

[3] Estimate The big O for  $T(n)$

$$a. T(n) = \begin{cases} 1 & n=0 \\ 3T(n-1) + n & n>0 \end{cases}$$

Method  $\rightarrow$  using Master theorem (Decreasing Function)

$$T(n) = aT(n-b) + f(n) \rightarrow \text{Form}$$

where  $a > 0$

$b > 0$

$f(n) = O(n^k)$  where  $k \geq 0$

$$\therefore T(n) = 3T(n-1) + n \rightarrow \begin{cases} a=3 \\ b=1 \\ k=1 \end{cases}$$

$$\therefore a > 1 \rightarrow O(a^n b n^k) \rightarrow \boxed{O(3^n * n)}$$

$$b. T(n) = \begin{cases} 1 & n=1 \\ T(n/2) + 1 & n>1 \end{cases}$$

Method  $\rightarrow$  using Master Theorem (dividing Functions)

$$T(n) = aT(n/b) + f(n) \rightarrow \text{where } a \geq 1, b > 1$$

$$f(n) = O(n^k \log^p n)$$

$$\therefore T(n) = T(n/2) + 1 \rightarrow \begin{matrix} a=1 & k=0 \\ b=2 & p=0 \end{matrix}$$

$$\log_b^a = \log_2^1 = 0 \quad \therefore \log_b^a = k \rightarrow \therefore p=0 > -1$$

$$\therefore O(n^k \log^{p+1} n) = \boxed{O(\log n)}$$