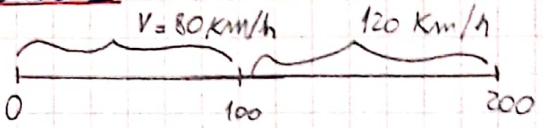
UNIDAD Nº 1 — CINEMATICA

VELOCIDAD MEDIA

EJERCICIO 1



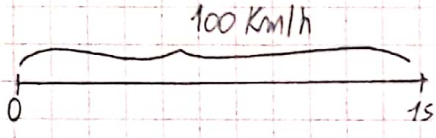
$t_1 = \frac{d_1}{V_1}$ $\quad t_2 = \frac{d_2}{V_2}$

$t_1 = \frac{100\,Km}{80\,Km/h}$ $\quad t_2 = \frac{100\,Km}{120\,Km/h}$

$t_1 = 1{,}25\,h$ $\quad t_2 = 0{,}83\,h$

$V_m = \frac{d_F - d_i}{t_F - t_i} = \frac{200 - 0}{2{,}8 + 0} = \boxed{96\ Km/h}$

EJERCICIO 2



$d_0 = 150\,km$ $\quad t_1 = 3h$ $\quad d_2 = 100\,Km$

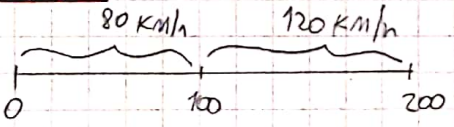
$V_0 = 100\,km/h$ $\quad V_1 = 80\,Km/h$ $\quad t_3 = 0{,}83\,h$

$t_0 = \frac{d_0}{V_0}$ $\quad d_0 = V_1 . t_1$

$t_0 = 1{,}5\,h$ $\quad d_0 = 240\,Km$

$V_m = \frac{d_F - d_i}{t_F - t_i} = \frac{490}{5{,}66} = \boxed{86{,}57\ km/h}$

EJERCICIO 3



$d_1 = 100\,km$ $\quad d_2 = 100\,km$

$V_1 = 80\,km/h$ $\quad V_2 = 120\,Km/h$

$t_1 = \frac{d_1}{V_1} = 1{,}25\,h$ $\quad t_2 = \frac{d_2}{V_2} = 0{,}83\,h$

$V_m = \frac{d_F - d_i}{t_F - t_i} = \frac{200}{2{,}08} = \boxed{96{,}15\ Km/h}$

EJERCICIO 4

$X_1 = 700\,cm$ $\quad X_2 = -900\,cm$ $\quad X_3 = 300\,cm$

$t_1 = 0\,s$ $\quad t_2 = 6\,s$ $\quad t_3 = 8\,s$

$V_m = \frac{-900 - 700}{6 - 0} = \boxed{-266{,}6\ cm/s}$ $\quad V_m = \frac{300 + 900}{8 - 6} = \boxed{600\ cm/s}$ $\quad V_m = \frac{300 - 700}{8} = \boxed{-50\ cm/s}$

EJERCICIO 5

$V_{m1} = 1200\,cm/s$ $\quad V_{m2} = 400\,cm/s$

$t_1 = 9\,s$ $\quad t_2 = 7\,s$

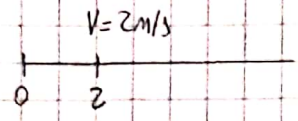
$V_{m1} = \frac{d_{F1} - d_i}{t_{F1} - t_1}$ $\quad V_{m2} = \frac{d_{F2} - d_{F1}}{t_2 - t_1}$ $\quad V_{mt} = \frac{14160 - 0}{16 - 0}$

$1200\,cm/s = \frac{d_{F1} - 0}{9 - 0}$ $\quad 400\,cm/s = \frac{d_{F2} - 10800}{16 - 7}$ $\quad V_{mt} = \boxed{885\ cm/s}$

$d_{F1} = 10800\,cm$ $\quad d_{F2} = \boxed{14160\ cm}$

MRU

EJERCICIO 6



$$X(t) = X_0 + v_0 (t - t_0)$$

$$X(t) = 2m + 2m/s\ (t - 0)$$

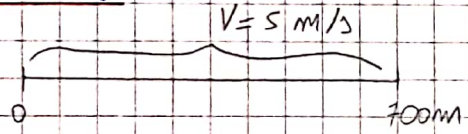
EJERCICIO 7

$X(t) = X_0 + v_0(t - t_0)$ $\quad X_0 = 20m \quad t_0 = 5s \quad v_0 = 20 m/s$

$X(t) = 20m + 20m/s\ (t - 5s)$

$X(10) = 20m + 20 m/s\ (10s - 5s) = \boxed{120 m}$

EJERCICIO 8



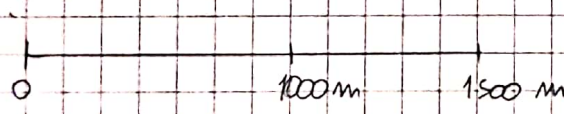
$$X(t) = X_0 + v_0 (t - t_0)$$

$$X(t) = 0 + 5 (t - 0)$$

Ⓐ $700 m = 5 . t \Rightarrow \boxed{t = 140 s}$

Ⓑ $X(20) = 5 . 20 = \boxed{100 m}$

EJERCICIO 9



Ⓐ $X(t) = x_0 + v_0 (t - t_0)$

$X(t) = -1000 m + v_0 (t - 0)$

Ⓑ $500 = -1000 + v_0 . 50$ $\quad 0 = -1000 + 30 t$

$\boxed{v_0 = 30 m/s}$ $\quad \boxed{t = 33,33 s}$

Ⓒ $X = -1000 + 30 (58,33)$

$\boxed{X = 750 m}$

EJERCICIO 10

(A) $V_m = \frac{1000 - 600}{16 - 0} = \boxed{25\ m/s}$

(B) $X(t) = x_0 + v_0 (t - t_0)$

$X(30) = 600m + 25 m/s\ (30 s - 0)$

$\boxed{X(30) = 1350\ m}$

(C) Si $x = 1570m$

$1570m = 600m + 25 m/s\ (t - 0)$

$\boxed{t = 38,8 s}$

MRUV

EJERCICIO 11

(A) $x = 2m$, $V = 2 m/s$, $a = 5 m/s^2$, $t_0 = 0$

$X(t) = X_0 + v_0 (t - t_0) + \frac{1}{2} a (t - t_0)^2$

$X(t) = 2m + 2 m/s \cdot t + 2,5 m/s^2 \cdot t^2$

EJERCICIO 12

$X(t) = x_0 + v_0 (t - t_0) + 1/2\ a (t - t_0)^2$

$a = \frac{V_F - V_i}{t_F - t_i} = \frac{40 - 20}{15 - 5} =$

$X(10) = 20 + 20 \cdot 10 + 1/2 \cdot 2 \cdot 10^2$

$\boxed{X(10) = 320\ m}$

EJERCICIO 13

$a = 2 m/s^2$

$v = v_0 + a_0 (t - t_0) \Rightarrow v = 0 + 2 \cdot (15 - 0) = \boxed{30 m/s}$

$X(15) = X_0 + v_0 (t - t_0) + 1/2\ a (t - t_0)^2 \Rightarrow X(15) = 0 + 0 + 1/2 \cdot 2 \cdot 15^2 = \boxed{225\ m}$

$216 km/h = 60 m/s \Rightarrow 60 m/s = 2 \cdot t \Rightarrow t = 30 s$

$X(30) = 0 + 0 + 1/2 \cdot 2 \cdot 30^2 = \boxed{900\ m}$

EJERCICIO 14

$V_0 = 10\ m/s$ $\quad V_F = 72\ Km/h = 20\ m/s$ $\quad t_F = 40\ s$

$$V_m = \frac{V_i + V_F}{2} = \frac{20 + 10}{2} = \boxed{15\ m/s}$$

EJERCICIO 15

$a = 8\ m/s^2$

$$v = v_0 + a_0 (t - t_0)$$

$$\left.\begin{array}{l} \text{i) } t=0,\ v_0 = 0 \Rightarrow v = 0 \\ \text{ii) } t=5,\ v = 0 \Rightarrow v = 40 \end{array}\right\} \quad V_m = \frac{40 + 0}{2} = \boxed{20\ m/s}$$

EJERCICIO 17

$V_i = 72\ km/h = 20\ m/s$

$V_F = 18\ km/h = 5\ m/s$, $t = 5\ s$

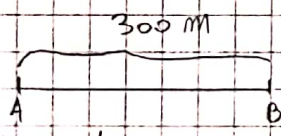
(A) $v = v_0 + a_0 (t - t_0)$

$5\ m/s = 20\ m/s + a_0 (5 - 0) \Rightarrow \boxed{a_0 = -3}$

(B) $X(t) = x_0 + v_0 (t - t_0) + \frac{1}{2} a_0 (t - t_0)^2$

$X(5) = 0 + 20 \cdot 5 + \frac{1}{2} (-3) \cdot 5^2 \Rightarrow \boxed{X(5) = 62{,}5\ m}$

EJERCICIO 18



$$\begin{cases} v = v_0 + a_0 (t - t_0) \\ X(t) = x_0 + v_0 (t - t_0) + \frac{1}{2} a_0 (t - t_0)^2 \end{cases} \quad \begin{cases} v = 30 + a \cdot t \rightarrow (1) \ \text{i) } v = 0 \\ 300 = 30t + \frac{1}{2} a t^2 \rightarrow (2) \end{cases}$$

(1) $50 = 30 + a \cdot t \Rightarrow a = \frac{50 - 30}{t} = \frac{20}{t} = \boxed{2{,}6\ m/s^2}$

(2) $300 = 30t + \frac{1}{2} \left(\frac{20}{t}\right) t^2 \Rightarrow \boxed{t = 7{,}5\ s}$

$v = 30 + 2{,}6 \cdot 20$

$\boxed{v = 83{,}3\ m/s}$

EJERCICIO 19

$t_1 = 4\,s$
$t_2 = 8\,s$
$\Big\} \Delta t = 4\,s \Rightarrow x = 48\,m$

$x(t) = x_0 + v_0(t - t_0) + \frac{1}{2} a (t - t_0)^2$

$x(t) = 0 + 0 + \frac{1}{2} a t^2$

Ⓐ

EJERCICIO 20

$a_0 = 0{,}25\ m/s^2$ $\quad v_0 = 0\ m/s$ $\quad v_1 = 90\ km/h = 25\ m/s$ $\quad a_F = 0{,}5\ m/s^2$ $\quad v_F = 0\ m/s$ $\quad \Delta x = 10\ km = 10000\ m$

$v_1 = v_0 + a_0 t \Rightarrow 25 = 0{,}25 \cdot t_1 \Rightarrow t_1 = 100\,s$

$v_F = v_1 + a_F t \Rightarrow 0 = 25 + (-0{,}5) t \Rightarrow t_2 = 50\,s$

$x(100) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \Rightarrow x(100) = 1250\,m$

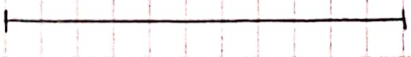
$x(50) = x_0 + v_1 t + \frac{1}{2} a_F t^2 \Rightarrow x(50) = 25 \cdot 50 - \frac{1}{2} \cdot 0{,}5 \cdot 50^2 = 625\,m$

$x(t_F) = x(100) + x(50) + x(t_3) \Rightarrow 10000 = 1250 + 625 + x(t_3) \Rightarrow x(t_3) = 8125\,m$

$x(t_3) = v_1 t_3 \Rightarrow 8125 = 25 \cdot t_3 \Rightarrow t_3 = 325\,s$

$t_F = t_1 + t_2 + t_3 \Rightarrow \boxed{t_F = 475\,s}$

EJERCICIO 21



$x_0 = 0$ $\quad x_F = 50\,m$
$t_0 = 0$
$v_0 = 20\,m/s$

Ⓑ
$\begin{cases} v = v_0 + a_0 t & \Rightarrow ① \\ x(t) = x_0 + v_0 t + \frac{1}{2} a t & \Rightarrow ② \end{cases}$

① $a = \frac{-20}{t} \Rightarrow \boxed{a = -4\,m/s}$

② $50 = 20t + \frac{1}{2}\left(\frac{-20}{t}\right) t^2$

$50 = 20t - 10t$

$t = 5\,s$

Ⓒ $x = x_0 + v(t - t_0) \Rightarrow x = v \cdot t \Rightarrow x = 10\,m$

$\begin{cases} 0 = 20 + a_0 t & \Rightarrow Ⓐ \\ 40 = 20t + 1/2\, a t^2 & \Rightarrow Ⓑ \end{cases}$

Ⓐ $a = \frac{-20}{t} \Rightarrow \boxed{a = -5\,m/s}$

Ⓑ $40 = 20t + \frac{1}{2}\left(\frac{-20}{t}\right) t^2$

$40 = 20t - 10t$

$t = 4\,s$

EJERCICIO 22

AC (MRUV)

$X(t) = x_0 + V_0 (t - t_0) + \frac{1}{2} a (t - t_0)^2$

$X(12,5) = 0 + (-10)(12,5) + \frac{1}{2} \cdot 2 (12,5)^2$

$\boxed{X(12,5) = 31,25 \text{ m}}$

$x_0 = 0$
$V_0 = -10 \text{ m/s}$
$t_0 = 0$
$t = 12,5$
$V_F = 0 \text{ m/s}$

$a = \frac{V_F - V_i}{t_F - t_i} = \frac{0 - (-10)}{5 - 0} = \boxed{\frac{2 \text{ m}}{\text{s}^2}}$

$V_F = V_0 + a t$
$V_F = 0 + 2 \cdot 12,5$
$\boxed{V_F = 25 \text{ m/s}}$

CD (MRU)

$X(t) = x_0 + v_0 (t - t_0)$

$X(20) = 0 + 25 \cdot (20 - 12,5)$

$\boxed{X(20) = 187,5 \text{ m}}$

$x_0 = 0$
$V_0 = 25 \text{ m/s}$
$t_0 = 12,5$
$t_F = 20$

DF (MRUV)

$X(30) = 0 + 25 \cdot 10 + \frac{1}{2} \cdot (-5) \cdot 10^2$

$X(30) = 250 - 250$

$X(30) =$

$x_0 = 0$
$V_0 = 25 \text{ m/s}$
$t_0 = 20$
$t_F = 30$

$a = \frac{V_F - V_i}{t_F - t_i} = \frac{0 - 25}{25 - 20} = \frac{-5 \text{ m}}{\text{s}}$

$V_F = V_0 + a \cdot (t - t_0)$
$V_F = 25 + (-5) \cdot 10$
$\boxed{V_F = -25}$

FG (MRUV)

