2021 IM²C
Problem: Who is the Greatest? Maradona or Pelé? Biles or Khorkina?

We read all the time in the sports pages about an athlete being called the **G.O.A.T.** - the *Greatest Of All Time*. What does that really mean and how can that truly be determined?

For the purpose of this IM²C problem, we consider two types of "**sports**" – and, we allow "sports" to be defined broadly.

1. Individual Sports. An individual person competes against one or more other players either **"one-on-one"** or against an **inanimate** standard (highest or lowest ranking or score or time measure). Competitors do not necessarily physically interact with each other (e.g. golf, marathon running, swimming, chess, and table tennis), but they could interact physically with their opponent (e.g. boxing, wrestling). Winners in these sports competitions are individuals and not teams.

2. Team Sports. A group of individuals competes against another group of individuals and the competition includes multiple interactions between any and all players physically, strategically, or through equipment. Examples of team sports include basketball, hockey, American football, International football (soccer), and water polo. Winners in team sports competitions are teams. Although team sports incorporate a variety of position players, individuals, like Michael Jordan in basketball, may stand out.

How is the *greatest* determined? Sometimes we judge sports figures based on an accumulation of records and results over several years such as Palmer or Nicholas in golf, Bonnie Blair in women's speed skating, Ma Long in table tennis, or Tom Brady in American football. Sometimes we call them great because of one athletic feat such as Bob Beamon's world record long jump in the 1968 Olympics or Nadia Comaneci's perfect 10s in gymnastics during the 1976 Olympics. Some athletes come up at a time of rich competition or establish a famous rivalry like Evert and Navratilova in women's tennis or Ali and Frasier in boxing.

Top Sport, a sports network, has hired your team to consider models to measure "greatness." Your first assignment is to develop a model for individual sports and use your model to determine the "G.O.A.T." of one individual sport.

Given that there may be different divisions in sports having restrictions for membership, such as separate competitions for men and women, or various weight classes in boxing, wrestling, and weight lifting, you should consider each of these divisions a single sport (e.g. women's gymnastics, men's gymnastics, featherweight boxing, lightweight boxing) each able to have its own G.O.A.T.

Requirements

1. Warm Up. Consider the individual sport of singles women's tennis over the period of one year in 2018. The best women's tennis players play in the four Grand Slam tournaments (Australian Open, French Open, Wimbledon, and US Open). We provide results of these tournaments on pages 4-6 (2018 Grand Slam Results). Who among these players was the greatest?

> a. Develop a mathematical model for determining the greatest woman tennis player in 2018 on the basis of Grand Slam tournament results. Discuss your choice of factors/variables and the development of your model.

> b. Use your model to choose the greatest woman tennis player of 2018. Analyze your result.

2. Finding the G.O.A.T. of any Individual Sport. Note that in #1 you only looked at one year of a particular individual sport. Now, consider determining the *Greatest Of All Time* (G.O.A.T.) of ANY individual sport.

> a. Choose one example of a individual sport (other than Women's Tennis), and develop a mathematical model (or models) from any factors and data you find significant, measurable, and obtainable for determining the G.O.A.T. in that sport. Analyze your result. Ensure you document any resources used in gathering data and information about your sport.

> b. Discuss any changes your G.O.A.T. model from #2.a. would require to determine the G.O.A.T. of any individual sport. You DO NOT need to develop a new model, but address and explain how models for other individual sports would differ.

3. Extending Your G.O.A.T Model from #2. Now think about team sports.
Discuss any changes your G.O.A.T. models from #2 would require to determine the G.O.A.T. of a team sport. You DO NOT need to develop a new model, but address and explain how a model for team sports would differ from your models for individual sports.

4. Letter. Write a one-page letter to the Director of *Top Sport* describing your team's model and your example of the G.O.A.T. for your selected individual sport. The Director is an executive who understands little about math modeling and science, but is interested in general principles and your key findings.

Note that IM^{2}C is aware of available resources and references that address and discuss this question. It is not sufficient to simply re-present any of these models or discussions, even if properly cited. Any successful paper MUST include development and analysis of your own team's model and clearly explaining the difference between your model and any referenced existing ranking system.

