The production of $a_0(980)$

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1. $\eta\pi$ scattering

The elastic scattering amplitude is

$$f_{el} = -\frac{1}{2q_\eta}\frac{\Gamma_P}{E - E_{BW} + \frac{i\Gamma_P}{2} + i\bar{g}_K\frac{k}{2}} = -\frac{1}{2}\frac{\bar{g}_\eta}{E - E_{BW} + \frac{i\Gamma_P}{2} + i\bar{g}_K\frac{k}{2}}. \tag{1}$$

For $E > 0$, the elastic cross section is

$$\sigma_{el} = 4\pi|f_{el}|^2 = \frac{\pi\bar{g}_\eta^2}{|E - E_{BW}|^2 + \frac{(\Gamma_P + \bar{g}_K k)^2}{4}}. \tag{2}$$

For $E < 0$, $k = i\kappa$, so that

$$\sigma_{el} = \frac{\pi\bar{g}_\eta^2}{|E - E_{BW} - \frac{\bar{g}_K\kappa}{2}|^2 + \frac{\Gamma_P^2}{4}}. \tag{3}$$

Neglecting terms which are of higher order than k, the σ_{el} in $E > 0$ can be approximated by

$$\begin{aligned}\sigma_{el} &\simeq \frac{\pi\bar{g}_\eta^2}{E_{BW}^2 + \frac{(\Gamma_P + \bar{g}_K k)^2}{4}} \\ &= \frac{4\pi\Gamma_P^2}{q_\eta^2[4E_{BW}^2 + (\Gamma_P + \bar{g}_K k)^2]} \\ &= \frac{4\pi}{q_\eta^2[\left(\frac{2E_{BW}}{\Gamma_P}\right)^2 + (1 + \frac{\bar{g}_K k}{\bar{g}_\eta q_\eta})^2]} \\ &= \frac{4\pi}{q_\eta^2}\frac{1}{\alpha^2 + (1 + Rk/q_\eta)^2}.\end{aligned} \tag{4}$$

Similarly, for $E < 0$, the σ_{el} can be approximated by

$$
\begin{aligned}
\sigma_{el} &\simeq \frac{\pi\Gamma_P^2}{q_\eta^2[(E_{BW}+\frac{\bar{g}_K\kappa}{2})^2+\frac{\Gamma_P^2}{4}]}\\
&= \frac{4\pi}{q_\eta^2[\left(\frac{2E_{BW}}{\Gamma_P}+\frac{\bar{g}_K\kappa}{\bar{g}_\eta q_\eta}\right)^2+1]}\\
&= \frac{4\pi}{q_\eta^2}\frac{1}{1+(\alpha+R\kappa/q_\eta)^2}. \qquad (5)
\end{aligned}
$$

Therefore, the line-shape of elastic channel is only dependent on $\alpha = 2E_{BW}/\Gamma_P$ and $R = \bar{g}_K/\bar{g}_\eta$.

2. $K\bar{K}$ scattering

The elastic scattering amplitude of $K\bar{K} \to K\bar{K}$ is

$$
f_{el} = -\frac{1}{2k}\frac{\Gamma_K}{E-E_{BW}+\frac{i\Gamma_P}{2}+i\bar{g}_K\frac{k}{2}} = -\frac{1}{2}\frac{\bar{g}_K}{E-E_{BW}+\frac{i\Gamma_P}{2}+i\bar{g}_K\frac{k}{2}}. \qquad (6)
$$

For $E > 0$, the elastic cross section is

$$
\begin{aligned}
\sigma_{el} &= \frac{\pi\bar{g}_K^2}{|E-E_{BW}|^2+\frac{(\Gamma_P+\bar{g}_K k)^2}{4}}\\
&\simeq \frac{4\pi\bar{g}_K^2}{4E_{BW}^2+(\Gamma_P+\bar{g}_K k)^2}\\
&= \frac{4\pi}{\left(\frac{2E_{BW}}{\bar{g}_K}\right)^2+\left(k+\frac{\Gamma_P}{\bar{g}_K}\right)^2}. \qquad (7)
\end{aligned}
$$

The line-shape is dependent only on $\Gamma_P/\bar{g}_K$ and $2E_{BW}/\bar{g}_K$.

$X(3872)$ in $\underline{D^0\bar{D}^0}\pi^0$ channel

The propagator of $X_(3872)$ reads

$$\frac{1}{E - E_X + \frac{igk}{2} + \frac{i\Gamma_0}{2}}, \; k = \sqrt{2\mu E}. \tag{8}$$

The $X(3872)$ is produced by its short range core, so that the differential cross section is proportional to g, but not g^2, i.e.

$$\begin{aligned}\frac{d\sigma}{dE} &\sim \frac{gk}{(E-E_X)^2 + \frac{(gk+\Gamma_0)^2}{4}} \\ &\simeq \frac{k}{\frac{E_x^2}{g} + \frac{g}{4}(k + \frac{\Gamma_0}{g})^2}.\end{aligned} \tag{9}$$

If charged $D^*\bar{D}$ channel is considered. the inverse propagator reads

$$E - E_X + \frac{igk}{2} + \frac{i\Gamma_0}{2} + \frac{ig}{2}\sqrt{2\mu(E-\delta)}. \tag{10}$$

Since $\delta \simeq 7\text{MeV} \gg E$,

$$\sqrt{2\mu(E-\delta)} = i\sqrt{2\mu(\delta - E)} \simeq i\sqrt{2\mu\delta}(1 - \frac{E}{2\delta}). \tag{11}$$

Therefore the inverse propagator becomes

$$(1 + \frac{g\sqrt{2\mu\delta}}{4\delta})E - (E_X - \frac{g\sqrt{2\mu\delta}}{2}) + \frac{igk}{2} + \frac{i\Gamma_0}{2} \tag{12}$$

The differential cross section can be approximated by

$$\frac{d\sigma'}{dE} \simeq \frac{k}{\frac{(E_X - \frac{g\sqrt{2\mu\delta}}{2})^2}{g} + \frac{g}{4}(k + \frac{\Gamma_0}{g})^2}. \tag{13}$$

In both cases, the expression includes g explicitly, so that the line-shape does not process scaling invariance. However, the $D^*\bar{D}$ scattering amplitude t_{22} does process scaling invariance:

$$t_{22} = \frac{1}{8\pi^2\mu_p}\frac{g}{E - E_X + \frac{igk}{2} + \frac{i\Gamma_0}{2}}. \tag{14}$$

The cross section of $D^*\bar{D} \to D^*\bar{D}$ reads

$$\sigma \sim |t_{22}|^2 \sim \frac{g^2}{(E-E_X)^2 + \frac{(gk+\Gamma_0)^2}{4}} \sim \frac{4}{(\frac{2E_X}{g})^2 + (k + \frac{\Gamma_0}{g})^2}. \tag{15}$$