

# Computability and Complexity

## COSC 4200

### Pushdown Automata

## Definition

A (*nondeterministic*) *pushdown automaton* (*PDA*) is a 6-tuple  $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$  where

- $Q$  is a finite set of *states*
- $\Sigma$  is the *input alphabet*
- $\Gamma$  is the *stack alphabet*
- $\delta : Q \times \Sigma_{\epsilon} \times \Gamma_{\epsilon} \rightarrow \mathcal{P}(Q \times \Gamma_{\epsilon})$  is the *transition function*
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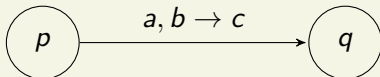
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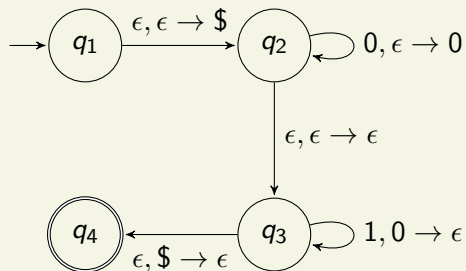
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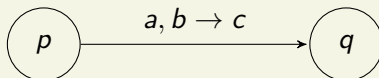
For each  $(q, c) \in \delta(p, a, b)$ , we have the following transition in a diagram for the PDA:



# Example: PDA for $\{0^n 1^n \mid n \geq 0\}$

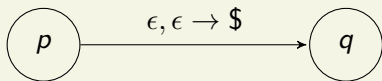


# Transitions

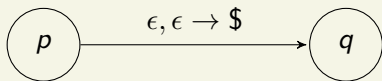


This transition means that when the PDA is in state  $p$ , if the input symbol is  $a$  and the top of the stack is a  $b$ , then the PDA may transition to state  $q$  and replace  $b$  by  $c$  on the top of the stack.

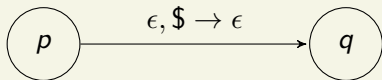
When  $a = \epsilon$ , no input symbol is read. When  $b = \epsilon$ , no stack symbol is read or replaced.



“push a \$ on the stack without reading any input”

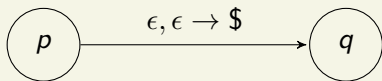


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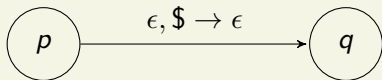


“pop a \$ on the stack without reading any input”

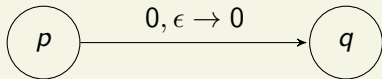




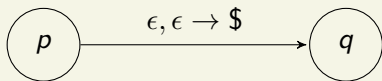
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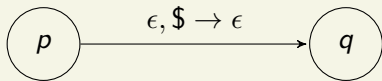
“pop a \$ on the stack without reading any input”



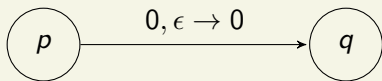
“read a 0 from the input, push a 0 on the stack”



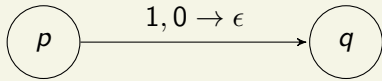
“push a \$ on the stack without reading any input”



“pop a \$ on the stack without reading any input”



“read a 0 from the input, push a 0 on the stack”



“read a 1 from the input, pop a 0 from the stack”

# Configurations

A *configuration* of  $M$  is a 3-tuple

$$(q, w, z) \in Q \times \Sigma^* \times \Gamma^*.$$

Here  $q$  denotes the current state,  $w$  denotes the remaining input, and  $z$  denotes the current stack.

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A configuration  $(q, w, z)$  is *accepting* if  $q \in F$  and  $w = \epsilon$ , that is the state is accepting and no input remains.

$$(q, \epsilon, z)$$

# Computation of PDA

Let  $(q_1, w_1, z_1)$  and  $(q_2, w_2, z_2)$  be two configurations.

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$$(q_1, w_1, z_1) \rightarrow (q_2, w_2, z_2)$$

if

$$\begin{aligned}w_1 &= aw & w_2 &= w \\ z_1 &= b_1z & z_2 &= b_2z\end{aligned}$$

for some  $a \in \Sigma_\epsilon$ ,  $w \in \Sigma^*$ ,  $b_1, b_2 \in \Gamma_\epsilon$ ,  $z \in \Gamma$ , so that

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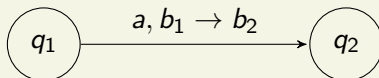
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$$(q_1, w_1, z_1) = (q_1, aw, b_1z) \rightarrow (q_2, w, b_2z) = (q_2, w_2, z_2)$$





# Language of PDA

We say that  $(q_1, w_1, z_1)$  *yields*  $(q_2, w_2, z_2)$  if there is a sequence of configurations  $(p_1, x_1, y_1), \dots, (p_k, x_k, y_k)$  such that

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An input  $w$  is *accepted* by  $M$  if the initial configuration  $(q_0, w, \epsilon)$  yields some accepting configuration.