$X(50) = 0 + (-25) \cdot 20 + \frac{1}{2} (1,25) \cdot 20^2$

$X(50) = -500 + 250$

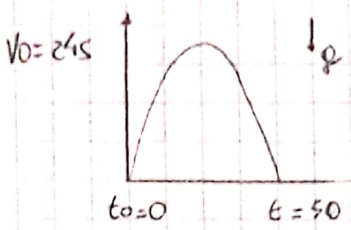
$\boxed{X(50) = -250 \text{ m}}$

$x_0 = 0$
$V_0 = -25$
$t_0 = 30$
$t_F = 50$

$a = \frac{0 + 25}{50 - 30} = \frac{1,25 \text{ m}}{\text{s}}$

TIRO VERTICAL

EJERCICIO 32



$$\begin{cases} v = v_0 - g t \\ y = y_0 + v_0 t - \frac{1}{2} g t^2 \end{cases} \Rightarrow \begin{cases} v = 245 - 9{,}8 \cdot t \\ y = 0 + 245 t - \frac{1}{2} \cdot 9{,}8\, t^2 \end{cases}$$

(A) $h_{MAX} \Rightarrow v = 0 \Rightarrow 0 = 245 - 9{,}8 \cdot t \Rightarrow t = 25$

$y = 245 \cdot 25 - \frac{1}{2} \cdot 9{,}8 \cdot (25)^2 \Rightarrow \boxed{y = 3062{,}5\ m}$

(B) $VF = -245\ m$

(C) $\begin{cases} v = 245 - 9{,}8\, t \Rightarrow \boxed{v = \pm 201\ m/s} \\ 1000 = 245 t - \frac{1}{2} \cdot 9{,}8\, t^2 \end{cases} \Rightarrow -4{,}9 t^2 + 245 t - 1000 = 0 \Rightarrow t_1 = 45{,}51,\ t_2 = 4{,}48$

EJERCICIO 34

$$\begin{cases} v = v_0 - g \cdot t \\ y = y_0 + v_0 t - \frac{1}{2} g t^2 \end{cases}$$

(A) $\begin{cases} 15 = v_0 - g\, t \Rightarrow (1) \\ 300 = 0 + v_0 t - 4{,}9\, t^2 \Rightarrow (2) \end{cases}$

(1) $v_0 = 15 - g \cdot t \Rightarrow v_0 =$

(2) $300 = (15 - 9{,}8t)t - 4{,}9 t^2$

$300 = 15t - 9{,}8 t^2 - 4{,}9 t^2$

$0 = -14{,}7 t^2 + 15t - 300$

$t_1 = \frac{25}{49} + 4{,}4 = 4{,}9$

$t_2 = \frac{25}{49} - 4{,}4 = -3{,}8$

EJERCICIO 35

$V_0 = 20\ m/s$
$h = 1000\ m$

$$y(t) = y_0 + v_0(t - t_0) + \frac{1}{2} g t^2$$

$$0 = 1000 + 20t - 4{,}9t^2$$

$$\boxed{t = 16{,}47\ s}$$

CAÍDA LIBRE

EJERCICIO 37

$h = 49\ m$

(A) $y(t) = y_0 + v_0 t - 4{,}9t^2$

$0 = 49 + 0 - 4{,}9t^2$

$t^2 = \frac{-49}{-4{,}9}$

$\boxed{t = 3{,}16\ s}$

(B) $v = v_0 + a \cdot t$

$v = -9{,}8 \cdot 3{,}16$

$\boxed{v = -30{,}96\ m/s}$

(C)

$20 = 49 - 4{,}9t^2$

$t^2 = \frac{20 - 49}{-4{,}9}$

$t = 2{,}43$

$v = -9{,}8 \cdot 2{,}43$

$\boxed{v = -23{,}814}$

(D) $29 = 49 - 4{,}9t^2$

$t^2 = \frac{29 - 49}{-4{,}9}$

$t = 2{,}02$

$v = -9{,}8 \cdot 2{,}02$

$\boxed{v = -19{,}80\ m/s}$

EJERCICIO 38

40m { $t = 1$

(A) $y(t) = y_0 + v_0 t + \frac{1}{2} g t^2$

$0 = 40 + v_0 t - 4{,}9t^2$

$v_{40} = -35{,}1\ m/s$

$v_{40} = v_0 - g \cdot t$

$-35{,}1 = 0 - g \cdot t$

$t = 3{,}58\ seg$

$\boxed{t_{TOTAL} = 4{,}58\ seg}$

(B) $0 = y_0 - 4{,}9t$

$\boxed{y_0 = 102{,}85\ m}$

(C) $v = v_0 - g \cdot t$

$v = -g \cdot 4{,}58$

$\boxed{v = -44{,}88\ m/s}$

TIRO OBLICUO

EJERCICIO 42

$\alpha = 30°$
$V_0 = 100\ m/s$

$V_{0x} = v_0 \cos\theta = 86,60\ m/s$
$V_{yx} = v_0 \operatorname{sen}\theta = 50\ m/s$

$$\begin{cases} x(t) = x_0 + v_{0x}\, t \\ y(t) = y_0 + v_{0y}\, t - \frac{1}{2} g\, t^2 \end{cases}$$

(A) $y(t) = 0$

$0 = 0 + 50.t - 4,9t^2$

$0 = t\,(50 - 4,9\,t)$

$\boxed{t = 10,20}$ → (C)

$x(t) = 0 + 86,60.t$

$\boxed{X(t) = 883,7\ m}$

(B) $V_Y = V_{0Y} - g.t$

$0 = 50 - 4,9\,t$

$t = 5,10\ s$

$y(t) = 0 + 50t - 4,9.t^2$

$\boxed{y(t) = 127,5\ m}$

EJERCICIO 43

$d_{corte} = 100\ m$
$h_{max} = 100\ m$

$x(t) = x_0 + V_{0x}\, t$

$\frac{100}{9} = V_{0x}$

$V_{0x} = 11,1\ m/s$

↓ $y_0 = 0$

$y(t) = y_0 + V_{0Y}\, t - 4,9\, t^2$

$0 = V_{0Y}\, t - 4,9\, t^2$

$0 = t\,(V_{0Y} - 4,9\, t)$

$V_{0Y} = 4,9\, t$

$V_{0Y} = 44,1\ m/s$

↓ $y_0 = 100$, $V_{0Y} = 0$

$y(t) = 100 - 4,9\, t^2$

$0 = 100 - 4,9\, t^2$

$t = \sqrt{\frac{-100}{-4,9}}$

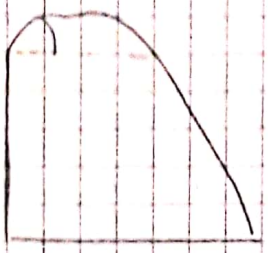
$t = 4,5\ seg$

$t_{TOTAL} = 9\ seg$

$v_0 = \sqrt{V_{0x}^2 + V_{0Y}^2} = \boxed{45,47\ m/s}$

$\alpha = \operatorname{arctg} \frac{V_{0Y}}{V_{0x}} = \boxed{76°}$

EJERCICIO 44



$\alpha = 37°$, $V_0 = 20\ m/s$, $d = 64\ m$

$V_{0x} = V_0 \cos\theta = 15,97\ m/s$

$V_{0y} = V_0 \operatorname{sen}\theta = 12,03\ m/s$

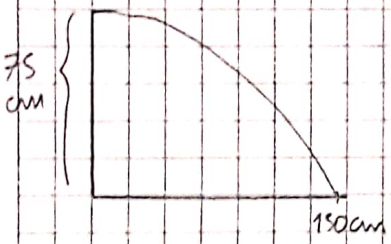
$x(t) = x_0 + V_{0x} t \Rightarrow 64 = 0 + V_{0x} t \Rightarrow \boxed{t = 4\ seg}$

$y(t) = y_0 + V_{0y} t - \frac{1}{2} g t^2$

$0 = y_0 + 48,12 - 78,4$

$\boxed{y_0 = 30,28\ m}$

EJERCICIO 45



$h = 75\ cm$, $d = 150\ cm$ $\quad V_{0y} = 0$

$0,75\ m$, $1,5\ m$

(A) $y(t) = y_0 + V_{0y} t + \frac{1}{2} g t^2$

$0 = 0,75 - 4,9\ t^2$

$\boxed{t = 0,39\ seg}$

(B) $x(t) = x_0 + V_{0x} t$

$1,5 = 0 + V_{0x} \cdot 0,39$

$\boxed{V_{0x} = 3,84\ m/s}$

(C) $V_y = V_{0y} - g t$

$V_y = -3,82\ m$

$|V| = \sqrt{V_{0x}^2 + V_{0y}} = \boxed{5,41\ m/s}$

$\alpha = \operatorname{arctg} \frac{V_y}{V_x} = -44,85$

EJERCICIO 46

$\theta = 60°$

$x(t) = x_0 + V_{0x} t$

$1000 = V_0 \cos\theta\ t$

$V_0 = \frac{1000}{\cos\theta\ t} = \boxed{117\ m/s}$

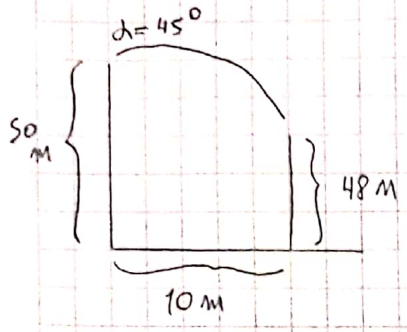
$y(t) = y_0 + V_{0y} t - \frac{1}{2} g t^2$

$300 = V_0 \operatorname{sen}\theta\ t - 4,9\ t^2$

$300 = 1000 \frac{\operatorname{sen}\theta\ t}{\cos\theta\ t} - 4,9\ t^2$

$t = \sqrt{\frac{300 - 1000 \operatorname{tg}\theta}{-4,9}} = 17,09\ seg$

EJERCICIO 47



(A) $y(t) = y_0 + V_{0y} t - \frac{1}{2} g t^2$

$48 = 50 + \frac{10 \operatorname{sen}\theta \, t}{\cos\theta \, t} - \frac{1}{2} g t^2$

$48 = 50 + 10 \operatorname{tg}\theta - \frac{1}{2} g t^2$

$t = \sqrt{\frac{48 - 50 - 10 \operatorname{tg}\theta}{-4{,}9}}$

$t = 1{,}56 \text{ seg}$

$x(t) = x_0 + V_x t$

$10 = V_0 \cos\theta \, t$

$V_0 = \frac{10}{\cos\theta \, t}$

$\boxed{V_0 = 9{,}06 \text{ m/s}}$

(B) $V_x = V_{0x} = V_0 \cos\theta = 6{,}39 \text{ m/s}$

$V_y = V_{0y} - g t = V_0 \operatorname{sen}\theta - g t = -8{,}89 \text{ m/s}$

$|VF| = \sqrt{V_x^2 + V_y^2} = \boxed{10{,}94 \text{ m/s}}$

(C)

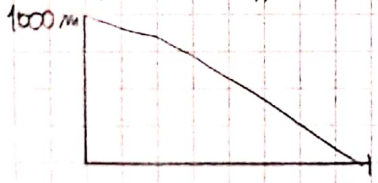
$\alpha = \operatorname{arctg} \frac{V_y}{V_x} = -54{,}29$ $\qquad \beta = -54{,}29 + 90 = 35{,}71$

$a_N = |g| \operatorname{sen}\beta = 5{,}72 \text{ m/s}$

$a_t = |g| \cos\beta = 7{,}95 \text{ m/s}$

EJERCICIO 48

$V_1 = 2000 \text{ km/h} = 555{,}5 \text{ m/s}$



20 km = 5,5 m

$V_2 = 100 \text{ km/h} = 27{,}7 \text{ m/s}$

PROYECTIL

$x_P(t) = V_1 t$

$y_P(t) = y_0 - 1/2 g t$

BARCO

$x_B(t) = x_0 + V_2 t$

$y_B(t) = 0$

$x_P(t) = x_B(t)$

$y_P(t) = y_B(t)$

$$\begin{cases} V_1 t = x_0 + V_2 t \Rightarrow t = d / V_1 - V_2 \\ y_0 - 1/2 g t = 0 \end{cases}$$

$y_0 - 1/2 g \left(\frac{d}{V_1 - V_2}\right)^2 = 0$

$\frac{d^2}{(V_1 - V_2)^2} = \frac{2h}{g}$

$d = \sqrt{\frac{2h}{g}} (V_1 - V_B) = 7{,}96 \text{ km}$

MOVIMIENTO CIRCULAR UNIFORME

EJERCICIO 53

$r = 8\ m$, $v_0 = 45\ Km/h = 12,5\ m/s$

$V = w \cdot r \Rightarrow w = \frac{V}{r} = \frac{12,5}{8} = \boxed{1,56\ s^{-1}}$

EJERCICIO 54

$V = 120\ RPM$

$diam = 3\ m$, $r = 1,5$

(A) $120\ RPM = \frac{120 \cdot 2\pi}{60\ seg} = 12,56 = \boxed{4\pi\ s^{-1} = w}$

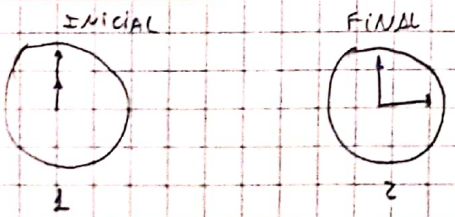
$T = \frac{2\pi}{w} = \frac{1}{2} = \boxed{0,5\ s}$

(B) $V = w \cdot r = 4\pi \cdot 0,47 = \boxed{18,84\ m/s}$

(C) $ac = w \cdot v = w^2 R = (4\pi)^2 \cdot 1,5 = \boxed{236,8\ m/s^2}$

(D) $F = \frac{w \cdot t}{2\pi} = \frac{4\pi^2 \cdot 20}{2\pi} = \boxed{40\ vueltas}$

EJERCICIO 55



$90^\circ \left(\frac{\pi}{2}\right)$

$w_1 = \frac{2\pi}{3600\ seg}$

$w_2 = \frac{2\pi}{43200\ seg}$

$\theta_1 = \theta_0 + w_1 (t - t_0)$ } $\theta_1 = w_1 t$

$\theta_2 = \theta_0 + w_2 (t - t_0)$ } $\theta_2 = w_2 t$

$\theta_2 - \theta_1 = \frac{\pi}{2} \Rightarrow \left| (w_2 - w_1) t \right| = \frac{\pi}{2} \Rightarrow \boxed{t = 981,81\ seg}$

MOVIMIENTO CIRCULAR UNIFORMEMENTE VARIADO

EJERCICIO 56

$F_1 = 100$ RPM
$F_2 = 1200$ RPM
} 10 seg

(A) $100 \text{ RPM} = \frac{100 \cdot 2\pi}{60 \text{ seg}} = 10,47 = 3,33\pi$

$1200 \text{ RPM} = \frac{1200 \cdot 2\pi}{60 \text{ seg}} = 125,66 = 40\pi$

$\alpha = \frac{\Delta w}{\Delta t} = \frac{40\pi - 3,33\pi}{10 \text{ seg}} = \boxed{11,52 \text{ s}^{-2}}$

(B) $\theta(t) = \theta_0 + W_1(t - t_0) + \frac{1}{2}\alpha(t - t_0)^2$

$\theta(10) = 0 + W_1 10 + \frac{1}{2}\alpha \cdot 10^2 = 680,21 \text{ rad} = \boxed{108 \text{ vueltas}}$

