FacetSizeVsDetectionDepth

February 17, 2026

1 How deep does a target need to be for a 100 % probability of detection?

[1]:
```python
import numpy as np
import matplotlib.pyplot as plt
```

1.1 Method 1: Area Only

In this method we assume that the probability of detection is 1 when the area where the target would be illuminated exceeds that of the area where a specific facet is the closest.

Assuming a square grid of facets, the area where a given facet is closest can be described as follows, where l is the side length of the facet:

$$A_f = l^2$$

Then, the area where the target would be illuminated can be described as follows:

$$A_t = \pi(d\sin\theta)^2$$

Where d and θ are target depth and the target "specular" beam width.

So the probability of detection here can be written as:

$$P = \frac{A_t}{A_f} = \frac{\pi(d\sin\theta)^2}{l^2}$$

So for a probability of 1, the depth of the target must be:

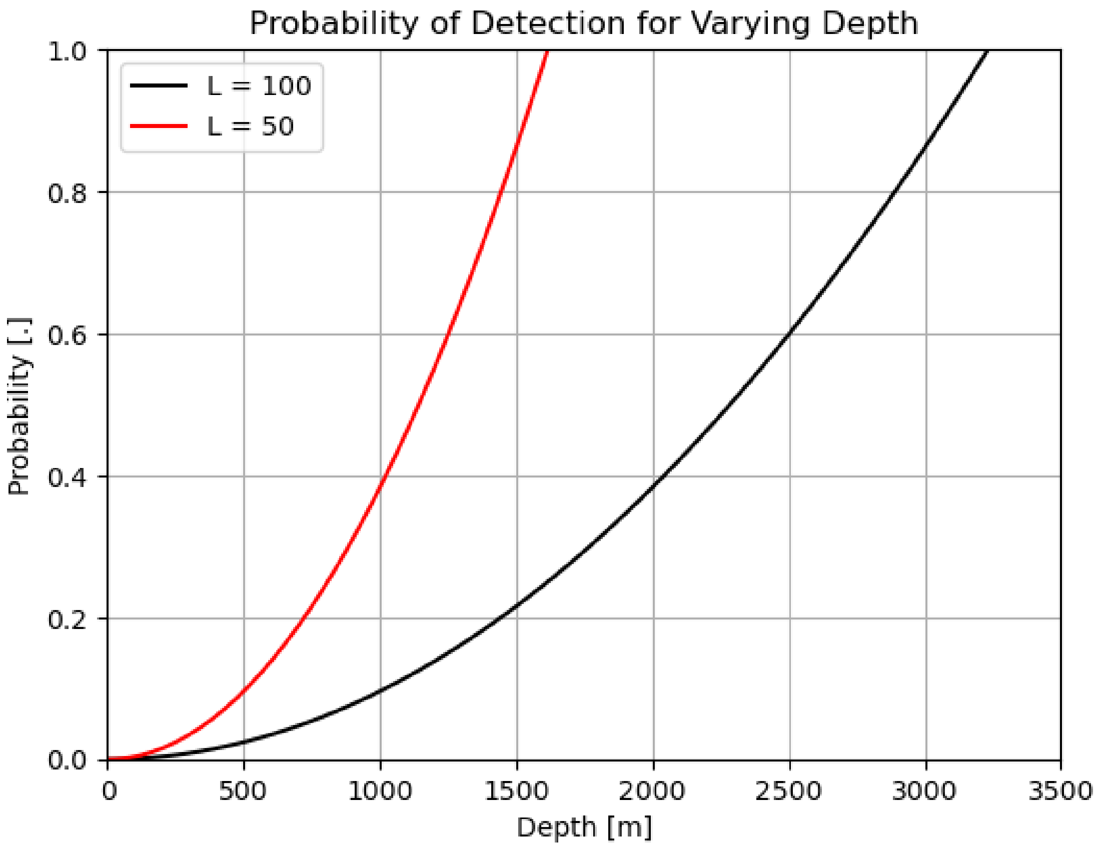
$$d = \frac{l}{\sin\theta\sqrt{\pi}}$$

1.1.1 1a. How does probability of detection change with depth?