Your submission should consist of:
- One-page Summary Sheet
- Table of Contents
- One page letter to the Director of *Top Sport*.
- Your solution of no more than 20 pages (A4 or letter size), for a maximum of 23 pages with your summary, table of contents, and letter. Note that your font must be no smaller than 12-point type.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.

Glossary

G.O.A.T. – An abbreviation for *The Greatest of All Time* referring to the person considered the best ever to compete, perform, or participate in a specific sport or activity.

Inanimate – A description of something that is not alive.

One-on-One – Playing directly against a single opposing player.

Sport – An activity usually undertaken competitively, governed by a set of rules or customs, that involves particular skills or physical exertion.

IM²C Attachment: 2018 Grand Slam Results

All four Grand Slam Tournaments start the first round with 128 women competitors. Each tournament **seeds (ranks)** the top 32 players. Other players are **unseeded**. The partial brackets in this document show the Fourth through Final Rounds of all four 2018 Grand Slam Tournaments. In each match, women play to win two out of three sets. If one player wins the first two sets, the match is over and that player wins. To win a set, the player must win at least 6 games AND also win by at least 2 games (e.g. 7-5, 6-4, 6-3). If a set reaches 6-6 (no player winning by 2 games), the set goes to a tiebreaker. The player winning at least 7 points AND also winning by 2 points wins the tiebreaker. In some cases, a player may win by default because her opponent is unable to play (called a walkover, w/o) or her opponent retires (r) during a match due to illness or injury.

The following examples illustrate the scoring.

Example 1: Bertens (seeded #20 in the tournament) won the first set 6 games to 3 games. Gorges (seeded #13 in the tournament) won the second set 7 games to 5 games. She had to win a 7th game in order to win by 2 games. Because players had won 1 set each, they played a 3rd set and Gorges won the 3rd set 6 games to 1 game. Gorges won two out of the three sets and so won the match.

Seed/Rank	Player	Set 1	Set 2	Set 3
20	K. Bertens	6	5	1
13	J. Gorges	3	7	6

Example 2: Kerber (seeded #11 in this tournament) played unseeded Bencic and won the first set 6 games to 3 games. In the second set the two players were tied at 6 games each and had to play a tiebreaker. Kerber won the tiebreaker 7 points to 5 points and so won the set 7 to 6. Because Kerber won the first 2 sets, no 3rd set was needed and Kerber won the match.

Seed/Rank	Player	Set 1	Set 2	Set 3
11	A. Kerber	6	7^7	
	B. Bencic	3	6^5	

Seed/Rank: You might notice that some players have various entries in the Seed/Rank box. Examples:

11	Seed/Rank – this player is seeded 11th in the tournament (32 of these per tournament).
PR	Protected Ranking – this player was inactive recently due to injury or returning from pregnancy leave, but qualified for the tournament based on her previous world ranking.
25/PR	This player qualified by her previous world ranking, and is such a highly ranked player that she also was seeded 25th in the tournament.
Q	Qualifier – this player qualified through a qualification event (16 of these players per tournament).
	Blank – this player's was among the top 104 players signed up for the tournament, but does not fall into any of the above categories. This player is not "seeded" in the tournament.

