It is argued that the branch cut associated with the open channels $\Gamma(E)$, e,g, $X(3872) \to J/\psi\rho \to J/\psi\pi\pi$, cannot be chosen to lie along the negative real axis for the following reasons:

The scattering amplitude satisfies the relation

$$f(E^*) = f^*(E), \tag{1}$$

which leads to

$$Disc[f(E)] = f(E+i\epsilon) - f(E-i\epsilon) = 2i\mathrm{Im}\left\{f(E+i\epsilon)\right\}. \tag{2}$$

Further more, based on the Optical theorem, $\mathrm{Im}\left\{f(E+i\epsilon)\right\} > 0$*, when* $E > 0$ *, which meas that there must be the non-vanishing discontinuity when E is above the branch point, i.e. the branch cut must be taken to go from the threshold toward the larger energy E.*

However, this statement is on the contrary to the arbitrariness of the choice of branch cut in the course of complex analysis, and must be clarified.

The key point is that :

The relation (1) is not a necessary condition for scattering amplitude. Whether the relation (1) is true or not in the domain of $f(E)$ depends on the choice of branch cut. This is a common phenomenon in complex analysis.

To illustrate this, we take the following complex functions as an example:

$$f_a(E) = \sqrt{-E-i\epsilon} = \begin{cases} -i\sqrt{E} & ,E>0 \\ \sqrt{|E|} & ,E<0 \end{cases} \tag{3}$$

$$f_a(E^*) = \sqrt{-E+i\epsilon} = \begin{cases} i\sqrt{E} & ,E>0 \\ \sqrt{|E|} & ,E<0 \end{cases} \tag{4}$$

For f_a , the branch cut is along the positive real axis, and the relation (1) is true for f_a. However, if the branch cut of f_a is turned to lie along the negative real axis, it becomes f_b , i.e.

$$f_b(E) = -i\sqrt{E+i\epsilon} = \begin{cases} -i\sqrt{E} & ,E>0 \\ \sqrt{|E|} & ,E<0 \end{cases}. \tag{5}$$

f_b is equivalent to f_a on the real axis, but (1) is not true for f_b, i.e.

$$f_b(E^*) = -i\sqrt{E-i\epsilon} = \begin{cases} -i\sqrt{E} & ,E>0 \\ -\sqrt{|E|} & ,E<0 \end{cases} \Rightarrow f_b(E^*) \neq f_b^*(E). \tag{6}$$

Apart from this, $\ln E$ is also a good example.

Therefore, it is still free to choose the branch cut without breaking unitarity. Namely, the unitarity requires that the imaginary part of $f(E)$ is non-zero above the threshold, but an improper choice of branch cut may makes the amplitude continuous across the real axis above the threshold.

As a natural consequence, in the decay of $X(3872)$, the branch cuts associated with channels other than $D^*\bar{D}$ can be turned to lie along the negative real axis. If so, only the unitary cut of $D^*\bar{D}$ channel exists on the right hand side.