EJERCICIO 57

$V_0 = 1500 \text{ RPM} = \frac{1500 \cdot 2\pi}{60 \text{ seg}} = 157,07 = 50\pi$

$N = 150$

$W_F = W_0 + \alpha t$
$0 = 50\pi + \alpha t$
$\alpha = \frac{-50\pi}{t} \Rightarrow \alpha = -13,08$

$\theta(t) = \theta_0 + W(t - t_0) + \frac{1}{2}\alpha(t - t_0)^2$

$2\pi \cdot 150 = Wt + \frac{1}{2}\left(\frac{-50\pi}{t}\right)t^2$

$2\pi \cdot 150 = 50\pi t - 25\pi t \Rightarrow \boxed{t = 12 \text{ seg}}$

EJERCICIO 58

$V = 6,28 \text{ rad/s}$ $\alpha = 1,57 \text{ rad s}^2$ $t = 16$

(A) $W(t) = W_0 + \alpha_0(t - t_0)$

$W(16) = 6,28 + 1,57 \cdot 16 = \boxed{31,4 \text{ s}^{-1}}$

(B) $\theta(t) = \theta_0 + W(t - t_0) + \frac{1}{2}\alpha(t - t_0)^2$

$\theta(16) = 0 + 6,28 \cdot 16 + \frac{1}{2} 1,57\, 16^2$

$\theta(16) = 301,44 = \boxed{48 \text{ vueltas}}$

(C) $40 \text{ cm} = 0,4 \text{ m} = r$

$V = W \cdot r$

$V = 31,4 \cdot 0,4 = \boxed{12,56 \text{ m/s}}$

EJERCICIO 59

$d = 2{,}4\ m$

$F_1 = 100\ RPM \Rightarrow \frac{100 \cdot 2\pi}{60\ seg} = 10{,}47 = 3{,}33\pi = W_0$

$t = 4\ seg$

$t_1 = 2 \qquad t_2 = 3$

$W = W_0 + \alpha t$

$0 = 3{,}33\pi + \alpha t$

$\alpha = \frac{-3{,}33\pi}{4} = -2{,}61\ s^{-1}$

$a_t = \alpha \cdot R = \boxed{-3{,}132\ m/s^2}$

$\Downarrow$

$t_1 a_t = t_2 a_t$

(A) t_1

$W_F = 3{,}33\pi + \alpha t_1$

$W_F = 5{,}24\ s^{-1}$

$a_n = W^2 \cdot R$

$\boxed{a_n = 32{,}94\ m/s^2}$

t_2

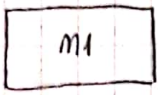
$W_F = 3{,}33\pi + \alpha t_2$

$W_F = 2{,}63\ s^{-1}$

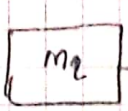
$a_n = W^2 R$

$\boxed{a_n = 8{,}30\ m/s^2}$

EJERCICIO 5



$a_1 = 2\ m/s^2$



$a_2 = 3\ m/s$

$F = m_1 a_1$

$m_1 = F/a_1$

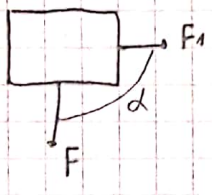
$F = m_2 a_2$

$m_2 = F/a_2$

$$(m_1 + m_2) = \frac{F}{a_1} + \frac{F}{a_2}$$

$$(m_1 + m_2) = \frac{5}{6} F \Rightarrow F = (m_1 + m_2)\frac{6}{5} \Rightarrow a = \frac{6}{5} = 1{,}2\ m/s^2$$

EJERCICIO 6



$a_1 = 5\ m/s^2$

(A) $\alpha = 90^\circ$, $\theta = 45$

$F_1 = F \cos\theta$

$m a_1 = m a \cos\theta$

$a = \frac{a_1}{\cos\theta} \Rightarrow$ **$a = 7{,}01\ m/s$**

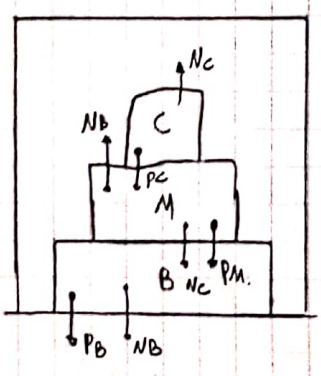
(B) $\alpha = 45$, $\theta = 22{,}5$

$F_1 = F \cos\theta$

$m a_1 = m a \cos\theta$

$a = \frac{a_1}{\cos\theta} \Rightarrow a =$

EJERCICIO 7



$P_M = m_m g$

$P_C = m_c g$

$P_B = m_B g$

MATIAS

$N_B - N_C - P_M = m_m a$

si $N_B = 960\ N$, $M_C = 0$, $N_C = 0$

$$① \begin{cases} N_B - P_M = m_m a \end{cases}$$

$$① \begin{cases} 960N - m_m g = m_m a \\ 1200N - N_C - m_m g = m_m a \\ N_C - 196\ N = 20\ a \end{cases}$$

CAJA

$N_C - P_C = m_c a$

$m_c = 20kg$

si $N_B = 1200\ N$, $M_C = 20kg$

$$② \begin{cases} N_B - N_C - P_M = m_m a \\ N_C - P_C = m_c a \end{cases}$$

① $960N - m_m g = m_m a$

② $1200N - N_c - m_m g = m_m a$ (2+3) →

③ $N_c - 196N = 20\,a$

$$\begin{cases} 960N - m_m g = m_m a & \text{(A)} \\ 1004N - m_m g - a(m_m + 20) & \text{(B)} \end{cases}$$

(B-A) → $44N = a \cdot 20kg \Rightarrow \boxed{a = 2,2\ m/s}$

REEMPLAZO EN 1

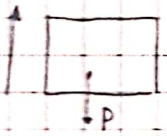
$960N - m_m g = m_m a$

$960N = m_m a + m_m g$

$960N = m_m (a+g)$

$m_m = \frac{960N}{(a+g)} \Rightarrow \boxed{m_m = 80kg} \Rightarrow P_M = m_m g = \boxed{784N}$

EJERCICIO 9



$m = 50kg$

$a = 3m/s$

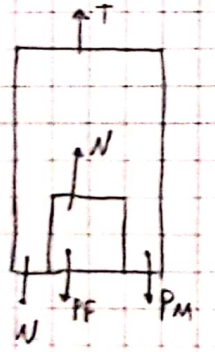
$P - F = m\,a$

$mg - F = m\,a$

$-F = ma - mg$

$\boxed{F = 340N}$

EJERCICIO 10



$m_m = 1200kg$

$v = 4m/s$

$m_F = 600kg$

FARDO

x) –

y) $N - m_F g = m_F a$

MONTACARGAS

x) –

y) $T - N - m_M g = m_m a$

$v = v_0 + a(t - t_0)$

$4 = 0 + a(1 - 0)$

$a_S = 4\ m/s$ // SUBIENDO

$a_B = -4\ m/s$ // BAJANDO

Ⓐ $N - m_F g = m_F a$

$N = m_F a + m_F g$

$\boxed{N = 8280\,N}$

Ⓑ si $v = cte \Rightarrow a = 0$

$N - m_F g = 0$

$N = m_F g$

$\boxed{N = 5880\,N}$

Ⓒ $a = -4\,m/s$

$N - m_F g = m_F a$

$N = m_F a + m_F g$

$\boxed{N = 3480\,N}$

Ⓓ $T - N - m_M g = m_M a$

$T = m_M a + m_M g + N$

$\boxed{T = 24840\,N}$

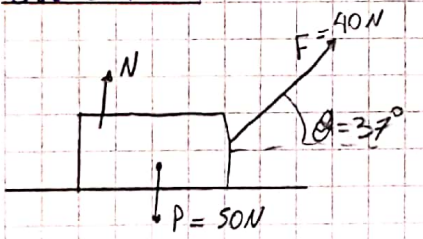
Ⓔ $N = 0$, $a = -4\,m/s$

$T - m_M g = m_M a$

$T = m_M (a + g)$

$\boxed{T = 6960\,N}$

EJERCICIO 11



x) $F \cos\theta = m a$

y) $N - P - F \operatorname{sen}\theta = 0$

Ⓐ $N = P + F \operatorname{sen}\theta$

$\boxed{N = 26\,N}$

Ⓑ $F \cos\theta = m a$

$a = \frac{F \cos\theta}{m}$

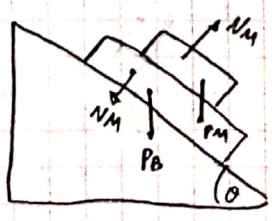
$\boxed{a = 6{,}26\,m/s^2}$

$P = m g$

$50 = m g$

$m = 5{,}1\,kg$

EJERCICIO 12



$m_M = 65\,kg$

$\theta = 30$

MUCHACHA

x) $P_M \operatorname{sen}\theta = m a$

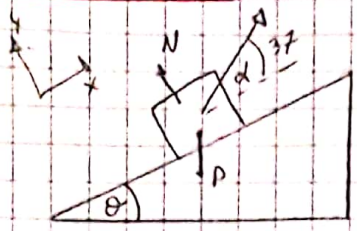
y) $N_M - P \cos\theta = 0$

$N_M = P \cos\theta$

$N_M = m_M g \cos\theta$

$\boxed{N_M = 552\,N}$

EJERCICIO 13



$m = 60\ kg$

$F = 500\ N$

$x) -P \operatorname{sen}\theta + F\cos\alpha = m\,a$

$y) N + F\operatorname{sen}\alpha - P\cos\theta = m\,a$

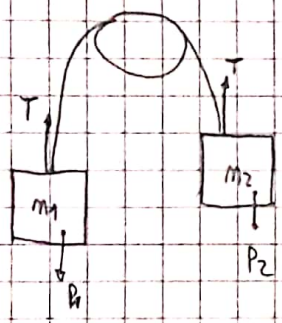
$F\cos\alpha - P\operatorname{sen}\theta = m\,a$

$F\cos\alpha - m\,g\operatorname{sen}\theta = m\,a$

$$a = \frac{F\cos\alpha - m\,g\operatorname{sen}\theta}{m}$$

$$a = \frac{500\cdot\cos(37) - 60\cdot 9{,}8\operatorname{sen}(37)}{60} \Rightarrow \boxed{a = 0{,}75\ m/s^2}$$

EJERCICIO 14



$m_1 + m_2 = 10\ kg$ $\quad a = 2\ m/s^2$ $\quad a_1 = -a_2$

m_1

$x) -$

$y)\ T + m_1 g = m_1 a_1$

m_2

$x) -$

$y)\ T - m_2 g = m_2 a_2$

$$\left.\begin{cases} T - m_1 g = m_1 a \\ T - m_2 g = -m_2 a \end{cases}\right\}^{1-2}$$

$$-m_1 g + m_2 g = m_1 a + m_2 a$$

$$(m_2 - 10)g + m_2 g = (10 - m_2)a + m_2 a$$

$$2m_2 g - 10g = 10a$$

$$2m_2 = \frac{10a + 10g}{g}$$

$$m_2 = \frac{10(a+g)/g}{2}$$

$$\boxed{m_2 = 6\ kg}$$

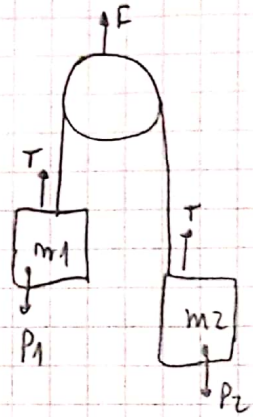
$m_1 + m_2 = 10\ kg$

$m_1 = 10 - m_2$

$\boxed{m_1 = 4\ kg}$

EJERCICIO 15

$m_1 = 5kg$, $m_2 = 8kg$, $F = 200\,N$, $T_1 = T_2 = \frac{F}{2} = 100N$



m1

x) -

y) $T - m_1 g = m_1 a$

$\Downarrow$

$a_1 = \frac{T - m_1 g}{m_1}$

$\boxed{a_1 = 10,2\ m/s^2}$ ↑

m2

x) -

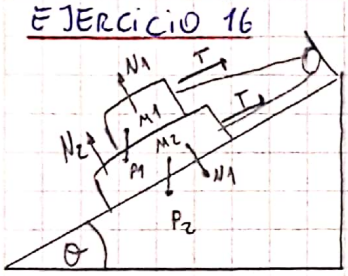
y) $T - m_2 g = m_2 a$

$\Downarrow$

$a_2 = \frac{T - m_2 g}{m_2}$

$\boxed{a_2 = 2,7\ m/s^2}$ ↓

EJERCICIO 16



$m_1 = 20\,kg$, $m_2 = 10\,kg$

$\theta = 30$

$a_1 = -a_2$

m1

① x) $T - m_1 g \operatorname{sen}\theta = m_1 a_1$

y) $N_1 - m_1 g \cos\theta = 0 \Rightarrow N_1 = m_1 g \cos\theta$

$N_1 = 169,74\,N$

m2

② x) $T - m_2 g \operatorname{sen}\theta = m_2 a_2$

y) $N_2 - N_1 - m_2 g \cos\theta = 0 \Rightarrow N_2 = N_1 + m_2 g \cos\theta$

$N_2 = 254,61\,N$

1-2

$-m_1 g \operatorname{sen}\theta + m_2 g \operatorname{sen}\theta = m_1 a_1 - m_2 (-a_1)$

$-98 + 49 = a_1 (m_1 + m_2)$

$a_1 = \frac{-98 + 49}{(m_1 + m_2)} =$ $\boxed{a_1 = -1,63 \quad a_2 = 1,63}$

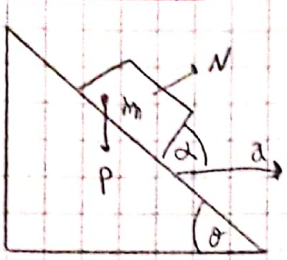
$T = m_2 a_2 + m_2 g \operatorname{sen}\theta$