2018 Wimbledon Championships – Women's Singles

Fourth Round

	S-w Hsieh	4	1	
	D Cibulková	6	6	

12	J Ostapenko	7⁷	6	
	A Sasnovich	6⁴	0	

	A Van Uytvanck	7⁸	3	2
14	D Kasatkina	6⁶	6	6

11	A Kerber	6	7⁷	
	B Bencic	3	6⁵	

7	Ka Plíšková	3	6¹	
20	K Bertens	6	7⁷	

13	J Görges	6	6	
	D Vekić	3	2	

25/PR	S Williams		6	6
Q	E Rodina		2	2

	C Giorgi	6	6	
	E Makarova	3	4	

Quarterfinals

	Dominika Cibulková	5	4	
12	Jeļena Ostapenko	7	6	

14	Daria Kasatkina	3	5	
11	Angelique Kerber	6	7	

20	Kiki Bertens	6	5	1
13	Julia Görges	3	7	6

25/PR	Serena Williams	3	6	6
	Camila Giorgi	6	3	4

Semifinals

12	Jeļena Ostapenko	3	3	
11	Angelique Kerber	6	6	

13	Julia Görges	2	4	
25/PR	Serena Williams	6	6	

Final

11	Angelique Kerber	6	6	
25/PR	Serena Williams	3	3	

2018 French Open – Women's Singles

Fourth Round

1	S Halep	6	6	
16	E Mertens	2	1	

12	A Kerber	6	6	
7	C Garcia	2	3	

3	G Muguruza	2		
	L Tsurenko	0ʳ		

PR	S Williams			
28	M Sharapova	w/o		

26	B Strýcová	4	3	
	Y Putintseva	6	6	

13	M Keys	6	6	
31	M Buzărnescu	1	4	

25	A Kontaveit	2	0	
10	S Stephens	6	6	

14	D Kasatkina	7⁷	6	
2	C Wozniacki	6⁵	3	

Quarterfinals

1	Simona Halep	6²	6	6
12	Angelique Kerber	7⁷	3	2

3	Garbiñe Muguruza	6	6	
28	Maria Sharapova	2	1	

	Yulia Putintseva	6⁵	4	
13	Madison Keys	7⁷	6	

10	Sloane Stephens	6	6	
14	Daria Kasatkina	3	1	

Semifinals

1	Simona Halep	6	6	
3	Garbiñe Muguruza	1	4	

13	Madison Keys	4	4	
10	Sloane Stephens	6	6	

Final

1	Simona Halep	3	6	6
10	Sloane Stephens	6	4	1

2018 US Open – Women's Singles

Fourth Round

	K Kanepi	0	6	3
17	S Williams	6	4	6
18	A Barty	4	4	
8	Ka Plíšková	6	6	
3	S Stephens	6	6	
15	E Mertens	3	3	
19	A Sevastova	6	1	6
7	E Svitolina	3	6	0
30	C Suárez Navarro	6	6	
22	M Sharapova	4	3	
14	M Keys	6	6	
29	D Cibulková	1	3	
26	A Sabalenka	3	6	4
20	N Osaka	6	2	6
	M Vondroušová	7^7	5	2
	L Tsurenko	6^3	7	6

Quarterfinals

17	Serena Williams	6	6
8	Karolína Plíšková	4	3
3	Sloane Stephens	2	3
19	Anastasija Sevastova	6	6
30	Carla Suárez Navarro	4	3
14	Madison Keys	6	6
20	Naomi Osaka	6	6
	Lesia Tsurenko	1	1

Semifinals

17	Serena Williams	6	6
19	Anastasija Sevastova	3	0
14	Madison Keys	2	4
20	Naomi Osaka	6	6

Final

17	Serena Williams	2	4
20	Naomi Osaka	6	6

2018 Australian Open – Women's Singles

Fourth Round

1	S Halep	6	6	
	N Osaka	3	2	
20	B Strýcová	7^7	3	2
6	Ka Plíšková	6^5	6	6
	S-w Hsieh	6	5	2
21	A Kerber	4	7	6
17	M Keys	6	6	
8	C Garcia	3	2	
	P Martić	6^5	5	
	E Mertens	7^7	7	
Q	D Allertová	3	0	
4	E Svitolina	6	6	
32	A Kontaveit	6	4	6
	C Suárez Navarro	4	6	8
19	M Rybáriková	3	0	
2	C Wozniacki	6	6	

Quarterfinals

1	Simona Halep	6	6	
6	Karolína Plíšková	3	2	
21	Angelique Kerber	6	6	
17	Madison Keys	1	2	
	Elise Mertens	6	6	
4	Elina Svitolina	4	0	
	Carla Suárez Navarro	0	7^7	2
2	Caroline Wozniacki	6	6^3	6

Semifinals

1	Simona Halep	6	4	9
21	Angelique Kerber	3	6	7
	Elise Mertens	3	6^2	
2	Caroline Wozniacki	6	7^7	

Final

1	Simona Halep	6^2	6	4
2	Caroline Wozniacki	7^7	3	6