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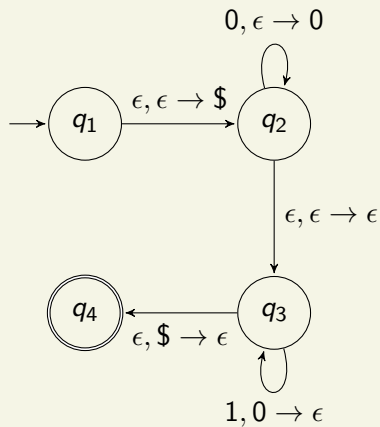
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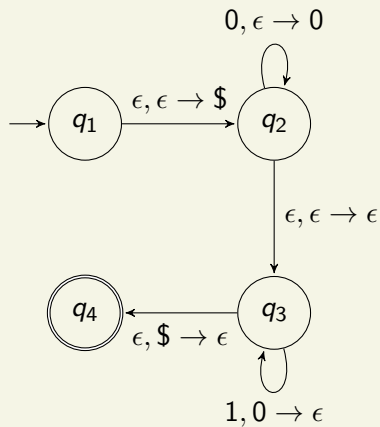
The language of  $M$  is

$$L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}.$$

Computation on 000111:

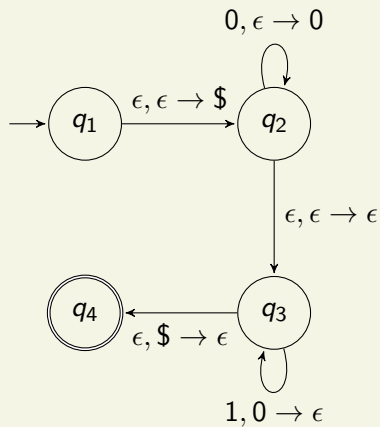
$(q_1, 000111, \epsilon)$





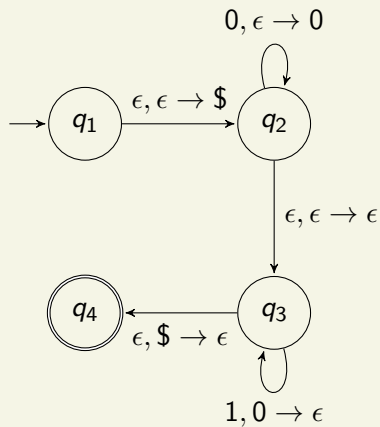
Computation on 000111:

$(q_1, 000111, \epsilon) \rightarrow (q_2, 000111, \$)$



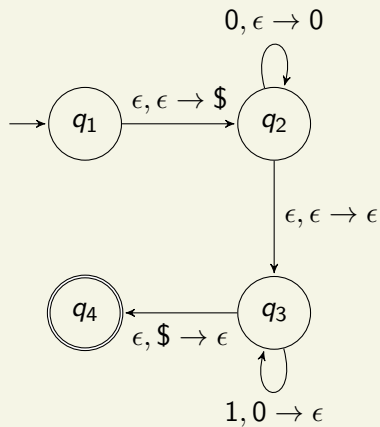
Computation on 000111:

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 (q_1, 000111, \epsilon) &\rightarrow (q_2, 000111, \$) \\
 &\rightarrow (q_2, 00111, 0\$)
 \end{aligned}$$



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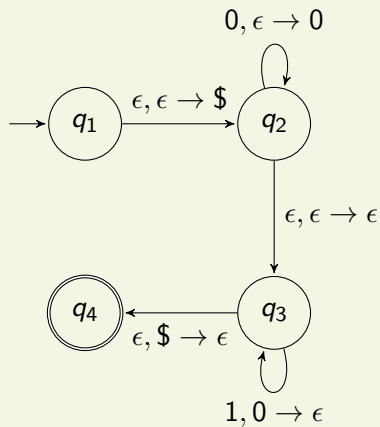
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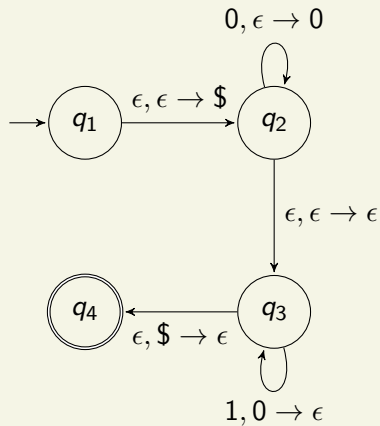
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 &\rightarrow (q_2, 0111, 00\$) \\
 &\rightarrow (q_2, 111, 000\$)
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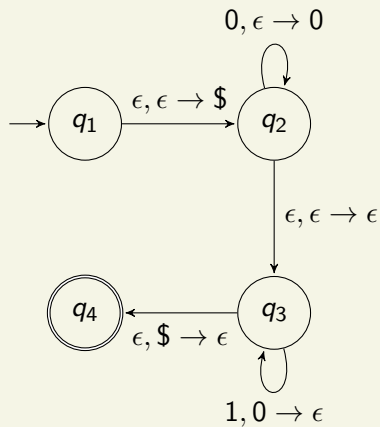
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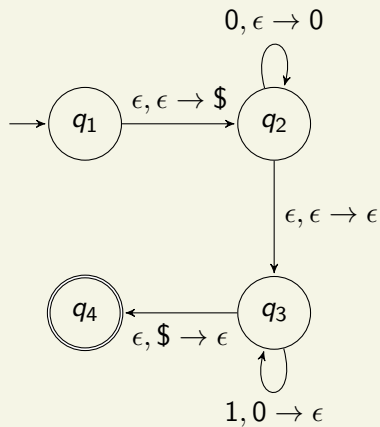
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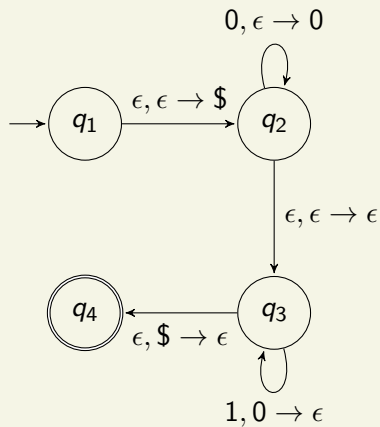
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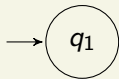


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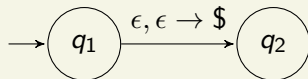
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 $\rightarrow (q_4, \epsilon, \epsilon)$