$T = m_2 (a_2 + g \operatorname{sen}\theta)$

$\boxed{T = 65,3\,N}$

EJERCICIO 17

$m = 2kg$, $\theta = 60$, $\alpha = 30$

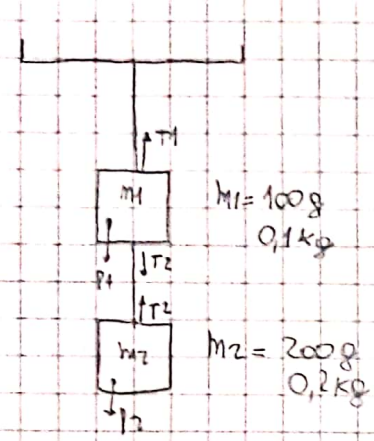


x) $mg \operatorname{sen}\theta = m1\ a \operatorname{sen}\alpha \Rightarrow a = \dfrac{mg \operatorname{sen}\theta}{m \operatorname{sen}\alpha}$

y) $N - mg\cos\theta = 0$

$\boxed{a = 16,97\ m/s^2}$

EJERCICIO 20



$m_1 = 100g$ $0,1kg$

$m_2 = 200g$ $0,2kg$

m_1	m_2
x) -	x) -
y) $T_1 - T_2 - m_1 g = m_1 a$	y) $T_2 - m_2 g = m_2 a$

Ⓐ Si $a = 0$

$T_1 = T_2 + m_1 g$

$\boxed{T_1 = 2,94\ N}$

$T_2 = m_2 g$

$\boxed{T_2 = 1,96\ N}$

Ⓑ Si $a = 1\ m/s$

$T_1 = T_2 + m_1 (a + g)$

$\boxed{T_1 = 3,29\ N}$

$T_2 = m_2 (a + g)$

$\boxed{T_2 = 2,16\ N}$

EJERCICIO 21

MUJER	PLATAFORMA
x) -	x) -
y) $N - m_c g + T = m_c a$	y) $T - N - m_p g = m_p a$

Ⓐ
$$\begin{cases} N - m_c g + T = m_c a \\ T - N - m_p g = m_p a \end{cases} + \begin{cases} 2T - m_c g - m_p g = a(m_c + m_p) \end{cases}$$

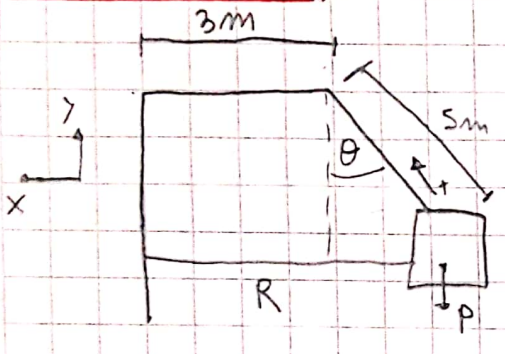
$2T = a(m_c + m_p) + g(m_c + m_p)$

$T = \dfrac{(a+g)(m_c + m_p)}{2} \Rightarrow \boxed{T = 325\ N}$

Ⓑ Si $v = cte \Rightarrow a = 0$

$T = \dfrac{g(m_c + m_p)}{2} \Rightarrow \boxed{T = 318,5\ N}$

EJERCICIO 22



$\theta = 30^\circ$, $R = 3 + 5 * \operatorname{sen}\theta = 5{,}5\ m$

X) $T \operatorname{sen}\theta = m a_c = w^2 R m = \left(\frac{2\pi}{w}\right)^2 R\ m$

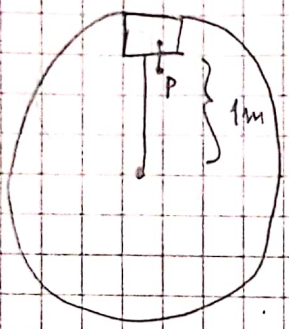
Y) $T\cos\theta - mg = 0 \Rightarrow T = \frac{mg}{\cos\theta}$

(A) $mg\ tg\ \theta = m\ \frac{4\pi^2}{T^2}\ R$

$T^2 = \frac{4\pi^2}{tg\,\theta\, g/R}$

$T = \sqrt{\frac{4\pi^2}{tg\,\theta\, g/R}} \Rightarrow \boxed{T = 6{,}19\ s}$

EJERCICIO 25

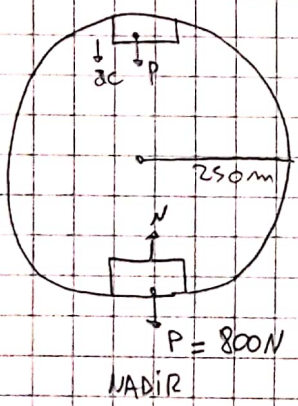


X) -

Y) $mg = m\,ac$

$ac = g \Rightarrow \frac{V^2}{r} = g \Rightarrow V = \sqrt{g \cdot r} = \boxed{3,13\ m/s}$

EJERCICIO 26



CENIT

$P = 800N$

$m = \frac{P}{ac} = 81,63kg$

X) -

Y) $-mg = -m\,ac$

Ⓐ $ac = g$

$\frac{V^2}{r} = g$

$V = \sqrt{gr} = \boxed{49,49\ m/s}$

NADIR

$ac = \frac{V^2}{r} = 19,26$

X) -

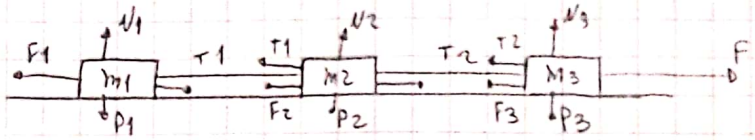
Y) $N - mg = m\,ac$

Ⓑ $N = m(ac + g)$

$\boxed{N = 2372,16\ N}$

ROZAMIENTO

EJERCICIO 28



$F = 1000\,N$ $\quad m_1 = 100$

$\mu = 0{,}30$ $\quad m_2 = 50$

$m_3 = 100$

$F_1 = F_2 = F_3 = \mu N$

PARA 1 $\quad T_1 - F_1 = m_1 a$

PARA 2 $\quad T_2 - F_2 - T_1 = m_2 a$

PARA 3 $\quad F - T_2 - F_3 = m_3 a$

$\sum$ 1+2+3

$F - F_1 - F_2 - F_3 = (m_1 + m_2 + m_3) a$

$F - \mu m_1 g - \mu m_2 g - \mu m_3 g = (m_1 + m_2 + m_3) a$

$F - \mu g (m_1 + m_2 + m_3) = (m_1 + m_2 + m_3) a$

$\frac{F - \mu g (m_1 - m_2 - m_3)}{(m_1 + m_2 + m_3)} = a \Rightarrow \boxed{a = 1{,}06\ m/s^2}$

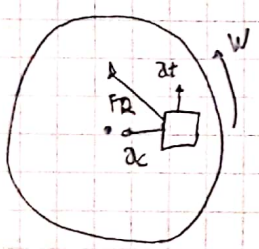
$T_1 = m_1 a + \mu m_1 g$

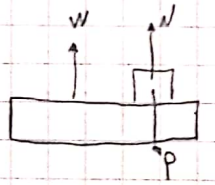
$\boxed{T_1 = 400\,N}$

$T_2 = m_2 a + T_1 + \mu m_2 g$

$\boxed{T_2 = 600\,N}$

EJERCICIO 29





$R = 60\ cm = 0{,}6\ m$, $\alpha = 1\,s^{-1}$, $\mu_e = 0{,}25$

x) $F_R = m a \Rightarrow \mu_e m g = m a \Rightarrow a = \mu_e g$

y) $N - mg = 0$

Ⓐ $\sum a = \sqrt{a_t^2 + a_c^2}$

$\sqrt{(\alpha R)^2 + (w^2 R)^2} \leq \mu_e g$

$w^4 R^2 \leq (\mu_e g)^2 - (\alpha R)^2$

$w \leq \sqrt{\frac{(\mu_e g)^2 - (\alpha R)^2}{R^2}}$

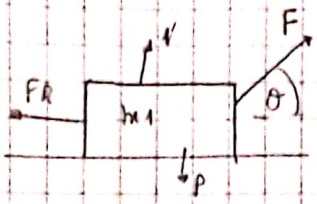
$w \leq 2\,s^{-1}$

Ⓑ $f = \frac{w}{2\pi} = 0{,}32\,Hz = \boxed{19\ RPM}$

$1\,Hz \longrightarrow 60\ RPM$

$0{,}32 \longrightarrow 19\ RPM$

EJERCICIO 30



$\theta = 40°$, $\mu_e = 0{,}20$, $\mu_d = 0{,}15$, $m = 50\,kg$

x) $F\cos\theta - FR = m\,a$

y) $N + F\operatorname{sen}\theta - mg = 0$

(A) Si $F = 100\,N$

$N = mg - F\operatorname{sen}\theta$

$N = 425\,N$

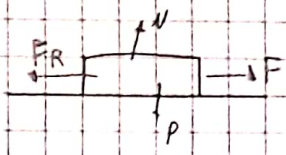
$FR = \mu N$ $\quad a = \dfrac{F\cos\theta - FR}{m} =$

(B) Si $F = 110$

$N = 400\,N$

$FR = \mu_d N = \boxed{60\,N}$ $\quad a = \dfrac{F\cos\theta - FR}{m} = \boxed{0{,}99\,m/s^2}$

EZERCICIO 31



$m = 136\,kg$, $F = 412\,N$

x) $F - FR = m\,a$

y) $N - mg = 0 \Rightarrow N = mg \Rightarrow FR = \mu mg$

(A) Si $\mu_e = 0{,}37$

$F < FR_{MAX} = \mu_e mg \Rightarrow F < \mu_e mg \Rightarrow \boxed{412 < 493}$

(B)

x) $F - FR = m\,a$

y) $F' + N - mg = 0 \Rightarrow N = mg - F'$

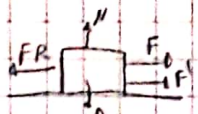
NO SE MUEVE

$F < FR_{MAX} = \mu_e N \Rightarrow F < \mu_e (mg - F') \Rightarrow \dfrac{F}{\mu_e} < mg - F' \Rightarrow F' < mg - \dfrac{F}{\mu_e}$

SE MUEVE

$F' \geq mg - \dfrac{F}{\mu_e} \Rightarrow \boxed{F' \geq 219\,N}$

(C)



x) $F + F' - FR = m\,a$

y) $N - mg = 0 \Rightarrow N = mg$

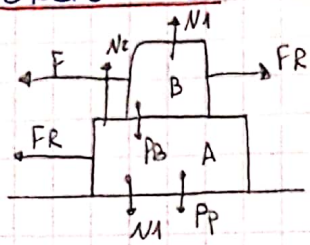
NO SE MUEVE

$F < FR_{MAX} = \mu_e N \Rightarrow F < \mu_e mg$

SE MUEVE

$F_r \geq \mu_e mg \Rightarrow F + F' \geq \mu_e mg \Rightarrow F' \geq \mu_e mg - F \Rightarrow \boxed{F' \geq 81\,N}$

EJERCICIO 32



$m_P = 42\,kg$ $\quad \mu_e = 0,53$ $\quad F = 110\,N$

$m_B = 9,7\,kg$ $\quad \mu_d = 0,35$ $\quad FR = \mu N_1 = \mu m_B g$

PLACA

x) $-FR = m_P a_P$

y) $N_2 - N_1 - m_P g = 0$

$N_2 = N_1 + m_P g$

BLOQUE

x) $FR - F = m_B a_B$

y) $N_1 - m_B g = 0$

$N_1 = m_B g$

PLACA

$a_P = \dfrac{-FR}{m_P}$

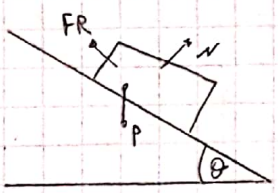
$a_P = -\dfrac{\mu m_B g}{m_P} \Rightarrow \boxed{a_P = -0,86\ \frac{m}{s^2}}$

BLOQUE

$a_B = \dfrac{FR - F}{m_B}$

$a_B = \dfrac{\mu m_B g - F}{m_B} \Rightarrow \boxed{-a_B = -7,6\ \frac{m}{s^2}}$

EJERCICIO 33



$\theta = 28°$, $FR = \mu N = \mu mg\cos\theta$

x) $mg\,\text{sen}\,\theta - FR = m a$

y) $N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$

$x(t) = x_0 + v_0(t - t_0) + \frac{1}{2} a (t - t_0)$

$2,53\,m = 0 + 0t + \frac{1}{2} a t^2$

$a = \dfrac{2,53 \cdot 2}{t^2} \Rightarrow a = 0,32\ m/s^2$

μ_e

$a = 0$

$FR = mg\,\text{sen}\,\theta$

$\mu m g\cos\theta = m g\,\text{sen}\,\theta$

$\mu_e = \dfrac{\text{sen}\,\theta}{\cos\theta} = tg\,\theta$

$\boxed{\mu_e = 0,53}$

μ_d

$a = 0,32$

$mg\,\text{sen}\,\theta - \mu_d mg\cos\theta = ma$

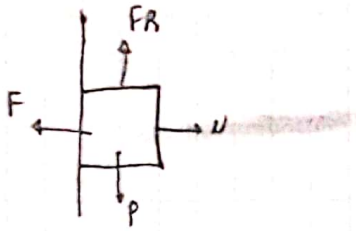
$\mu_d mg\cos\theta = -ma + mg\,\text{sen}\,\theta$

$\mu_d = \dfrac{-ma + mg\,\text{sen}\,\theta}{mg\cos\theta}$

$\boxed{\mu_d = 0,49}$

EJERCICIO 34

$F = 200\ N$, $m = 5\ kg$, $\mu = 0,4$



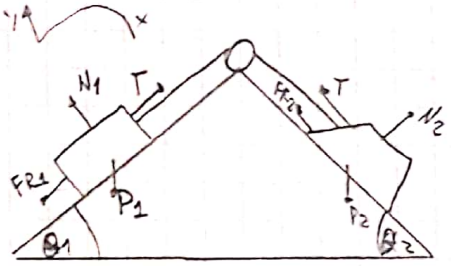
x) $N - F = 0 \Rightarrow N = F$

y) $FR - mg = 0 \Rightarrow FR = mg \Rightarrow FR = 49\ N$

$F_R \leq F_{RMAX} = \mu N = \mu F \Rightarrow mg \leq \mu F \Rightarrow F \leq \frac{mg}{\mu}$

$F \geq \frac{mg}{\mu} \Rightarrow \boxed{F \geq 122,5\ N}$

EJERCICIO 36



M1

x) $T - FR1 - P1 \operatorname{sen}\theta_1 = m_1 a$

y) $N1 - P1 \cos\theta_1 = 0$
$N1 = P1 \cos\theta_1$

M2

x) $P_2 \operatorname{sen}\theta_2 - T - FR2 = m_2 a$

y) $N_2 - P_2 \cos\theta_2 = 0$
$N_2 = P_2 \cos\theta_2$

$P_1 = 1000N$ $P_2 = 800N$
$\theta_1 = 30$ $\theta_2 = 60$
$\mu_1 = \mu_2 = 0,1$

(A) $FR1 = \mu N_1 = \mu P_1 \cos\theta_1$, $FR2 = \mu N_2 = \mu P_2 \cos\theta_2$

$\begin{cases} T - FR1 - P1 \operatorname{sen}\theta_1 = m_1 a \\ P_2 \operatorname{sen}\theta_2 - T - FR2 = m_2 a \end{cases}$ 1+2 $\begin{cases} P_2 \operatorname{sen}\theta_2 - P_1 \operatorname{sen}\theta_1 - FR1 - FR2 = a m_1 + a m_2 \end{cases}$

$P_2 \operatorname{sen}\theta_2 - P_1 \operatorname{sen}\theta_1 - FR1 - FR2 = a(m_1 + m_2)$

$a = \frac{P_2 \operatorname{sen}\theta_2 - P_1 \operatorname{sen}\theta_1 - FR1 - FR2}{(m_1 + m_2)}$

