MOSHOOD ABIOLA POLYTECHNIC
ABEOKUTA. OGUN STATE.
DEPARTMENT OF STATISTICS & MATHEMATICS

1ˢᵗ SEMESTER EXAMINATION
2022/2023 SESSION

COURSE TITTLE: LOGIC AND LINEAR ALGEBRA
COURSE CODE: MTH 111
LEVEL: ND I   COMPUTER SCIENCE (Full Time & Part Time)
UNIT: 2
TIME ALLOWED: 2HRs

INSTRUCTIONS: (i) Answer Question 1 and any other THREE (3) questions.
(ii) Write ONLY your matriculation number on your question paper.

1. (a) Logic can best be described as the science of true, good and valid reasoning. Explain.
   (b) Differentiate between simple statements and compound statements.
   (c) Let p be the statement "He is funny" and q be the statement "He is intelligent".
   Write each of the following compound statements in symbolic form using p and q.
   (i) He is not funny but intelligent.
   (ii) Only if he is not intelligent, is he funny.
   (iii) He is not funny if he is intelligent, and he is not funny only if he is intelligent.
   (d) Construct the truth tables for your answers in (c).

2. (a) Differentiate between permutation and combination.
   (b) There are 3 copies of Harry Potter and the Philosopher's Stone, 4 copies of the Lost Symbol 5 copies of the Secret of the Unicorn. In how many ways can you arrange these books on a shelf?
   (c) From 6 boys and 4 girls, a committee of 6 is to be formed. In how many ways can this be done if:
   (i) no restriction is imposed?
   (ii) the committee contains exactly 2 girls?
   (iii) the committee contains at least 2 girls?

3. (a) Mathematical induction is a proof technique that can be applied to establish the veracity of mathematical statements. Explain.
   (b) Prove by induction that:
   $$1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n \cdot 2^n = (n - 1) \cdot 2^{n+1} + 2$$

4. (a) Using the binomial theorem, expand $\left(x - \dfrac{2}{x}\right)^5$.

   (b) Find the middle term in the expansion of $\left(2ax - \dfrac{b}{x^2}\right)^{12}$.

5. The matrices X, Y, Z are given below.

$$X = \begin{bmatrix} 0 & -1 \\ 2 & -1 \end{bmatrix}, \quad Y = \begin{bmatrix} -3 & -2 \\ -1 & 1 \end{bmatrix}, \quad Z = \begin{bmatrix} 3 & -3 \\ 1 & 2 \end{bmatrix}$$

Write out the following matrices.
(a) $2(X + Y)$
(b) $2X + 2Y$
(c) $2Y + Z$
(d) $2(X + Y) + Z$
(e) $2X - 3(Y - Z)$

6. (a) Translate the following compound statements into symbolic notation.
(i) Either the fire was produced by arson or it was produces by spontaneous combustion.
(ii) If the water was clear, then either Janet can see the bottom of the pool or she is a nincompoop.
(iii) Either Seyi is not here or Esther is, and Opeyemi certainly is.
(iv) If there are more cats than dogs, then there are more horses than dogs and there are fewer snakes than cats.
(v) The man in the room is a fake, and if the same is true of Santa Claus, then many children are deceived.
(vi) If either red-heads are lovely or blondes do not have freckles, then logic is confusing.

(b) Suppose $p(x, y, z)$ is the statement "$x^2 + y - 2z = 5$", $q(x, y, z)$ is the statement "$xyz < -3$", and $r(x, y, z)$ is the statement "$(y+z)/x = -1$". Find the truth values for the following statements when $x = -2$, $y = 1$, and $z = 1$.

(i) $\sim r(x, y, z) \lor [p(x, y, z) \leftrightarrow q(x, y, z)]$

(ii) $q(x, y, z) \land [p(x, y, z) \rightarrow r(x, y, z)]$

. (iii) $[\sim p(x, y, z) \rightarrow q(x, y, z)] \lor [r(x, y, z) \land p(x, y, z)]$

(iv) $[\sim q(x, y, z) \leftrightarrow q(x, y, z)] \leftrightarrow [p(x, y, z) \lor \sim r(x, y, z)]$

(v) $[p(x, y, z) \lor r(x, y, z)] \rightarrow [\sim r(x, y, z) \leftrightarrow q(x, y, z)]$