[2]:
```python
# first lets do an example with the probability of detection
theta = np.radians(1)             # target beamwidth is 1 degree
l1    = 100                       # facet size [m]
l2    = 50                        # facet size [m]
d     = np.linspace(0, 4e3, 1000) # target depth [m]
```

[3]:
```
P1 = (np.pi * (d * np.sin(theta))**2) / l1**2
P2 = (np.pi * (d * np.sin(theta))**2) / l2**2
```

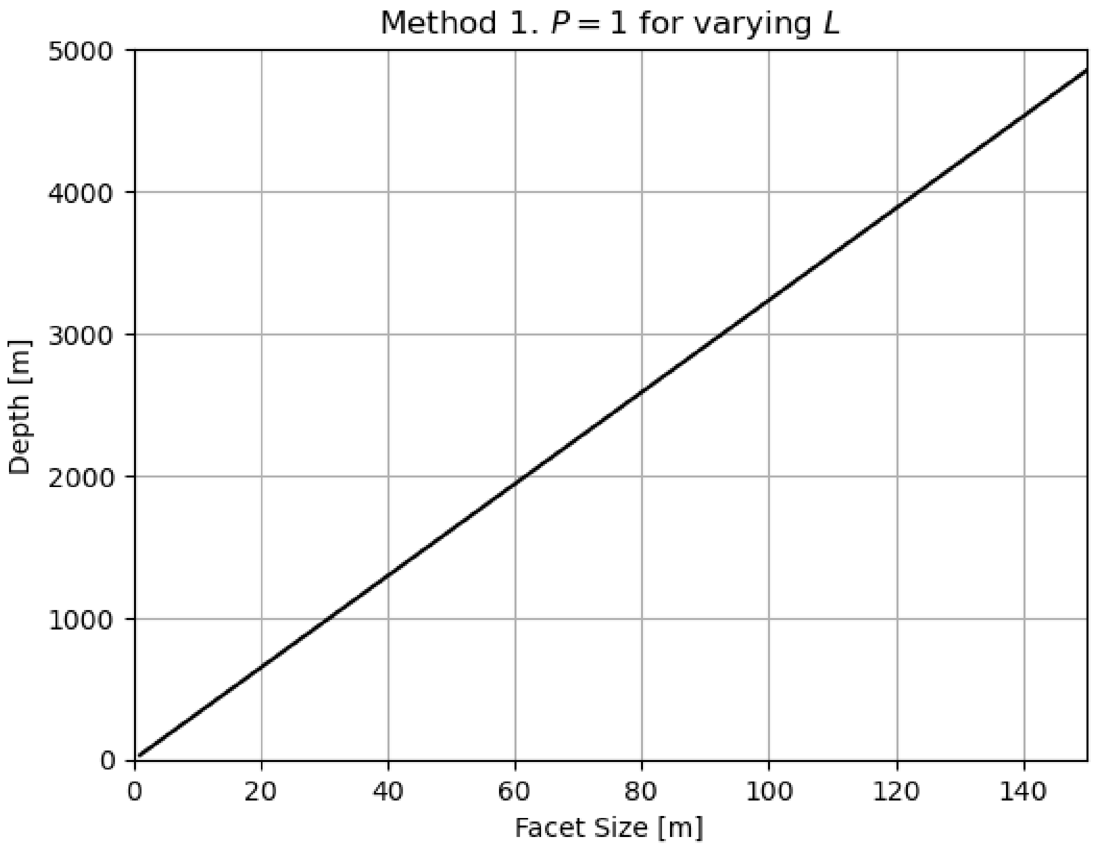
[4]:
```
plt.plot(d, P1, color="black", label=f"L = {l1}")
plt.plot(d, P2, color="red", label=f"L = {l2}")
plt.ylim(0, 1)
plt.xlim(0, 3500)
plt.legend()
plt.title("Probability of Detection for Varying Depth")
plt.ylabel("Probability [.]")
plt.xlabel("Depth [m]")
plt.grid()
plt.show()
```



1.1.2 1b. When is detection guaranteed? $(P = 1)$

[5]:
```
l      = np.linspace(1, 150, 500) # facet size [m]
depth1 = l / (np.sin(theta) * np.sqrt(np.pi))
```

```
[6]: plt.plot(l, depth1, color="black")
     plt.xlim(0, 150)
     plt.ylim(0, 5000)
     plt.title("Method 1. $P=1$ for varying $L$")
     plt.ylabel("Depth [m]")
     plt.xlabel("Facet Size [m]")
     plt.grid()
     plt.show()
```



Unfortunately this method is relaxed, ignoring the corners which are still not considered even when $P = 1$ using this method.