$P_1 = m_1 g$ $P_2 = m_2 g$
$m_1 = \frac{P_1}{g}$ $m_2 = \frac{P_2}{g}$
$m_1 = 102,04$ $m_2 = 81,63$

$a = \frac{692,82 - 500 - 86,60 - 40}{183,67}$

$\boxed{a = 0,36\ m/s^2}$

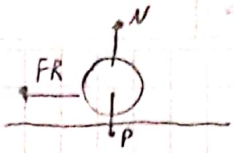
(B) $T = m_1 a + FR1 + P_1 \operatorname{sen}\theta_1$

$T = 36,73 + 86,60 + 500$

$\boxed{T = 623,33\ N}$

EJERCICIO 37

90 Km/h = 25 m/s



$V_F = V_0 + a t$

$a = \frac{V_F}{t} \Rightarrow a = \frac{25}{12} \Rightarrow a = 2{,}08\ m/s^2$

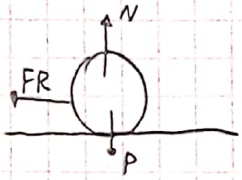
X) $FR = m\,a$

Y) $N - mg = 0 \Rightarrow N = mg$

$FR = ma \leq FR_{max} = \mu N = \mu mg \Rightarrow ma \leq \mu mg \Rightarrow \mu \geq \frac{a}{g} \Rightarrow \boxed{\mu = 0{,}21}$

EJERCICIO 38

$\mu = 0{,}6$



X) $FR = m\,a$

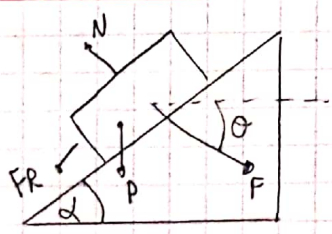
Y) $N - P = 0 \Rightarrow N = P$

(A) $FR = ma \leq FR_{MAX} = \mu N = \mu mg \Rightarrow ma \leq \mu mg \Rightarrow a \leq \mu g \Rightarrow a \leq 5{,}88\ m/s^2$

$\boxed{a_{MAX} = 5{,}88\ m/s^2}$

(B)

EJERCICIO 40



$\alpha = 30°$, $\theta = 7°$, $\mu = 0{,}25$, $m = 100kg$

X) $F \operatorname{sen}\theta - mg \operatorname{sen}\alpha - FR = m\,a$

Y) $N - F\cos\theta - mg\cos\alpha = 0$

$N = F\cos\theta + mg\cos\alpha$

i) $a = 0$

$FR = F\operatorname{sen}\theta - mg\operatorname{sen}\alpha \leq FR_{MAX} = \mu N = \mu(F\cos\theta + mg\cos\alpha)$

$F\operatorname{sen}\theta - mg\operatorname{sen}\alpha \leq \mu F\cos\theta + \mu mg\cos\alpha$

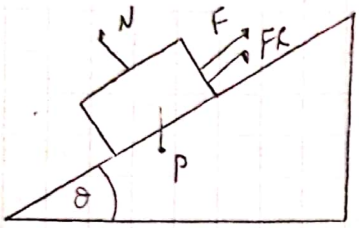
$F\operatorname{sen}\theta - \mu F\cos\theta \leq \mu mg\cos\alpha + mg\operatorname{sen}\alpha$

$F \leq \frac{\mu mg\cos\alpha + mg\operatorname{sen}\alpha}{(\operatorname{sen}\theta - \mu\cos\theta)}$ → $\theta = 53$

$F \geq$

NOTA

EJERCICIO 41



$\theta = 22$, $m = 7{,}95\ kg$, $\mu_e = 0{,}25$, $\mu_d = 0{,}15$

x) $F + FR - mg \operatorname{sen}\theta = m a$

y) $N - mg \cos\theta = 0 \Rightarrow N = mg \cos\theta$

(A) Si $a = 0$

$$FR = mg \operatorname{sen}\theta - F \leq FR_{MAX} = \mu N = \mu mg \cos\theta$$

$$mg \operatorname{sen}\theta - F \leq \mu mg \cos\theta$$

$$F \geq mg \operatorname{sen}\theta - \mu mg \cos\theta = mg(\operatorname{sen}\theta - \mu \cos\theta)$$

$$\boxed{F \geq 11{,}12\ N}$$

(B) Si $a = 0$, mueve hacia arriba

x) $F - FR - mg \operatorname{sen}\theta = m a$

$$FR = F - mg \operatorname{sen}\theta \leq FR_{MAX} = \mu N = \mu mg \cos\theta$$

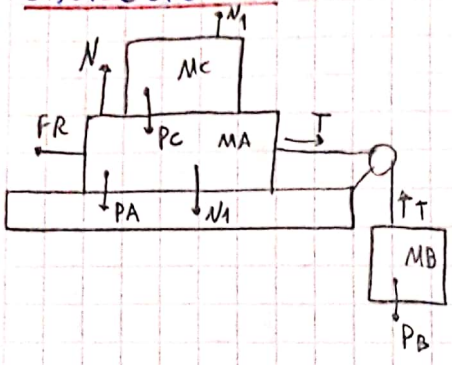
$$F - mg \operatorname{sen}\theta \leq \mu mg \cos\theta$$

$$F \leq \mu mg \cos\theta + mg \operatorname{sen}\theta = mg(\mu \cos\theta + \operatorname{sen}\theta)$$

$$\boxed{F \geq 40{,}02\ N}$$

EJERCICIO 42

$m_A = 20\ kg$, $m_B = 10\ kg$, $\mu_e = 0{,}2$, $\mu_d = 0{,}1$



MA

X) $T - FR = m_A a$

Y) $N - m_A g - N_1 = 0$

$N = N_1 + m_A g$

MB

X) -

Y) $T - m_B g = 0$

$T = m_B g$

MC

X) -

Y) $N_1 - m_C g = 0 \Rightarrow N_1 = m_C g$

(A) Si $a = 0$

$$FR = T = m_B g \leq FR_{MAX} = \mu N = \mu(N_1 + m_A g) = \mu(m_C g + m_A g)$$

$$m_B g \leq \mu (m_C + m_A) g$$

$$\frac{m_B - \mu m_A}{\mu} \leq m_C \Rightarrow m_C \geq \frac{m_B - \mu m_A}{\mu} \Rightarrow m_C \geq 30\ kg$$

(B)

MA

X) $T - FR = m_A a$

Y) $N - m_A g = 0$

$N = m_A g$

MB

X) -

Y) $T - m_B g = m_B a_b \Rightarrow T = m_B a_b + m_B g$

$a_B = -a_b$

$$m_B g - m_B a - \mu_d m_A g = m_A a$$

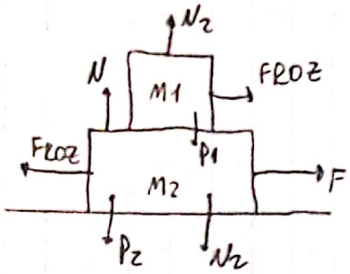
$$m_B g - \mu_d m_A g = m_A a + m_B a$$

$$\frac{m_B g - \mu_d m_A g}{(m_A + m_B)} = a \Rightarrow \boxed{a = 2{,}61\ m/s^2}$$

EJERCICIO 93

$m_1 = 1\,kg$, $m_2 = 4\,kg$ $\quad \mu_E = 0{,}3$

CASO A



m1

x) $FROZ = m_1 a_1$

y) $N_2 - m_1 g = 0$

$N_2 = m_1 g$

m2

x) $F - FROZ = m_2 a_2$

y) $N - N_2 - m_2 g = 0$

$N = g(m_1 + m_2)$

(A) $FROZ = m_1 a$

$\mu_E m_1 g = m_1 a$

$a = \mu_E g$

$FROZ = F - m_2 a \leq FROZMAX = \mu_E N_2$

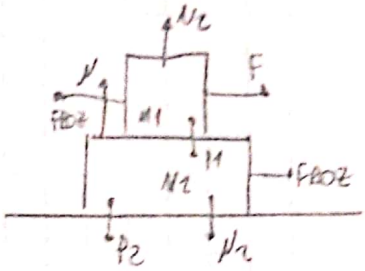
$F - m_2 a \leq \mu_E m_1 g$

$F \leq \mu_E m_1 g + m_2 \mu_E g$

$F \leq \mu_E (m_1 g + m_2 g)$

$\boxed{F \leq 19{,}7\,N}$

CASO B



m1

x) $F - FROZ = m_1 a_1$

y) $N_2 - m_1 g = 0$

$N_2 = m_1 g$

m2

x) $FROZ = m_2 a_2$

y) $N - N_2 - m_2 g = 0$

$N = g(m_1 + m_2)$

(B) $FROZ = m_2 a_2$

$\mu_E m_1 g = m_2 a$

$a = \dfrac{\mu_E m_1 g}{m_2}$

$FROZ = F - m_1 a \leq FROZMAX = \mu_E N_2$

$F - m_1 a \leq \mu_E m_1 g$

$F \leq \mu_E m_1 g + m_1 \dfrac{\mu_E m_1 g}{m_2}$

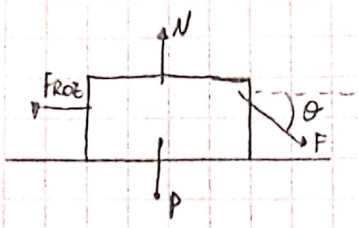
$F \leq 2{,}94 + 0{,}735$

$\boxed{F \leq 3{,}675\,N}$

EJERCICIO 47

$\theta = 30$, $\mu_E = 0{,}6$, $m = 50\,kg$

CASO A



x) $F\cos\theta - FROZ = m\,a$

y) $N - mg - F\,\mathrm{sen}\,\theta = 0$

$N = mg + F\,\mathrm{sen}\,\theta$

$FROZ = F\cos\theta \leq \mu N$

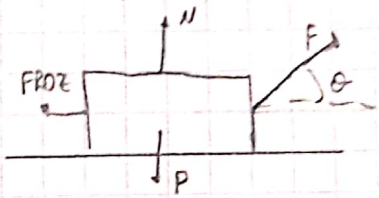
$F\cos\theta \leq \mu_E (mg + F\,\mathrm{sen}\,\theta)$

$F\cos\theta - F\,\mathrm{sen}\,\theta\,\mu_E \leq \mu_E\, mg$

$F \leq \dfrac{\mu_E\, mg}{(\cos\theta + \mathrm{sen}\,\theta\,\mu_E)}$

$F \geq 519\,N$

CASO B



x) $F\cos\theta - FROZ = m\,a$

y) $N - mg + F\,\mathrm{sen}\,\theta = 0$

$N = mg - F\,\mathrm{sen}\,\theta$

$FROZ = F\cos\theta \leq \mu N$

$F\cos\theta \leq \mu_E (mg - F\,\mathrm{sen}\,\theta)$

$F\cos\theta + F\,\mathrm{sen}\,\theta\,\mu_E \leq \mu_E\, mg$

$F \leq \dfrac{\mu_E\, mg}{(\cos\theta + \mathrm{sen}\,\theta\,\mu_E)}$

$F \geq 252\,N$

GRAVITACION

EJERCICIO 2.49

$R_T = 6{,}37\ 10^6$

$\frac{G M_T \cancel{M}}{(R+h)^2} = \cancel{M} g_h$

$g_h = g \frac{R_T^2}{(R_T+h)^2}$

$\frac{g_h}{g} = \frac{R_T^2}{(R_T+h)^2}$

$g_h = 9.8 - 0{,}98 = 8{,}82$

$(R_T+h)^2 = R_T^2 \frac{g}{g_h}$

$R_T + h = \sqrt{R_T^2 \, g/g_h}$

$h = \sqrt{R_T^2 \, g/g_h} - R_T$

$\boxed{h = 344569{,}5651}$

EJERCICIO 2.50

$h = 20\,200\,000\ m$

$R = R_T + h \Rightarrow R = 26510000$

Ⓐ $\frac{G M_T \cancel{M}}{R^2} = mg = m a_c = \cancel{m} \frac{v^2}{\cancel{R}}$

$v = \sqrt{\frac{g R_T^2}{R}} \Rightarrow \boxed{v = 3868{,}625525}$

$\omega = \frac{v}{R} \Rightarrow \omega = 1{,}45 \times 10^{-4}$

$T = \frac{2\pi}{\omega} = \boxed{43153{,}37128}$

Ⓑ $m = 770$

$\frac{G M_T \cancel{M}}{R^2} = \cancel{m} a_c$

$a_c = \frac{g R_T^2}{R^2} \Rightarrow \boxed{a_c = 0{,}563}$

$F_G = mg = m a_c = \boxed{433{,}51}$

EJERCICIO 2.51

$\omega_{SAT} = \omega_{TIERRA} = \frac{2\pi}{24\,3600} = 7{,}27 \times 10^{-5}$

$a_{c\,SAT} = \omega^2 (R_T+h)$

$\frac{G M_T \cancel{M}}{(R_T+h)^2} = mg = \cancel{m} a_{c\,SAT}$

$\frac{g R_T^2}{(R_T+h)^2} = \omega^2 (R_T+h)$

$(R_T+h)^3 \omega^2 = g R_T^2$

$(R_T+h)^3 = \frac{g R_T^2}{\omega^2}$

$R_T + h = \sqrt[3]{g R_T^2/\omega^2}$

$h = \sqrt[3]{g R_T^2/\omega^2} - R_T$

$\boxed{h = 35846175{,}96}$

TRABAJO Y ENERGÍA

EJERCICIO 54

$\Delta \vec{r} = (3\hat{i} + 3\hat{j})m$

$m = 2kg$

$\vec{F} = (2\hat{i} - 1\hat{j})N$

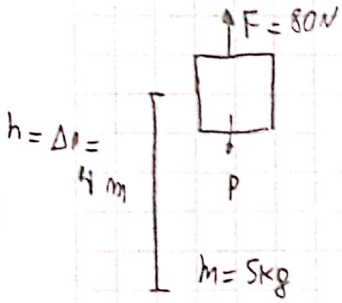
(A) $W_F = F \cdot \Delta r \cos\theta$

$W_F = (F_x \Delta r + F_y \Delta r)\cos\theta$

$W_F = 2N - 3m - 1N + 3m$

$\boxed{W_F = 3J}$

EJERCICIO 55



(A) $W_F = F\ \Delta r \cos\theta$

$W_F = 80N \cdot 4m \cdot \cos(0)$

$\boxed{W_F = 320J}$

(B) $W_F = P\ \Delta r \cos\theta$

$W_F = mg \cdot h \cdot \cos(180)$

$\boxed{W_F = -196J}$

(C) $W_{F\,TOTAL} = E_{cF} - E_{c=}$, $V_i = 0$

$(F-P)\,h = \frac{1}{2} m V_F^2 - \frac{1}{2} m V_i^2$

$\sqrt{\frac{2.124}{m}} = VF \Rightarrow \boxed{VF = 7{,}04\ m/s}$

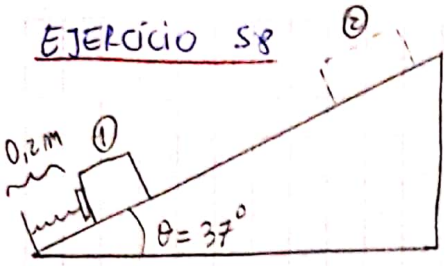
EJERCICIO 56

$m = 2K \qquad V_{x=0} = 3\ m/s$

(A) Ec en $x = 0$

$EC = \frac{1}{2} m V^2 = 9J$

EJERCICIO 58



$m = 2\,kg \qquad K = 500\,N/m$

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_1 = E_2$

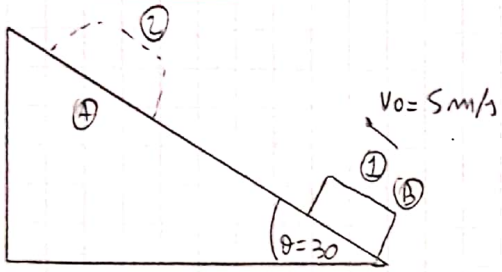
, energía potencial elástica

$E(1) = E(2) \Rightarrow \frac{1}{2} K x^2 = mgh$ → energía potencial

$\frac{1}{2} \frac{K x^2}{mg} = h \Rightarrow \boxed{h = 0,51\,m}$ $\qquad h = d \operatorname{sen}\theta$