Example: PDA for  $\{a^i b^j c^k \mid i = j \text{ or } i = k\}$

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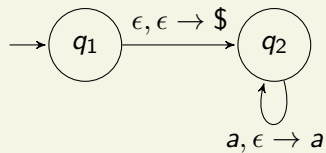


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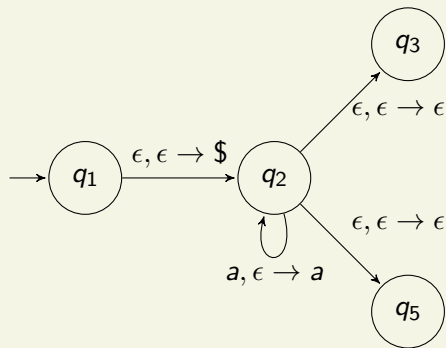




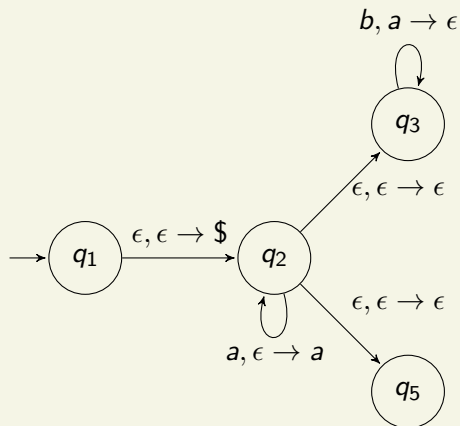
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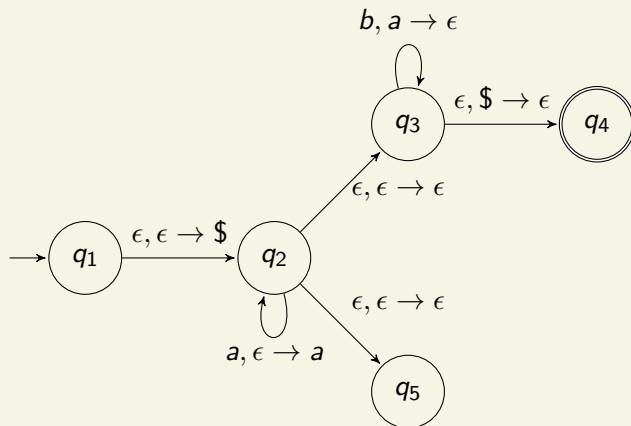
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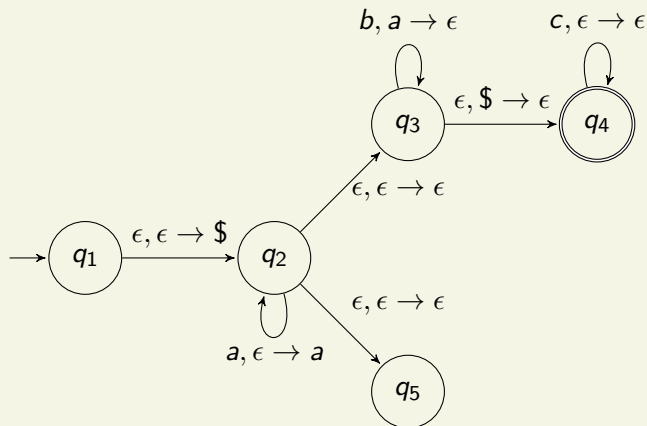
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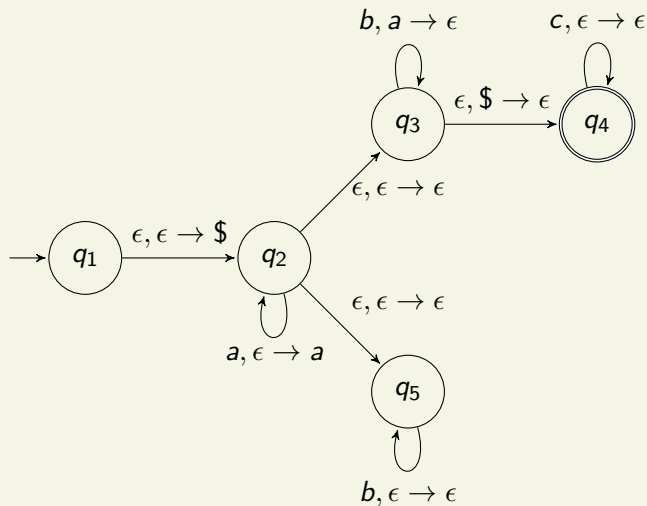
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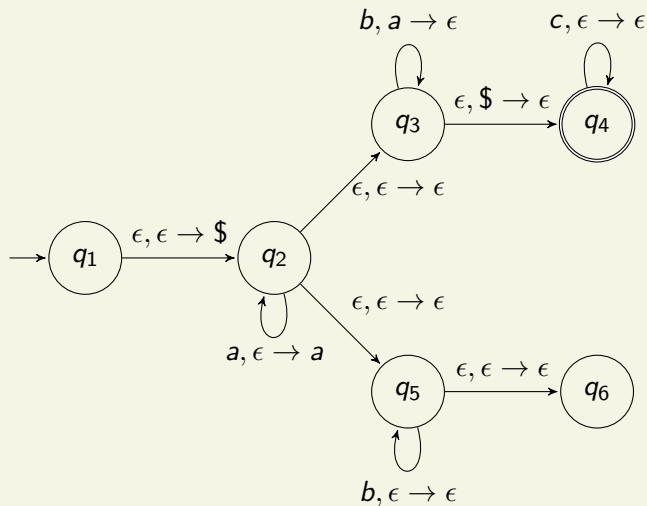
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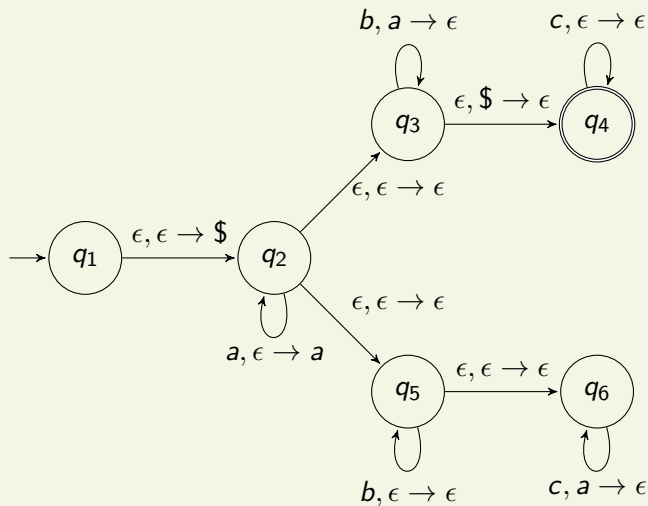
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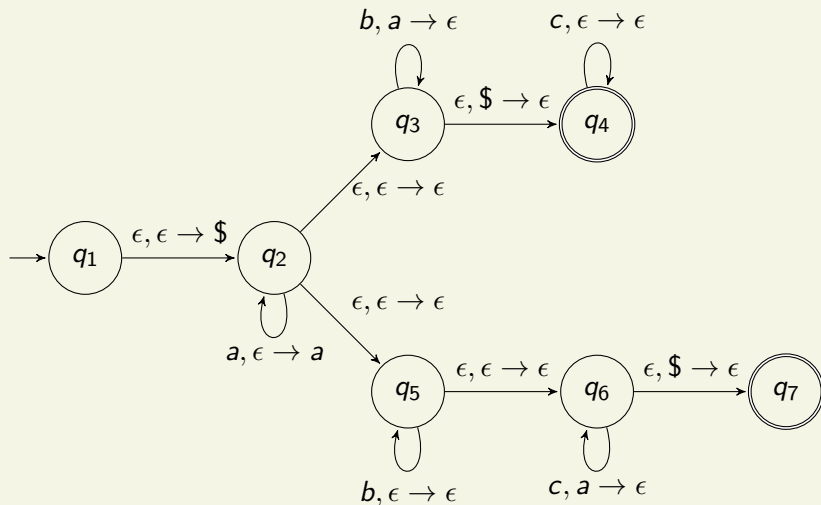