1.2 Method 2: True Guaranteed Detection

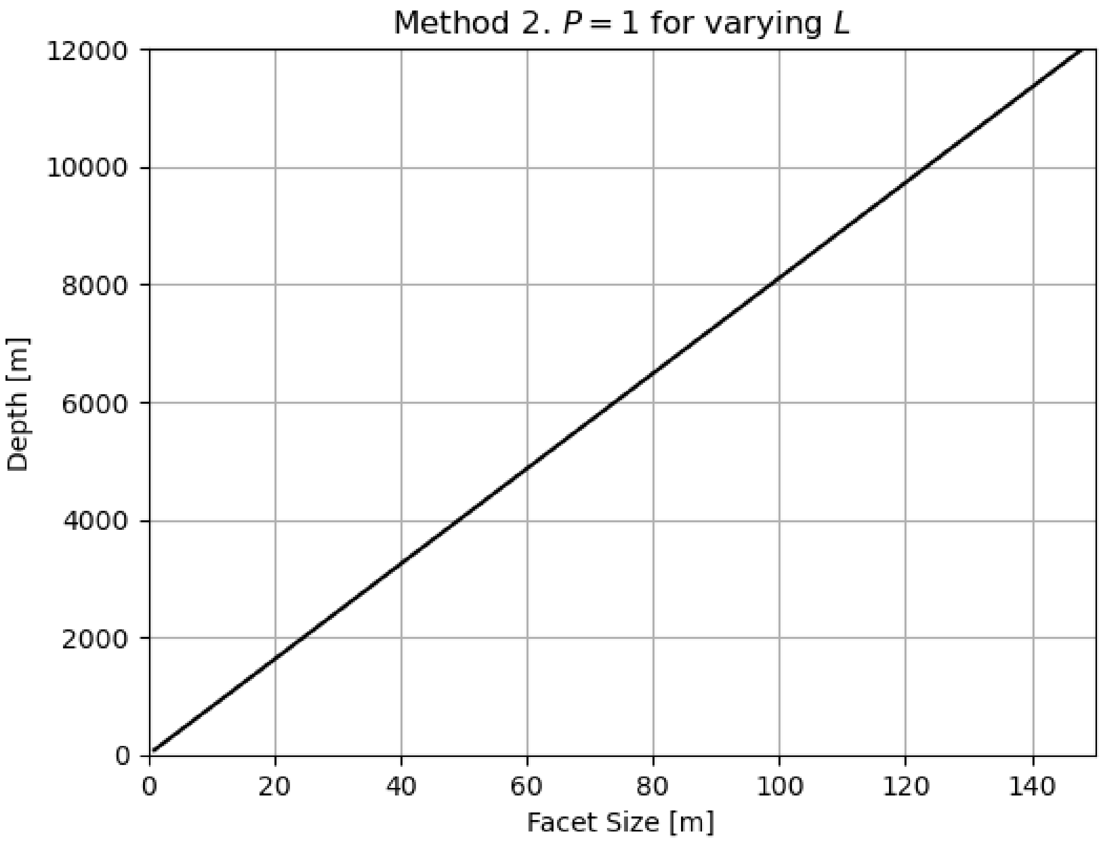
This method corrects for the flaws of the previous one by saying that $P = 1$ when the radius of the circle of illumination for a given depth is greater than the diagonal. We can solve for the depth at which detection is guaranteed as follows:

$$d \sin\theta = l\sqrt{2}$$

$$d = \frac{l\sqrt{2}}{\sin\theta}$$

[7]:
```
l      = np.linspace(1, 150, 500) # facet size [m]
depth2 = (l * np.sqrt(2)) / np.sin(theta)
```

