

①

## \* Learning Method: Baum Welch algorithm \*

- \* When The label of dataset is available  $\rightarrow$  maximum likelihood
- \* but if The label isn't available How we learn The parameters of The system ?? using a type of Expectation Maximization called ((Baum Welch algorithm))

We will use Baum Welch algorithm To find maximum likelihood estimate of The parameters of hidden markov chain

HMM parameters:  $\lambda = (P, E, \pi)$

$P = \{P_{ij}\} = P(X_t = j | X_{t-1} = i)$  is (Transition matrix)

$\pi = \{\pi_i\} = P(X_1 = i)$  is (The initial state distribution)

$E = \{e_j(o_t)\} = P(O_t = o_t | X_t = j)$  is The emission matrix

$X_t \rightarrow$  hidden state

$O_t \rightarrow$  observable var.

((I have To learn These parameters))

**Problem** :- if we have The corpus not labeled but The word sequence are given

given observations :  $O_1, O_2, \dots, O_T$

The algorithm Tries To find The parameter  $\lambda$  That maximize The probability of The (observation)

$\downarrow$   
optimal

**Aim** :- To find out what are The optimal parameter  $\lambda$  That maximize The prob. of likelihood of The observation

optimal  $\lambda$  :  $\lambda$  الذي يحدّد أقصى احتمال لحدوث الـ sequence

The algorithm idea:-

is to start with some random initial conditions on the parameter  $\lambda$ , estimate best values of state paths  $x_t$  to compute state path probabilities, then re-estimate the parameters  $\lambda$  using the just computed values of  $x_t$  iteratively (until it converges)

use some par.  $\lambda$  to get some likelihood on the hidden states once you have these likelihood or probabilities use that to compute my  $\lambda$

- (intuition) :-
- ① Choose some random initial values for  $\lambda$
  - ② Determine Probable (State) Paths  $x_{t-1}=i, x_t=j$
  - ③ Count the expected number of transitions  $b_{ij}$  as well as the expected number of times various emissions  $b_j(O_t)$
  - Count the number of paths: because we are not finding the actual paths
  - ④ Re-estimate  $\lambda = (P, \pi, E)$  using  $b_{ij}$  and  $b_j(O_t)$ s
  - Repeat ②, ③, ④ until convergence

Forward and backward algo. are used for finding Probable Paths



③

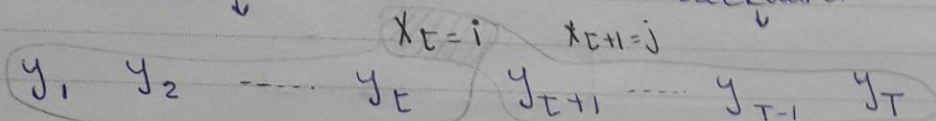
# Forward-Backward algo.

① Forward Procedure:-

② Backward Procedure

Forward  
↓

backward  
↓



The prob. of observing  $y_1, \dots, y_t$  &  $x_t = i$  given  $\lambda$

$\alpha_t^i$

$$P(Y_1=y_1, Y_2=y_2, \dots, Y_t=y_t, x_t=i | \lambda)$$

We want to compute  $\alpha_t^i$  for all possible  $i$ s & all possible values of  $t$

$$\alpha_1^i = P(Y_1=y_1, x_1=i | \lambda)$$

$$\pi_i b_i(y_1)$$

For all possible values

$$\alpha_{t+1}^j = \sum_{i=1}^N \alpha_t^i b_{ij}(y_{t+1})$$

The probability of observing this given  $x_t = i$  "state"

$\beta_t$

$$P(Y_{t+1}=y_{t+1}, \dots, Y_T=y_T | x_t=i, \lambda)$$

$$\beta_T^i = 1$$

end of sequence

• Probability of ending the sequence with  $y_{t+1}$  to  $y_T$  given the state  $i$  and  $\lambda$

(This is my back ward.)

$$\beta_T^i = 1 \text{ (end of the sequence)}$$

$$\beta_t^i = \sum_{j=1}^N \beta_{t+1}^j b_{ij}(y_{t+1})$$

\* We want to calculate these probabilities of various paths

$$P(x_t=i | Y, \lambda)$$

$$P(x_t=i, x_{t+1}=j | Y, \lambda)$$

مقدار  
حساب  
π  
لید نو

Transition Probability

\* Finding probabilities of Paths:-

$$\textcircled{1} \delta_T^i = P(X_T=i | Y, h) = \frac{P(X_T=i, Y | h)}{P(Y | h)} = \frac{P(X_T=i, Y | h)}{P(\sum_{t=1}^T X_t=j | h)}$$

(all possible states at Time)

$$\therefore \delta_T^i = \frac{\alpha_T^i \beta_T^i}{\sum_{j=1}^n \alpha_T^j \beta_T^j}$$

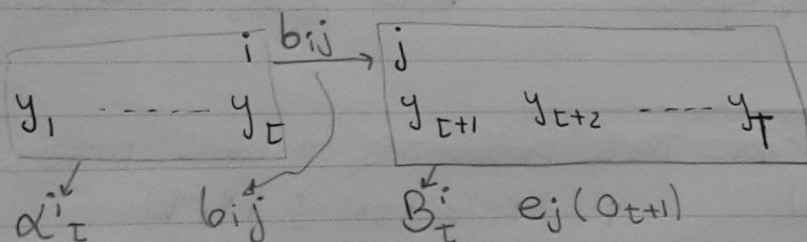
\textcircled{2} Probability of being in state i & j at Time T and T+1 respectively given Y and h

$$\tau_{ij}(t) = P(X_T=i, X_{T+1}=j | Y, h)$$

$$= \frac{P(X_T=i, X_{T+1}=j, Y | h)}{\sum_{i,j} P(X_T=i, X_{T+1}=j, Y | h)}$$

\because زوا فيها  
شكنا كدة هتستخرجها  
عشان نجيب زوا b  
(\smiley)

all possible i & j



$$\therefore \tau_{ij}(t) = \frac{\alpha_T^i b_{ij} \beta_{T+1}^j e_j(o_{T+1})}{\sum_{i=1}^n \sum_{j=1}^n \alpha_T^i a_{ij} \beta_{T+1}^j e_j(o_{T+1})}$$

\* I Found Some of The Probabilities of Possible State Paths



⑤

\* How To re-estimate The Parameters again:-

$\pi_i = \sum_t \delta_i(t)$  expected no of Times state  $i$  was Seen at Time  $t$

\* Transition probability:

$$b_{ij} = \frac{\sum_t \tau_{ij}(t)}{\sum_t \delta_i(t)}$$

State  $i$  is visited  $T$  times overall as follows

$e_j(o_k)$

How many Times we are in state  $j$  by the observation we can divide by # of Times you are in state  $j$

$$e_j(o_k) = \frac{\sum_{t=1}^T 1_{y_t=o_k} \delta_j(t)}{\sum_{t=1}^T \delta_j(t)}$$

indicator

$$1_{y_t=o_k} = \begin{cases} 1 & \text{if } o_k = o_t \\ 0 & \text{otherwise} \end{cases}$$