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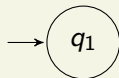




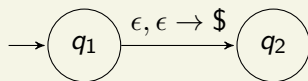
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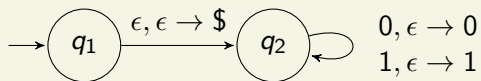
Example: PDA for  $\{ww^R \mid w \in \{0,1\}^*\}$



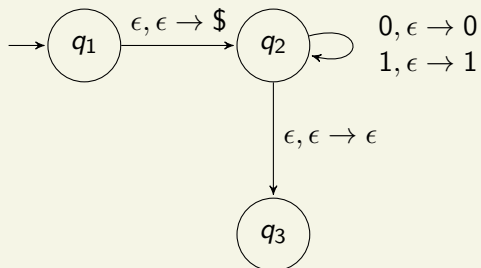
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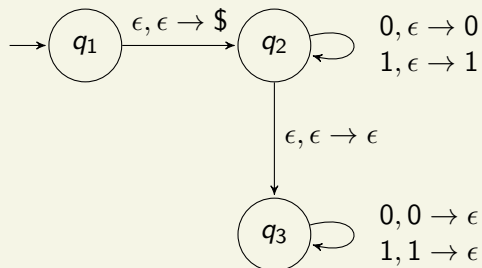
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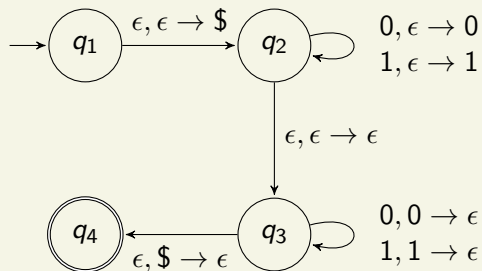
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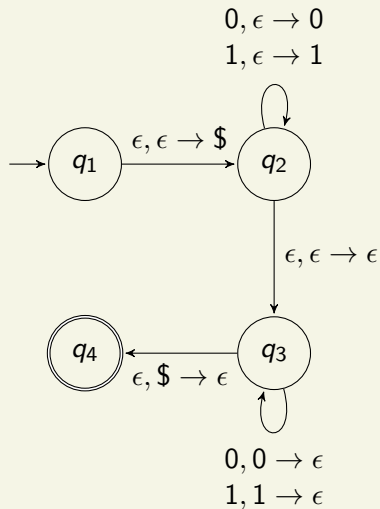
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# Example Computation

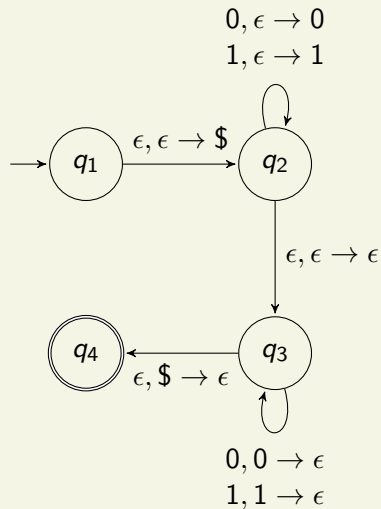
Computation on 01011010:

$(q_1, 01011010, \epsilon)$





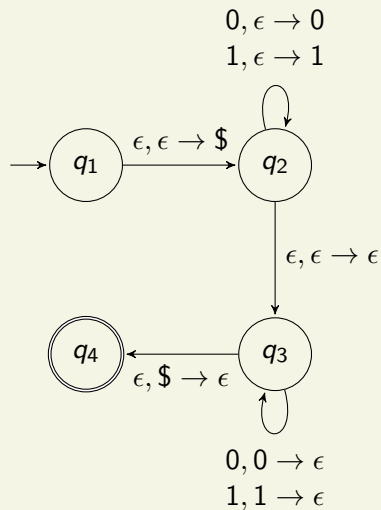
# Example Computation



Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$

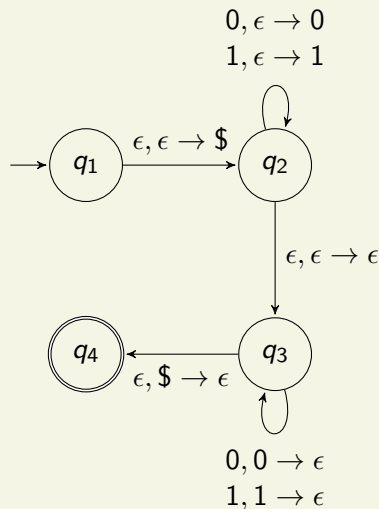
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Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$

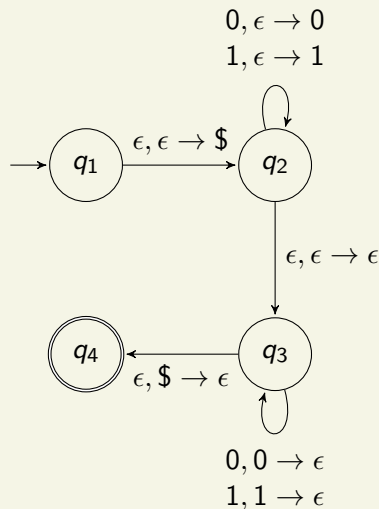
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Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$

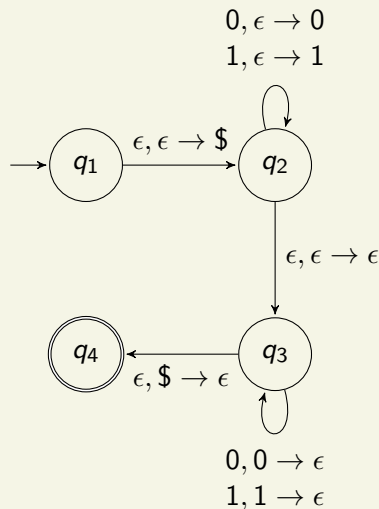
# Example Computation



Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$   
 $\rightarrow (q_2, 11010, 010\$)$

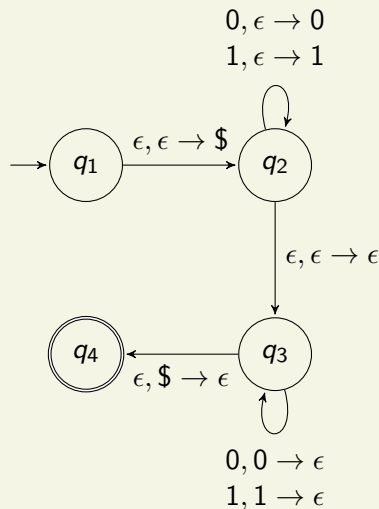
# Example Computation



Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$   
 $\rightarrow (q_2, 11010, 010\$)$   
 $\rightarrow (q_2, 1010, 1010\$)$

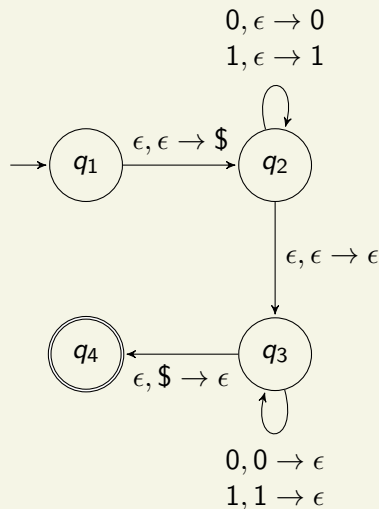
# Example Computation



Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$   
 $\rightarrow (q_2, 11010, 010\$)$   
 $\rightarrow (q_2, 1010, 1010\$)$   
 $\rightarrow (q_3, 1010, 1010\$)$

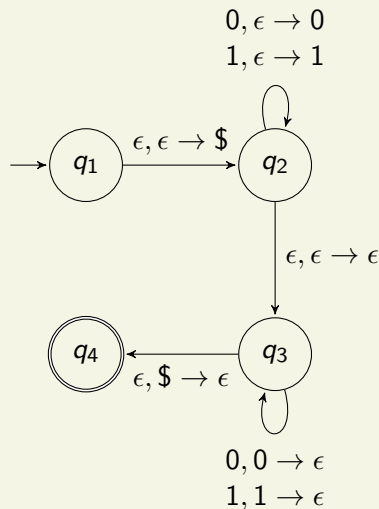
# Example Computation



Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$   
 $\rightarrow (q_2, 11010, 010\$)$   
 $\rightarrow (q_2, 1010, 1010\$)$   
 $\rightarrow (q_3, 1010, 1010\$)$   
 $\rightarrow (q_3, 010, 010\$)$