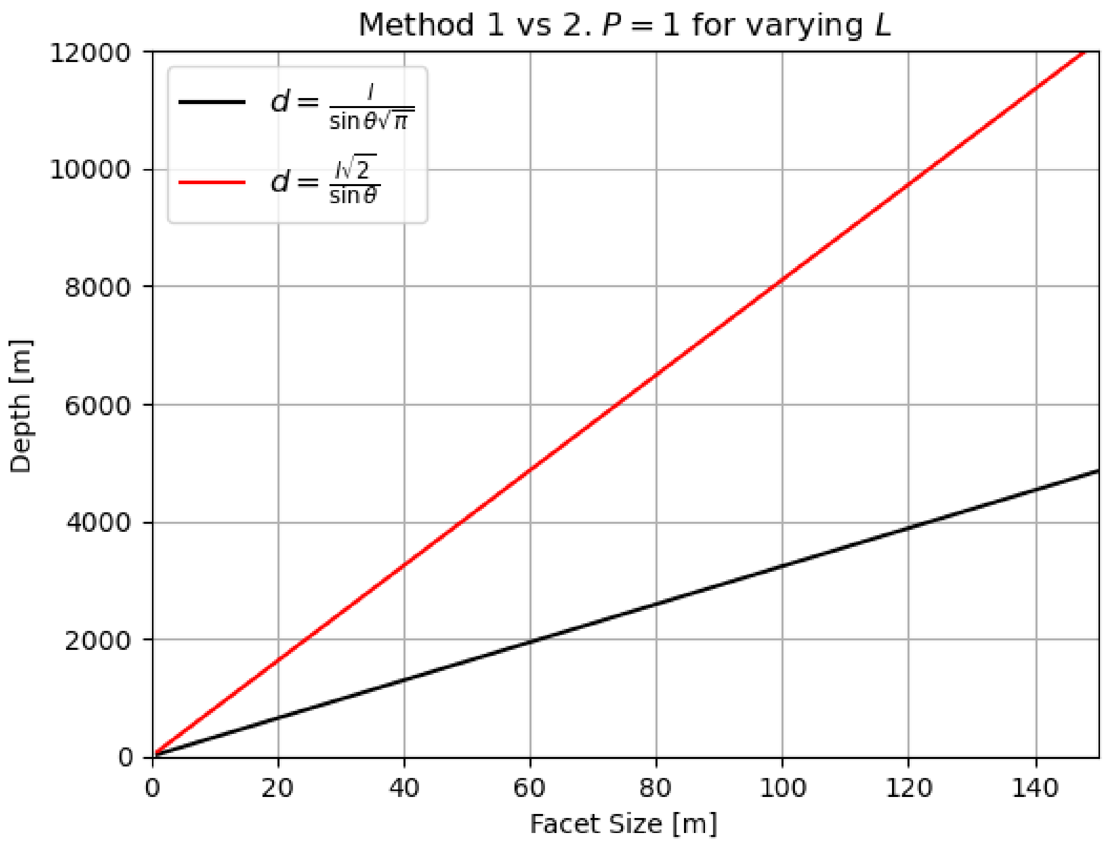
[8]:
```
plt.plot(l, depth2, color="black")
plt.xlim(0, 150)
plt.ylim(0, 12000)
plt.title("Method 2. $P=1$ for varying $L$")
plt.ylabel("Depth [m]")
plt.xlabel("Facet Size [m]")
plt.grid()
plt.show()
```



1.3 Comparison of Method 1 vs 2:

Rate of depth increase per facet size:
Method 1: 32.32
Method 2: 81.03

```
[9]: plt.plot(l, depth1, color="black", label=r"$d=\frac{l}{\sin\theta\sqrt{\pi}}$")
     plt.plot(l, depth2, color="red", label=r"$d=\frac{l\sqrt{2}}{\sin\theta}$")
     plt.xlim(0, 150)
     plt.ylim(0, 12000)
     plt.title("Method 1 vs 2. $P=1$ for varying $L$")
     plt.ylabel("Depth [m]")
     plt.xlabel("Facet Size [m]")
     plt.legend(fontsize=12)
     plt.grid()
     plt.show()
```