$d = \frac{h}{\operatorname{sen}\theta} \Rightarrow \boxed{d = 0,85}$

EJERCICIO 59



(A)

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E1 = E2$

$E(1) = E(2) \Rightarrow \frac{1}{2} m V^2 = mgh$

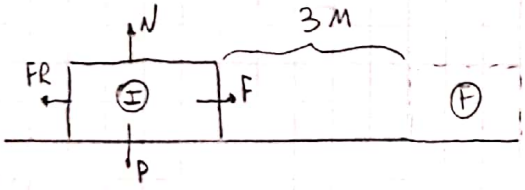
$\frac{1/2\, V^2}{g} = h \Rightarrow h = 1,27$

$d = \frac{h}{\operatorname{sen}\theta} \Rightarrow \boxed{d = 2,54\,m}$

(B) Cuando regreso la veo de nuevo

$E(A) = E(B) \Rightarrow mgh = \frac{1}{2} m V^2 \Rightarrow V = \sqrt{2gh} \Rightarrow \boxed{V = 4,98\,m/s}$

EJERCICIO 61



$F = 25\,N$, $m = 4kg$, $\mu d = 0,35$, $d = 3\,m$, $Vi = 0$

$\begin{cases} F - FR = ma \\ N - P = 0 \Rightarrow N = mg \end{cases}$

$FR = \mu N = \mu d\, mg = 13,72$

$EF = \frac{1}{2} m V_F^2 \qquad Ei = \frac{1}{2} m V_i^2 = 0$

$W_F = \Delta EC$

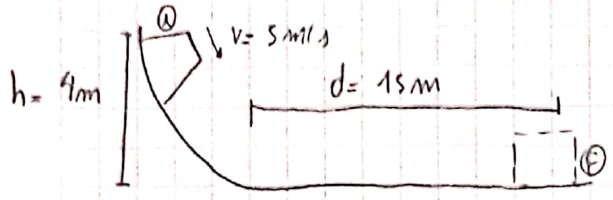
$(F - FR)\Delta x = EF - Ei$

$(F - FR).d = \frac{1}{2} m V_F^2 - 0$

$\sqrt{\frac{2(F - FR)\,d}{m}} = VF$

$\boxed{VF = 4,11\,m/s^2}$

EJERCICIO 62



$W_{FNC} = \Delta E_{MEC}$ $\quad h_i = 4m$, $h_F = 0m$, $V_i = 5m/s$, $V_F = 0\ m/s$

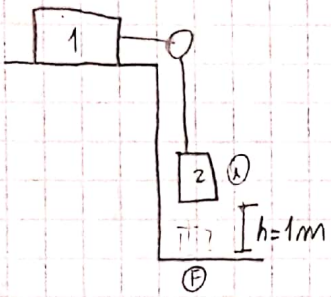
$W_{FROZ} = E_F - E_i$

$-\mu_d\, m g \cdot d = \left[\frac{1}{2} m V_F^2 - m g h_F\right] - \left[\frac{1}{2} m V_i^2 - m g h\right]$

$-\mu_d\, m g\, d = \frac{1}{2} m V_i^2 - m g h$

$\mu_d = \frac{\frac{1}{2} m V_i^2 + m g h}{m g d} \Rightarrow \boxed{\mu_d = 0{,}35}$

EJERCICIO 63



$m_1 = 150kg$, $m_2 = 40kg$

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_1 = E_2$

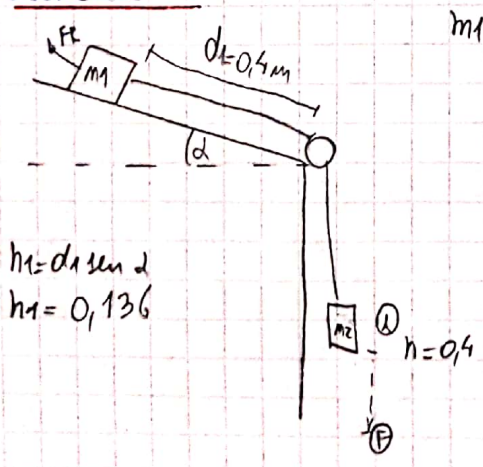
$E_1 = E_2$

$m_2 g h = \frac{1}{2} m_1 V_F^2 + \frac{1}{2} m_2 V_F^2$

$m_2 g h = \frac{1}{2} V_F^2 (m_1 + m_2)$

$V_F = \sqrt{\frac{2 \cdot m_2 g h}{(m_1 + m_2)}} \Rightarrow \boxed{V_F = 2\ m/s^2}$

EJERCICIO 64



$m_1 = 40kg$, $m_2 = 30kg$, $\alpha = 20$, $\mu_d = 0{,}15$, $d = 0{,}4$, $h = d \operatorname{sen} \alpha$

$h_1 = d \operatorname{sen} \alpha$

$h_1 = 0{,}136$

$W_{FNC} = \Delta E_{MEC}$ $\quad h_F = 0$, $V_i = 0$

$-F_{ROZ} \cos\alpha \cdot d = E_F - E_i$

$-\mu_d m_1 g \cos\alpha \cdot d = \left[\frac{1}{2}(m_1 + m_2) V_F^2 + m g h_F\right] -$

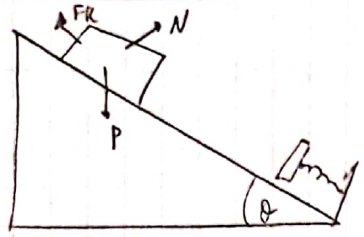
$\left[\frac{1}{2}(m_1 + m_2) V_i^2 + m_1 g h_1 + m_2 g h_2\right]$

$-\mu_d m_1 g \cos\alpha \cdot d = 1/2 (m_1 + m_2) V_F^2 - m_1 g h_1 - m_2 g h_2$

$\frac{2(-\mu_d m_1 g \cos\alpha\, d + m_1 g h_1 + m_2 g h_2)}{(m_1 + m_2)} = V_F^2$

$\boxed{2{,}06 = V_F}$

EJERCICIO 66



$m = 10\,kg$, $\theta = 15°$, $d = 0,25\,m$, $k = 1600\,N/m$, $\Delta x = 0,05\,m$

$d = 0,25 + 0,05 = 0,30\,m$ $\quad h = d \cdot \operatorname{sen}\theta = 0,07\,m$

$\mu e \quad a = 0$

x) $mg \operatorname{sen}\theta - FR = 0 \Rightarrow \mu_e\, mg \cos\theta = mg \operatorname{sen}\theta$

y) $N - P = 0 \Rightarrow N = mg\cos\theta$ $\quad \mu e = tg\,\theta$

$$\boxed{\mu e = 0,27}$$

μd

$W_{FNC} = EF - Ei$

$$-\mu d\, mg\cos\theta \cdot d = \left[\frac{1}{2} m V_F^2 + mg.h + \frac{1}{2} k \Delta x^2\right] - \left[\frac{1}{2} m V_i^2 + mgh\right]$$

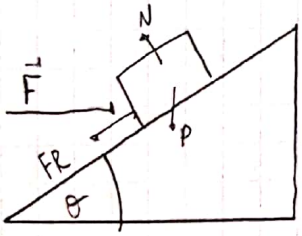
$$-\mu d\, mg\cos\theta . d = [0 + 0 + \frac{1}{2} k \Delta x^2] - [0 + mgh]$$

$$-\mu d\, mg\cos\theta\, d = \frac{1}{2} k \Delta x^2 - mg.h$$

$$\mu d = \frac{-1/2\, k \Delta x^2 + mg.h}{mg\cos\theta . d}$$

$$\boxed{\mu d = 0,2}$$

EJERCICIO 67



$m = 4kg$, $\theta = 37°$, $F = 60N$, $VF = 1,2\,m/s$, $d = 1\,m$

$h = d.\operatorname{sen}\theta = 0,6\,m$

(A)

$W_{FNETA} = EF - Ei$

$$(W_{FROZ} + W_F).d = \left[\frac{1}{2} m V_F^2 + mgh\right] - \left[\frac{1}{2} m V_i^2 + mgh\right]$$

$$(W_{FROZ} + W_F).d = \frac{1}{2} m V_F^2 + mg.h$$

$$W_{FROZ} = \frac{1/2\, m V_F^2 + mg.h - F\cos\theta . d}{d}$$

$$\boxed{W_{FROZ} = -21,51\,J}$$

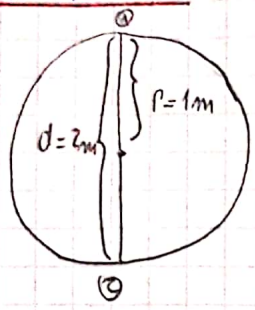
(B)

$$-\mu d\,(F \operatorname{sen}\theta + mg\cos\theta) = -21,51$$

$$\mu d = 21,51 \,/\, (F\operatorname{sen}\theta + mg\cos\theta)$$

$$\boxed{\mu d = 0,31}$$

EJERCICIO 68



$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_1 = E_2$

$E_1 = E_2$

$\frac{1}{2} m V_F^2 + mg\,h = \frac{1}{2} m V_i^2 + mg\,h$ (h = 0)

$\frac{1}{2} V_F^2 + g.h = \frac{1}{2} V_i^2$

$\sqrt{2\left(\frac{1}{2} V_F^2 + g.h\right)} = V_i$

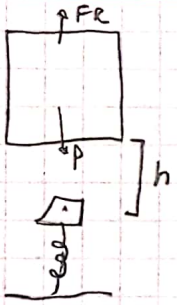
$\boxed{V_i = 7\ m/s}$

$P = 2c$

$g = \frac{V_F^2}{r} \Rightarrow V_F = \sqrt{g.r}$

$\boxed{V_F = 3{,}13\ m/s}$

EJERCICIO 69



$m = 400kg$, $h = 6m$, $k = 20000\ N/m$, $FR = 2500N$

(A)

$W_{FNC} = EF - Ei$

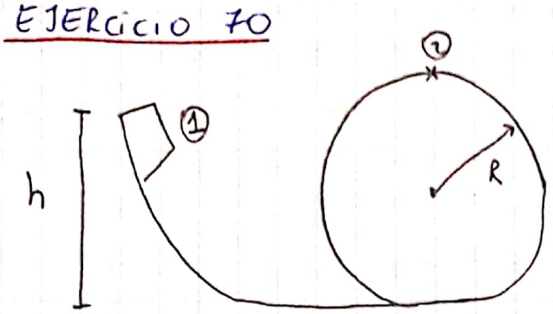
$-FR.h = \left[\frac{1}{2} m V_F^2 + mg.h\right] - \left[\frac{1}{2} m V_i^2 + mg.h\right]$ (h = 0, V_i = 0)

$-FR.h = \frac{1}{2} m V_F^2 - mg.h$

$V_F^2 = \frac{-FR.h + mg.h}{1/2\ m} \Rightarrow V_F^2 = 42{,}6 \Rightarrow \boxed{V_F = 6{,}52\ m/s}$

(B)

EJERCICIO 70



$R = 1m$

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_1 = E_2$

(A) $E_1 = E_2$

$$\frac{1}{2} m V_i^2 + mg.h = \frac{1}{2} m V_F^2 + mg.2R$$

(B) $N + mg = m\,ac \Rightarrow N = m\frac{V_F^2}{R} - mg \geq 0$

$$V_F = \sqrt{g.R}$$

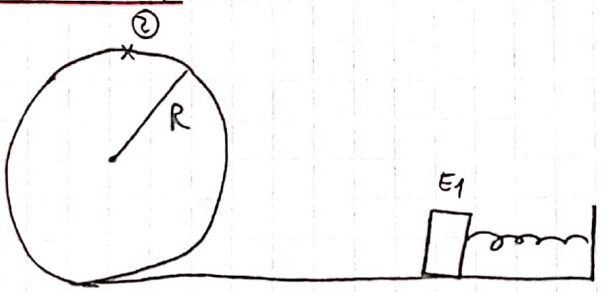
(A) $mg.h = \frac{1}{2} m V_F^2 + mg\,2R$

$mg.h = 1/2\ m g.R + mg.2R$

$h = 1/2.R + 2R$

$\boxed{h = 2,5\ m}$

EJERCICIO 71



$m = 1kg$, $K = 10000\ N/m$, $R = 1m$, $V_i = 0$

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_1 = E_2$

(A) $E_1 = E_2$

$$\frac{1}{2} m V_i^2 + \frac{1}{2} K \Delta x^2 = \frac{1}{2} m V_F^2 + mg.2R$$

(B) $N + mg = m\,ac \Rightarrow N = m\frac{V_F^2}{R} - mg \geq 0$

$$V_F = \sqrt{g.R}$$

(A) $\frac{1}{2} K \Delta x^2 = \frac{1}{2} mg.R + mg.2R$

$$\Delta x^2 = \frac{1/2\ mg.R + mg\,2R}{1/2\ K}$$

$\boxed{\Delta x = 0,07\ m = 7cm}$

EJERCICIO 75

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_1 = E_2$

Ⓐ $E_1 = E_2$

$$\frac{1}{2} m V_i^2 + mgR = \frac{1}{2} m V_F^2 + mgh$$

$R = 1,5$

$h = R\cos\theta$

Ⓑ $-N + mg\cos\theta = ma_c = m\frac{V^2}{R}$

Ⓐ $mgR = \frac{1}{2} m V_F^2 + mgh$

$mgR - mgh = \frac{1}{2} m V_F^2$

$V_F = \sqrt{2gR(1-\cos\theta)}$

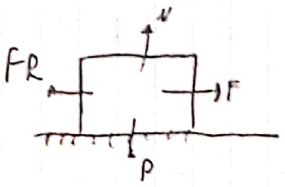
Ⓑ $N = mg\cos\theta - m\frac{2gR(1-\cos\theta)}{R} \geq 0$