# Example Computation

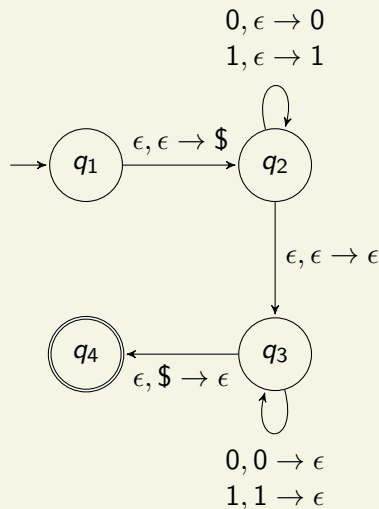


Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$   
 $\rightarrow (q_2, 11010, 010\$)$   
 $\rightarrow (q_2, 1010, 1010\$)$   
 $\rightarrow (q_3, 1010, 1010\$)$   
 $\rightarrow (q_3, 010, 010\$)$   
 $\rightarrow (q_3, 10, 10\$)$



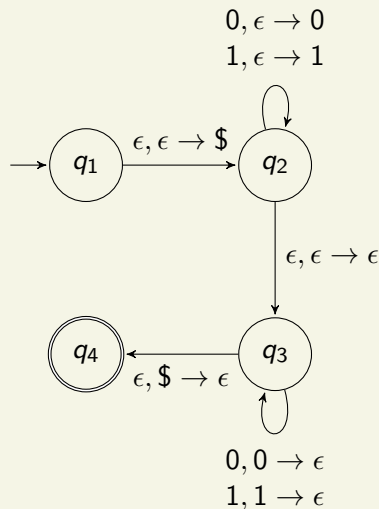
# Example Computation



Computation on 01011010:

$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$   
 $\rightarrow (q_2, 11010, 010\$)$   
 $\rightarrow (q_2, 1010, 1010\$)$   
 $\rightarrow (q_3, 1010, 1010\$)$   
 $\rightarrow (q_3, 010, 010\$)$   
 $\rightarrow (q_3, 10, 10\$)$   
 $\rightarrow (q_3, 0, 0\$)$

# Example Computation



Computation on 01011010:

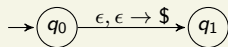
$(q_1, 01011010, \epsilon)$   
 $\rightarrow (q_2, 01011010, \$)$   
 $\rightarrow (q_2, 1011010, 0\$)$   
 $\rightarrow (q_2, 011010, 10\$)$   
 $\rightarrow (q_2, 11010, 010\$)$   
 $\rightarrow (q_2, 1010, 1010\$)$   
 $\rightarrow (q_3, 1010, 1010\$)$   
 $\rightarrow (q_3, 010, 010\$)$   
 $\rightarrow (q_3, 10, 10\$)$   
 $\rightarrow (q_3, 0, 0\$)$   
 $\rightarrow (q_4, \epsilon, \epsilon)$

Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$

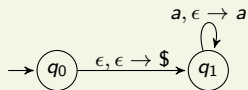
Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$



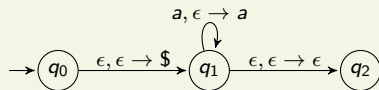
Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$



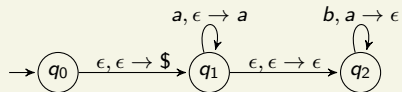
Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$



Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$

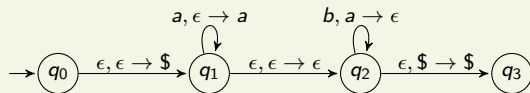


Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$

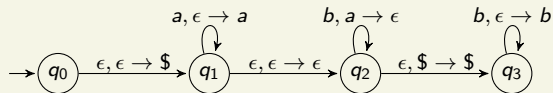




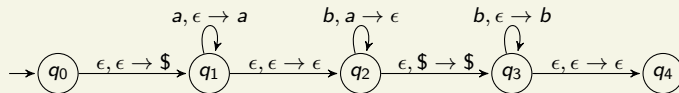
Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$



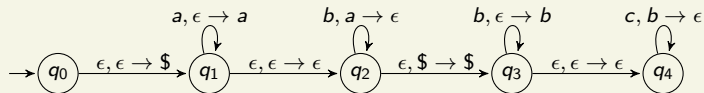
Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$



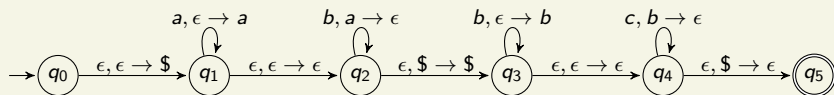
Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$

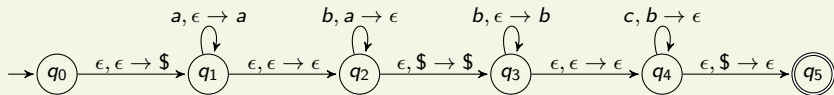


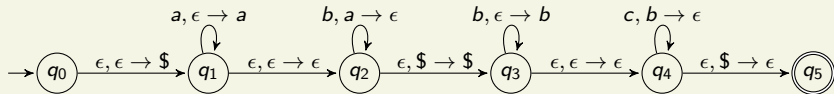
Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$



