed and number filtering.
- **Teaching Points:**
  - **The Leibniz Problem:** Students start by programming the classical series. They experiment by changing N to observe the sluggishness of convergence.
  - **The Modular Clock (6k):** Visual introduction. It is demonstrated that prime numbers (and the terms of π) only inhabit positions 1 and 5 of the clock ($6k \pm 1$).
  - **FAQ:** *Why do we include composite numbers like 25?*
  - **Answer:** In the sum, we look for numbers coprime to 6 (not divisible by 2 or 3), not necessarily absolute primes.

3.2 Session 2: The Connected Universe of π

Colab Link: [Insert Link Here]

- **Key Concept:** π as a universal constant beyond geometry.
- **Teaching Points:**
  - **Wallis Product:** Born from multiplying fractions.
  - **The Gaussian Bell Curve (Statistics):** Interdisciplinary connection. The modular series is used to calculate the area under the normal curve.
  - **Euler's Identity:** Introduction to complex numbers ($e^{i\pi} + 1 = 0$) validated with the modular structure.
  - **Ramanujan:** Comparison of Efficiency vs. Understanding.

3.3 Session 3: π in Action (Geometry and Chance)

Colab Link: [Insert Link Here]

- **Key Concept:** The impact of error and simulation.
- **Teaching Points:**
  - **The Butterfly Effect:** A small error in π, cubed (r^3) in the volume of the Earth, generates a massive error.
  - **Monte Carlo vs. Modular:** We pit Order (formula) against Chaos (chance). The student verifies that mathematical structure beats randomness.
  - **The Impossible:** Calculating the factorial of 0.5 using the Gamma Function and π.

3.4 Session 4: Tools of Infinity (Engineering)

Colab Link: [Insert Link Here]

- **Key Concept:** Applications in Data Science and Computing.
- **Teaching Points:**
  - **Riemann Zeta Function:** Solving the Basel Problem ($\sum 1/n^2 = \pi^2/6$) by squaring the modular series. Drastic improvement in convergence.
  - **Error Function (erf):** Application in telecommunications and engineering.
  - **Prime Search Algorithms:** Code optimization. It is demonstrated how discarding useless 6k channels (evens and multiples of 3) speeds up algorithms by $\approx 33\%$.

3.5 Session 5: The Source Code of Mathematics (Theory)

Colab Link: [Insert Link Here]

- **Key Concept:** Abstraction and paradigm shift.
- **Teaching Points:**
  - **Numerical DNA:** Visualization of the number line as 6 intertwined threads.
  - **Modular Fibonacci:** Discovery of the 24-step cycle (Pisano Period) in nature.
  - **Modular Differential Calculus:** Redefinition of the derivative as a discrete jump (Δ) instead of a continuous limit.
  - **Fourier and Matrices:** Decomposition of signals and matrices into 'interaction channels' (Primes vs. Composites).

4 Technical Instructions

- **Platform:** Google Colab (web browser, Google account).
- **Libraries:** Standard Python (‘math‘, ‘numpy‘, ‘matplotlib‘, ‘cmath‘, ‘sympy‘, ‘time‘). No local installation required.
- **Dynamics:** The student executes cells using `Shift + Enter`. It is strongly recommended to modify the variable N (iterations) to observe changes in precision and time.

5 Challenge Solutions

- **Challenge 1 (Precision of π):** To get 6 correct decimal places with the modular series, approximately 500,000 terms are required.
- **Challenge 3 (Monte Carlo):** Randomness converges with the square root of the number of attempts. To match the modular precision of 100 terms, Monte Carlo would need millions of points.

- **Challenge 4 (Algorithms):** Modular optimization in prime search reduces the search space by eliminating classes 0, 2, 3, and 4.

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Citing the original source: Peinador Sala, J. I. (2025). The Modular Spectrum of π.