$-2 + 2\cos\theta + \cos\theta \geq 0$

$\cos\theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}(2/3)$

$\boxed{\theta = 48,18}$

EJERCICIO 76



$m = 2kg$, $\mu_d = 0,3$, $x = 3 + t^2$, $t = 3$, $V_i = 0$

Ⓐ $W_{FNETO} = \Delta E_C$

$(F - FR)\,d = \Delta E_C$

Ⓑ x) $F - FR = ma \Rightarrow F = ma + FR$

y) $N - P = 0 \Rightarrow N = mg$

Ⓒ $x = 3 + t^2$

$V = 2t$

$a = 2\ m/s^2$

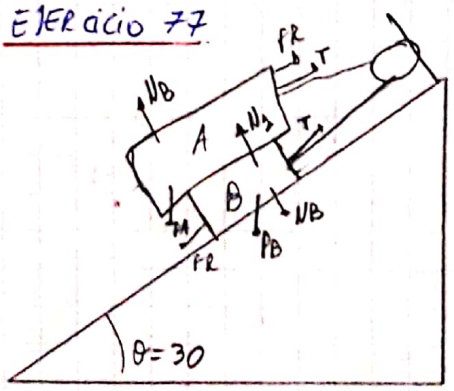
Ⓐ $FR = \mu_d N = \mu_d mg = 5,88$

$F = ma + FR = 9,88\ N$

$(F - FR)\,d = \Delta E_C$

$(9,88 - 5,88) \cdot 12 = \Delta E_C$

EJERCICIO 77



$m_A = 20kg$, $m_B = 10kg$, $\mu_d = 0,1$, $d = 0,5$

$W_{FNETA} = EF - Ei$

$$-2\mu_d m_A g\cos\theta\, d = \frac{1}{2} m_A V_F^2 + \frac{1}{2} m_B V_F^2 - m_A g h + m_B g h$$

$$-2\mu_d m_A g\cos\theta\, d = \frac{1}{2}(m_A + m_B) V_F^2$$

$$-2\mu_d m_A g\cos\theta\, d + m_A g h - m_B g h = \frac{1}{2}(m_A + m_B) V_F^2$$

$$V_F^2 = -2\mu_d m_A g\cos\theta\, d + m_A g h - m_B g h \,/\, \frac{1}{2}(m_A + m_B)$$

$V_F^2 = -16,97 + 49 - 24,5 / 15$

$h = d\cdot\cos\theta$

$h = 0,43$

$\boxed{V_F = 0,70\ m/s}$

UNIDAD 3 DINAMICA DE SISTEMAS DE PARTICULAS

CENTRO DE MASA

EJERCICIO 1

$$X_{cm} = \frac{11 + 2.2 + 7.5}{8} = 5cm$$

EJERCICIO 2

$$\vec{r}_{cm} = x_{cm} = \frac{-3m + 0m + 3m}{3m} = 0$$

$$y_{cm} = \frac{0m + 0m + \sqrt{3}a}{3m} = \frac{\sqrt{3}}{3}a$$

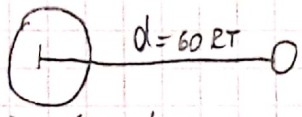
$\Big\}\ (0, \sqrt{3}/3\,a)$

$4a^2 = a^2 + y^2$

$y^2 = 4a^2 - a^2$

$y = \sqrt{3}\,a$

EJERCICIO 4

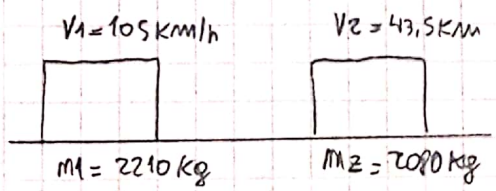


$R_T = 6370\ km$

$\frac{MT}{ML}$

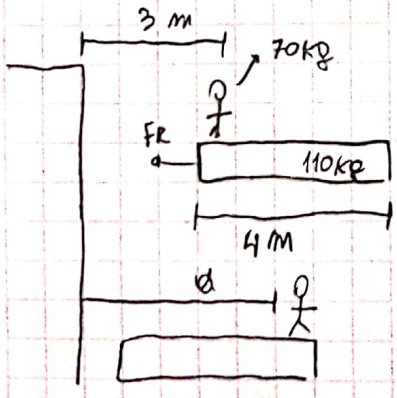
$$x_{cm} = \frac{MT \cdot 0 + 60\,R_T\,ML}{MT + ML} = \frac{60 \cdot 6370 \cdot ML}{81\,ML} = 4720\,km$$

EJERCICIO 5



$$V_{cm} = \frac{2210 \cdot 29,16 + 2080 \cdot 12,08}{4290} = \boxed{20,87\ \frac{m}{1}}$$

EJERCICIO 8

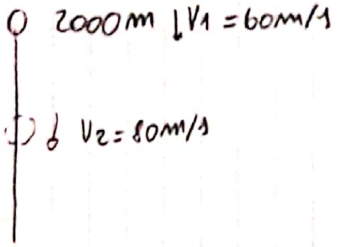


$$X_{cm} = \frac{70 \cdot 3m + 110 \cdot 5}{180} = 4,2 \Rightarrow cte$$

$$X_{cm} = \frac{70 \cdot d + 110\,(d-2)}{180}$$

$$\boxed{d = 5,44\ m}$$

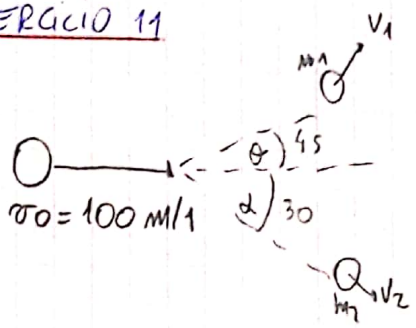
EJERCICIO 9



$$y_{cm} = y_0 - V_0 t - \frac{1}{2} g t^2$$

$$y_{cm} = 2000 - 60 \cdot 10 - \frac{1}{2} g \cdot 10^2 = 910 m$$

EJERCICIO 11

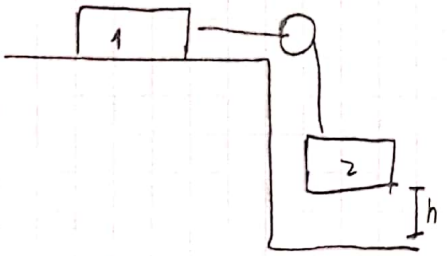


$$x) \quad m_1 V_1 \operatorname{sen}\theta - m_2 v_2 \operatorname{sen}\alpha = 0 \Rightarrow V_1 = \frac{-m_2 V_2 \operatorname{sen}\alpha}{m_1 (\operatorname{sen}\theta)} = \boxed{69 m/s}$$

$$y) \quad m_1 V_1 \cos\theta + m_2 v_2 \cos\alpha = m V_0$$

$$V_2 = \frac{M V_0}{m_2 (\cos\theta - \operatorname{sen}\alpha)} = \boxed{293 m/s}$$

EJERCICIO 13



$m_1 = 150 kg \quad m_2 = 40 kg \quad \mu_d = 0,15 \quad h = 1m = d$

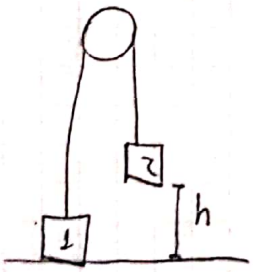
$$W_{FNC} = E_F - E_i$$

$$F_R \cdot d = \left[\frac{1}{2}(m_1 + m_2) V_F^2\right] - [m_2 g h]$$

$$-\mu_d m_1 g \cdot d = 1/2 (m_1 + m_2) V_F^2 - m_2 g h$$

$$\sqrt{\frac{2(-\mu_d m_1 g \cdot d + m_2 g h)}{(m_1 + m_2)}} = V_F \Rightarrow \boxed{V_F = 1,35 m/s}$$

EJERCICIO 14



$m_1 = 2 kg \quad m_2 = 3 kg \quad h = 0,2 m$

$$W_{FNC} = 0 \Rightarrow E = ctte \Rightarrow E_i = E_F$$

$$E_i = E_F$$

$$m_2 g h = \frac{1}{2} m_2 V_F^2 + \frac{1}{2} m_1 V_F^2 + m_1 g h$$

$$m_2 g h = 1/2 (m_1 + m_2) V_F^2 + m_1 g h$$

$$\sqrt{\frac{2(m_2 g h - m_1 g h)}{(m_1 + m_2)}} = V_F \Rightarrow \boxed{V_F = 0,89 m/s}$$

Km → m/s %3,6

← ×3,6

CHOQUE

EJERCICIO 20

CHOQUE PLASTICO

(A) $P = cte \Rightarrow P_{antes} = P_{despues}$

$$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V'$$

$$V' = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2} \Rightarrow \boxed{V' = 4,2 \text{ m/s}}$$

(B)

$$\Delta E_C = E_F - E_i$$

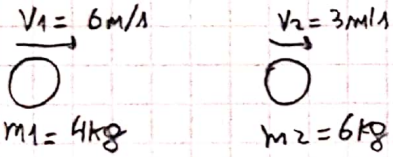
$$\Delta E_C = \frac{1}{2}(m_1 + m_2) V'^2 - \left[\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2\right]$$

$$\Delta E_C = 88,2 - 72 - 27$$

$$\boxed{\Delta E_C = -10,8 \text{ J}}$$

EJERCICIO 21

CHOQUE ELASTICO $\Rightarrow K = 1$



$P_{antes} = P_{despues}$

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2' \quad (1)$$

$$K = -\frac{(V_2' - V_1')}{V_2 - V_1} = 1 \quad (2)$$

$$m_1 V_1 + m_2 V_2 = m_1 V_2 - m_1 V_1 + m_1 V_2' + m_2 V_2'$$

$$V_1' = V_2 - V_1 + V_2'$$

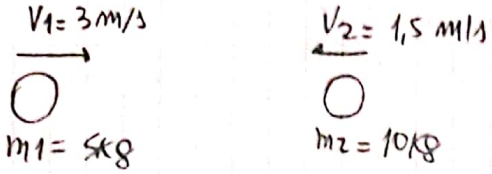
$$\boxed{V_1' = 2,4 \text{ m/s}}$$

$$m_1 V_1 + m_2 V_2 = m_1 (V_2 - V_1) + (m_1 + m_2) V_2'$$

$$\frac{m_1 V_1 + m_2 V_2 - m_1 (V_2 - V_1)}{(m_1 + m_2)} = V_2'$$

$$\boxed{V_2' = 5,4 \text{ m/s}}$$

EJERCICIO 22.



A) PLASTICO

Power = Pdespues

$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V'$ (↓ -1,5)

$V' = 0$ m/s

(B) ELASTICO

Power = Pdespues

$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$

$m_1 V_1 - m_2 V_2 - m_1 (V_2 - V_1) = V_2' (m_1 + m_2)$

$V_2' = 1,5$ m/s

$K = -\frac{(V_2' - V_1')}{V_2 - V_1} = 1$

$V_1' = V_2 - V_1 + V_2'$

$V_1' = -3$

EJERCICIO 23

$V_1 = 12$ km/h $= 3,3$ m/s $\quad V_2 = 0$ m/s

CHOQUE PLASTICO

$FR = (m_1 + m_2) \cdot g \cdot 0,05$

$FR = 39.200$

1) Power = Pdespues

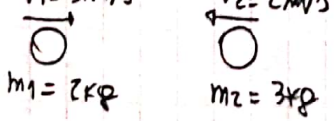
$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V' \Rightarrow V' = 2,06$ m/s

2) $W_{FNC} = E_F - E_I$

$-FR \cdot d = \frac{1}{2} m V_F^2 - \frac{1}{2} (m_1 + m_2) V'^2$ ($V_F \to 0$)

$d = \frac{-1/2 (m_1 + m_2) V'^2}{-FR} \Rightarrow d = 4,33$ m

EJERCICIO 24



CHOQUE ELASTICO, $V_2' = 2$ m/s

(A) Power = Pdespues

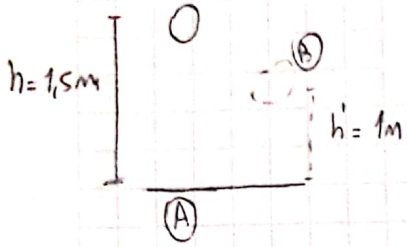
$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$

$V_1' = \frac{m_1 V_1 - m_2 V_2 - m_2 V_2'}{m_1}$

$V_1' = -1$ m/s

(B) $K = -\frac{(V_2' - V_1')}{V_2 - V_1} = 0,42$

EJERCICIO 25



$W_{FNC} = 0 \Rightarrow E = CTE \Rightarrow E_F = E_i$

(A) $E_i = E_F$

$mgh = \frac{1}{2} m V_1^2 \Rightarrow V_1 = \sqrt{2gh} = V_1 = 5,1\,m/s$

(B) $E_i = E_F$

$\frac{1}{2} m V_1'^2 = mgh \Rightarrow V_1' = \sqrt{2gh} = 4,42\,m/s$

$$K = -\frac{(V_2' - V_1')}{V_2 - V_1} = \frac{V_1'}{-V_1} = \boxed{0,87}$$

EJERCICIO 26

$V_1 = 10\,m/s$ $\quad$ $V_2 = 4\,m/s$ $\quad$ CHOQUE PLASTICO, $K = 0,7$

$m_1 = 2kg$ $\quad$ $m_2 = 1kg$

(A) Power = Pdespues

$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$

$m_1 V_1 + m_2 V_2 = m_1 \cdot 0,7 (V_2 - V_1) + m_1 V_2' + m_2 V_2'$

$m_1 V_1 + m_2 V_2 - m_1 \cdot 0,7 (V_2 - V_1) = V_2' (m_1 + m_2)$

$\boxed{V_2' = 10,8\,m/s}$

$$K = -\frac{(V_2' - V_1')}{V_2 - V_1} = 0,7$$

$V_1' = 0,7 (V_2 - V_1) + V_2'$

$\boxed{V_1' = 6,6\,m/s}$

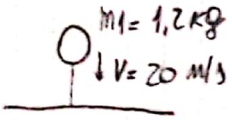
(B) $\Delta E_C = E_F - E_i$

$$\Delta E_C = \left[\frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 V_2'^2\right] - \left[\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2\right]$$

$\Delta E_C = 43,56 + 58,32 - 100 - 8 = \boxed{-6,12\,J}$

EJERCICIO 28

$K = 0{,}9 \Rightarrow K = \dfrac{-(V_2' - V_1')}{V_2 - V_1} = 0{,}9 \quad v_{r2} = v_2' = 0$