Example: PDA for  $\{a^i b^j c^k \mid i + k = j\}$

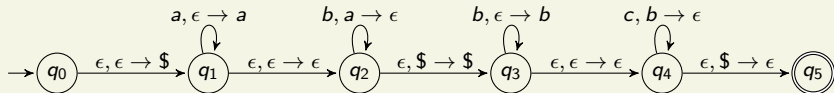






Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$

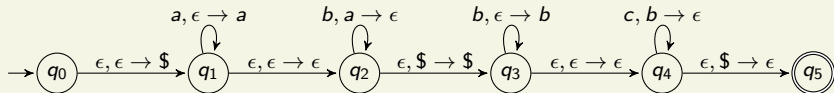


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$

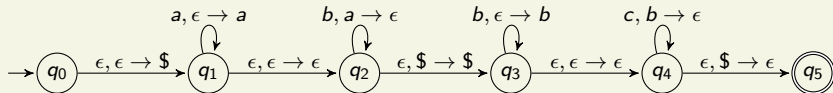
$\rightarrow (q_1, aabbbbcc, \$)$





Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$



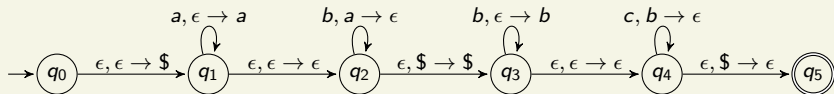
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$

$\rightarrow (q_1, aabbbbcc, \$)$

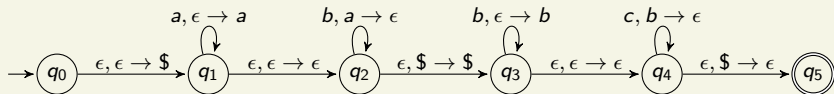
$\rightarrow (q_1, abbbbcc, a\$)$

$\rightarrow (q_1, bbbbcc, aa\$)$



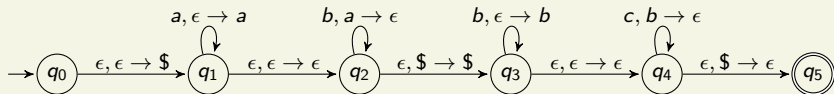
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$



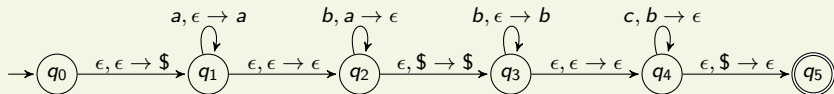
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$



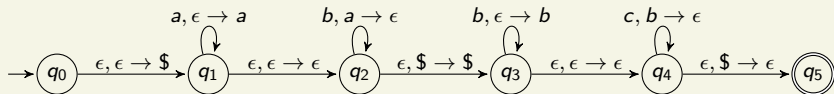
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$



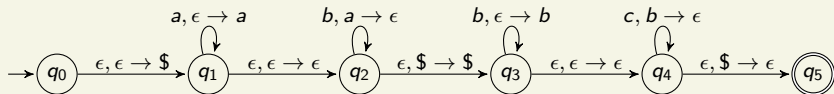
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$



Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$



Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$

$\rightarrow (q_1, aabbbbcc, \$)$

$\rightarrow (q_1, abbbbcc, a\$)$

$\rightarrow (q_1, bbbbcc, aa\$)$

$\rightarrow (q_2, bbbbcc, aa\$)$

$\rightarrow (q_2, bbbcc, a\$)$

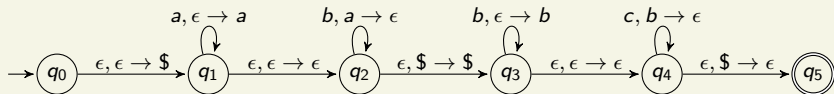
$\rightarrow (q_2, bbcc, \$)$

$\rightarrow (q_3, bbcc, \$)$

$\rightarrow (q_3, bcc, b\$)$

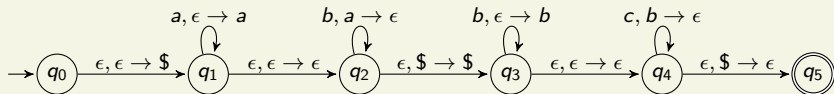
$\rightarrow (q_3, cc, bb\$)$





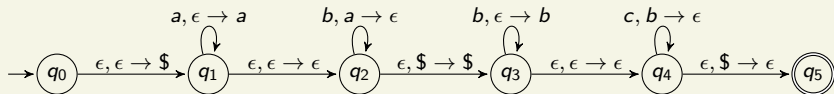
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$



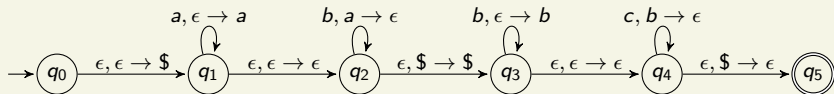
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$



Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$

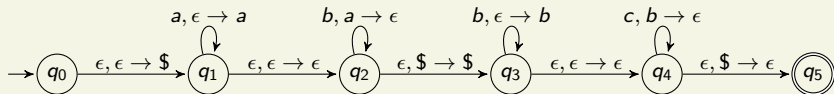


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$

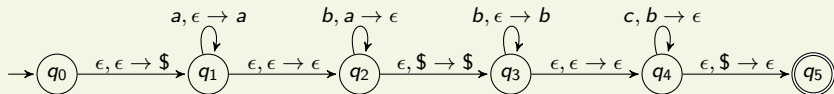


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbbc, \$)$

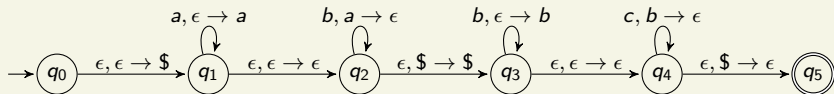


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbbc, \$)$   
 $\rightarrow (q_1, abbbbc, a\$)$