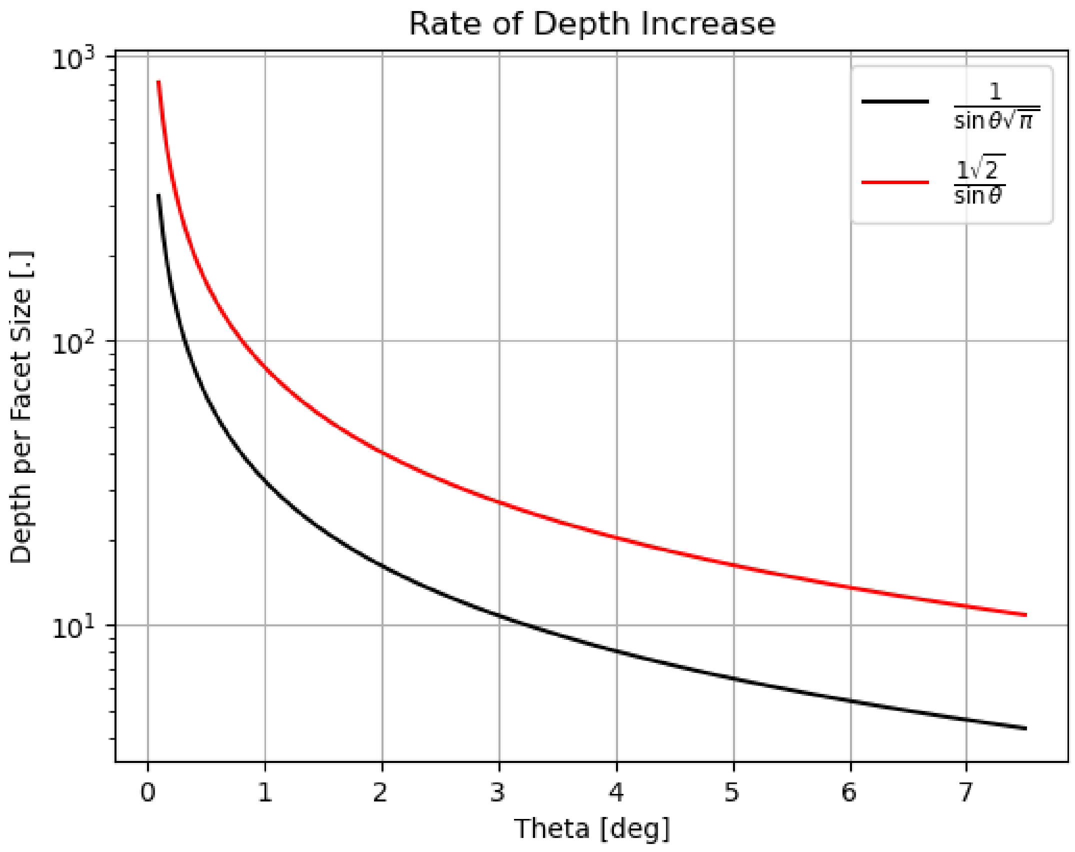


1.4 What is the rate of depth increase for each facet size in each method?

```
[14]: thetas = np.radians(np.linspace(0.1, 7.5, 200))
      rate1  = 1 / (np.sin(thetas) * np.sqrt(np.pi))
      rate2  = np.sqrt(2) / np.sin(thetas)
```

```
[19]: plt.plot(np.degrees(thetas), rate1, color="black",␣
       ↪label=r"$\frac{1}{\sin\theta\sqrt{\pi}}$")
```

```
plt.plot(np.degrees(thetas), rate2, color="red",␣
  ↪label=r"$\frac{1\sqrt{2}}{\sin\theta}$")
plt.title("Rate of Depth Increase")
plt.ylabel("Depth per Facet Size [.]")
plt.xlabel("Theta [deg]")
plt.yscale("log")
plt.legend(fontsize=12)
plt.grid()
plt.show()
```



```
[ ]:
```