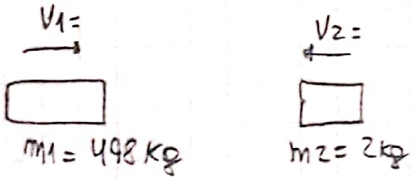
$K = -\dfrac{V_1'}{V_1}$

$V_1' = -K V_1 \Rightarrow \boxed{V_1' = 18 \text{ m/s}}$

$\Delta E_C = E_F - E_i$

$\Delta E_C = \frac{1}{2} m V_1'^2 - \frac{1}{2} m V_1^2 = \boxed{-45{,}6 \text{ J}}$

EJERCICIO 29



$\mu d = 0{,}2 \quad V_1 = 2 \text{ m/s} \quad V_2 = -98 \text{ m/s}$

CHOQUE PLASTICO

$F_R = \mu d\, mg = 976{,}02 \text{ N} \quad F_R = ma \quad a = \dfrac{F_R}{m} = 1{,}96$

(A) Pantera = Polenta

$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V' \Rightarrow V' = 1{,}6 \text{ m/s}$

$V_F = V_0 + a \cdot t \quad t = \dfrac{V_F - V_0}{a} = \boxed{0{,}8 \text{ s}}$

$W_{FNC} = E_F - E_i$

$-\mu d\, mg \cdot d = \frac{1}{2}(m_1 + m_2) V_F^2 - \frac{1}{2}(m_1 + m_2) V'^2$ ($V_F = 0$)

$d = \dfrac{-\frac{1}{2}(m_1 + m_2) V'^2}{-\mu d\, mg} \Rightarrow \boxed{d = 0{,}65 \text{ m}}$

EJERCICIO 30

$m_1 = 50 \text{ kg} \quad V_x = 2 \text{ m/s}$
$m_2 = 70 \text{ kg} \quad V_y = 1{,}5 \text{ m/s}$

X) $m_1 V_x = (m_1 + m_2) V_x' \Rightarrow V_x' = 0{,}83 \text{ m/s}$

y) $m_2 V_y = (m_1 + m_2) V_y' \Rightarrow V_y' = 0{,}87 \text{ m/s}$

(A) $|V'| = \sqrt{V_x'^2 + V_y'^2} = \boxed{1{,}20 \text{ m/s}}$

(B) $\Delta E_{Cx} = E_{Fx} - E_{ix}$

$\Delta E_{Cx} = \frac{1}{2}(m_1 + m_2) V_x'^2 - \frac{1}{2} m_1 V_x^2$

$\Delta E_{Cx} = -58{,}5 \text{ J}$

$\Delta E_{Cy} = E_{Fy} - E_{iy}$

$\Delta E_{Cy} = \frac{1}{2}(m_1 + m_2) V_y'^2 - \frac{1}{2} m_2 V_y^2$

$\Delta E_{Cy} = -33{,}33 \text{ J}$

$\boxed{\Delta E_C = -91{,}8 \text{ J}}$

EJERCICIO 31

CHOQUE PLASTICO $\theta = 60°$

$m_1 = 8\,kg$ $\quad V_1 = 10\,m/s\ \hat{i}$

$m_2 = 10\,kg$ $\quad V_2 = 10\,m/s$

(A)

x) $m_1 V_1 + m_2 V_2 \cos\theta = (m_1+m_2) V'_x \Rightarrow V'_x = 7,22\ m/s$

y) $m_2 V_2 \operatorname{sen}\theta = (m_1+m_2) V'_y \Rightarrow V_y = 4,81\ m/s$

$\Big\} V' = (7,22; 4,81)$

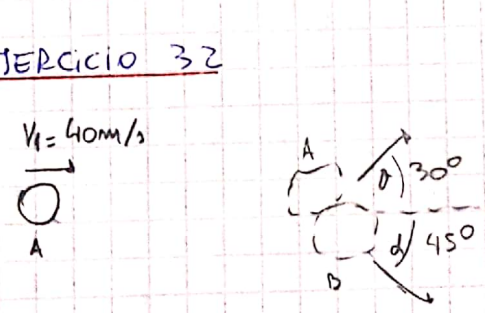
(B)

$\Delta EC = EF - Ei$

$\Delta EC = \frac{1}{2}(m_1+m_2)\left[V_x'^2 + V_y'^2\right] - \left(\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2\right)$

$\Delta EC = \quad 677,3805 \quad - 900 \quad = \boxed{-222,61}$

EJERCICIO 32



x) $m_A V_{Ax} + m_B V_B = m_A V'_A \cos\theta + m_B V'_B \cos\alpha$

y) $m_A V_{Ay} + m_B V_{By} = m_A V'_A \operatorname{sen}\theta - m_B V'_B \operatorname{sen}\alpha$

($V_{Ay} = 0$, V_B vertical $\Rightarrow V_{By} = 0$)

x) $m_A V_A = m_A V'_A \cos\theta + m_A V'_B \cos\alpha$

$V_A = V'_A \cos\theta + \frac{V'_A \operatorname{sen}\theta}{\operatorname{sen}\alpha} \cos\alpha$ (1)

$V'_A = V_A / (\cos\theta + \operatorname{sen}\theta) \Rightarrow \boxed{V'_A = 29,28\ m/s}$

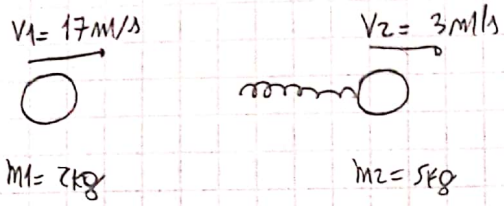
y) $0 = m_A V'_A \operatorname{sen}\theta - m_A V'_B \operatorname{sen}\alpha$

$V'_B \operatorname{sen}\alpha = V'_A \operatorname{sen}\theta$

$V'_B = \frac{V'_A \operatorname{sen}\theta}{\operatorname{sen}\alpha} \Rightarrow \boxed{V'_B = 20,70\ m/s}$

PROBLEMAS INTEGRADORES

EJERCICIO 33



K= 4480

CHOQUE ELASTICO ⇒ K=1

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_i = E_F$

$\vec{P} = cte$

(A) $E_i = E_F$

$$\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2}(m_1 + m_2) V'^2 + \frac{1}{2} K \Delta x^2$$

$$\frac{2\left(\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 - \frac{1}{2}(m_1 + m_2) V'^2\right)}{K} = \Delta x^2$$

$$\frac{2(289 + 22{,}5 - 171{,}5)}{4480} = \Delta x^2 \Rightarrow \boxed{\Delta x = 0{,}25 \text{ m/s}}$$

Power = P max comp

$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V' \Rightarrow V' = 7 m/s$

(B) Power = P despues

$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$

$m_1 V_1 + m_2 V_2 = m_1 V_2 - m_1 V_1 + m_1 V_2' + m_2 V_2'$

$m_1 V_1 + m_2 V_2 - m_1 (V_2 - V_1) = V_2' (m_1 + m_2)$

$$\frac{m_1 V_1 + m_2 V_2 - m_1 (V_2 - V_1)}{(m_1 + m_2)} = V_2' \Rightarrow \boxed{V_2' = 11 \text{ m/s}}$$

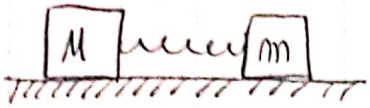
$$K = -\frac{(V_2' - V_1')}{V_2 - V_1} = 1$$

$V_1' = V_2 - V_1 + V_2'$

$\boxed{V' = -3 \text{ m/s}}$

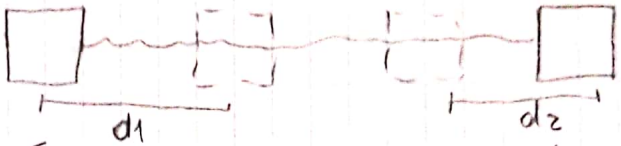
EJERCICIO 34

$M = 30\,kg$ $\quad m = 20\,kg$ $\quad d = 0{,}2\,m$ $\quad V_1 = V_2 = 0$



CHOQUE PLASTICO

$\vec{P} = cte$



$P_{antes} = P_{despues}$

$m V_1 + M V_2 = m V_1' + M V_2'$

$0 = m V_1' + M V_2'$

$m V_1' = M V_2'$

$\frac{V_1'}{V_2'} = \frac{M}{m}$

$W_{FNC} = E_F - E_i$

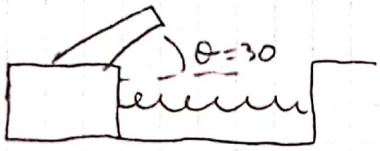
1) $-M d \not{m} g \cdot d_1 = \frac{1}{2} M V_F^2 - \frac{1}{2} M V_1'^2$ $\quad$ 2) $-M d \not{m} g \cdot d_2 = \frac{1}{2} m V_F^2 - \frac{1}{2} m V_2^2$

$V_F = 0$ $\qquad$ $V_F = 0$

1 ÷ 2 $\qquad \frac{-M d g\, d_1}{M d g\, d_2} = \frac{V_1'^2}{V_2'^2}$

$\frac{d_1}{d_2} = \left(\frac{V_1'}{V_2'}\right)^2 = \left(\frac{M}{m}\right)^2 \Rightarrow d_2 = \left(\frac{M}{m}\right)^2 d_1 = \boxed{0{,}45\,m}$

EJERCICIO 35



$K = 25\,k$ $\quad 25000$ $\qquad m_p = 200\,kg$ $\quad m_c = 5000\,kg$ $\qquad V_p = 200\,m/s$

$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_i = E_F$

(A)

$P_{antes} = P_{justo\ despues}$

$m_c V_c + m_p V_p = m_c V_c' + m_p V_p' \cos\theta$

($V_c = 0$, $V_p = 0$)

$m_c V_c' = -m_p V_p' \cos\theta$

$V_c' = -\frac{m_p V_p' \cos\theta}{m_c}$

$\boxed{V_c' = -6{,}9\,c}$

(B) $E_i = E_F$

$\frac{1}{2} m_c V_c'^2 = \frac{1}{2} m_c V_{cF}^2 - \frac{1}{2} k \Delta x^2$

$\frac{1}{2} m V_c'^2 = -\frac{1}{2} K \Delta x^2$

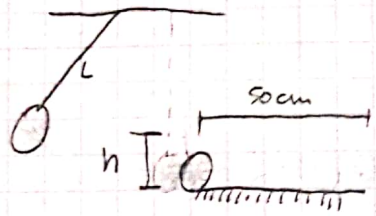
$\Delta x^2 = \frac{m V_c'^2}{-K}$

$\boxed{\Delta x = 3{,}09\,m}$

ETERCICIO 37

$m_1 = 2kg$ $m_2 = 1kg$ $d = 0,5\,m$ $\mu_d = 0,2$

$V_2 = 0$ CHOQUE ELASTICO $\Rightarrow k = 1$



(A) Pjunto antes = Pjunto despues

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2' \quad (1)$$

$$m_1 V_1 = -m_1 V_1 + m_1 V_2' + m_2 V_2'$$

$$m_1 V_1 + m_1 V_1 = V_2' (m_1 + m_2)$$

$$2 m_1 V_1 = V_2' (m_1 + m_2)$$

$$V_1 = \frac{V_2' (m_1 + m_2)}{2 m_1}$$

$$V_1 = 1,1\,m/s$$

$$K = -\frac{(V_2' - V_1')}{V_2 - V_1} = 1$$

$$V_1' = -V_1 + V_2' \quad (2)$$

$$V_1' = 0,3$$

$$W_{FNC} = E_F - E_i$$

$$-\mu_d m_2 g \cdot d = \frac{1}{2} m_2 V_F^2 - \frac{1}{2} m_2 V_2' \quad (3)$$

($V_F = 0$)

$$V_2' = \sqrt{2 \mu_d g d}$$

$$V_2' = 1,4\,m/s$$

$E_i = E_F$

$$\frac{1}{2} m_1 V_i^2 + m_1 g h = \frac{1}{2} m_1 V_1^2 + m_1 g h'^{0}$$

($V_i = 0$)

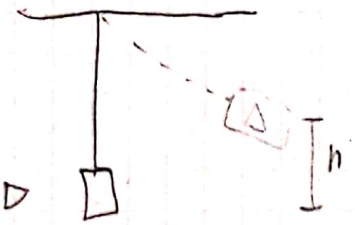
$$m g h = 1/2\, m V_1^2$$

$$h = 1/2 \frac{V_1^2}{g} \Rightarrow \boxed{h = 0,06\,m}$$

(B) $\boxed{V_1 = 1,1\,m/s}$

(C) $W = \frac{V}{R} = \frac{V_1}{R} = \boxed{1,1\,s^{-1}}$

EJERCICIO 38



Ⓐ $E_{CA} = \frac{1}{2} m_1 V_1^2$

$E_{CA} = 346\ J$

$E_{CD} = \frac{1}{2}(m_1+m_2) V'^2$

$E_{CD} = 2,05\ J$

Potencia = Pdespues

$m_1 V_1 = (m_1+m_2) V'$

$V' = \frac{m_1 V_1}{m_1+m_2} \Rightarrow V' = 1,43\ m/s$

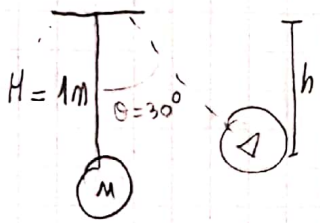
$W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_i = E_F$

$\frac{1}{2}(m_1+m_2) V'^2 = (m_1+m_2) g h$

$h = \frac{1/2\, V'^2}{g}$

$h = 0,10\ m$

EJERCICIO 39



$m_1 = 0,02 \quad m_2 = 1kg$

$h = d \cos\theta$

$H - h = 0,13\ m$

Ⓐ $W_{FNC} = 0 \Rightarrow E = cte \Rightarrow E_i = E_F$

$(m_1+m_2) g h = \frac{1}{2}(m_1+m_2) V'^2$

$V'^2 = 2gh$

$V' = \sqrt{2gh}$

$V' = 1,62\ m/s$

Potencia = Pdespues

$m_1 V_1 = (m_1+m_2) V'$

$V_1 = \frac{(m_1+m_2) V'}{m_1}$

$V_1 = 82,67\ m/s$

Ⓑ

$P = ma_c$

$mg = m \frac{V_F^2}{r}$

$V_F = \sqrt{g \cdot r}$

$V_F = 3,13\ m/s$

$\frac{1}{2}(m_1+m_2) V'^2 = \frac{1}{2}(m_1+m_2) V_F^2 + (m_1+m_2) g h_2$

$m_1 V'^2 + m_2 V'^2 - m_1 g h_2 - m_2 g h_2 = m_1 V_F^2 + m_2 V_F^2$

$m_1 V'^2 - m_1 g 2h - m_1 V_F^2 = m_2 V_F^2 - m_2 V'^2 + m_2 g 2h$

$m_1 (V'^2 - 2gh - V_F^2) = m_2 (V_F^2 - V'^2 + 2gh)$

$m_1 = \frac{m_2 (V_F^2 - V'^2 + 2gh)}{(V'^2 - 2gh - V_F^2)}$

$m_1 V = m_1 V' + m_2 V'$

$m_1 = \frac{m_2 V'}{V - V'}$

$m_1 = 0,03\ kg$