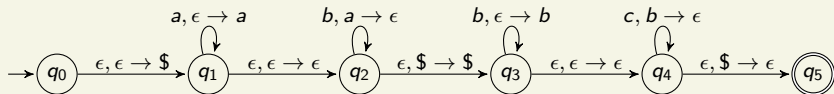


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbbc, \$)$   
 $\rightarrow (q_1, abbbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$



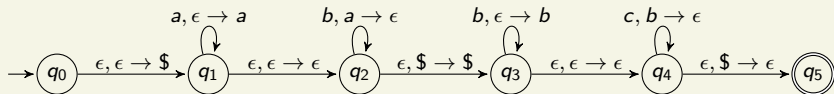
Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbbc, \$)$   
 $\rightarrow (q_1, abbbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$



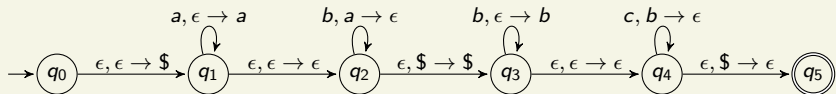


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbc*:

$(q_0, aabbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbc, \$)$   
 $\rightarrow (q_1, abbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$   
 $\rightarrow (q_2, bbc, a\$)$

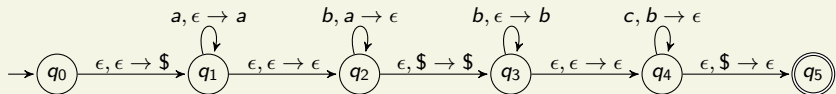


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbbc, \$)$   
 $\rightarrow (q_1, abbbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$   
 $\rightarrow (q_2, bbc, a\$)$   
 $\rightarrow (q_2, bc, \$)$

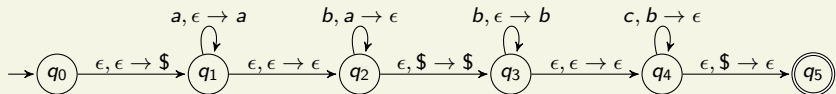


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbbc, \$)$   
 $\rightarrow (q_1, abbbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$   
 $\rightarrow (q_2, bbc, a\$)$   
 $\rightarrow (q_2, bc, \$)$   
 $\rightarrow (q_3, bc, \$)$

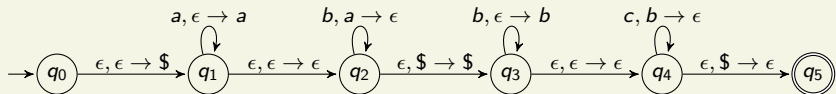


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbc*:

$(q_0, aabbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbc, \$)$   
 $\rightarrow (q_1, abbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$   
 $\rightarrow (q_2, bbc, a\$)$   
 $\rightarrow (q_2, bc, \$)$   
 $\rightarrow (q_3, bc, \$)$   
 $\rightarrow (q_3, c, b\$)$

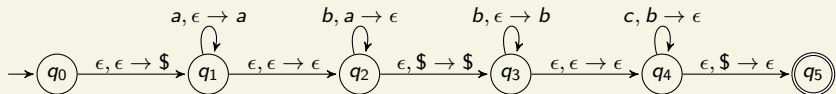


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbc*:

$(q_0, aabbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbc, \$)$   
 $\rightarrow (q_1, abbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$   
 $\rightarrow (q_2, bbc, a\$)$   
 $\rightarrow (q_2, bc, \$)$   
 $\rightarrow (q_3, bc, \$)$   
 $\rightarrow (q_3, c, b\$)$   
 $\rightarrow (q_4, c, b\$)$

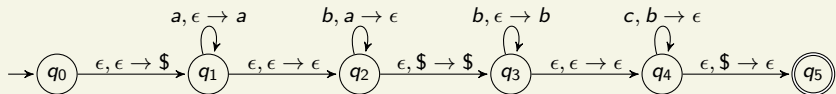


Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbc*:

$(q_0, aabbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbc, \$)$   
 $\rightarrow (q_1, abbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$   
 $\rightarrow (q_2, bbc, a\$)$   
 $\rightarrow (q_2, bc, \$)$   
 $\rightarrow (q_3, bc, \$)$   
 $\rightarrow (q_3, c, b\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$



Computation on *aabbbbcc*:

$(q_0, aabbbbcc, \epsilon)$   
 $\rightarrow (q_1, aabbbbcc, \$)$   
 $\rightarrow (q_1, abbbbcc, a\$)$   
 $\rightarrow (q_1, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbbcc, aa\$)$   
 $\rightarrow (q_2, bbbcc, a\$)$   
 $\rightarrow (q_2, bbcc, \$)$   
 $\rightarrow (q_3, bbcc, \$)$   
 $\rightarrow (q_3, bcc, b\$)$   
 $\rightarrow (q_3, cc, bb\$)$   
 $\rightarrow (q_4, cc, bb\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$

Computation on *aabbbbc*:

$(q_0, aabbbbc, \epsilon)$   
 $\rightarrow (q_1, aabbbbc, \$)$   
 $\rightarrow (q_1, abbbbc, a\$)$   
 $\rightarrow (q_1, bbbc, aa\$)$   
 $\rightarrow (q_2, bbbc, aa\$)$   
 $\rightarrow (q_2, bbc, a\$)$   
 $\rightarrow (q_2, bc, \$)$   
 $\rightarrow (q_3, bc, \$)$   
 $\rightarrow (q_3, c, b\$)$   
 $\rightarrow (q_4, c, b\$)$   
 $\rightarrow (q_4, \epsilon, \$)$   
 $\rightarrow (q_5, \epsilon, \epsilon)$